Operating a fleet of electric taxis

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OPERATING A FLEET OF ELECTRIC TAXIS

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Abstract. The deployment of electric taxi fleets is highly desirable from a sustainable point of view. Nevertheless, the weak autonomy of this kind of vehicles requires a careful operation. The way of managing such a fleet and the question of locating charging terminals for the vehicles are addressed in this paper. Methods for dealing with these tasks are proposed and their efficiency is proved through simulations.

1. Introduction

1.1. **Context.** *Centrale OO*[1] is a pioneering project aiming to deploy in Paris a fleet of 100 % electric taxis. The company in charge of the management of the fleet is the *Société du Taxi Electrique Parisien (STEP)*. The deployment of such fleets finds is main motivation in sustainable issues: electric vehicles release almost no air pollutants at the place where they are operated and have less noise pollution than internal combustion engine vehicles. However, the main drawback of an electric vehicle is its weak autonomy – 80 km in the case of the *Centrale OO* project. In taxi fleet management, two kinds of requests can be differentiated: *booking requests* and *opportunistic requests*. The first ones can be immediate or in advance of travel and have to be processed by the taxi dispatching system which assigns the request to a taxi. The opportunistic requests correspond to the traditional taxi services picking up passengers at cab-ranks or from the side of the road. Of course, this kind of requests is not processed by the dispatching system. The constraints of the management, as expressed by the *STEP*, are

- A taxi must never break down
- An opportunistic demand inside Paris and its suburbs must always be satisfied (legal environment of Paris)
- The number of booking demands accepted has to be maximized

The charging problem of the taxis must therefore be carefully addressed. At a tactical level, a good assignment of the trips to the taxis is crucial. We propose an efficient way to manage the electric fleet in real-time while taking into account the charging tasks.

At a strategic level, the charging problem includes the determination of the best location for the charging terminals. The significant initial investment (the cost of an electrical charging terminal is about 20.000 euros) and the restricted vehicle autonomy give a high relevancy to the charging terminal location task. Indeed, a wrong placement may in effect lead to a poor fleet management with vehicles having difficulties to charge the batteries due to charging terminals saturation or even with vehicles constantly running out of charge to keep operating. Our purpose is to propose a practical way for computing the "best" locations for the charging terminals.

1.2. **Model.** We describe now formally the model we deal with in this paper. We derive also some elementary relations, which gives some informations on the capacity of a given system (in terms of number of trips that can be realized by unit of time).

1.2.1. *Input description.* A complete directed graph $G = (V, A)$ models the network. The vertices are points in the city at which trips start and finish. They can moreover be used to locate vehicle charging terminals. The arcs model the possible trips. The duration of a trip is a random variable T_a of expectation τ_a. The

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[1]See the website http://taxioo.com/index.html for an artistic view.

demand for each possible trip $a \in A$ is assumed to follow a Poisson process of rate λ_a. Actually, this demand is split between a *booking demand* and an *opportunistic demand*, see Section 5 for a more accurate description.

There are n taxis. A taxi consumes γ Wh by unit of time when it is moving. It stores ρ Wh by unit of time when it is charging.

The number of charging terminals is denoted by r. Several terminals can be located at the same vertex.

1.2.2. *Elementary relations.* Let us denote by $\tilde{\lambda}_a$ the average number of demands for a trip a that are accepted by unit of time. We have $\tilde{\lambda}_a \leq \lambda_a$.

Let $\tilde{\lambda} = \sum_{a \in A} \tilde{\lambda}_a$ be the average number of trips accepted by unit of time and let $\tau = \frac{1}{\tilde{\lambda}} \sum_{a \in A} \tilde{\lambda_a} \tau_a$ be the average duration of an accepted trip.

The energy consumption of the system by unit of time is $\gamma \tilde{\lambda} \tau$. The maximal rate of supply in energy is ρr. Therefore, we have the following inequality

$$\gamma \tilde{\lambda} \tau \leq \rho r \tag{1}$$

A second inequality can be derived by considerations on the time needed to perform the different tasks. Let us consider a taxi over a time window of sufficiently large duration T. Denote by x the time during which it stores energy at a charging terminal. Over the time window, it spends in average $\frac{T \tilde{\lambda} \tau}{n}$ unit of time with a customer on board. Therefore, we have

$$\frac{T \tilde{\lambda} \tau}{n} + x \leq T$$

During this duration x, it stores a quantity of energy that must cover in average the consumption over the time window. Hence

$$\frac{\gamma T \tilde{\lambda} \tau}{n} \leq \rho x$$

Combining these two inequalities leads to

$$(\gamma + \rho) \tilde{\lambda} \tau \leq n \tag{2}$$

Equations (1) and (2) can be summarized in the following inequality.

$$\tilde{\lambda} \leq \min \left(\frac{n}{(\gamma + \rho)\tau}, \frac{\rho r}{\gamma \tau} \right) \tag{3}$$

Knowing the number of taxis, their efficiency (encoded by γ), the number of charging terminals, and their efficiency (encoded by ρ), then an upper bound of the number of trips that can be accepted by unit of time can be calculated.

1.3. **Plan.** Section 2 is devoted to the literature review for the two problems addressed in this paper, namely fleet management and charging terminal location. The following sections – Section 3 and Section 4 – detail the approaches proposed for each of these problems. Next, we describe a simulator that has been implemented for the evaluation of the proposed approaches (Section 5). The results of the experiments are described in Section 6. The paper ends with concluding remarks (Section 7).

2. Literature review

2.1. **Taxi dispatching.** Traditional taxi dispatching systems are characterized by two principles. First, simple rules such as for example "nearest vehicle first" or "least utilized first" are used for dynamic vehicle assignment and second, the geographical space is usually divided into zones. In the literature, most of works on the topic basically focus on customer waiting time minimization by proposing improved methods for rule-based systems. In this context, Shrivastava et al. [SCMK97] describe a fuzzy model for rule selection and Alshamsi et al. [AAR09] propose a new technique for dynamically divide the dispatch areas.

