



HAL
open science

Taxonomy of models for tolerance analysis in assembling.

Wilma Polini

► **To cite this version:**

Wilma Polini. Taxonomy of models for tolerance analysis in assembling.. International Journal of Production Research, 2011, pp.1. 10.1080/00207543.2011.576275 . hal-00720576

HAL Id: hal-00720576

<https://hal.science/hal-00720576>

Submitted on 25 Jul 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Taxonomy of models for tolerance analysis in assembling.

Journal:	<i>International Journal of Production Research</i>
Manuscript ID:	TPRS-2010-IJPR-0552.R1
Manuscript Type:	Original Manuscript
Date Submitted by the Author:	07-Jan-2011
Complete List of Authors:	Polini, Wilma; Università degli Studi di Cassino, Dipartimento di Ingegneria Industriale
Keywords:	TOLERANCE ANALYSIS, TOLERANCING
Keywords (user):	models taxonomy

SCHOLARONE™
Manuscripts

Taxonomy of models for tolerance analysis in assembling

Wilma Polini

Dipartimento di Meccanica, Strutture, Ambiente e Territorio, Università degli Studi di Cassino, Cassino, Italy

Via Gaetano di Biasio 43, 03043 Cassino, Italy, e-mail: polini@unicas.it

(Received XX Month Year; final version received XX Month Year)

The dimensional and geometrical variations of each part have to be limited by tolerances in order to guarantee quality and, in the same time, a decrease of the manufacturing costs. Therefore, how to allocate the tolerances among the different parts of an assembly is the fundamental tool to ensure assemblies that work rightly at lower costs. Tolerance analysis allows to evaluate the cumulative effect that the single tolerances assigned on the components has to satisfy the functional requirements of the whole assembly. The results of the tolerance analysis are meaningfully conditioned by the adopted mathematical model.

The purpose of this work is to analyse the most significant models for tolerance analysis. The first part of the paper describes a description of the most common criteria used to categorize existing models for tolerance analysis. A taxonomy is then suggested that may be a useful tool to help evaluate, compare and select such models. Five of the most representative models are explained and discussed in detail in order to identify their strong points and their limitations.

Keywords: tolerance analysis; models taxonomy; assembling

1. Introduction

Mechanical products are usually made by assembling many parts. Their quality is guaranteed by the respect of some functional requirements assigned to the whole assembly. It is needed to limit the deviations from nominal due to the manufacturing process of each assembly's component inside the assigned tolerances. Moreover, the respect of the functional requirements of the assembly depends on the effect of the tolerances assigned to the single parts and on the assembly constraints.

The aim of the tolerance analysis is to study the dimensional and/or geometric variations of the assembly due to a stack of dimensions and tolerances that are applied

1
2
3 to the assembly's components. Therefore, tolerance analysis is the fundamental tool to
4
5 allocate the tolerances on the single components by solving the trade-off between the
6
7 quality of a product and its cost.
8
9

10 11 **1.1 Tolerance analysis**

12 The aim of the tolerance analysis of an assembly is to evaluate the cumulative effect
13
14 of the tolerances, that are assigned to the assembly components, on the functional
15
16 requirements of the whole assembly. Each functional requirement is schematized
17
18 through an equation, that is usually called stack-up function, whose variables are the
19
20 model parameters that are function of the dimensions and the tolerances assigned to
21
22 the assembly components. It looks like
23
24
25

$$26 \quad FR = f(p_1, p_2, \dots, p_n) \quad (1)$$

27
28 where FR is the considered functional requirement, p_1, \dots, p_n are the model parameters
29
30 and $f(p)$ is the stack-up function, that is usually not linear.
31
32

33
34 A functional requirement is a usually characteristic that relates two features. Its
35
36 analytical expression is obtained by applying the equations of the Euclidean geometry
37
38 to the features that define the functional requirement or to the points of the features
39
40 that define the functional requirement.
41
42

43
44 A stack-up function has to model two possible assembly variations. The first variation
45
46 is due to the tolerances assigned to the features of the assembly components. The
47
48 obtained model (that is called "*local model*") has to be able to schematize all the
49
50 tolerance kinds, i.e. dimensional, form, and so on, but in the same time it has to be
51
52 able to represent the Envelope Principle (Rule # 1 of ASME standard) or the
53
54 Independence Principle (according with ISO 8015 standard) applied to different
55
56 dimensions of the same part. The local model has to define the range of variation of
57
58 the model's parameters from the assigned tolerances and it has to schematize the
59
60

1
2
3 interaction among the assigned tolerance zones. The second variation is due to the
4 contact among the assembly components. The variability of the coupled features, by
5 which the link among the parts is made, gives a deviation in the location among the
6 coupled parts. The resulting model (that is called “*global model*”) has to be able to
7 schematize the joints with contact and the joints with clearance between the coupled
8 features. Moreover, the global model has to be able to approach to the joints which
9 give a linear structure of the FR equation (stack-up function) and to the joints which
10 carry out to a complex structure of the FR equation (network function), as shown in
11 Figure 1.
12
13
14
15
16
17
18
19
20
21
22
23

24 Once modeled the stack-up functions, they may be solved by means of a worst case or
25 a statistical case approaches (Creveling 1997). To carry out a worst case approach, it
26 is needed to define the worst configurations of the assembly (i.e. those configurations
27 due to the cumulative effect of the smallest and the highest values of the tolerances
28 assigned to the assembly components) that satisfy its assigned tolerances. This means
29 to solve a problem of optimization (maximization and/or minimization) under
30 constraints due to the assigned tolerances. Many are the methods developed by the
31 literature to carry out a worst case approach (see Luenberger 2003). To carry out a
32 statistical approach, it is needed to translate each tolerance assigned to an assembly
33 components into one or more parameters of the stack-up function. Therefore, a
34 Probability Density Function (PDF), that is usually a Gaussian Function, is assigned
35 to each parameter. The variation of the FR is obtained by means of a Monte Carlo
36 simulation technique (Nigam and Turner 1995, Nassef and ElMaraghy 1996); it is
37 usually calculated as \pm three times the estimated standard deviation (three sigma
38 paradigm of Creveling 1998).
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1.2 State of the art on tolerance analysis

The result of the tolerance analysis is meaningfully conditioned by the adopted mathematical model. Some are the models proposed by the literature to carry out a tolerance analysis of an assembly, but there does not exist in the literature a paper that compares the different analytical models on the basis of a case study that underlines in a clear way all the advantages and the weakness. In the literature, Chase and Parkinson compare a number of analytical models existing with varying levels of sophistication for 1D stack-up functions that involve only dimensions (Chase and Parkinson 1991). The authors deal with the 2D and 3D tolerance analysis by means of the vector loop model only. Hong and Chang classified the 3D tolerance propagation scheme according to two things that are closely related each other: the representation of tolerance zones and the spatial tolerance propagation scheme (Hong and Chang 2002). Shen et al. explain the differences among tolerance charting, a commercial CAT (CETOL) and T-Map method that is developed by the authors (Shen et al. 2005). Some of the criteria used for the comparison are quantitatively and very interesting: the analysis type, the tolerance types, the bonus/shift tolerances, the datum precedence, the tolerance zone interaction. Prisco and Giorleo compare the five commercial CATs by using some of the parameters introduced by Shen et al. Other studies compare the main commercial CATs that implement some of the models of the tolerance analysis (Turner and Gangoit 1991, Salomons *et al.* 1998, Prisco and Giorleo); they use few parameters to describe the approach to the tolerance analysis, such as the variation sources (dimensional, geometric or small kinematic adjustments), the simplifying assumptions (rigid body and so on), the tolerance analysis kind (worst case or statistical analysis) and the sensitivity analysis. However, a complete comparison of the models proposed to solve the tolerance analysis involving all the aspect of a tolerance analysis problem does not exist in the

1
2
3 literature and, therefore, no guidelines exist to select the method more appropriate to
4
5 the specific aims.
6
7

8 9 **2. Development of the taxonomy**

10
11
12 The purpose of this paper is to provide a reference framework of the major models for
13 tolerance analysis. A taxonomy to evaluate and compare them is also suggested. The
14 first part of the paper provides a description of the most common criteria to categorize
15 tolerance analysis models (taxonomy) and, then, the case study used to compare the
16 models. Subsequently, five of the most representative models are independently
17 described and set in the suggested taxonomy (see Table 1), in order to identify their
18 common features as well as those set them apart. Considering the great abundance of
19 models presented in the literature, those discussed here were selected owing to their
20 originality and spread. The five chosen model have been completely developed for 3D
21 applications that involve geometrical tolerances too. All models are accompanied by
22 explanatory representation schemes.
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39

