Estimation of cascade failure probability in electrical transmission networks by Subset Simulation

Enrico Zio, L. R. Golea, Nicola Pedroni, Giovanni Sansavini

Estimation of cascade failure probability in electrical transmission networks by Subset Simulation

E. Zio
Ecole Centrale Paris- Supelec, Paris, France
Politecnico di Milano, Milan, Italy

L. R. Golea, N. Pedroni, G. Sansavini
Politecnico di Milano, Milan, Italy

ABSTRACT: In this work, standard Monte Carlo Simulation (MCS) and Subset Simulation (SS) are applied for assessing the probability of cascading failures in network systems. The latter method has been originally developed to solve structural reliability problems [Au and Beck, 2001], based on the idea that a small failure probability can be expressed as a product of larger conditional probabilities of some intermediate events. A model of cascading failure dynamics previously presented in the literature is taken as reference to predict the size of cascading failures in network systems, under realistic loading. In the application, SS is shown to be more efficient than MCS.
The sensitivity of the system response to uncertain parameters is also studied.

1 INTRODUCTION

Engineered critical infrastructures, e.g. distribution, communication and/or transportation networks are large scale, spatially distributed, complex systems. These systems are made of "a large number of interacting components, show emergent properties difficult to anticipate from the knowledge of single components, are characterized by a large degree of adaptability to absorb random disruptions and are highly vulnerable to widespread failure under adverse conditions" [Duenas-Osorio and Vemuru, 2009]. Small, unnoticed perturbations can trigger large scale consequences in critical infrastructures, as well as disruptions may also be caused by targeted attacks. While in general most failures in these systems emerge and dissolve locally, remaining largely unnoticed at the global level, a few occurrences trigger avalanche mechanisms (‘cascading failures’) that can have catastrophic effects over the entire networks. The vulnerability analysis of a CI must then duly account for the cascading failures phenomenon [Strogatz, 2001, Dorogovtesev and Mendes, 2003].

Typical examples of CIs suffering from cascading failures are the electrical power transmission systems, which are subject to multiple hazards and threats that must be identified and analyzed for optimal protection. All the electrical components have limits on their currents and voltages; if these limits are exceeded, automatic protection devices or the system operators disconnect the components from the system. The failure of a component causes a transient in which the power flow is redistributed to other components according to circuit laws and to the above mentioned automatic and manual reconfiguration actions.

The interactions between component failures which may lead to cascading failures are stronger when components are highly loaded. For example, if a more highly loaded transmission line fails, it produces a larger transient, there is a larger amount of power to redistribute to other components and failures in nearby protection devices are more likely. Moreover, if the overall system is highly loaded, components have smaller margins so they can tolerate smaller increases in load before failure, the system nonlinearities and dynamical couplings increase and the system operators have fewer options, less safety margins and more stress.

Functional models have been developed to capture the basic features of CI networks within topological analysis frameworks. These models analyze the extent to which the failure propagation process could affect the network system and allow identifying the actions that can be performed in order to prevent or mitigate the undesired effects [Motter, 2004, Dobson et al., 2005]. In case the consequences of a cascading event are underestimated, there is a risk that the system is not adequately protected and is operated at regimes for which the occurrence of cascading failures is highly likely: thus, the accuracy of the estimates provided by such models must be verified and quantified in particular in informed-based analyses and safety-oriented applications.

In this paper, a previously developed model for predicting the size of a cascading failure in network systems under varying uniform loading is considered [Zio and Sansavini, 2008]. The probability of cascading failure under certain stressing, normal loading conditions is evaluated by Monte Carlo Simulation (MCS). This, however, requires considerable computational efforts because of the long model calculations and the small values of the failure probability to be estimated [Schueller, 2007].

To overcome the MCS computational problem, the Subset Simulation (SS) method is applied. The SS method, originally developed to solve structural reliability problems [Au and Beck, 2001], is founded on the idea that a small failure probability can be expressed as a product of larger conditional probabilities of some intermediate (more probable) events.

The efficiency of the method is demonstrated by comparison to the standard MCS in an application concerning the Italian high-voltage electrical transmission network. Furthermore, the sensitivity of the electrical network system to uncertain system input parameters is studied through the examination of the deviations of the conditional sample distributions at different levels, from the unconditional one.

The contents of the paper are organized as follows: in Section 2 the model of failure propagation in the network system is presented; in Section 3 the SS method is briefly described; in Section 4 the results of the application of the SS method to the estimation of the cascade probability are compared with those obtained using MCS; conclusions are drawn in Section 5.