The recent apparition of transportation technologies (GPS, EDI, GIS) has widely increased the opportunities for fleet management optimization. It is also the case for taxi dispatching. For example, Seow et

al. [SDL10] propose a collaborative model for taxi dispatching where a set of n taxis of the same zone are defined as the agents of the model and a set of n customers as the service-requests. The objective is then to maximize the total service quality solving a collaborative linear assignment problem. However, taxi dispatching is not the only aspect that can be optimized. For example, Lee et al. [LSP08] and Jia et al. [Jia08] use real-time vehicle information to propose a model for taxi relocation recommendation based on demand forecasting and a probability model for the design of taxi stops, respectively.

Another approach for fleet management optimization consists in modeling the problem as a variant of the Pick-up and Delivery Vehicle Routing Problem with Time Windows (PDVRPTW). The idea is to plan a set of routes satisfying known in advance customer requests. In the taxi management context, Wang et al. [WLC09] propose a bi-criteria two-phase method with an initial feasible assignment first and a tabu search improvement later in order to minimize the number of vehicles and the sum of travel times for advanced bookings. However, the idea to block some vehicles only for advanced bookings might in some cases yield to a fleet underutilization. Horn and al. [Hor02] and Meng et al. [MMYH10] try to fill the gap between simple non-optimized rule-based taxi dispatching systems and static routing approaches. The second paper describes a genetic network programming in order to find the optimal balance between the waiting time and the detour time. The work of Horn [Hor02] is of particular interest in relation to the present work, proposing a taxi dispatching system architecture similar to our fleet management system. He proposes a system for vehicle travel time minimization composed by a set of insertion algorithms to decide whether a new customer is accepted or not and a set of optimization mechanisms in order to improve the solution. However, some important differences exist between our work and these last two fleet management systems. The first difference is that in our case, the constraints related to the restricted autonomy of the vehicles have also to be taken into account by scheduling charging tasks in the routes of the vehicles. The second difference is that, unlike us, both articles deal with the multi-customer problem authorizing customers to share the same vehicle at the same time.

2.2. **Location issue.** The location problem was originally defined by Webber when he considered how to position a single warehouse minimizing the total distance between the warehouse and a set of customers [Web29]. In 1964, Hakimi [Hak64] defines the *P-median problem*, the problem consists in determining the best location for a set of limited facilities in order to minimize the sum of the weighted distances between the clients and the facilities serving these clients.

The problem increases its relevance during the last decades. High costs related to property acquisition and facility construction make facility location projects a critical aspect of strategic planning for a wide range of private and public firms. Indeed, the fact that facility location projects are long-term investments leads the researchers to focus on dynamic and stochastic location problems (see [OD98] for a review of this extension of the problem). Another important variant of the problem is the *Capacitated Facility Location Problem (CFLP)* where facilities have a constraining upper limit on the amount of demand they can satisfy. An extension of the CFLP closely to our problem is the *Capacitated Facility Location Problem with Multiple facilities in the same site (CFLPM)*. In charging terminal location, the positions of the terminals are not the only decision variables, the number of terminals at each position have to be fixed too.

However, in some real-world applications, selecting the best location for distance minimization is not the best suitable choice. For example, in electric vehicle charging terminal location, like in other critical applications such as ambulance and fire terminal location, the interest is to guarantee that the different geographic zones are covered by a facility (closer than a previously fixed covering distance). This class of problems are known as *Covering Location Problems* (see [WC74], [SVB93] and more recently [VP10] for a complete review of covering problems). In that context, the covering issue can be sometimes modelled as a problem constraint. However, if the covering distance is fixed to a small value the problem might become unfeasible. The *Maximal Covering Location Problem (MCLP)* [CR74] locates the facilities in order to maximize the number of covered customers (customers with a distance to the nearest facility smaller than an initial fixed distance). An extension of the problem is the maximal covering with mandatory closeness problem which imposes a maximal distance (less stringent than the covering distance) between the geographical zones and the nearest facility [CR74]. These covering models implicitly assume that if a geographical zone is covered by a facility then the facility will be always available to serve the demand. However, in some applications, when facilities have a fixed capacity, being covered is not sufficient to guarantee the demand satisfaction. We find

in the literature some models attempting to overcome this issue by maximizing the number of geographical zones covered by multiple facilities [DS81, HR86, GLS97].

3. Fleet management

We describe in this section two ways for managing the fleet, a classical and rule-based one (Subsection 3.1), and an improved one trying to address explicitly the charging issue (Subsection 3.2). Let us first introduce some notations. Let CR_i be a booking customer request. Each customer request CR_i is defined by a start time S_i and an origin-destination pair $O_i - D_i$. The S_i is fixed by the customer when the customer request arrives. The completion time of a trip is $C_i = S_i + \tau_{O_i D_i}$, where $\tau_{O_i D_i}$ is the travel time between the origin and destination of the customer request CR_i. Finally, let $R : CT_j$ be a taxi charging task scheduled on the charging terminal CT_j.

3.1. **A classical rule-based taxi dispatching system.** A taxi dispatching system based on the principles of the most common real-world systems (see for example [SCMK97], [LWCT04] or [AAR09]) is described in this section. The architecture of the current taxi dispatching systems are very similar to the system illustrated in Figure 1. The two main components of the system are (1) a customer acceptation mechanism deciding for each new customer if it is accepted (the accepted customers are inserted into a queue of customers) or rejected and (2) a rule-based mechanism assigning accepted customer requests (trips) to the free taxis. For each accepted trip i, the assigning process has to start a few minutes (Θ) before the fixed start time (S_i) in order to maximize the chances to find a taxi to attend the demand. Once a trip is assigned to a taxi, the vehicle is automatically blocked and the taximeter begins counting.