40 **2.1 Taxonomy description**

41 In this section we propose a taxonomy to benchmark tolerance analysis models.
42 Taxonomy is a useful tool for evaluating and comparing tolerance analysis models,
43 depending on their peculiarities. In the next section, five of the most representative
44 tolerance analysis models are illustrated and classified in detail. The evaluation
45 criteria has been defined by starting from those proposed by the literature and
46 introducing all the criteria needed to describe all the steps involved in a tolerance
47 analysis problem: the translation of the applied tolerances and the kinematic joints
48 into model's parameters, the building of the stack-up functions, the strategy adopted to
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3 solve the stack-up functions. Those steps may be found in all the methods proposed
4
5 by the literature. Therefore, the proposed evaluation criteria are:
6
7

- 8 • the tolerance kind: the models allows to deal with the dimensions and their
9 related tolerances or with the form tolerances or with all the geometrical
10 tolerances (with the exception of form);
11
- 12 • the Envelope and the Independence: the model allows to deal with the
13 Envelope or the Independence Principle applied to a dimensional tolerance;
14
- 15 • the tolerance zones' interaction: the model allows or it does not allow to
16 schematize more tolerances applied on the same feature;
17
- 18 • the precedence among datum: the model allows or it does not allow to deal
19 with an assigned precedence among the datum;
20
- 21 • the Material Modifier Condition: the model allows or it does not allow to deal
22 with assigned material modifier conditions;
23
- 24 • the model's parameters from tolerances: the model allows or it does not allow
25 to assign a probability density function to the model's parameters starting
26 from the assigned tolerances;
27
- 28 • the joint type: the model may deal with joints with contact only or it may deal
29 with joints with clearance too;
30
- 31 • the functional requirement organization: the functional requirement is
32 represented through features or through points belonging to the features;
33
- 34 • the tolerance stack-up function: the model may solve linear stack-up function
35 only or network stack-up function too.
36
- 37 • the analysis type: the model takes into account the worst-case approach or the
38 statistical approach to solve the stack-up functions.
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

The values of the proposed criteria are shown in Table 2.

2.1.1 Development of a case study

To compare the tolerance analysis models, the case study shown in Figure 2 is introduced too. The two-dimensional geometry of the example assembly is made of a rectangular box containing two disk-shaped parts. The width g of the gap between the top disk and the upper surface of the box is assumed as the functional requirement to be investigated by the analysis. Goal of the tolerance analysis problem is to identify the tolerance stack-up function that defines the variability of g , and describes it as a function of the geometries and tolerances of the components involved in the assembly.

Tolerance analysis is based on the dimensional and geometrical tolerances illustrated in Figure 2.

Both the worst-case and the statistical approaches are considered when solving the tolerance analysis problem. The case study is representative of all the main aspects and critical issues involved in a typical tolerance analysis problem, while at the same time being computationally simple enough to allow for a geometrical resolution procedure in order to have an exact solution that is a term to compare the results due to all five tolerance analysis models. The example is adapted from a real-life industrial application and properly simplified to make it easier to be presented and discussed in this context. The applied tolerancing scheme, which may not appear as entirely rigorous under the viewpoint of a strict application of standardized tolerancing rules, is directly derived from the current practice adopted for the actual industrial product.

As a quantitative reference to be used when comparing results obtained by applying the tolerancing models, a first computation is now illustrated which makes use of the worst-case conditions to derive the exact geometrical solution for the variability of the

gap width. The dimensional tolerances have been considered, while the geometry of the components has been considered nominal.

The maximum value of the gap has been calculated by considering the maximum height and width of the box, together with the minimum value of the radius of the disks:

$$g_{\max_{\text{dim}}} = 80.5 - 19.95 - \sqrt{(19.95 \cdot 2)^2 - (504 - 19.95 - 19.95)^2} - 19.95 = 2.1064 \text{mm} \quad (2)$$

In the same way the minimum value of the gap is due to:

$$g_{\min_{\text{dim}}} = 79.5 - 20.05 - \sqrt{(20.05 \cdot 2)^2 - (49.80 - 20.05 - 20.05)^2} - 20.05 = 0.4909 \text{mm} \quad (3)$$

The variability of the gap is the difference between the maximum or the minimum values and the nominal one:

$$\begin{aligned} \Delta g_{\text{dim}_1} &= +(g_{\max_{\text{dim}}} - g_N) = (2.1064 - 1.2702) \cong +0.84 \text{mm} \\ \Delta g_{\text{dim}_2} &= -(g_N - g_{\min_{\text{dim}}}) = -(1.2702 - 0.4909) \cong -0.78 \text{mm} \end{aligned} \quad (4)$$

In the following paragraphs five of the most significant tolerance analysis model are described in detail, following the criteria on taxonomy presented previously.

3. Vector loop model

The vector loop model uses vectors to represent relevant dimensions in an assembly (Chase *et al.* 1995, Chase *et al.* 1996, Chase *et al.* 1997). Each vector represents either a component dimension or an assembly dimension. Vectors are arranged in chains or loops to reproduce the effects of those dimensions that stack together to determine the resultant assembly dimensions. Three types of variations are modelled in the vector loop model: *dimensional variations*, *kinematic variations* and *geometric variations*.

In a vector loop model the magnitude of a geometric dimension is mapped to the length (L_i) of the corresponding vector. Dimensional variations defined by dimensional tolerances are incorporated as +/- variations in the length of the vector.

1
2
3 Kinematic variations describe the relative motions among mating parts, i.e. small
4 adjustments that occur at assembly time in response to the dimensional and geometric
5 variations of the components. In the vector loop model, kinematic variations are
6 modelled by means of *kinematic joints*, i.e. schematizations such as the *slider*, etc. In
7 vector loop models there are 6 common joint types available for 2-D assemblies and
8 12 common joints for 3-D assemblies. At each kinematic joint, assembly adjustments
9 are turned into ranges for the motions allowed by the joint (i.e. degrees of freedom). A
10 local datum reference frame (DRF) must be defined for each kinematic joint.

11
12 Geometric variations capture those variations that are imputable to geometric
13 tolerances. These are modelled by adding additional degrees of freedom (DOFs) to
14 the kinematic joints illustrated above. This introduces a simplification: although
15 geometric tolerances may affect an entire surface, in vector loop models they are
16 considered only in terms of the variations they induce at mating points, and only in
17 the directions allowed by the type of kinematic joint. Depending on what type of
18 geometric variation is represented by the tolerance and what motions are allowed at
19 the kinematic joint, a geometric tolerance is typically modelled as an additional set of
20 translational and rotational transforms (e.g. displacement vectors, rotation matrices) to
21 be added at the joint.

22
23 To better understand the vector loop model, the basic steps for applying it to a
24 tolerance analysis problem are provided below (Gao et al 1998, Chase 1999, Faerber
25 1999, Nigam and Turner 1995).

- 26
27 (1) *Create the assembly graph* - The first step is to create an assembly graph. The
28 assembly graph is a simplified diagram of the assembly representing the parts,
29 their dimensions, the mating conditions and functional requirements, i.e. the
30 final assembly dimensions that must be measured in order to verify that the

1
2
3 product is capable of providing the required functionality. An assembly graph
4
5 assists in identifying the number of vector chains and loops involved in the
6
7 assembly.
8
9

10 (2) *Define the datum reference frame (DRF) for each part* - The next step is to
11
12 define the DRF for each part. DRFs are used to locate relevant features on each
13
14 part. If there is a circular contact surface, its center is considered as a DRF too.
15
16

17 (3) *Define kinematic joints and create datum paths* - Each mating relation among
18
19 parts is translated into a kinematic joint. Kinematic joints are typically located
20
21 at contact points between parts. Datum paths are geometric layouts specifying
22
23 the direction and orientation of vectors forming the vector loops, they are
24
25 created by chaining together the dimensions that locate the point of contact of a
26
27 part with another, with respect to the DRF of the part itself.
28
29
30

31 (4) *Create vector loops* - Using the assembly graph and the datum paths, vector
32
33 loops are created. Each vector loop is created by connecting datums; vector
34
35 loops may be open or closed; an open loop terminates with a functional
36
37 requirement, which can be measured in the final assembly (it could either be
38
39 the size of a relevant gap in the final assembly, or any other functionally-
40
41 relevant assembly dimension); a closed loop indicates the presence of one or
42
43 more adjustable elements in the assembly.
44
45
46
47

