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To cite this version:

Thomas Lux. Rational Forecasts or Social Opinion Dynamics? Identification of Interaction Effects in a Business Climate Survey. Journal of Economic Behavior and Organization, 2009, 72 (2), pp.638. 10.1016/j.jebo.2009.07.003 . hal-00720175

HAL Id: hal-00720175

https://hal.science/hal-00720175v1

Submitted on 24 Jul 2012

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PII: S0167-2681(09)00184-X
DOI: doi:10.1016/j.jebo.2009.07.003
Reference: JEBO 2421

To appear in: *Journal of Economic Behavior & Organization*

Received date: 30-5-2008
Revised date: 17-7-2009
Accepted date: 17-7-2009

Please cite this article as: Lux, T., Rational Forecasts or Social Opinion Dynamics? Identification of Interaction Effects in a Business Climate Survey, *Journal of Economic Behavior and Organization* (2008), doi:10.1016/j.jebo.2009.07.003

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Rational Forecasts or Social Opinion Dynamics? Identification of Interaction Effects in a Business Climate Survey

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revised version: July 16, 2009

Abstract

This paper develops a methodology for estimating the parameters of dynamic opinion or expectation formation processes with social interactions. We study a simple stochastic framework of a collective process of opinion formation by a group of agents who face a binary decision problem. The aggregate dynamics of the individuals' decisions can be analyzed via the stochastic process governing the ensemble average of choices. Numerical approximations to the transient density for this ensemble average allow the evaluation of the likelihood function on the base of discrete observations of the social dynamics. This approach can be used to estimate the parameters of the opinion formation process from aggregate data on its average realization. Our application to a well-known business climate index provides strong indication of social interaction as an important element in respondents' assessment of the business climate.

JEL classification numbers: C42, D84, E37

Keywords: business climate, business cycle forecasts, opinion formation, social interactions

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*Financial support from the EU STREP ComplexMarkets (contract number 516446) and from the Volkswagen foundation is gratefully acknowledged. Helpful remarks and comments by Simone Alfarano, Reiner Franke, Cars Hommes, Alan Kirman, Axel Leijonhufvud, Claas Prelle, Mark Salmon, Detlef Seese, Jamsheed Shorish, Enrico Zaninotti and two anonymous referees are gratefully acknowledged.

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1 Introduction

Recent literature has started to consider the role of social interdependencies between individual decisions. The potential importance of social interactions has been highlighted in analyses of such diverse phenomena as human capital acquisition (Bénabou, 1993), social pathologies due to peer group effects (Glaeser, Sacerdote and Scheinkman, 1996), or herding in financial markets (Kirman, 1993; Lux, 1995). Existence and uniqueness of equilibria in large economies with both local or global interactions have been studied recently by Horst and Scheinkman (2006).

Empirical work on social interactions has mostly been based on an adaptation of the discrete choice framework allowing for social spillovers in agents' utility functions. Brock and Durlauf (2001a,b) provide an introduction into the econometric implementation of this approach. While the discrete choice approach typically studies social interactions in cross-sectional data and assumes that the configuration of choices represents a self-consistent equilibrium, we are interested in a *dynamic* process of ongoing opinion formation within a group of agents. While our incorporation of social influences is very close (both in its spirit and its formal implementation) to Brock and Durlauf's more static approach to social interaction, we do not necessarily impose that agents have settled at an equilibrium. Another difference is that we do not model social interaction effects as due to spillovers in utility or payoff functions. Due to the nature of the time series we wish to model, we are profoundly ignorant about the relevant underlying incentives of agents. In fact, there might be no incentive component of any importance in our particular setting.

One area in which a dynamic process of opinion formation could arguably be of some relevance is survey data on business expectations or so-called sentiment indices that are published by academic and private institutes in most developed countries. While these indices attract quite some public attention upon their regular compilation, they have found only scarce consideration in the macroeconomics literature. Due to the underlying motivation for collecting such data, much of the limited body of available literature focuses on the predictive power for macroeconomic activity of these surveys (cf. Hüfner and Schröder, 2002; Gelper et al., 2007, Taylor and McNabb, 2007). However, as far as we know, attempts at formulating behavioral models for the underlying data-generating process of these surveys are practically non-existent. While social interactions have been hypothesized to be of some importance in expectation formation (Caroll, 2003), such factors have to my knowledge not been incorporated explicitely in the small sample of papers testing positive models of expectation formation. The hypothesis underlying our present study is that these survey data might be viewed as the result of a social process of opinion formation among the respondents. If these data could be explained via social interactions, they would represent behavioral components of macroeconomic activity quite different from rational attempts at forecasting the future development of the business cycle.