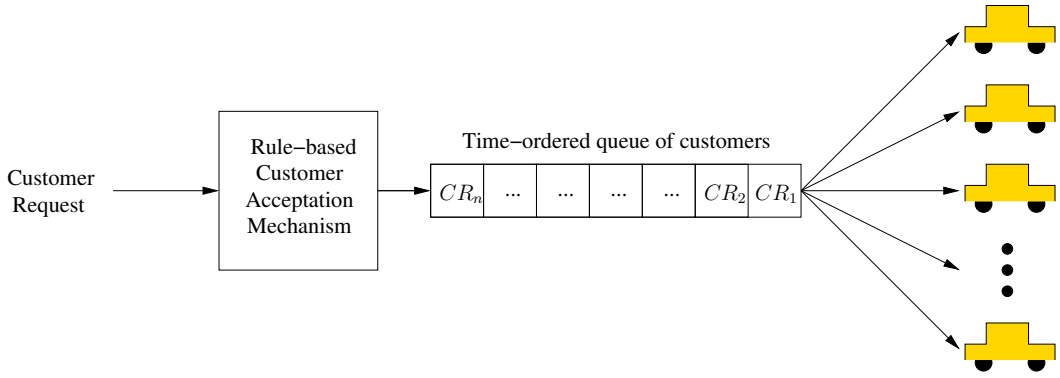

Figure 1. Rule-based taxi dispatching system

A rule for customer acceptation using the time windows for the trips already accepted is proposed. The idea is to limit the trips that have to be performed at the same time in order to minimize the number of not served customers and to establish a margin of k vehicles to attend opportunistic customers. For each new customer request CR_{new} the Algorithm 1 determines if it is accepted or not.

Algorithm 1: Rule-based checking for customer acceptation for a margin of k vehicles

$L = \{CR_1, CR_2, \ldots, CR_n\}$, list of already accepted customers
CR_{new}, new booking customer request
$nC \leftarrow 0$, number of trips performed at the same time than CR_{new}
foreach CR_i *of* L **do**
  if CR_i *is executed at the same time than* CR_{new} *(*$(S_i \leq S_{new} < C_i)$ *or* $(S_{new} \leq S_i < C_{new})$*)* **then**
    Step 1: Increase the number of customers performed at the same time than CR_{new}
    $(nC \leftarrow nC + 1)$
if *condition to accept the customer* $(nC < n - k)$ **then**
  Step 2: Insert CR_{new} to the list of accepted customers L

Once the customer request CR_i is accepted, it remains in the queue of customers until $S_i - \Theta$ (Θ is usually fixed around 20 minutes). At that moment, the system automatically starts looking for a free taxi having sufficient charge to operate the trip. If different taxis are available, the system assigns the trip to the taxi minimizing the customer waiting time (a parameterizable not announced customer waiting time can be authorized). In the case of no vacant taxis are available, the system waits for a vehicle to become available. If the waiting time for any request exceeds the authorized maximal customer waiting time Δ, the customer request is then canceled. Note that the number of unsatisfied customers can be reduced by using a more restrictive rule for the customer acceptation mechanism.

The main advantage of such a system where no future work is planned is the high degree of independence for taxi drivers. On the other hand, the drawbacks are the underutilization of the fleet and the lost of efficiency during the peak hours when most of the companies have to close their booking requests systems in order to avoid unsatisfied customers. Indeed, some real-world systems do not integrate a customer acceptation mechanism leading, in rush hours, to unsatisfied customers who had been initially accepted and they are finally served with an unannounced and, sometimes, intolerable delay or, eventually, never served at all. Furthermore, the charging tasks of the vehicles cannot be controlled leading to a poor fleet management with vehicles having difficulties to charge the batteries due to charging terminals saturation.

3.2. **The improved electric vehicle management system.** An improved fleet management system aiming to overcome the weakness of the rule-based taxi dispatching system is proposed in this section. The main objectives of the system are to maximize the number of accepted customers and to minimize the customer waiting time. One of the major issues is how to deal with opportunistic demand. Indeed, this kind of demand is unpredictable and must always be satisfied, so free taxis must be at any moment able to satisfy the longest trip without running out of charge. This constraint makes the problem considerably more complex forcing the system to provide a mechanism ensuring the feasibility of the already accepted trips each time an opportunistic demand is accepted.

The approach proposed consists in maintaining continuously a feasible planning for the taxis and the charging terminals (see Figure 2). Each time a customer asks for a trip, a simple *insertion algorithm* is run, at the end of which either the trip has been successfully inserted or not. The objective is to assign the customer to the taxi minimizing the customer waiting time (a parameterizable announced customer waiting time can be authorized). If none of the tried delays on the pick-up time leads to a feasible planning, a *rescheduling algorithm* allowing to reallocate the already accepted customers to the taxis is run.

In all these processes, a key routine is often called, namely the *charging task manager*, which schedules the charging tasks of a taxi, given a planning for the other taxis and the charging terminals.

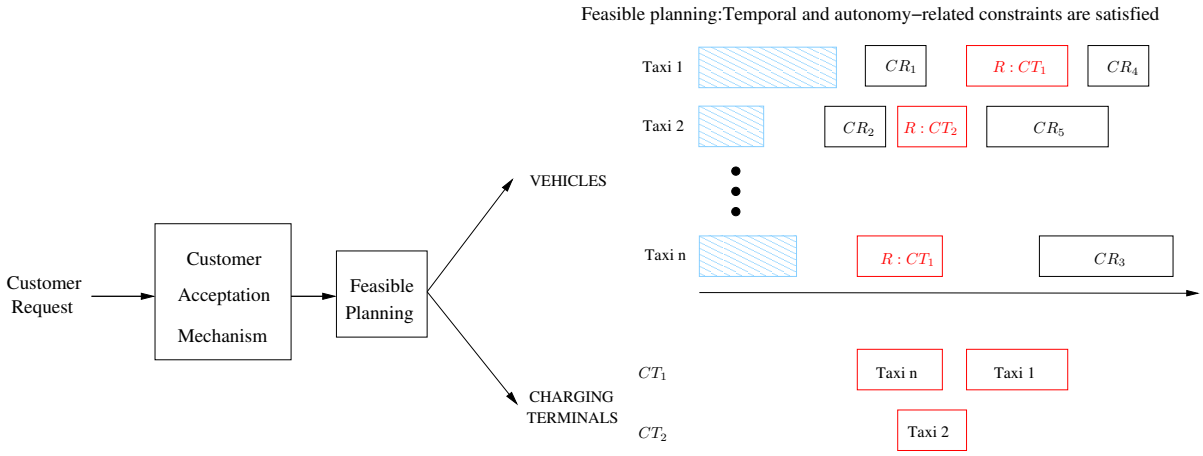

FIGURE 2. Customer acceptation mechanism of the electric vehicle management system

In the case of an opportunistic demand, which is necessarily accepted, we follow exactly the same scheme except that there is no degree of freedom in the insertion process: the trip is inserted at the front of the planning of the taxi stopped by the customer, and the rescheduling algorithm is also run if it is necessary.