48 (5) *Derive the stack-up equations* - The assembly constraints defined within vector
49
50 loop-based models may be mathematically represented as a concatenation of
51
52 homogeneous rigid body transformation matrices. \mathbf{H} is the resultant matrix. If
53
54 the assembly is described by a closed loop of constraints, \mathbf{H} is equal to the
55
56 identity matrix, otherwise \mathbf{H} is equal to the \mathbf{g} vector representing the resultant
57
58 transformation that will lead to the identification of a functional requirement.
59
60

1
2
3 (6) *Tolerance Analysis* - assuming an assembly as made of p-parts. Each part is
4 represented by a \mathbf{x} -vector of its relevant dimensions and by an α -vector
5 containing additional dimensions, added for taking into account geometric
6 tolerances. When parts are assembled together, the resulting product is
7 characterized by a \mathbf{u} -vector of the assembly variables and by a \mathbf{g} -vector of
8 measurable functional requirements. It is possible to calculate \mathbf{g} -vector from the
9 closed or open loop of constraints by means of the Direct Linearization Method
10 (DLM). The DLM is a very simple and rapid method, but it is approximated
11 too. When an approximated solution is not acceptable, it is possible to use
12 alternative approaches, such as numerical simulation by means of a Monte
13 Carlo technique (Gao *et al.* 1998, Boyer and Stewart 1991).
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29

30 **3.1 Taxonomy descriptors**

31 *Vector loop* model may be solved by means of both the worst case and the statistical
32 approaches. It allows to take into account the dimensional and all the geometrical
33 tolerances (the form too). If more tolerances are applied to the same feature, each
34 tolerance is considered separately. Therefore, the model is not able to take into
35 account the Envelope or the Independence principle applied to the dimensional
36 tolerances and the interaction among the tolerance zones, since the *vector loop* models
37 a dimensional tolerance by a vector with only one parameter that can change (its
38 length). The model allows easily to assign a probability density function to the
39 model's parameters starting from the assigned tolerances. The model may deal with
40 linear stack-up function and network stack-up functions. All the considered joints
41 provide the contact among the mating parts. The functional requirements of the
42 assembly may be represented through both features or points. The model is not able to
43 distinguish the precedence among the datum. It may not take into account the MMC,
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3 since the Envelope principle is not included. A commercial Computer Aided
4 Tolerancing (CAT) software, known as Cetol 6 σ [®] of Sigmatrix, is based on vector
5 loop model.
6
7
8
9

10 Table 3 shows the results of the application of the *vector loop* model to the case study,
11 when only the dimensional tolerances or both the dimensional and the geometrical
12 tolerances are applied. All the mathematical steps are reported in [Marziale and Polini
13 2009]. If only the dimensional tolerances are applied, the worst case approach gives
14 small under estimated results of about 4%, when compared with the geometrical exact
15 solution. This is probably due to the same way the dimensional tolerances are
16 schematize (i.e. the first datum is nominal, the variability due to the dimensional
17 tolerance is considered applied only on one of the two features delimiting the
18 dimension). If both dimensional and geometrical tolerances are applied, the worst
19 case approach gives as result an increase of the range of the *g-dimension*. This is due
20 to the fact that the *vector loop* model considers the effect of a set of tolerances applied
21 to a surface as the sum of the effects due to each single tolerance applied to the same
22 surface. The effects of the different tolerances are considered independent. Therefore,
23 increasing the number of tolerances applied to the same surface increases the
24 variability of the functional requirement. This means that the interaction among
25 tolerances defined on the same surface are not properly handled. The statistical
26 approach gives similar results, when only dimensional tolerances or both dimensional
27 and geometrical tolerances are applied.
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52

53 **4. Variational model**

54
55
56
57
58 A mathematical foundation of this model has been proposed by Boyer and Stewart
59 (Boyer et al 1991) first, and then by Gupta and Turner (Gupta et al 1993). Later,
60

1
2
3 several additional variants have been proposed as well, and nowadays commercial
4
5 CAT software packages are based on this approach, such as eM-TolMate of UGS[®], 3-
6
7 DCS of Dimensional Control Systems[®], VisVSA of UGS[®].
8
9

10 The basic idea of the variational model is to represent the variability of an assembly,
11
12 due to tolerances and assembly conditions, through a parametric mathematical model.
13
14

15 To create an assembly, the designer must define the nominal shape and the
16
17 dimensions of each assembly component (this information is usually retrieved from
18
19 CAD files). Then, the designer identifies the relevant features of each component and
20
21 assigns dimensional and geometrical tolerances to them. Each feature has its local
22
23 Datum Reference Frame (DRF), while each component and the whole assembly have
24
25 their own global DRF. In nominal conditions, a homogeneous transformation matrix
26
27 (called **TN**) is defined that identifies the position of the feature DRF with respect to
28
29 the part DRF. In real conditions (i.e. manufactured part), the feature will be
30
31 characterized by a roto-translational displacement with respect to its nominal position.
32
33 This displacement is modeled to summarize the complete effects of the dimensional
34
35 and geometric variations affecting the part by means of another matrix: the
36
37 differential homogeneous transformation matrix (called **DT**). The variational model
38
39 may take into account the precedence among the datums by setting the parameters of
40
41 **DT** matrix.
42
43
44
45
46
47

48 The variational model is not able to deal with the form tolerances, such as vector loop
49
50 model does; this means that the actual feature shape is assumed unchanged, i.e.
51
52 feature shape variations are neglected. The position of the displaced feature in the
53
54 part DRF can be simply obtained by matrix multiplication as a change of DRF.
55
56

57 The model is parametric because different types and amounts of variations can be
58
59 modeled by simply altering the contents (parameters) of the **DT** matrix. In some
60

1
2
3 cases, the localization of a feature affected by variation may be defined by a transform
4 with respect to another feature in the same part, which is affected by variations as
5 well. Therefore, the material modifier condition are modeled by setting the parameters
6 of DT matrix.
7
8
9

10
11
12 Once the variabilities of the parts are modeled, they must be assembled together.
13
14 Another set of differential homogeneous transformation matrices is introduced to
15 handle the roto-translational deviations introduced by each assembly mating relation.
16
17 Such matrices are named **DA**, with the letter A (=assembly) to distinguish them from
18 the matrices that have been used for parts. Those matrices are hard to evaluate, since
19 they depend by both the tolerances imposed on the parts in contact and by the
20 assembly conditions. This model is not able to represent mating conditions with
21 clearance. The problem of evaluating the differential matrix is analyzed in several
22 literature works. A possible strategy consists in modeling the joint between the
23 coupled parts by reconstructing the coupling sequence between the features (Berman
24 2005). Another possibility is to impose some analytical constraints on the assembly
25 parameters (Whitney 2004).
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40

41 When all the transformation matrices are obtained, it is possible to express all the
42 features in the same global DRF of the assembly. Finally, the functional requirements
43 can be modelled in the form of equation (1), obtained from the matrix multiplications
44 described above. This model may be applied to assemblies involving joints which
45 makes a linear structure among the parts (linear stack-up function, see Figure 1a) and
46 joints which makes a complex structure among the parts (networks of stack-up
47 functions, see Function 1b), such as vector loop does.
48
49
50
51
52
53
54
55
56

57 Once the stack-up functions are modelled, there are two approaches to solve them: the
58 worst-case approach and the statistical approach. The worst-case analysis, consists in
59
60

1
2
3 identifying the extreme configurations of the assembly under a given set of tolerances.
4
5 In the variational approach, the problem is generally handled as an optimization
6
7 (maximization and/or minimization) problem, under constraints defined by the
8
9 tolerances themselves. The statistical approach is generally handled by assigning
10
11 predefined probability density functions, e.g. Gaussian, to the parameters identifying
12
13 the main elements that contribute to the variation of each feature (often assumed
14
15 independent, by simplification), and then solving the stack-up functions accordingly
16
17 (Salomons et al 1996).
18
19
20
21

22 To better illustrate the variational method, its basic steps are illustrated in the
23
24 following:
25
26

- 27 (1) *Create the assembly graph* - The first step is to create an assembly graph. The
28
29 assembly graph is a simplified diagram of the assembly representing the parts,
30
31 the features, the mating conditions and the functional requirements.
32
33
- 34 (2) *Define the DRF of each feature, of each part, and of the assembly* - The next
35
36 step is to identify the local DRF of each feature, the global DRF of each part
37
38 and of the assembly (usually the DRF of the assembly coincides with the DRF
39
40 of the first part). DRFs are positioned depending on surface type; from the
41
42 DRFs, local parameters and the differential homogeneous transformation
43
44 matrices **DT** are defined.
45
46
47
- 48 (3) *Transform the features* – Once the transformation matrices are known, each
49
50 feature of a part is transformed in the global DRF of the part.
51
52
- 53 (4) *Create the assembly* - Using the assembly graph and the transformed features,
54
55 the assembly conditions are extracted, i.e. the assembly parameters included
56
57 into the matrix **DA** are calculated.
58
59
60

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

(5) *Derive the equations of the functional requirements* – Once known the assembly parameters, all the features can be expressed in the same global DRF of the assembly. At this point, the functional requirements are defined in terms of functions, that can be solved by means of the previously described worst-case and/or statistical approaches.