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Rather than representing rational forecasts of future economic developments they could be interpreted as manifestations of *animal spirits*. Our paper can, thus, be seen as an attempt to operationalize the alleged role of animal spirits in macroeconomic fluctuations (cf. Akerlof and Shiller, 2009, for a prominent example of the recent surge of interest in 'non-rational' fluctuations of expectations and confidence).

Fig. 1 gives an intuitive preview on our subsequent results. The figure contrasts the monthly observations of the ZEW Business Climate Index for the German economy compiled by the Centre for European Economic Research (German acronym: ZEW) at the University of Mannheim from about 350 respondents with a frequent measure of real economic activity (HP filtered industrial production). The business climate index is computed so that it is bounded by +1 and -1 from above and below (see sections 2 and 4 for details). Quite obvious, positive (negative) values are meant to indicate an optimistic (pessimistic) majority among respondents. The higher the absolute value, the more pronounced the positive (negative) outlook for the German economy. A glance at the lower panel shows a striking contrast to the real thing: while the output gap as measured by the residuals from the HP filter appears quite noisy, the climate index has much more obvious swings between low and high values. It appears that there is a much clearer image of the business cycle dynamics in the eyes of the observers compared to what can be extracted from real economic activity. The ZEW index is also characterized by very abrupt and drastic switches between more optimistic or more pessimistic majorities than any switch between positive or negative realizations of the output gap.

Fig. 1 about here

The pronounced swings of our sentiment series is quite typical of such data. While, in principle, these swings could be caused by the revelation of important news about the subsequent development, the hypothesis we are going to explore in this paper is that these swings are imprints of a process of social interaction among respondents. It is not difficult to imagine that respondents' changing assessments of the economic outlook are at least in part influenced by the evolution of the opinion of their peers. Interpersonal effects might come into play via private exchange of opinions but probably even more so via the influence of a 'social field' of the average mood of their peer group of which they learn through a variety of professional and private channels of communication.

To test for social interaction in opinion formation, we will adopt a formalization along the lines of Weidlich and Haag (1983) and Lux (1995). While the basic goal is to identify potential interaction effects, this framework is general enough to allow us to also cover exogenous factors of influence on the opinion dynamics. Naturally enough, macroeconomic data would be our candidate explanatory variables. Including both these exogenous forces and an intrinsic

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feedback allows us to study their interplay in the formation of group expectations.

There is another important issue we explore in our study: while we have a relatively constant number of respondents in our survey (about 350), it is not clear whether all these participants would, in fact, act as independent decision makers. This issue is quite subtle: apart from the overall hypothesized interaction effect, there might be coherence within subgroups of the entire pool of respondents that is so strong as to lead to entirely or highly correlated synchronized behavior among subgroups. The behavior of synchronized subgroups would simply collapse onto that of a single agent (and any member of the group would be a representative agent of it). The dynamics of the opinion formation process would look differently if certain subgroups would always move together. The framework to be formalized below allows us to cope with this phenomenon: first, we start by specifying the opinion dynamics for a given number of independent actors, equal to the average number of respondents in the survey. Since the number of agents explicitly appears as a variable in our model, we may, however, also adopt an agnostic view and let the model speak on the number of *effectively independent* agents. As it turns out, endogeneizing the number of active groups of agents allows a huge improvement in the goodness-of-fit of the model. Subsequent statistical analyses confirm that this specification covers the salient features of the data much better than alternative specifications.

Further explanatory power is obtained by allowing for a 'momentum' effect in addition to the baseline social interaction. In contrast to these refinements of the social part of the dynamics, allowing for an additional feedback from macroeconomic data (e.g., industrial production) improves only slightly the goodness-of-fit with a more modest increase of the likelihood. Our simple stochastic model also allows to compute confidence bounds for future observations from the transient density. We use these to assess whether the empirical series could be a likely realization of the process of social interaction given the initial condition and the macro influence. We also explore whether any single entry would be a probable realization conditional on last month's entry and the contemporaneous macro feedback. As it turns out, in both cases the empirical data hardly ever move out of the pertinent 95 percent confidence intervals which nicely confirms the explanatory power of the model.