3.2.1. *Insertion algorithm.* This algorithm is the first step in order to decide if a new trip CR_{new} is accepted or not. The objective is to assign the trip to the taxi minimizing the delay on the pick-up time (see Algorithm 2). The algorithm increasely tests the different authorized pick-up times. Once the start time is fixed, we sequentially try for each vehicle to insert the new request. First the scheduled charging tasks are removed. Then the new request is accepted only if it can be inserted with no constraint violation (the pick-up times of the rest of customers are respected and the current autonomy of the vehicle, without any charging task, is sufficient). In the case that the vehicle autonomy-related constraint is violated, a greedy algorithm trying to schedule a charging task between each pair of trips is proposed. After the charging tasks are inserted, if the taxi is able to perform the trips without running out of charge, then the customer request is also accepted.

Algorithm 2: New request insertion algorithm for a maximal authorized delay of Δ minutes

```
V = {V_1, V_2, ..., V_r}, list of taxis
CR_new, new booking customer request
accepted ← false, variable indicating if the new request is accepted
st ← S_new, start time of the trip
while st ≤ S_new + Δ and accepted = false do
    foreach v_i of V do
        Step 1: Delete the charging tasks of the vehicle v_i
        if CR_new starting at st can be inserted in the route of the vehicle v_i then
            if the vehicle autonomy-related constraint is satisfied then
                Step 2: CR_new starting at st is inserted in the route of the vehicle v_i (accepted ← true)
            else
                Step 3: Insert charging tasks for v_i between each pair of trips
                if the vehicle autonomy-related constraint is satisfied then
                    Step 2: CR_new starting at st is inserted in the route of the vehicle v_i
                    (accepted ← true)
    if accepted = false then
        Step 4: Increase the pick-up time for the CR_new (st ← st + 1)
```

3.2.2. *Rescheduling algorithm.* The rescheduling algorithm is proposed when the new customer is still not accepted after the insertion algorithm. As for the insertion algorithm, the goal is to find a new feasible planning for the vehicles integrating the new request CR_{new}. The main difference is that the trips can be reassigned to different vehicles.

The problem without taking into account the autonomy-related constraints can be solved in polynomial time [NSZ02]. The idea is to convert the schedule of trips (without the charging tasks) into a graph and to verify using a max flow computation that all trips can be performed by the taxis. To construct the network two vertices are considered for each customer request CR_i, the first one v_i represents the pick-up time and the second one v_i' the completion time of the customer request. Four dummy vertices are required: 0, $0'$, a source s and a sink t. The arcs of the graph are $(s, 0)$, $(0', t)$, all the (s, v_i), all the (v_i', t), all the (v_i, v_i'), and all the (v_i', v_j) such that the customer request CR_j can be performed by the same taxi than the customer request CR_i and after CR_i, that means if $S_j \geq C_i + \tau_{D_iO_j}$. Except the arcs $(s, 0)$ and $(0', t)$, they all have a capacity equal to 1. The arcs $(s, 0)$ and $(0', t)$ have a capacity equal to n. A maximum flow computation in this directed graph determines the schedule feasibility and also proposes a new planning for the vehicles respecting the customers pick-up times.

The max flow computation is integrated in the rescheduling algorithm in order to check the feasibility of the schedule for a given pick-up time $st \in [S_{new}, S_{new} + \Delta]$ and, if it is the case, to find a reference planning (planning without charging tasks). A local search explores the neighborhood of the reference planning defined by the *swap* and the *reallocation* operators [Sav92]. Finally, for each explored planning respecting temporal constraints, the greedy algorithm for charging task scheduling is sequentially applied to the taxis that do not satisfy autonomy-related constraints (that is, taxis whose current charge is not enough to perform all

the trips assigned to them without adding charging tasks). If a feasible solution is found, the new customer is then accepted.

3.2.3. *Charging task manager.* As we have already seen, the *insertion* and the *rescheduling algorithm* constantly runs a greedy algorithm aiming to insert a charging task between each pair of successive trips of the same route. The algorithms proposed to determine if a new charging task can be integrated in a specific charging terminal planning are described in this subsection. The main feature of our problem is that the processing time of the new charging task is not fixed, instead it is a decision variable defined between the interval limited by the minimal charging time for a vehicle p^{min} (customizable parameter) and the maximal charging time corresponding to the time necessary for a full charge.

The problem to be solved by the charging terminal manager can be then formally stated as follows. A *charging task* R_i is defined by its time window $[r_i, d_i]$, where r_i is the earliest start time (earliest arrival time to the terminal) and d_i the latest end time (latest departure time from the terminal). Let p_i be the decision variable corresponding to the processing time of the task R_i, then $r_i \leq S_i$ and $S_i + p_i \leq d_i$, where S_i is the effective start time of R_i.

Given a feasible schedule of n charging tasks $S^n = \{S_1, S_2, \ldots, S_n,\}$ for the charging terminals located at the same geographical position. We are given a new charging task R_{n+1} with a time window $[r_{n+1}, d_{n+1}]$ and a processing time p_{n+1} inside the interval $p^{min} \leq p_{n+1} \leq p^{max}_{n+1}$. The problem consists in finding a new feasible schedule $S^{n+1} = \{S_1, S_2, \ldots, S_n, S_{n+1}\}$ maximizing the processing time of the task R_{n+1} (whence without changing the processing time of the other tasks).