4.1 Taxonomy descriptors

Variational model may be solved by means of both the worst case and the statistical approaches. It deals with the dimensional and the geometrical tolerances, form tolerance excepted. In fact, it considers the substitute feature as the actual feature. The model is not able to take into account the Envelope or the Independence principle applied to the dimensional tolerances and the interaction among the tolerance zones. The model allows easily to assign a probability density function to the model's parameters starting from the assigned tolerances. The model may deal with linear stack-up function and network stack-up functions. All the considered joints provide the contact among the mating parts. The functional requirements of the assembly may be represented through both features or points. The model is able to distinguish the precedence among the datum. It may take into account the MMC.

Table 3 shows the results of the application of the *variational* model to the case study, when only the dimensional tolerances or both the dimensional and the geometrical tolerances are applied. All the mathematical steps are reported in (Marziale and Polini 2009). If only the dimensional tolerances are applied, the worst case approach gives small under estimated results of about 4%, when compared with the geometrical exact solution. This is probably due to the same way the dimensional tolerances are schematize (i.e. the first datum is nominal, the variability due to the dimensional tolerance is considered applied only on one of the two features delimiting the

1
2
3 dimension). If the geometrical tolerances are applied too, the worst case approach
4
5 gives the same result. This is probably due to the fact that the *variational* model does
6
7 not consider the effect of the form tolerances and the three applied orientation
8
9 tolerances are few to have a significant effect on the *g-dimension*. The statistical
10
11 approach gives similar results, when only dimensional tolerances or both dimensional
12
13 and geometrical tolerances are applied. In this case the variability range is smaller
14
15 than that of the worst case approach, as it is foreseen.
16
17
18
19
20

21 **5. Matrix model**

22
23
24 Instead of deriving equations that model a specific displacement a part or assembly
25
26 may be subjected to as a function of given set of geometric dimensions (parameters)
27
28 assuming specific values within the boundaries defined by tolerances (like in the
29
30 variational approach), the matrix model aims at deriving an explicit mathematical
31
32 representation of the boundary of the entire spatial region that encloses all possible
33
34 displacements due to one or more variability sources. In order to do that, homogenous
35
36 transform matrices are again considered as the foundation of the mathematical
37
38 representation. A displacement matrix **DT** is used to describe any roto-translational
39
40 variation a feature may be subjected to; the matrix is defined with respect to a local
41
42 DRF. Since the goal is to represent the boundaries of the region of possible variations
43
44 (i.e. extreme values), the approach is intrinsically a worst-case approach. No
45
46 statistical approach may be implemented, such as vector loop and variational models
47
48 do. To represent boundaries, constraints must be added to the displacements modelled
49
50 within the **DT** matrices. Displacement boundaries resulting from complex series of
51
52 tolerances are solved by modelling the effects of each tolerance separately and by
53
54 combining the resulting regions. Analogously, gaps/clearances are represented as if
55
56
57
58
59
60

1
2
3 they were tolerance regions. Finally by classifying the surfaces into several classes,
4 each characterized by some type of invariance with respect to specific displacement
5 types (e.g. a cylinder is invariant to any rotation about its axis) displacements – and
6 the resulting displacement matrix- can be simplified (Clément *et al.* 1994).
7
8
9

10
11
12 A similar approach is followed to model the dimensions acting as functional
13 requirements of the assembly; since in this case the resulting region (of possible
14 values) is essentially contained in a segment, segment boundaries must be computed
15 by means of a worst-case approach (min-max distances between the two points). The
16 two points defining the boundaries of the segment must be defined as the result of
17 stack-up functions (Desrochers *et al.* 1997).
18
19
20
21
22
23
24
25

26
27 The matrix model is based on the positional tolerancing and the Technologically and
28 Topologically Related Surfaces (TTRS) criteria (Clément *et al.* 1998). Geometric
29 features are assumed as ideal, i.e. the form tolerances are neglected, such as
30 variational model does. To better understand the matrix method for tolerance analysis,
31 its basic steps are provided below.
32
33
34
35
36
37

38
39 (1) *Transform the tolerances applied to the drawing* - The first step is to transform
40 the tolerances applied to the drawing to make them compliant to the positional
41 tolerancing and the TTRS criteria.
42
43
44

45
46 (2) *Create the assembly graph* - The second step is to create an assembly graph.
47 The assembly graph allows for identifying the global DRF and the linkages
48 among the features to which the tolerances are assigned. The assembly parts
49 should be in contact, the joints with clearance may not be considered.
50
51
52
53

54
55 (3) *Define the local DRF of each part feature* - a DRF must be assigned to each
56 part feature.
57
58
59
60

- 1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
- (4) *Identify the measurable points for each functional requirement* – Points that locate the boundaries of each functional requirement must be identified and the path that connects them to the global DRF must be defined, taking into account all the tolerances stacking up along the way.
- (5) *Define the contributions of each single displacement and the related constraints* - It is necessary to define the contribution of each displacement to the total displacement region, and the constraints necessary to identify its boundaries. Each surface can be classified into one of the seven classes of invariant surfaces; this allows to discard some displacements and to obtain a simplified displacement matrix. Additional information is necessary to specify the constraints ensuring that the feature remains inside the boundaries of the tolerance zone.
- (6) *Apply the superimposition principle and run the optimization* - If more than one tolerance is applied on the same part, the total effect is computed through the superimposition principle. For example, if n tolerances are applied to the same feature, in the local DRF, the displacement of a generic point belonging to the feature is simply defined as a sum of single contributions. The aggregation of expressions obtained for each toleranced feature results in a constrained optimization problem, which can be solved with known, standard approaches. This model has been developed for assemblies involving joints which makes a linear structure among the parts (linear stack-up function), while it is not able to deal with joints which makes a complex structure among the parts (network stack-up function). The worst case approach may be applied to the matrix model, since the statistical one has not been developed yet.

5.1 Taxonomy descriptors

Matrix model may be solved only by means of the worst case approaches. It deals with the dimensional and the geometrical tolerances, form tolerance excepted. In fact, it considers the substitute feature as the actual feature. The model is not able to take into account the Envelope or the Independence principle applied to the dimensional tolerances. It considers the interaction among the tolerance zones. The model does not allow to assign a probability density function to the model's parameters starting from the assigned tolerances. The model may deal only with linear stack-up function. The considered joints may consider contact or clearance among the mating parts. The functional requirements of the assembly may be represented only through features. The model is not able to distinguish the precedence among the datum. It may not take into account the MMC, since the Envelope principle is not included.

Table 3 shows the results of the application of the *matrix* model to the case study, when only the dimensional tolerances or both the dimensional and the geometrical tolerances are applied. All the mathematical steps are reported in (Marziale and Polini 2009). If only the dimensional tolerances are applied, the worst case approach gives small under estimated results of about 14%, when compared with the geometrical exact solution. This is probably due to the translation of the tolerance into the scheme of the TTRS and the application of superimposition principle. If the geometrical tolerances are applied too, the worst case approach gives the same result. This is due to the fact that the *matrix* model does not consider the effect of the form tolerances and the three applied orientation tolerances are few to have a significant effect on the *g-dimension*. The statistical approach may not be applied for this model.