2 THE MODEL FOR CASCADING FAILURES DUE TO DISTRIBUTED RANDOM DISTURBANCES

In the following, a description of the model used for the propagation of cascading failures in a network system is given [Zio and Sansavini, 2008]. A simulation framework is developed which abstracts the physical details of the services provided by the infrastructure while at the same time capturing its essential operating features.

To study the propagation of cascading failures, the system is modeled as a network of N identical components with random initial loads, L_j, sampled from a uniform load distribution ranging between a minimum value L^{min} and a maximum value L^{max}.

All components have the same limit of operation L^{fail}, beyond which they are failed. Upon component failure, the positive amount of overload P is propagated locally, to first-neighbors of the failed node in the network structure. If there is no working node in the neighborhood of a failed component, the cascade spreading in that “direction” is stopped.

To start the cascade, an initial disturbance imposes on each component an additional load D. If the sum of the initial load L_j of component j and the disturbance D is larger than a component load threshold L^{fail}, component j fails. This failure occurrence leads to the redistribution of the additional load P on the neighboring nodes which may, in turn, get overloaded and thus fail in a cascade which follows the connection pattern of the network system. As the components become progressively more loaded, the cascade proceeds.

The MCS algorithm for simulating the cascading failures proceeds in successive stages as follows:

0 At stage $i = 0$, all N components are initially working under independent uniformly random initial loads $L_1, L_2, \dots, L_N \in [L^{min}, L^{max}]$, with $L^{max} < L^{fail}$.

1 An initial disturbance D is added to the load of each component.

2 Each unfailed component is tested for failure: for $j = 1, \dots, N$, if component j is unfailed and its load $> L^{fail}$ then component j fails.

3 The components loads are incremented taking into account the network topology, i.e. the failed component neighborhood: for each failed node, the load of its first-neighbors is incremented by an amount P. If the neighborhood set of the failed node is empty, the associated failure propagation comes to an end.

4 The stage counter i is incremented by 1 and the algorithm is returned to step 2.

The algorithm stops when failures are no further propagated. In order to compute the probability of failure, the cascading failure process is repeated N_T times, with N_T being the number of samples of the MCS algorithm.

For accurate estimation, MCS requires a large number of samples; in order to improve the simulation efficiency, the SS method, described in the following Section, is employed.

3 SUBSET SIMULATION

Subset Simulation (SS) is an adaptive stochastic simulation method for efficiently computing small failure probabilities, originally developed for the reliability analysis of structural systems [Au and Beck, 2001]. Structural reliability problems are naturally formulated within a functional failure framework of analysis in which the system fails when the applied load exceeds the structure capacity [Lewis, 1994; Schueller and Pradlwarter, 2007].

For application to the analysis of the network systems here of interest, the cascading failure process must be evaluated with respect to one or more variables crossing specified threshold values.
The idea underlying the SS method is to express a (small) failure probability as a product of (larger) probabilities conditional on some intermediate events. This allows converting a rare event simulation into a sequence of simulations of more frequent events. During simulation, the conditional samples are generated by means of a Markov chain designed so that the limiting stationary distribution is the target conditional distribution of some adaptively chosen intermediate event; by so doing, the conditional samples gradually populate the successive intermediate regions up to the target (rare) failure region [Au and Beck, 2001, 2003].

4 CASE STUDY: THE ITALIAN HIGH-VOLTAGE (380 KV) ELECTRICAL TRANSMISSION NETWORK

The model for cascading failures described in Section 2 has been applied to the Italian high-voltage (380 kV) electrical transmission network (HVIET) [Rosato et al., 2007]. The network consists of 127 bus locations (also called nodes) connected by 171 lines and transformers (Figure 1).

Suppose that the system is operated so that the normalized, initial component loadings vary from L_{min} to L_{max}=L_{fail}=1. A uniform distribution is assumed, without loss of generality; however, note that the modeling assumption of uniform initial load distribution may not be realistic and more complicated distributions for the loads in the grid buses can be anticipated from power network management. The average initial component loading L=(L_{min}+1)/2 can be increased by increasing L_{min}. The initial disturbance D is set to a value of 0.01; the load transfer amount P is taken equal to 0.05.

Figure 1. The 380 kV Italian power transmission network [TERNA 2002, Rosato et al. 2007].

In Figure 2, the results of the propagation of the cascading failures in the HVIET network obtained with the model of Section 2 are presented. The average number of failed components over N_T propagation simulations, i.e. the average cascade size $\bar{S}$, is reported as a function of the average initial load L.