The rest of the paper is structured as follows: in section 2, the basic stochastic framework of social interactions will be introduced together with a review of its properties. Section 3 contemplates the problem of estimating the parameters of such a stochastic framework with an ensemble of interacting agents. Section 4 provides some results on Monte Carlo experiments with small samples to arrive at insights on the reliability and accuracy of our subsequent estimates. Section 5 then contains the application to the ZEW index of the business climate, and section 6 provides a detailed analysis of the statistical properties of Monte Carlo replications of the estimated models to explore their explanatory power together

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with an assessment of their goodness-of-fit. Section 7 concludes.

2 A 'canonical' stochastic model of social interaction

As a simple formalization for the process of social opinion formation, we adapt an approach that goes back at least to Weidlich and Haag (1983) and that had been used in behavioral finance models by Lux (1995, 1997). The model deals with a binary choice problem and stochastic transitions of agents between both alternatives due to exogenous factors and group pressure. Let the two groups have occupation numbers n_+ and n_- respectively, with the overall population size being $2N$ (multiplication by 2 simply serves to avoid the case of an odd number of individuals).
The aggregate outcome of this choice process at any point in time can be described via the difference between the number of individuals in the "+" and "−" groups:

$$n = \frac{1}{2}(n_+ - n_-), \tag{1}$$

or an equivalent *opinion index*:

$$x = \frac{n}{N} = \frac{n_+ - n_-}{2N} \qquad \text{with } x \in [-1, 1]. \tag{2}$$

Agents' beliefs are either optimistic or pessimistic; they change their beliefs in continuous time, with a Poisson process describing the changes from the "+" to the "−" group or *vice versa* within the next instant. We denote the pertinent transition rates by $w_\uparrow$ and $w_\downarrow$ and assume that they are the same for all agents within each group.

Following the earlier literature quoted above we assume an exponential functional form of the transition rates $w_\uparrow$ and $w_\downarrow$:

$$w_\uparrow = v\, exp(U), \qquad w_\downarrow = v\, exp(-U). \tag{3}$$

The function U might be labeled the 'forcing function' for transitions and is analogous to the utility function in a discrete choice setting. It is assumed to consist of a constant factor (bias) α_0 and a second component formalizing group pressure in favor or against homogeneous decisions, $\alpha_1 x$:

$$U = \alpha_0 + \alpha_1 x. \tag{4}$$

The parameters of the model are, thus: v which determines the frequency (time scale) of moves between groups, α_0 which generates a bias towards the choice of "+" ("−") opinions if positive (negative) and α_1 which formalizes the

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degree of group pressure (if it is positive, if negative it would rather imply a tendency of non-conformity). With this set-up the opinion dynamics is described as the aggregate outcome of $2N$ coupled jump Markov processes for agents' choices in continuous time. During small time increments Δt, the probability of an agent to switch from his previous group (decision) to the other alternative, is approximately equal to $w_{\uparrow}\Delta t$ and $w_{\downarrow}\Delta t$, respectively. Interpersonal differences are covered in the stochasticity of the process, i.e. by the very fact that individual choices are not deterministic, but are only determined in expectation.

A few comments on the underlying 'philosophy' of this framework are in order: First, it might be important to emphasize that we do not view this model so much as a description of microscopic behavior, but rather as a phenomenological model for the dynamics resulting from the supposedly more multifaceted social interaction of our respondents. Since the 'true' interactions might be much more complex, our interest is, therefore, less in the verisimilitude rather than the empirical performance of the model. As a consequence, the viewpoint here is not that this simple story of social interaction should be taken at face value concerning its assumed decision processes, but rather that it might provide an empirically successful approximation to the social component of the dynamics of the ZEW survey (and maybe other surveys as well).[1] Because of this phenomenological interpretation, we also abstain from a strict consideration of information sets. As one referee noted, the continuous time dynamics with transition rates given by eqs. (3) and (4) implies that agents also react to changes of sentiment between the discrete monthly observations which are not available public information. However, our interpretation is that there is an ongoing public opinion formation process among agents so that they have some feeling for the change of business climate even between the discrete points in time when the survey results are published (we could try to model this communication between survey publications via a noisy information transmission, but our concern here is to have a compact model of the outcome of a social opinion dynamics rather that an exact choice-theoretic description on the micro level). We have estimated a version of the model in which we had frozen x in eq. (4) at its last public observation x_{t_i} during $t\epsilon[t_i, t_{i+1}]$ with t_i the points of time at which the survey results are published. Results are virtually identical to those reported below (details are available on request).