6 Jacobian model

1
2
3 In the terminology adopted by the *jacobian model* approach, any relevant surface
4 involved in the tolerance stack-up is referred to as *functional element* (FE). In the
5 tolerance chain, FEs are considered in pairs: the two paired surfaces may belong to
6 the same part (internal pair), or to two different parts, and paired since they interact as
7 mating elements (kinematic pair, also referred to as external pair). The parts should
8 be in contact to be modeled by this model.
9

10 Transform matrices can be used to locate a FE of a pair with respect to the other: such
11 matrices can be used to model the nominal displacement between the two FEs, but
12 also additional small displacements due to the variabilities modeled by the tolerances.
13 The form tolerance are neglected. The main peculiar aspect of the *jacobian* approach
14 is how such matrices are formulated, i.e. by means of an approach derived from the
15 description of kinematic chains in robotics. The transform that links two FEs
16 belonging to a pair, and that includes both nominal displacement and small deviations
17 due to tolerances, can be modeled by a set of six virtual joints, each associated to a
18 datum reference frame. Each virtual joint is oriented so that a FE may have either a
19 translation or a rotation along its z-axis. The aggregation of the six virtual joints gives
20 origin to the transformation matrix linking one FE to the other FE of the pair
21 (Laperrière and Lafond 1999, Laperrière and Kabore 2001). The position of a point
22 laying on the second FE of a pair, which may be assumed as depicting the FR
23 (functional requirement) under scrutiny, with respect to the DRF of the first FE
24 (assumed as the global DRF) may be expressed by considering the three small
25 translations and the three small rotations of the point in the global DRF through the
26 product of a Jacobian matrix associated with the FE with tolerances of all the involved
27 FE pairs (internal or kinematic) and a vector of small deviations associated with the
28 FE with tolerances of all the involved FE pairs, expressed in the local DRF. The main
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3 element of the expression is the Jacobian matrix, which is relatively easy to compute,
4
5 starting from the nominal position of the geometric elements involved. The tricky
6
7 part, however, is to turn the assembly tolerances into displacements to assign to the
8
9 virtual joints defined for each FE pair in the chain.
10
11

12 The main steps of the approach are described below.
13
14

- 15 (1) *Identify the FE (functional element) pairs* - The first step is the identification of
16
17 the functional element pairs (i.e. pairs of relevant surfaces). The FEs are
18
19 arranged in consecutive pairs to form a stack-up function aimed at computing
20
21 each functional requirement.
22
23
- 24 (2) *Define the datum reference frame (DRF) for each FE and the virtual joints* -
25
26 The next step is to define a DRF for each FE, and to create the chain of virtual
27
28 joints representing the transformation that links the pair of FEs. Once such
29
30 information is available, the transformation matrix for each FE can be obtained.
31
32
- 33 (3) *Create the chain and obtain the overall Jacobian matrix* – The transformation
34
35 matrices can be chained to obtain the stack-up function needed to evaluate each
36
37 FR. This model has been developed for assemblies involving joints which
38
39 makes a linear structure among the parts (linear stack-up function), while it is
40
41 not able to deal with joints which make a complex structure among the parts
42
43 (network of stack-up functions), such as the matrix model does.
44
45
46
47
- 48 (4) Once obtained the required stack-up function, it may be solved by the usual
49
50 methods of the literature (Salomons *et al.* 1996) for the worst-case or statistical
51
52 case approaches.
53
54
- 55 (5) Finally, it is necessary to observe that this model is based on the
56
57 Technologically and Topologically Related Surfaces (TTRS) criterion
58
59 (Clément *et al.* 1998) and on the positional tolerancing criterion (Legoff *et al.*
60

1
2
3 1999). Therefore, the tolerances of a generic drawing need to be converted in
4
5 accordance with the previously defined criteria, before carrying out the
6
7 tolerance analysis.
8
9

10 11 **6.1 Taxonomy descriptors**

12 *Jacobian* model may be solved only by means of both the worst case and the
13
14 statistical approaches. It deals with the dimensional and the geometrical tolerances,
15
16 form tolerance excepted. In fact, it considers the substitute feature as the actual
17
18 feature. The model is not able to take into account the Envelope or the Independence
19
20 principle applied to the dimensional tolerances. It considers the interaction among the
21
22 tolerance zones. The model does not allow to assign a probability density function to
23
24 the model's parameters starting from the assigned tolerances. The model may deal
25
26 only with linear stack-up function. The considered joints may consider contact or
27
28 clearance among the mating parts. The functional requirements of the assembly may
29
30 be represented through features and points. The model is not able to distinguish the
31
32 precedence among the datum. It may not take into account the MMC, since the
33
34 Envelope principle is not included.
35
36
37
38
39

40
41 Table 3 shows the results of the application of the *Jacobian* model to the case study,
42
43 when only the dimensional tolerances or both the dimensional and the geometrical
44
45 tolerances are applied. All the mathematical steps are reported in []. If only the
46
47 dimensional tolerances are applied, the worst case approach gives small under
48
49 estimated results of about 4%, when compared with the geometrical exact solution.
50
51 This is probably due to the same way the dimensional tolerances are schematize (i.e.
52
53 the first datum is nominal, the variability due to the dimensional tolerance is
54
55 considered applied only on one of the two features delimiting the dimension). If the
56
57 geometrical tolerances are applied too, the worst case approach gives the same result,
58
59
60

1
2
3 since it has been adopted the simplification to consider fixed the angles of the box in
4 order to avoid the network. In fact, to solve the stack-up function, it is needed to relate
5 the virtual joints displacements to the tolerances assigned on the components.
6
7 However, the form tolerances (the planar one applied on the bottom side of the box
8 and the two circularities applied on the circles) do not produce any effect because in
9 the *jacobian model* the features are considered with nominal shape; the other ones
10 (the perpendicularity applied on the left side of the box, and the two parallelisms
11 applied on the other sides of the box) cannot produce any orientation deviation, since
12 the angles of the box are considered fixed. The statistical approach gives similar
13 results, when only dimensional tolerances or both dimensional and geometrical
14 tolerances are applied. In this case the variability range is smaller than that of the
15 worst case approach, as it is foreseen.
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32

33 **7. Torsor model**

34
35
36 The torsor model uses screw parameters to model three dimensional tolerance zones
37 (Chase *et al.* 1996). Screw parameters are a common approach adopted in kinematics
38 to describe motion, and since a tolerance zone can be seen as the region where a
39 surface is allowed to move, screw parameters can be used to describe it. Each real
40 surface of a part is modelled by a substitution surface. A substitution surface is a
41 nominal surface characterized by a set of screw parameters that model the deviations
42 from the nominal due to the applied tolerances. Seven types of tolerance zones are
43 defined. Each one is identified by a subset of nonzero screw parameters, while the
44 remaining ones are set to zero as they leave the surface invariant. The screw
45 parameters are arranged in a particular mathematical operator called *torsor*, hence the
46 name of the approach. Considering a generic surface, if u_A , v_A , w_A are the translation
47
48
49
50
51
52
53
54
55
56
57
58
59
60

components of its point A, and α , β , γ are the rotation angles (considered small) with respect to the nominal, the corresponding torsor is:

$$\mathbf{T}_A = \left\{ \begin{array}{cc} \alpha & u_A \\ \beta & v_A \\ \gamma & w_A \end{array} \right\}_{\mathbf{R}} \quad (5)$$

where \mathbf{R} is the DRF that is used to evaluate the screw components.

To model the interactions between the parts of an assembly, three types of torsors (or Small Displacement Torsor SDT) are defined (Ballot and Bourdet 1997): a *part SDT* for each part of the assembly to model the displacement of the part; a *deviation SDT* for each surface of each part to model the geometrical deviations from nominal; a *gap SDT* between two surfaces linking two parts to model the mating relation. The form tolerances are neglected and they are not included in the deviation SDT.

A union of SDTs is used to obtain the global behavior of the assembly. The aggregation can be done by considering that the worst-case approach computes the cumulative effect of a linear stack-up function of n-elements by adding the single components of the torsors. This is not true for a network of stack-up functions that has not been developed by the torsor model yet. The torsor method does not allow to apply a statistical approach, since the torsor's components are intervals of the small displacements; they are not parameters to which it is possible to assign easily a probability density function.

The torsor model operates under the assumption that both the TTRS and the positional tolerancing criteria are adopted, which means that the tolerances in the drawing may need to be updated before carrying out the tolerance analysis. The solution of stack-up functions arranged in a network is not completely developed. Finally, it is worth pointing out that, in relevant literature, the use of small displacement torsors for modelling tolerance analysis problems tends to follow two main approaches: on one

1
2
3 hand, SDTs are used to develop functions for computing the position of geometric
4 elements (belonging to the assembly) as they are subjected to displacement allowed
5 by tolerances (e.g. see Chase *et al.* 1996); on the other hand SDTs are used to model
6 entire spatial volumes that encapsulate all the possible points in space that may be
7 occupied by geometric elements during their variations (e.g. see Laperrière *et al.*
8 2002). In the analysis of the case study, only the second approach has been
9 considered, since it looks more promising.