The mechanism tests first a task insertion aiming to find quickly a feasible solution. The complexity of the algorithm for task insertion maximizing the processing time of the new task is $O(n)$ where the start times and completion times of the scheduled jobs are non-decreasing ordered. If no solution is found after the task insertion algorithm, a *dichotomous algorithm* allowing to reschedule the tasks is proposed in order to find a solution maximizing the processing time of the new task. For each iteration of the algorithm, a *satisfiability test* based on constraint propagation involving *energetic reasoning* is first triggered. The goal of the feasibility test is to detect an inconsistency indicating that it is not possible to find a feasible schedule integrating the new task. Finally, if the energetic reasoning is not conclusive a local search algorithm is proposed in order to find a solution.

Satisfiability test: Energetic reasoning. A satisfiability test based on constraint propagation involving energetic reasoning is proposed [LE96]. A fictitious energy (which has nothing to do with the electricity) is produced by the charging terminals and it is consumed by the charging tasks. We determine the fictitious energy consumed by the tasks ($E_{consumed}$) over a time interval $\delta = [t_1, t_2]$ and we compare this fictitious energy with the available fictitious energy ($E_{produced} = m \times (t_2 - t_1)$). The minimal fictitious energy consumed by the tasks in an interval $\delta = [t_1, t_2]$ is:

$$E_{consumed} = \sum_{i=1}^{n+1} \max\{0, \min\{p_i, t_2 - t_1, r_i + p_i - t_1, t_2 - d_i + p_i\}\} \tag{4}$$

If $E_{consumed} > E_{produced}$, it is then impossible to find a feasible schedule S^{n+1} integrating the new task. The relevant intervals δ for a complete satisfiability analysis can be enumerate in $O(n^2)$. The test is restricted to the intervals $[t_1, t_2]$ specified by $\{r_i\} \cup \{d_i\} \cup \{r_i + p_i\} \cup \{d_i - p_i\}$ where the new task R_{n+1} may consume ($t_1 \leq d_{n+1}$ and $t_2 \geq r_{n+1}$).

Dichotomous algorithm. A dichotomous algorithm maximizing the processing time of the new task is described in this section (see Algorithm 3). A dichotomy is run on the processing time p as follows. For processing times $p \in [p^{min}, p^{max}_{n+1}]$, the satisfiability test based one the energetic reasoning indicates whether the necessary conditions are satisfied or not. If it is the case, a local search mechanism tries to find a feasible schedule. The parallel machine scheduling problem with time windows can be solved by a list scheduling algorithm. It means there exists a total ordering of the jobs (i.e., a list) that, when a given machine assignment rule is applied, reaches the optimal solution. For our problem, this rule consists in allocating each task to the machine that allows it to start at the earliest (Earliest Start Time or EST rule). The local search mechanism proposed to solve the problem is based on this result. First, the tasks are ordered in a non-decreasing order of their due dates (Earliest Due Date or EDD rule), then the local search consists in

exploring different permutations of the list defined by the insertion neighborhood ($O(n^2)$). For each list of task, the machines are assigned according to the EST rule in order to reach a feasible solution. If no feasible schedule is eventually found, the request is rejected.

Algorithm 3: Dichotomous algorithm for processing time maximization

```
min ← p^min
max ← p^max_{n+1}
S^{n+1}_best ← ∅
while min ≤ max do
    Step 1: Fix the processing time p of the new task R_{n+1} (p ← ⌊(min+max)/2⌋)
    if SatisfiabilityTest() then
        Step 2: Sort the tasks according to the EDD rule
        Step 3: Local search using the insertion operator
        if a feasible schedule S^{n+1} = {S_1, S_2, ..., S_n, S_{n+1}} is found then
            Step 4: Update the lower limit (min ← p + 1)
            Step 5: Update the best solution (S^{n+1}_best ← S^{n+1})
        else
            Step 6: No solution exists, update the upper limit (max ← p − 1)
    else
        Step 7: No solution exists, update the upper limit (max ← p − 1)
if S^{n+1}_best = ∅ then
    Step 8: No solution is found (return ∅)
else
    Step 9: A feasible solution is found (return S^{n+1}_best)
```

4. Electric vehicles charging terminal location

The EV charging terminal location problem consists in determining the best locations of the charging terminals. The linear programming model has to take into account two important aspects. First, the charging terminals have to be conveniently spread over the geographical area in order to avoid remote geographical zones which difficult taxi operability and fleet management. The second aspect is that the model has to determine the number of charging points facilitating the charging process of the taxis by minimizing the risks of terminals saturation. For these purposes, we propose two models, one called the *P-median model*, the other the *Demand-based model*.

V is the set of geographical points of the problem and $J \subseteq V$ is the set of potential locations where the charging terminals can be located. The number of terminals is limited to r.

4.1. *P*-median model.

Following Hakimi [Hak64], we define x_j to be the decision variables indicating if a facility is located to the point j and y_{ij} to be the variables indicating that the geographical point i is assigned to the facility located in j. The linear program minimizing the sum of the distances between clients and facilities can be written as follows.

$$\min \sum_{i \in V} \sum_{j \in J} dist_{ij} y_{ij} \tag{5}$$

s.t.

$$\sum_{j \in J} y_{ij} = 1 \text{ for all } i \in V \tag{6}$$

$$y_{ij} \leq x_j \text{ for all } i \in V,\ j \in J \tag{7}$$

$$\sum_{j \in J} x_j \leq r \tag{8}$$

$$x_j \in \{0,1\} \text{ for all } j \in J \tag{9}$$

$$y_{ij} \in \{0,1\} \text{ for all } i \in V,\ j \in J \tag{10}$$

4.2. **Demand-based model.** Another approach consists in defining a model with two distances β_{far} and β_{close} as proposed by Church and ReVelle [CR74]. The idea is then to spread the terminals by fixing a maximal distance (β_{far}) between the different geographical zones and the nearest charging terminal and, at the same time, trying to maximize the demand that will be covered by a nearby charging terminal (β_{close}).

We can then define J_i^{far} (resp. J_i^{close}) as the subset of points in J at distance less than β_{far} (resp. β_{close}) from $i \in V$. Conversely, V_j^{close} is the set of points at distance less than β_{close} from the point $j \in J$.

Let x_j be the decision variable indicating the number of terminals located at point $j \in J$ and y_{ij} to be the fraction of the demand d_i for $i \in V$ covered by a charging terminal located in j at distance less than β_{close} from i.