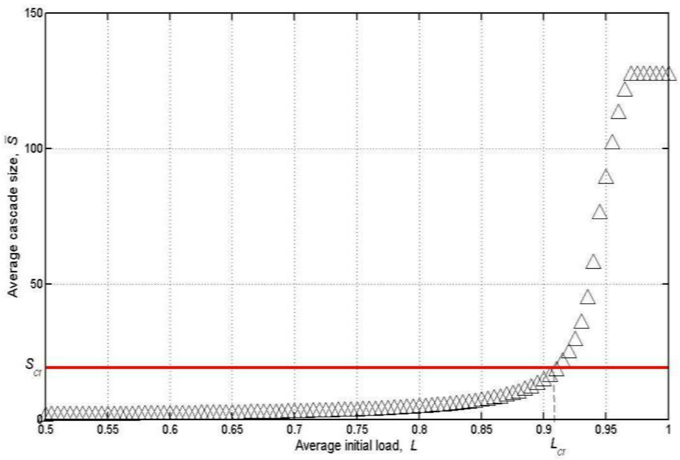

Figure 2. Cascade propagation in the HVIET network. The number of failed components averaged over N_T propagation simulations, i.e. the average cascade size $\bar{S}$ is reported as a function of the average initial load L. The initial load on the components is distributed as N(0.9, 0.025). The system is considered to be failed when more than S_{cr}=15% (19 components) fail (solid line).

To quantitatively assess the effects of the cascading failures at the system level, a threshold representing the maximum allowable cascade size, S_{cr}, can be set. This is interpreted as the maximum number of components which can be lost in the system without affecting the global service provided. This threshold can vary from system to system and is a distinguishing feature of the provided service. The cascade size threshold identifies the critical average initial load, L_{cr}, beyond which cascade effects are felt at the system level. In the following, a value of S_{cr}=15% of the total number of components is assumed (i.e. 19 components), which gives L_{cr}=0.92 for the system. The sharp change in gradient at the critical loading L_{cr}=0.92 corresponds to the increasing probability of all components failing and the consequent onset of a cascading failure phenomenon. As loading is increased past the critical value, an increasingly significant probability of all components failing and a significant risk of large cascading failures is to be expected.

These results can be used to estimate the cascading failure probability $P(\bar{S} > S_{cr})$ under realistic operating conditions. It is assumed that the buses are designed to all have the same amount of load but statistical fluctuations around the design load may occur during operation; this situation is here modeled by a truncated normal distribution of the components initial loads, L_j. The mean of the distribution is set equal to 0.92 and the standard deviation to 0.025 (Figure 3), to describe a stressed system operating on the very edge of the cascading failure regime. From the previous results of the predictive model with uniform distribution of the initial loads, a very low probability of entering in the critical zone of operation can be anticipated, i.e. the number of failed components is expected to go beyond the maximum allowable cascade size, S_{cr} with very low probability. Nonetheless, underestimation of the cascade size is to be avoided due to the critical consequences.

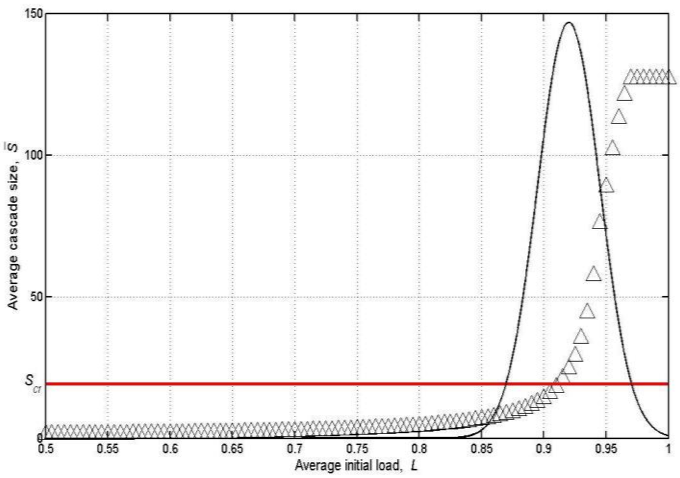

Figure 3. Cascade propagation in the HVIET network. The number of failed components averaged over N_T propagation simulations, i.e. the average cascade size $\bar{S}$ is reported as a function of the average initial load L. The initial load on the components is distributed as N(0.92, 0.025). The system is considered to be failed when more than S_{cr}=15% (19 components) components fail (solid line).