10
11 The basic steps of *torsor model* are described in the following (Villeneuve et al 2001,
12 Teissandier et al. 1999).

- 13
14
15
16
17
18
19
20
21
22
23
24
25 (1) *Identify the relevant surfaces of each part and the relations among them* - The
26 first step is to identify the relevant surfaces belonging to each part and the
27 relationships among them; this information is usually collected in a *surfaces*
28 *graph*. In this step the chains to relate the FRs to the relevant surfaces are
29 identified.
30
31
32
33
34
35
36
37 (2) *Derive the SDTs* - A deviation SDT needs to be associated to each relevant
38 surface of each part. This leads to the evaluation of a global SDT for each part.
39 Finally, the shape of the gap SDT is associated to each joint according to the
40 functional conditions of the assembly.
41
42
43
44
45
46 (3) *Obtain the FR stack-up functions*: compute the cumulative effects of the
47 displacements and obtain the final linear stack-up function of each FR.
48
49
50

51 **7.1 Taxonomy descriptors**

52 *Torsor* model may be solved only by means of the worst case approach. The *torsor*
53 *model* does not allow to apply a statistical approach, since the torsor's components are
54 intervals of the small displacements; they are not parameters to which it is possible to
55 assign easily a probability density function. It deals with the dimensional and the
56
57
58
59
60

1
2
3 geometrical tolerances, form tolerance excepted. In fact, it considers the substitute
4 feature as the actual feature. The model is not able to take into account the Envelope
5 or the Independence principle applied to the dimensional tolerances. It considers the
6 interaction among the tolerance zones. The model does not allow to assign a
7 probability density function to the model's parameters starting from the assigned
8 tolerances. The model may deal only with linear stack-up function. The considered
9 joints may provide contact or clearance among the mating parts. The functional
10 requirements of the assembly may be represented through features and points. The
11 model is not able to distinguish the precedence among the datum. It may not take into
12 account the MMC, since the Envelope principle is not included.
13
14
15
16
17
18
19
20
21
22
23
24
25
26

27 Table 3 shows the results of the application of the *torsor* model to the case study,
28 when only the dimensional tolerances or both the dimensional and the geometrical
29 tolerances are applied. All the mathematical steps are reported in (Marziale and Polini
30 2009). If only the dimensional tolerances are applied, the worst case approach gives
31 small under estimated results of about 4%, when compared with the geometrical exact
32 solution. This is probably due to the same way the dimensional tolerances are
33 schematize (i.e. the first datum is nominal, the variability due to the dimensional
34 tolerance is considered applied only on one of the two features delimiting the
35 dimension). If the geometrical tolerances are applied too, the worst case approach
36 gives the same result, since it has been adopted the simplification to consider fixed the
37 angles of the box in order to avoid the network. The statistical approach may not be
38 applied for this model.
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

8. Summary for tolerance analysis models

Sections 3-7 provide a fine description of five significant models for tolerance analysis of an assembly. As discussed earlier, a model for tolerance analysis allows to evaluate the effects of the tolerances assigned to an assembly components on the functional requirements of the assembly.

In this section, the five models are compared according to the taxonomy presented in Table 2. Identified criteria can be useful to evaluate and compare different models for tolerance analysis in all the aspects involved in a typical tolerance analysis problem. The criteria adopted by the works of the literature, that are presented in paragraph 1.2, allow to underline only some of the steps of a tolerance analysis problem. The aim is to provide a selected scheme to select them, depending on model's characteristics (see Table 3). Considering the actual research issues related to tolerance analysis models, there is much room for improvement.

The *vector loop* model and the *variational* model appear more developed than the others; they are the only ones that provide support for solving tolerance stack-up functions involving networks. Moreover, they provide a method for assigning probably density functions to model parameters, once given the applied tolerances. However, the *vector loop* model and the *variational* model are not completely consistent with the actual ISO and ASME standards and they do not provide support for handling interactions among tolerance zones. The *vector loop* model is the only one providing actual support for modelling form tolerances; all the other models adopt the simplification consisting in considering the real features as coincident with their substitute ones. The *variational* model supports the inclusion of precedence constraints among datums, and also the presence of material modifiers conditions. The *matrix* model and the *torsor* model support only the worst-case approach for

1
2
3 solving the tolerance analysis problem. This is a limitation, but their formalization
4
5 allows them to handle joints with clearance, and interaction among tolerance zones.
6
7
8 The *Jacobian model* has the advantage that the Jacobian matrix can be easily
9
10 calculated from nominal conditions, while displacements of the functional
11
12 requirements can be directly related to displacements of the virtual joints; however, it
13
14 is difficult to derive such virtual joint displacements from the tolerances applied to the
15
16 assembly components. On the other hand, the *torsor model* may allow for an easy
17
18 evaluation of the ranges of the small displacements directly from the tolerances
19
20 applied to the assembly components, but then, it is very difficult to relate these ranges
21
22 to the ranges of the functional requirements of the assembly. These two
23
24 considerations have suggested the idea of a *unified jacobian-torsor model* to evaluate
25
26 the displacements of the virtual joints from the tolerances applied to the assembly
27
28 components through the torsors and, then, to relate the displacements of the functional
29
30 requirements to the virtual joint displacements through the Jacobian matrix
31
32 (Laperrière *et al.* 2002, Desrochers *et al.* 2003). Although this is theoretically
33
34 possible, since the deviations are usually small and, therefore, the equations can be
35
36 linearized, the actual feasibility of this approach is still subject of research.
37
38
39
40
41
42

43 Finally, the considered models have some common limits. The first deals with the
44
45 Envelope Rule: the models do not allow to apply the Envelope Rule and the
46
47 Independence Rule to different tolerances of the same part. The second is that it does
48
49 not exist any criteria to assign a probability density function to the model parameters
50
51 joined to the applied tolerances and that considers the interaction among the tolerance
52
53 zones. The last deals with the assembly cycle: the models are not able to represent all
54
55 the type of coupling with clearance between two parts.
56
57
58
59
60

1
2
3 The results obtained by considering only the dimensional tolerances show that all the
4 models give small under estimated results with the worst case approach, when
5 compared with the exact geometrical solution. The *matrix* model presents the highest
6 error (-14%), while all the other models provide the same result (-4%). This is
7 probably due to the same way the dimensional tolerances are schematize (i.e. the first
8 datum is nominal, the variability due to the dimensional tolerance is considered
9 applied only on one of the two features delimiting the dimension). Moreover, the
10 statistical approach gives similar results for all the considered models.
11
12

13
14
15 The results obtained by considering both dimensional and geometrical tolerances
16 show that all the models, except the *vector loop* model, give similar results with the
17 worst-case approach. This is probably due to the fact that the *vector loop* model
18 considers the effect of a set of tolerances applied to a surface as the sum of the effects
19 due to each single tolerance applied to the same surface. The effects of the different
20 tolerances are considered independent. Therefore, increasing the number of tolerances
21 applied to the same surface increases the variability of the functional requirement.
22 This means that the interaction among tolerances defined on the same surface are not
23 properly handled.
24
25

26 All the five models produce very similar results, when the statistical approach is
27 applied.
28
29

30 Moreover, the results of Tables 3 obtained from the *jacobian* model and from the
31 *torsor* model are basically identical. This is due to the fact that a simplification has
32 been adopted when modelling the problem, i.e. to consider fixed at 90° the angles of
33 the box. This assumption is due to the need to avoid the networks of stack-up
34 functions that the two models are not able to deal. It means that all the applied
35 tolerances may involves only translations of the sides of the box.
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

9. Conclusions

Many tolerance analysis models have been proposed and developed by many authors.

This paper suggests a new taxonomy to help evaluate, compare and select tolerance analysis models, depending on the model characteristics and the type of application.

The paper focused on five tolerance analysis models, due to their better accuracy and their better chances of being applied to many contexts. Models have been discussed in detail in order to summarise their characteristics and peculiarities.