The linear programming model proposed to solve the problem called *Demand-based model* is the following.

$$\max \sum_{j \in J} \sum_{i \in V_j^{close}} d_i y_{ij} \tag{11}$$

s.t.

$$\sum_{j \in J_i^{far}} x_j \geq 1 \text{ for all } i \in V \tag{12}$$

$$\sum_{j \in J_i^{close}} y_{ij} \leq 1 \text{ for all } i \in V \tag{13}$$

$$\sum_{i \in V_j^{close}} d_i y_{ij} \leq x_j \text{ for all } j \in J \tag{14}$$

$$\sum_{j \in J} x_j \leq r \tag{15}$$

$$x_j \in \mathbb{Z}_+ \text{ for all } j \in J \tag{16}$$

$$y_{ij} \in \mathbb{R}_+ \text{ for all } i \in V,\ j \in J_i^{close} \tag{17}$$

The objective function (Eq. (11)) consists in maximizing the pointwise demand covered by a charging terminal considering the distance β_{close}. Eq. (12) imposes that a geographical zone $i \in V$ must be covered at least for one charging terminal considering the distance β_{far}. Here the mandatory closeness is only required for the geographical zones closer than β_{far} from a potential charging terminal location in order to find a solution even if this constraint is violated for some geographical zones. We stress that an adequately β_{far} make possible to spread the charging terminals over the geographical area. Eq. (13) specifies that for each geographical zone $i \in V$ the sum of the fractions of demand covered by a charging terminal considering the distance β_{close} has to be less or equal to the unit. The idea here is that the demand of each geographical point can be satisfied by different charging terminals and our interest is to maximize the potential energy required being supplied by a terminal closer than β_{close}. Eq. (14) are the constraints linking the variables x_j with the variables y_{ij}. For a given potential charging terminal location J_j, this last set of variables can only

be positive if a charging terminal is finally located to J_j, that means $x_j > 0$. Besides, thanks to the definition of the pointwise demand d_i, Eq. (14) also imposes that the demand allocated to a charging terminal cannot exceed its capacity. The goal of this constraint is to assign a larger number of terminals on the geographical points with a great demand decreasing that way the risk of saturation for the charging terminals. Finally, Eq. (15) limits the number of terminals of the problem.

Estimating a pointwise demand in energy. The proposed charging location model take in input a pointwise demand. A way to build such a demand d_i attached to a vertex i consists in computing the energy needed by unit of time for the trips starting at i: it is precisely $\gamma \sum_{a \in \delta^+(i)} \lambda_a \tau_a$. Dividing this quantity by ρ provides the number of charging terminals ensuring this supply. It suggests to define

$$d_i^{out} = \frac{\gamma}{\rho} \sum_{a \in \delta^+(i)} \lambda_a \tau_a$$

In the last equation, the pointwise demand is calculated considering the trips starting at vertex i. This model presumes that taxis charging tasks take place mostly at the origin of the trips. However, other realistic strategies can be also envisaged. One of these strategies is consider that taxis charge the batteries at the end of the trips. A pointwise demand considering the energy consumed by unit of time for the trips arriving at i is also proposed. We can then define

$$d_i^{in} = \frac{\gamma}{\rho} \sum_{a \in \delta^-(i)} \lambda_a \tau_a$$

Finally, a third strategy is proposed considering a pointwise demand as a convex combination of d_i^{out} and d_i^{in}:

$$d_i^{mix} = \alpha d_i^{out} + (1 - \alpha) d_i^{in}$$

5. Taxi simulator

The taxi behavior simulator consists in a discrete-events simulator programmed in C++. It simulates the model described in Section 1.2. Each possible trip between the points of the city is characterized by two parameters, λ_{book} and λ_{opp}, representing the rates of the Poisson process for booked and opportunistic demand, respectively. For sake of simplicity, the duration and the distance of each trip are considered constant over the time. The inputs of the simulator are a description of the system state and the simulation parameters. The system is originally defined by:

- A set of points defined by their coordinates representing the geographical points of the city.
- The location of the charging terminals.
- The fleet of taxis. Each taxi is defined by the following parameters:
  - AUT: it is the current autonomy of the vehicle.
  - MAX_AUT: it is the autonomy of the vehicle when it is fully charged.
  - POS: it is the initial position of the vehicle.

Opportunistic and booking demands are treated differently. The booking demands are managed by the taxi dispatching system deciding whether a demand is accepted or not. The opportunistic demands always have to be satisfied, the simulator affects the trip to a free taxi located at the same geographical point. As we can note, an opportunistic demand is only considered when it exists spatial and temporal coincidence between the demand and a free taxi, otherwise the demand is simply ignored. It is worth recalling that the opportunistic demands may lead to unsatisfied booking demands initially accepted by the taxi dispatching system.

6. Computational experiments

The fleet management systems and the charging terminal location models are compared and evaluated by simulation on a set of randomly generated instances. Two set of instances have been generated considering different values for the average number of demands by unit of time. We consider then a first set of instances with a *weak demand* ($\lambda_{book}^{weak} \approx 0.4$ and $\lambda_{opp}^{weak} \approx 1.0$) and a second set of instances with a *strong demand*

($\lambda_{book}^{strong} \approx 0.8$ and $\lambda_{opp}^{strong} \approx 2.0$). Each set is composed of 60 instances (10 instances for each combination) generated from different values for the number of terminals ($r \in \{5, 20, 40\}$) and for the number of vehicles ($n \in \{100, 200\}$). The computation experiments consist on a 900 minutes simulation. The maximal authorized delay on pick-up time is fixed to $\Delta = 15$ minutes for both systems and the minimal charging time for a vehicle is fixed to 10 minutes.

6.1. **Fleet management.** The improved fleet management system has been compared with the rule-based taxi dispatching system presented in Section 3.1. Table 1, Table 2, and Table 3 display the results for the comparison between both systems for the instances with 5, 20 and 40 charging terminals, respectively. The first column (*NbTrips*) displays the average of served customers. *NbBooking* is the average of served booking requests. The number of unsatisfied customers (customers initially accepted by the system but they are never served) is indicated by *Unsatisfied Customers* at the third column. Finally, the last column indicates the average customer overcharge (number of minutes in the taximeter when the taxi arrives at the pick up point).