We also believe that it is important to model the opinion process in continuous time: agents will very likely have an opinion about the future prospects of the economy *at any point in time* and will also very likely often change their view at points in time between survey publication dates. The hypothesized opinion formation process (that in reality covers a collection of influences such as interpersonal communication, exposure to media influence etc.) will also not only be activated at discrete points in time. The discrete measurements will thus collect

[1] Subsequent applications (Ghonghadze and Lux, 2009; Lux, 2009) show that the model (or extended versions of it) can be used for out-of-sample forecasting of sentiment changes.

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the results of previous social interaction between publication dates. We, thus view the business climate survey as a discrete sample of a stochastic process in continuous time. This perspective is standard in many areas, e.g. models of the term structure of interest rates in finance or mathematical models in epidemiology (Iacus, 2008). Diffusion processes from epidemiology have been adapted and expanded in the social science quite some time ago (Bartholomew, 1973). In a sense, our approach here is similar in that we apply a kind of epidemic process to our sentiment data, but instead of a macroscopic diffusion process we start with an idealized microscopic formalisation of agents' interactions.[2]

Models with the above basic ingredients have been thoroughly investigated in the literature. The basic features of the model can be summarized by the following findings[3]:

i) For $\alpha_1 \leq 1$, the group dynamics defined by (3) and (4) is characterized by a stationary distribution with a unique maximum. If $\alpha_0 = 0$, this maximum is located at $x^* = 0$. It shifts to the right (left) for $\alpha_0 > 0$ (< 0).

ii) For $\alpha_1 > 1$ and α_0 not too large, the stationary distribution has two maxima $x_+ > 0$ and $x_- < 0$. If $\alpha_0 = 0$, the distribution is symmetric around 0. It becomes asymmetric if $\alpha_0 \neq 0$ with right-hand (left-hand) skewness and more concentration of probability mass in the right (left) maximum if $\alpha_0 > 0$ (< 0) holds.

iii) If $|\alpha_0|$ becomes very large, the smaller mode vanishes and the stationary distribution becomes uni-modal again. This happens if $|\alpha_0|$ increases beyond the bifurcation value $\overline{\alpha_0}$ given by:

$$\cosh^2(\overline{\alpha_0} - \sqrt{\alpha_1(\alpha_1 - 1)}) = \alpha_1 \tag{5}$$

with $cosh(.)$ denoting the hyperbolic cosine, $cosh(x) = \frac{1}{2}(e^x + e^{-x})$. One might note that these findings are perfectly analogous to those in models of discrete choice with social interactions, cf. Brock and Durlauf (2001b, propositions 1 through 3): Moderate influence of social interaction ($\alpha_1 \leq 0$) leads to a balanced distribution of the population on both alternative choices while strong interaction leads to the emergence of a majority in one alternative. A positive (negative) bias α_0 generates asymmetry as it introduces a preference for one of both alternatives.

In most applications, the first step towards an analysis of the above group dynamics consists in the derivation of a quasi-deterministic law of motion for the first moment of x:

[2]We could, in fact have estimated a diffusion model with similar dynamic behavior as our agent-based model (e.g. the double-well potential: $dx_t = a(x_t - x_t^3)dt + \sigma dW_t$) but the estimated parameters would have had no behavioral interpretation.

[3]cf. Weidlich and Haag (1983, chap. 2), Lux (1995); essentially the same jump-Markov process is used by Blume and Durlauf (2003) as a dynamic process of strategy revisions in a discrete choice framework with social interactions.

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$$\begin{aligned}\frac{d}{dt}\overline{x} = & \quad v(1-\overline{x})e^{\alpha_0+\alpha_1\overline{x}} - v(1+\overline{x})e^{-\alpha_0-\alpha_1\overline{x}} \\ = & \quad 2v[\tanh(\alpha_0+\alpha_1\overline{x}) - \overline{x}]\cosh(\alpha_0+\alpha_1\overline{x}).\end{aligned} \tag{6}$$

(6) is exact in the limit of an infinite population and provides a first-order approximation of the dynamics of x for finite populations. Convergence of our jump process to the ordinary differential equation (6) in the limit $N \to \infty$ can be demonstrated along the lines of Horst and Rothe (2008). Fluctuations around this first order approximation can be shown to follow an Ornstein-Uhlenbeck process. Since we are interested in finite populations, we will use a different kind of diffusion approximation as detailed in Appendix A.