None of the models proposed by the literature provides a complete and clear mechanism for handling all the requirements included in the tolerancing standards (Shen *et al.* 2004). This limitation is reflected also by the available commercial Computer Aided Tolerancing (CAT) software applications, which are based on the same models (Prisco and Giorleo 2002). Like already discussed in detail in previous work (Marziale and Polini 2009), the main limitations of the actual models are the following: they do not properly support the application of the Envelope Rule and of the Independence Rule to different dimensional tolerances on the same part as prescribed by the ISO and ASME standards; they do not handle form tolerances (except for the vector loop model); they do not provide mechanisms for assigning probably density functions to model parameters starting from tolerances and considering tolerance zone interactions; finally, they are not capable of representing all the possible types of part couplings that may include clearance.

The overcoming of those limits through a new model and its application to case studies is subject of current ongoing research.

Acknowledgements

This research was partially funded by the Italian Ministry of University and Research grant: PRIN 2005 – ‘Characterization of the multiscale geometry of manufactured surfaces for process control and tolerance analysis’

References

- 1
2
3
4
5
6
7 Ameta, G., J. K. Davidson, J.K. and Shah, J.J., 2007, Influence of form on Frequency
8 Distribution for 1-D clearance which is generated from Tolerance-Maps,
9 *Proceedings of 10th CIRP Conference on Computer Aided Tolerancing*, March
10 21st-23rd, Erlangen, Germany.
- 11 ASME Standard, 1994. Dimensioning and tolerancing, ASME Y14.5M. American
12 Society of Mechanical Engineering, New York.
- 13 Ballot, E. and Bourdet, P., 1997. A computational method for the consequences of
14 geometric errors in mechanisms. *Proceedings of the 5th CIRP Seminar on*
15 *Computer Aided Tolerancing*, Toronto, 27–29 April 1997
- 16 Berman, Y.O., 2005. *Shape and Position Uncertainty in Mechanical Assembly*. Ph.
17 Thesis. The Hebrew University. Jerusalem. Israel.
- 18 Boyer, M. and Stewart, N.F., 1991. Modeling spaces for toleranced objects.
19 *International Journal of Robotics Research*, 10, 470-582.
- 20 Chase, K.W., 1999. Multi-Dimensional Tolerance Analysis (Automated Method), *In*:
21 P.J.R. Drake, ed., *Dimensioning and Tolerancing Handbook*, New York: McGraw-
22 Hill.
- 23 Chase, K.W., Gao, J. and Magleby S.P., 1995. General 2-D tolerance analysis of
24 mechanical assemblies with small kinematic adjustments. *Journal of Design*
25 *Manufacturing*, 5, 263-274
- 26 Chase, K.W., Gao, J., Magleby S.P. and Sorensen, C.D., 1996. Including geometric
27 feature variations in tolerance analysis of mechanical assemblies. *IIE Transactions*,
28 28, 795-807
- 29 Chase, K.W., Gao, J. and Magleby, S.P., 1997. Tolerance analysis of 2-and 3D
30 mechanical assemblies with small kinematic adjustments, *In*: H.C. Zhang, ed.
31 *Advanced Tolerancing Techniques*, New York: Wiley.
- 32 Chase, K.W. and Parkinson, A.R., 1991. A Survey of Research in the Application of
33 Tolerance Analysis to the Design of Mechanical Assemblies. *Research in*
34 *Engineering Design*, 3, 23-37.
- 35 Clément, A., Rivière, A. and Temmerman, M., 1994 *Cotation tridimensionnelle des*
36 *systèmes mécaniques, théorie & pratique*, Yvry-Sur-Seine Cedex: PYC Edition (in
37 French).
- 38 Clément, A., Rivière, A., Serré, P. and Valade, C., 1998. The TTRSs: 13 Constraints
39 for Dimensioning and Tolerancing, *In*: H.A. ElMaraghy, ed. *Geometric design*
40 *tolerancing: theories, standards and applications*, London: Chapman & Hall.
- 41 Creveling, C.M., 1997. *Tolerance Design*, Boston: Addison-Wesley.
- 42 Davidson, J.K., Mujezinovic, A., and Shah, J.J., 2002, A new mathematical model for
43 geometric tolerances as applied to round faces, *ASME Trans, J Mech Des*, 124,
44 609-622.
- 45 Desrochers, A. and Rivière, A., 1997. A Matrix Approach to the Representation of
46 Tolerance Zones and Clearances. *The International Journal of Advanced*
47 *Manufacturing Technology*, 13, 630-636
- 48 Desrochers, A., Ghie, W. and Laperrière, L., 2003. Application of a Unified Jacobian-
49 Torsor Model for Tolerance Analysis. *Journal of Computing and Information*
50 *Science in Engineering*, 3, 2-14
- 51 Faerber, P.J., 1999. *Tolerance Analysis of assemblies using kinematically derived*
52 *sensitivities* [online]. ADCATS Report No. 99-3, Available from:
53 <http://adcats.et.byu.edu/reportsandpublications.php>

- 1
2
3 Gao, J., Chase, K.W. and Magleby, S.P., 1998. Generalized 3-D Tolerance Analysis
4 of mechanical assemblies with small kinematic adjustments. *IIE Transactions*, 30,
5 367-377
6
7 Gupta, S. and Turner, J.U., 1993. Variational solid modelling for tolerance analysis..
8 *IEEE Computer Graphics and Applications*, 13, 64-74.
9
10 Hong, Y.S. and Chang, T.C., 2002 A comprehensive review of tolerancing research.
11 *International Journal of Prouction Research*, 40 (11), 2425-2459.
12
13 ISO Standard, 1985. ISO 8015: Fundamental Tolerancing Principle
14
15 Laperrière, L. and Lafond, P., 1999. Modelling tolerances and dispersions of
16 mechanical assemblies using virtual joints. Proceedings of ASME Design
17 Engineering Technical Conferences, September 12-15, Las Vegas, Nevada, USA
18
19 Laperrière, L. and Kabore, T., 2001. Monte Carlo simulation of tolerance synthesis
20 equations. *International Journal of Production Research*, 39, 2395-2406
21
22 Laperrière, L., Ghie, W., and Desrochers, A., 2002. Statistical and Deterministic
23 tolerance analysis and synthesis using a Unified Jacobian-Torsor Model. *CIRP*
24 *Annals*, 51, 417-420
25
26 Legoff, O., Villeneuve, F. and Bourdet, P., 1999. Geometrical tolerancing in process
27 planning: a tridimensional approach. *Proceedings of the Institution of Mechanical*
28 *Engineers. Part B: Journal of Engineering Manufacture*, 213, 635-640
29
30 Luenberger, D.G., 2003, *Linear and Nonlinear Programming*. Boston: Addison-
31 Wesley
32
33 Marziale, M. and Polini, W., 2009. A review of two models for tolerance analysis:
34 vector loop and matrix. *International Journal of Advanced Manufacturing*
35 *Technology*, 43, 1106-1123
36
37 Marziale, M. and Polini, W., 2009. A review of variational models for tolerance
38 analysis of an assembly. Accepted for publication on *Journal of Engineering*
39 *Manufacture*.
40
41 Marziale, M. and Polini, W., 2009. A review of two models for tolerance analysis of
42 an assembly: jacobian and torsor. Submitted to *International Journal of Computer*
43 *Integrated Manufacturing*.
44
45 Mujezinovic, A., Davidson, J.K. and Shah, J.J., 2004, A new mathematical model for
46 geometric tolerances as applied to polygonal faces, *ASME Trans, J Mech Des*,
47 126, 504-518.
48
49 Nassef, A.O. and ElMaraghy, H.A., 1996. Probabilistic Analysis of geometric
50 tolerances. In: F. Kimura, ed. *Computer Aided Tolerancing*. London: Chapman &
51 Hall, 187-203
52
53 Nigam, S.D. and Turner J.U., 1995, Review of statistical approaches to tolerance
54 analysis. *Computer Aided Design*. 27, 6-15
55
56 Prisco, U. and Giorleo, G., 2002. Overview of Current CAT Systems. *Integrated*
57 *Computer Aided Engineering*, 9 (4), 373-397.
58
59 Salomons, O.W., Haalboom, F.J., Jonge Poerink, H.J., van Slooten, F., van Houten,
60 F.J.A.M. and Kals, H.J.J., 1996 A computer aided tolerancing tool II: Tolerance
analysis. *Computers in Industry*, 31, 175-186.
Salomons, O.W., van Houten, F.J.A.M. and Kals, H.J.J. 1998. Current Status of CAT
Systems. In: H.A. ElMaraghy, ed. *Geometric Design Tolerancing: Theories,*
Standards and Applications. London: Chapman & Hall, .438-452.
Shen, Z., Ameta, G., Shah, J.J. and Davidson, J.D., 2004. A comparative study of
tolerance analysis methods. *Journal of Computing and Information Science in*
Engineering, 5 (3), 247-256.