The results show that the proposed fleet management clearly improves the rule-based taxi dispatching system. The number of served customers is on average considerably more important with the improved fleet management systems. Besides the percentage of unsatisfied customers remains acceptable and almost constant with this system contrarily to the rule-based taxi dispatching for which the number of unsatisfied customers grows exponentially with the demand for a small number of charging terminals, as we can see for the instances with 50 and 100 vehicles and 5 charging terminals. This fact can be explained because the vehicle charging aspects are completely ignored in the customer acceptation rule of the rule-based taxi dispatching system. Finally, customer overcharge is very similar for both systems, for the instances with 20 and 40 resources here again the proposed fleet management is a better system than the rule-based taxi dispatching.

Concerning the time needed to compute the answer to a booking request, it is in general under 1 ms, and in less than 2% of the cases above 5 s. In any case, the computation time is always reasonable, and, with faster computers, the time needed to compute the answer could be reduced if required.

$r = 5$		*NbTrips*	*NbBooking*	*Unsatisfied Customers (%)*	*Overcharge (min)*
weak demand					
$n = 50$	*Rule-based TD*	428.4	353.3	19.5 (5.23 %)	6.56
	Improved FM	437.2	354.2	0.6 (0.17 %)	6.68
$n = 100$	*Rule-based TD*	498.8	361.8	11 (2.95 %)	6.07
	Improved FM	499.7	362.2	1.0 (0.28 %)	6.34
$n = 200$	*Rule-based TD*	568.9	368.3	4.5 (1.21 %)	5.78
	Improved FM	571.5	368.4	0.6 (0.16 %)	5.94
strong demand					
$n = 50$	*Rule-based TD*	659.1	614.3	144.8 (19.08 %)	5.89
	Improved FM	672.7	324.1	0.9 (0.14 %)	6.79
$n = 100$	*Rule-based TD*	778.9	671.6	87.5 (11.53 %)	5.96
	Improved FM	788.2	675.5	1.0 (0.15 %)	6.48
$n = 200$	*Rule-based TD*	943.6	723.4	35.7 (4.70 %)	5.87
	Improved FM	941.7	717	2.1 (0.29 %)	6.13

TABLE 1. Comparison between different fleet management systems for instances with 5 charging terminals

6.2. **Charging terminal location.** The demand-based linear model for charging terminals location presented in Section 4.1 has been compared with the P-median model model minimizing the distance between the geographical points and the nearest charging terminal. Table 4 shows the results for the comparison between both models for the 40 instances with 5 charging terminals. The first column (*NbTrips*) displays the average of accepted customers. *NbBooking* is the average of accepted booking requests. The average

$r = 20$		*NbTrips*	*NbBooking*	*Unsatisfied Customers (%)*	*Overcharge (min)*
weak demand					
$n = 50$	*Rule-based TD*	433.9	356.3	16.5 (4.43 %)	6.47
	Improved FM	433.4	354.9	0.9 (0.25 %)	6.37
$n = 100$	*Rule-based TD*	488.9	360.6	12.2 (3.27 %)	6.05
	Improved FM	492.9	361.4	1.0 (0.28 %)	6.01
$n = 200$	*Rule-based TD*	554.8	368.5	4.3 (1.15 %)	5.77
	Improved FM	549.9	365.7	1.5 (0.41 %)	5.56
strong demand					
$n = 50$	*Rule-based TD*	776.3	710.6	47.3 (6.24 %)	6.60
	Improved FM	781.9	710.1	2.3 (0.32 %)	6.42
$n = 100$	*Rule-based TD*	881.9	732.3	26.8 (3.53 %)	6.00
	Improved FM	887.3	730.9	2.8 (0.38 %)	5.72
$n = 200$	*Rule-based TD*	991.4	744.3	14.8 (1.95 %)	5.74
	Improved FM	994.0	744.8	2.8 (0.37 %)	5.40

TABLE 2. Comparison between different fleet management systems for instances with 20 charging terminals

$r = 40$		*NbTrips*	*NbBooking*	*Unsatisfied Customers (%)*	*Overcharge (min)*
weak demand					
$n = 50$	*Rule-based TD*	430.2	351.9	20.9 (5.61 %)	6.39
	Improved FM	432.4	355.7	1.2 (0.34 %)	6.45
$n = 100$	*Rule-based TD*	486.7	359.8	13 (3.49 %)	6.13
	Improved FM	498.8	361.3	0.6 (0.17 %)	5.89
$n = 200$	*Rule-based TD*	557.7	364.8	8.0 (2.15 %)	5.80
	Improved FM	560.3	366.0	1.1 (0.30 %)	5.55
strong demand					
$n = 50$	*Rule-based TD*	797.8	716.7	41.1 (5.42 %)	6.48
	Improved FM	792.2	711.9	2.4 (0.34 %)	6.29
$n = 100$	*Rule-based TD*	906.0	731.2	27.9 (3.68 %)	5.99
	Improved FM	905.4	729.5	2.3 (0.31 %)	5.72
$n = 200$	*Rule-based TD*	1003.8	742.0	17.1 (2.25 %)	5.64
	Improved FM	1014.2	738.4	3.5 (0.47 %)	5.39

TABLE 3. Comparison between different fleet management systems for instances with 40 charging terminals

percentage of operating time and the average percentage of time when a taxi is waiting for an available charging terminal are displayed on the last two columns.

When the number of charging terminals is equal to five, the demand-based model outperforms the P-median model, for any criterion. More customers are satisfied, the operating time of taxis is higher and the time waiting for an available charging terminal is drastically reduced. In some cases, this last value is even divided by two. The different ways proposed to estimate the pointwise demand have been also compared (for d_{mix}, we set $\alpha = 0.5$). We observe that no strategy overcomes clearly the others.

When the number of charging terminals is larger as in Table 5 and Table 6 (20 and 40 charging terminals), there is no real difference between the two models. This is not surprising since in those cases charging terminals are no longer a critical resource. This fact explains also the small waiting time for charging. In fact, for large number of charging terminals, we expect similar behaviours for any locations regularly spread over the area of experimentation.