To evaluate the probability of cascading failure, $P(\bar{S} > S_{cr}) = P(\bar{S} > 19)$, a MCS has been performed sampling from the normal distribution N(0.92, 0.025) of initial loading. The uncertain parameters of the model are the capacities of the 127 components of the network, each one normally distributed with a mean of 0.92 and a standard deviation of 0.025. The initial disturbance D and the load transfer amount P are the same as in the previous case.

However, a MCS requires considerable computational efforts for estimating small probabilities. Hence, in order to more efficiently estimate the small probability of failure of the system, i.e. the probability that more than 19 components fail, the SS algorithm described in Section 3 has also been applied.

The results obtained are summarized in Table 1. In order to obtain a satisfactory accuracy in the estimation of the failure probability, it is necessary to sample N_T=100000 realizations by MCS and only N_T=5550 by SS.

It is worth noting that the total number of samples N_T employed by SS can not be defined a priori since it depends on the number of conditional levels; this depends in turn on the failure probability $P(\bar{S} > 19)$ to be estimated, which is unknown a priori. Thus, for fair comparison, the efficiency of the MCS and SS methods is evaluated in terms of two indices which are independent of the total number N_T of samples drawn: the unitary Coefficient of Variation (c.o.v.), Δ, and the so-called Figure of Merit (FOM).

The unitary c.o.v. Δ is defined as

$$\Delta = \delta \cdot \sqrt{N_T} = \frac{\sigma}{\hat{P}(\bar{S} > 19)} \cdot N_T$$

where δ is the c.o.v., $\hat{P}(\bar{S} > 19)$ is the sample estimate of $P(\bar{S} > 19)$ and σ is the sample standard deviation of $\hat{P}(\bar{S} > 19)$. Since in all Monte Carlo type estimators the standard deviation σ (and, thus, the c.o.v. δ) decays with a rate $O(1 \backslash N_T)$, then $\Delta = \delta \cdot \sqrt{N_T}$ is independent of N_T [Koutsourelakis et al., 2004]. Notice that the lower is the value of Δ, the lower is the variability of the corresponding failure probability estimator and thus the higher is the efficiency of the simulation method adopted.

However, in addition to the precision of the failure probability estimator, also the computational time associated to the simulation method has to be taken into account. Thus, the FOM is introduced as

$$\text{FOM} = \frac{1}{\sigma^2 \cdot t_{comp}}$$

where t_{comp} is the computational time required by the simulation. Since $\sigma^2 \propto N_T$ and approximately $t_{comp} \propto N_T$, also the FOM is independent of N_T. Notice that the higher is the value of this index, the higher is the efficiency of the method.

Table 1 reports the values of the estimate of the failure probability $\hat{P}(\bar{S} > 19)$, the unitary c.o.v. Δ and the FOM obtained by standard MCS with N_T=100000 samples and SS with N_T=5550.

Table 1. Values of the estimate of the failure probability $\hat{P}(\bar{S} > 19)$ (per demand), unitary Coefficient of Variation (c.o.v.) Δ and Figure of Merit (FOM) obtained by standard MCS with N_T=100000 samples and SS with N_T=5550 samples in the reliability analysis of the system of Figure 1.

Method	N_T	Failure probability	Δ	FOM
MC	100000	$2.4*10^{-4}$	64.541	$2.37*10^{7}$
SS	5550	$1.78*10^{-4}$	23.838	$7.57*10^{7}$

The Markov chain samples generated by SS can be used not only for estimating the conditional probabilities, but also for sensitivity analysis. The sensitivity of the electrical transmission network performance to the individual uncertain input parameters can be studied by examining the change of the sample distributions at different conditional levels. The histograms of the conditional samples of one of the uncertain load parameters at different conditional levels for a single SS run are shown in Figure 4. The histograms correspond to the Ravenna Canala bus component of the HVIET network [Zio and Sansavini, 2010], characterized by the largest connectivity degree (seven buses directly connected to it) and therefore with large influence on the failure propagation process. The importance of this bus is confirmed by its sensitivity in Figure 4: a small deviation of the histogram at level 3 is noted with respect to the histogram at level 0. The fact that deviation is small is due to the fact that the number of parameters considered is very large, i.e. 127 nodes, and therefore the extent of the propagation is influenced mainly by the average degree of the network and not by the individual degrees of the nodes.