- 1
2
3 Shen, Z., Ameta, G., Shah, J.J. and Davidson, J.D., 2005, A comparative study of
4 tolerance analysis methods. *Journal of Computing and Information Science in*
5 *Engineering*, 5 (3), 247-256.
6
7 Teissandier, D., Couétard, Y. and Gérard, A., 1999. A computer aided tolerancing
8 model: proportioned assembly clearance volume. *Computer Aided Design*, 31, 805-
9 817.
10
11 Turner, J. and Gangoiti, A.B., 1991. Tolerance analysis approaches in commercial
12 software. *Concurrent Engineering*, 1/2, 11-23.
13
14 Villeneuve, F., Legoff, O. and Landon, Y., 2001. Tolerancing for manufacturing: a
15 three-dimensional model. *International Journal of Production Research*, 39, 1625-
16 1648
17
18 Whitney, D.E., 2004. *Mechanical Assemblies. Their Design, Manufacture and Role in*
19 *Production Development*. New York: Oxford University Press Inc.
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

Table 1. Tolerance analysis models of the literature

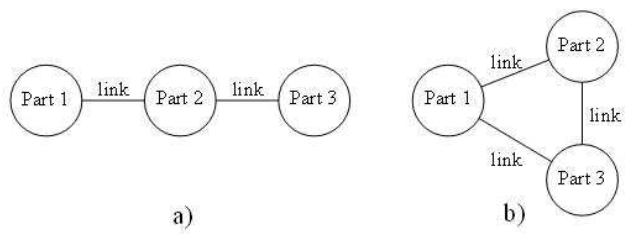
Table 2. Description of the suggested taxonomy

Table 3. Taxonomy of models for tolerance analysis

Figure 1. a) Linear stack-up function and b) network stack-up function.

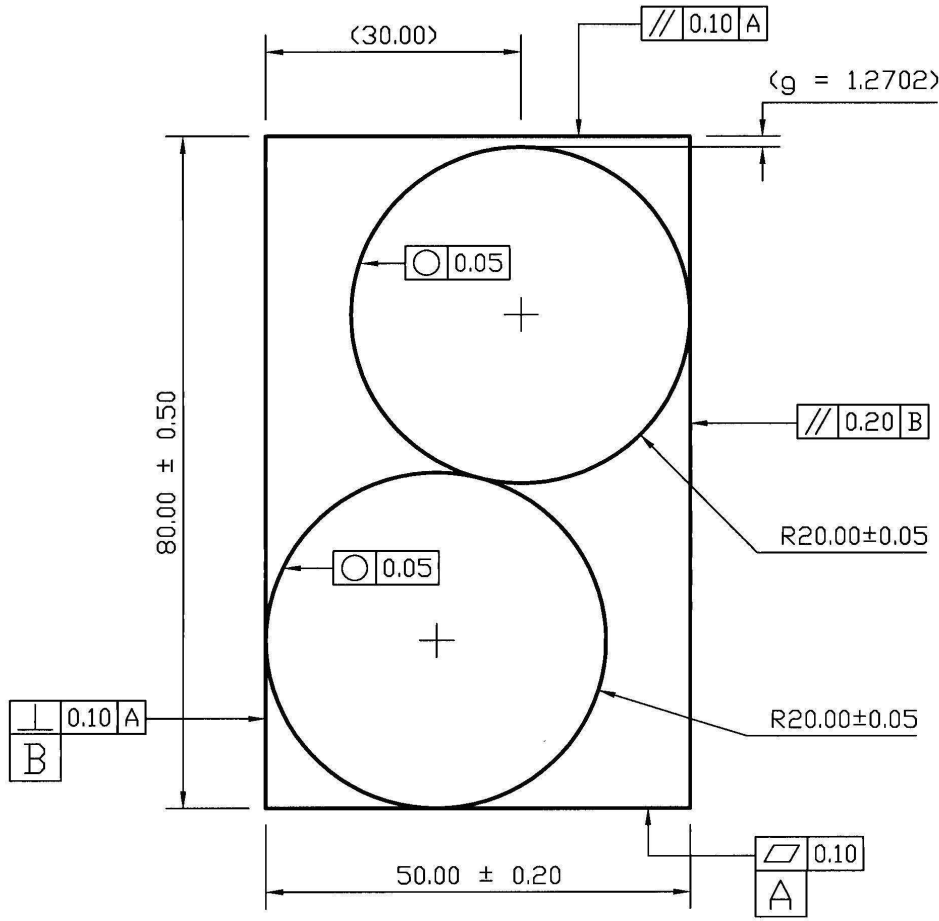
Figure 2. The case study

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



199x256mm (96 x 96 DPI)

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



524x508mm (96 x 96 DPI)

Only

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

Table caption

Table 1. Tolerance analysis models of the literature

Table 2. Description of the suggested taxonomy

Table 3. Taxonomy of models for tolerance analysis

For Peer Review Only

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

For Peer Review Only

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47

Models	References
Vector loop	(Chase <i>et al.</i> 1995, Chase <i>et al.</i> 1996, Chase <i>et al.</i> 1997, Gao et al 1998, Chase 1999, Faerber 1999, Nigam and Turner 1995).
Variational	(Boyer et al 1991, Gupta et al 1993, Berman 2005, Whitney 2004, Salomons et al 1996)
Matrix	(Clément <i>et al.</i> 1994, Desrochers <i>et al.</i> 1997, Clément et al 1998)
Jacobian	(Laperrière and Lafond 1999, Laperrière and Kabore 2001, Salomons <i>et al.</i> 1996, Clément <i>et al.</i> 1998, Legoff <i>et al.</i> 1999).
Torsor	(Chase <i>et al.</i> 1996, Ballot and Bourdet 1997, Chase <i>et al.</i> 1996, Laperrière <i>et al.</i> 2002, Villeneuve et al 2001, Teissandier et al. 1999).
Jacobian-Torsor	(Laperrière <i>et al.</i> 2002, Desrochers <i>et al.</i> 2003).
T-maps	(Davidson et al. 2002, Mujezinovic et al. 2004, Ameta et al. 2007)

Criteria	Values
Tolerance kind	dimensional/form/ geometrical(no form)
Envelope and Independence	Envelope /Independence
Tolerance zones' interaction	yes/no
Precedence among datum	yes/no
Material Modifier Condition	yes/no
Model parameters from tolerances	easy/hard
Joint type	contact/clearance
Tolerance stack-up function	linear /network
Functional requirement schematization	feature/points
Analysis kind	worst-case /statistical

	Models									
	vector loop		variational		matrix		jacobian		torsor	
Tolerance kind	dimensional/form / geometrical(no form)		dimensional/geometrical(no form)		dimensional/geometrical(no form)		dimensional/geometrical(no form)		dimensional/geometrical(no form)	
Envelope and Independence	Envelope		Envelope		Envelope		Envelope		Envelope	
Tolerance zones' interaction	no		no		yes		yes		yes	
Precedence among datum	no		yes		no		no		no	
Material Modifier Condition	no		no		no		no		no	
Model parameters from tolerances	yes		yes		no		no		no	
Tolerance stack-up function	linear /network		linear /network		linear		linear		linear	
Joint kind	contact		contact		contact/clearance		contact		contact/clearance	
Functional requirement schematization	feature/points		feature/points		feature		feature/points		feature/points	
Analysis kind	worst-case/statistical		worst-case/statistical		worst-case		worst-case/approach		worst-case	
Case study results	only dimensional tolerances	dimensional and geometrical tolerances	only dimensional tolerances	dimensional and geometrical tolerances	only dimensional tolerances	dimensional and geometrical tolerances	only dimensional tolerances	dimensional and geometrical tolerances	only dimensional tolerances	dimensional and geometrical tolerances
worst-case	±0.78 mm	±1.03 mm	±0.78 mm	±0.78 mm	±0.70 mm	±0.69 mm	±0.78 mm	±0.78 mm	±0.78 mm	±0.78 mm
statistical	±0.52 mm	±0.54 mm	±0.51 mm	±0.50 mm	-	-	±0.53 mm	±0.53 mm	-	-