$r = 5$		*NbTrips*	*NbBooking*	*%Operating*	*%Waiting for charging*
weak demand					
$n = 100$	*P-median model*	481.6	359.2	85.41 %	9.52 %
	Demand-based model (d_{out})	497.6	362.0	90.98 %	4.67 %
	Demand-based model (d_{in})	488.7	359.8	91.84 %	3.63 %
	Demand-based model (d_{mix})	488.5	358.9	91.17 %	4.41 %
$n = 200$	*P-median model*	551.6	366.9	90.16 %	7.06 %
	Demand-based model (d_{out})	567.9	368.3	94.97 %	2.80 %
	Demand-based model (d_{in})	563.3	365.8	94.99 %	2.70 %
	Demand-based model (d_{mix})	560.3	366.9	94.95 %	2.80 %
strong demand					
$n = 100$	*P-median model*	744.3	654.1	55.19 %	38.04 %
	Demand-based model (d_{out})	777.1	672.7	64.08 %	29.44 %
	Demand-based model (d_{in})	803.4	678.7	68.83 %	24.28 %
	Demand-based model (d_{mix})	789.9	674.3	65.63 %	27.67 %
$n = 200$	*P-median model*	886.0	703.9	66.01 %	30.33 %
	Demand-based model (d_{out})	943.5	725.0	75.04 %	21.48 %
	Demand-based model (d_{in})	943.0	718.5	77.30 %	19.09 %
	Demand-based model (d_{mix})	953.0	723.8	77.43 %	19.03 %

TABLE 4. Comparison between different programming models for instances with 5 charging terminals

$r = 20$		*NbTrips*	*NbBooking*	*%Operating*	*%Waiting for charging*
weak demand					
$n = 100$	*P-median model*	497.0	361.4	94.46 %	0.19 %
	Demand-based model (d_{out})	495.9	361.4	93.99 %	0.28 %
	Demand-based model (d_{in})	499.0	362.7	94.19 %	0.22 %
	Demand-based model (d_{mix})	495.1	363.4	93.98 %	0.31 %
$n = 200$	*P-median model*	566.7	367.0	96.79 %	0.11 %
	Demand-based model (d_{out})	564.5	367.9	96.49 %	0.17 %
	Demand-based model (d_{in})	561.2	368.5	96.57 %	0.16 %
	Demand-based model (d_{mix})	561.1	368.5	96.49 %	0.20 %
strong demand					
$n = 100$	*P-median model*	894.8	733.7	90.02 %	0.89 %
	Demand-based model (d_{out})	895.4	734.5	88.97 %	1.26 %
	Demand-based model (d_{in})	884.7	733.4	89.21 %	1.15 %
	Demand-based model (d_{mix})	895.5	737.1	88.59 %	1.36 %
$n = 200$	*P-median model*	994.9	741.2	93.85 %	0.73 %
	Demand-based model (d_{out})	1005.4	744.8	93.40 %	0.88 %
	Demand-based model (d_{in})	997.0	741.1	93.28 %	0.87 %
	Demand-based model (d_{mix})	1008.9	744.0	93.19 %	0.95 %

TABLE 5. Comparison between different programming models for instances with 20 charging terminals

7. CONCLUSION

In this paper, we deal with the management of a fleet of electric taxis. Two different problems have been solved: the taxi dispatching problem and the electric charging terminal location problem. Different approaches have been proposed to solve each problem.

For the taxi dispatching problem, two different approaches are proposed: a rule-based system inspired by most of the real-life taxi dispatching systems and an improved fleet management system adapted to

$r = 40$		*NbTrips*	*NbBooking*	*%Operating*	*%Waiting for charging*
weak demand					
$n = 100$	*P-median model*	492.6	362.1	94.88 %	0.06 %
	Demand-based model (d_{out})	493.8	361.2	94.71 %	0.06 %
	Demand-based model (d_{in})	493.3	362.5	94.85 %	0.04 %
	Demand-based model (d_{mix})	492.8	361.8	94.69 %	0.05 %
$n = 200$	*P-median model*	549.7	366.9	97.08 %	0.03 %
	Demand-based model (d_{out})	556.1	366.1	97.04 %	0.03 %
	Demand-based model (d_{in})	560.5	366.6	97.06 %	0.03 %
	Demand-based model (d_{mix})	559.1	367.9	96.99 %	0.02 %
strong demand					
$n = 100$	*P-median model*	902.1	731.6	91.30 %	0.24 %
	Demand-based model (d_{out})	897.6	732.9	91.66 %	0.13 %
	Demand-based model (d_{in})	896.4	732.7	91.44 %	0.14 %
	Demand-based model (d_{mix})	897.0	732.8	91.31 %	0.13 %
$n = 200$	*P-median model*	999.0	741.1	95.00 %	0.14 %
	Demand-based model (d_{out})	1009.3	741.4	94.96 %	0.09 %
	Demand-based model (d_{in})	1008.8	741.0	94.90 %	0.11 %
	Demand-based model (d_{mix})	1006.5	741.9	94.91 %	0.12 %

TABLE 6. Comparison between different programming models for instances with 40 charging terminals

the problem constraints. The idea of the improved fleet management system is to continuously maintain a feasible solution for the problem (a feasible planning for the taxis and for the charging terminals). Scheduling algorithms and a local search scheme are proposed to solve the problem. Both systems have been compared by simulation on randomly generated instances. The results show the efficiency of the proposed improved system and that the rule-based system does not suit the features of electric vehicles.

For the electric charging terminal location problem, the first approach is based on classical models of the literature to solve the facility location problem and minimizes the sum of the distances between the terminals and the points assigned to them. The second approach is a demand-based mixed integer linear programming model adapted to the characteristics of our problem, in which the "moving" demands of the taxis are approximated as static pointwise demands. Both models have been tested and compared on a set of realistic instances randomly generated. The results show that the proposed demand-based model outperforms the model minimizing the sum of distances when the number of charging terminals is small. When this number is large, charging terminals are no longer a critical resource and both models show similar behaviours.

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