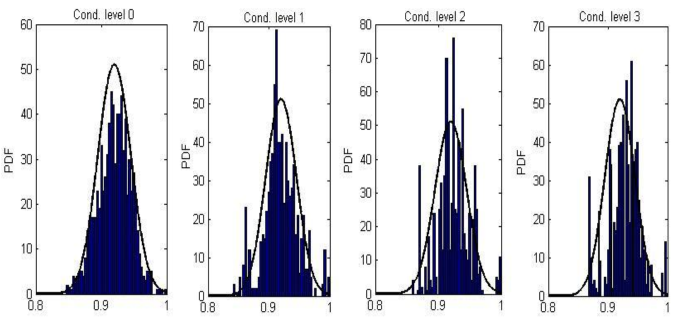

Figure 4. Empirical conditional distributions of the uncertain input load parameter of the Ravenna Canala bus

5 CONCLUSIONS

In this paper, the prediction power of a previously developed model for predicting the size of a cascading failure in network systems under varying working levels has been considered. The probability of cascading failure under certain stressing conditions has been evaluated by simulation. To overcome the computational problem of standard MCS due to the long calculations of the cascading failure model and the small values of the probabilities of failure involved, the SS method has been used. Such method circumvents the computational burden associated to the estimation of a small failure probability by computing it as a product of larger conditional probabilities of defined (more probable) intermediate events.

The results of SS have been compared to those of standard MCS for the estimation of probabilities as small as 10^{-4}. It has been shown that for small probabilities of failure, SS is more efficient than standard MCS.

Also, the sensitivity of the results to uncertain system input parameters has been studied through the examination of the conditional sample distributions at different levels. The analysis has shown that an informative measure of the importance of a given parameter in determining the failure of the system is the deviation of its conditional distribution from the unconditional one.

This work has been funded by the Foundation pour une Culture de Securite Industrielle of Toulouse, France, under the research contract AO2006-01.

6 REFERENCES

Au, S.K., Beck, J.L., Estimation of small failure probabilities in high dimensions by subset simulation. Probabilistic Engineering Mechanics 16 (4), 263–277, 2001.

Au, S.K., Beck, J.L., Subset Simulation and its application to seismic risk based on dynamic analysis. Journal of Engineering Mechanics 129 (8), 1–17, 2003.

Dobson, I., Carreras, B.A. and Newman, D.E., A loading-dependent model of probabilistic cascading failure, Probability in the Engineering and Informational Science, 19, pp. 15-32, 2005.

Dueňas-Osorio, L. and Vemuru ,S.-M.,Cascading failures in complex infrastructure systems, Structural Safety, Vol.31, pp. 157-167, 2009.

Dorogovtesev, S.N., Mendes, J.F.F., Evolution of Networks Oxford University Press, Oxford, 2003.

Koutsourelakis, P.S., Pradlwarter, H.J., Schueller, Reliability of structures in high dimensions. Part I: algorithms and application. Probabilistic Engineering Mechanics (19), 409–417, 2004.

Lewis, E.E., Introduction to Reliability Engineering, second ed. John Wiley and Sons, Inc., 1994.

Motter, A.-E., Cascade control and defense in complex Networks, Physical Review Letters, vol. 93, nr 9, pp. 098701(1-4), 2004.

Rosato, V., Bologna, et al., Topological properties of high-voltage electrical transmission networks. Electric Power Systems Research 77: 99-105, 2007.

Schueller, G.I., On the treatment of uncertainties in structural mechanics and analysis. Computers and Structures 85, 235–243, 2007.

Schueller, G.I., Pradlwarter, H.J., Benchmark study on reliability estimation in higher dimension of structural systems—an overview. Structural Safety (29), 167–182, 2007.

Strogatz, S. H., Exploring complex networks, Nature, Vol. 410, pp. 268-276, 2001.

TERNA. 2002. Dati statistici sull'energia elettrica in Italia. Techical Report. Terna S.p.A. - Rete Elettrica Nazionale. (in Italian) http://www.terna.it/LinkClick.aspx?fileticket=PUvAU57MlBY%3d&tabid=418&mid=2501

Zio, E. and Sansavini, G., Modeling failure cascades in networks systems due to distributed random disturbances and targeted intentional attacks, in Safety, Reliability and Risk Analysis: Theory, Methods and Applications – Martorell et al. (eds), Proceedings of ESREL 2008 and 17th SRA Europe Annual Conference, 22-25 September 2008, Valencia, Spain, Taylor & Francis Group, London, 2009.

Zio, E., Sansavini, G., Component criticality in failure cascade processes of network systems, submitted to Risk Analysis An International Journal, 2010.