3 Estimation: The Basic Framework

While the stochastic properties of population processes like the one depicted in sec. 2 have been studied in great detail (Weidlich and Haag, 1983; Aoki, 1996; Weidlich, 2000), this literature has not developed a systematic approach towards estimation of such models. In the following I will outline how such models can be estimated via a fairly conventional maximum likelihood procedure. The basic ingredient in our estimation procedure is the so-called Fokker-Planck equation for the time development of the transitional density of macroscopic observables of the process. The Fokker-Planck equation associated to a stochastic process is a parabolic partial differential equation that occupies a very prominent place in statistical physics (Risken, 1989; van Kampen, 2007).

For diffusion processes, the Fokker-Planck equation gives the exact law of motion for the transient density. Parameter estimation on the base of the Fokker-Planck equation, then, seems straight forward: if one has available discrete observations of a diffusion process and if the Fokker-Planck equation of the hypothesized process could be solved explicitly, the time-dependent solution to the transient density at the times of observations could be used to compute the likelihood of each observation conditional on the realization of the process in the previous period. Unfortunately, for many interesting models, a closed-form solution to the Fokker-Planck equation is not available. In this case, however, one can still resort to numerical approximations of the Fokker-Planck equation. Numerical integration of partial differential equations via finite difference of finite element methods is also a well developed field (Thomas, 1995) and has found important applications both in statistical physics and financial mathematics (Seydel, 2002, part III). Surprisingly, the approach outlined here has only found scarce applications in the literature so far. The first to propose approximate ML estimation on the base of a numerical integration of transitory densities has been Poulsen (1999) who applies this framework for estimation of models

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of the term structure of interest rules. His approach has been compared to alternative methods by Jensen and Poulsen (2002). Hurn *et al.* (2006) propose refinements using finite elements rather than finite differences. In their recent survey paper on estimation methods for stochastic differential equations Hurn et al. (2007) point out that numerical solution of the Fokker-Planck equation is the only completely generic estimation method.[4] Given that it is also very close to exact maximum likelihood, it seems surprising that it had been used only in the few papers mentioned in this paragraph.

For jump Markov processes like the present one, Fokker-Planck equations can be obtained in different ways as approximations to the law of motion for the transient density. The prevalent use of the Fokker-Planck equation for particle or molecular dynamics in physics and chemistry is typically based on the so-called Kramers-Moyal expansion (Risken, 1996, c. 4, van Kampen, 2007, c. 2). The Kramers-Moyal expansion is obtained as a Taylor series expansion of a continuous limit of the exact law of motion of the transient density. We provide a derivation of this heuristic approach in Appendix A. The resulting diffusion approximation consists in the stochastic differential equation with drift and diffusion terms equal to the first-order and second-order terms of the Kramers-Moyal expansion. The Appendix also provides details on the convergence of the original process to this diffusion limit.

In order to set the stage for our approximate ML estimation, consider a parabolic partial differential equation:

$$\frac{\partial f(x)}{\partial t} = \frac{\partial}{\partial x}(\mu(x,\theta)f(x)) + \frac{\partial^2}{\partial x^2}(g(x,\theta)f(x)). \tag{7}$$

If (7) refers to a Fokker-Planck equation, $f(x,t)$ is the transitory density of x, and $\mu(x,\theta) = -A(x,\theta)$, $g(x,\theta) = \frac{1}{2}D(x,\theta)$ with $A(x,\theta)$ and $D(x,\theta)$ the drift and diffusion functions of the process, and θ is a set of unknown parameters that one wants to estimate.

If no closed-form solution for $f(x,t)$ is available (which will mostly be the case), one can study the time development of the density via numerical integration of eq. (7). Various methods for discretisation of the stochastic equation (7) can be used. Applying a finite difference approach, the first and second derivatives on both sides of eq. (7) could be approximated either via forward differences of backward differences (called explicit or implicit methods). Higher accuracy of the approximation can be achieved by combining both forward and

[4]Other methods proposed in the literature either suffer from biases (such as the Euler method or discrete maximum likelihood, used as a benchmark below) or are applicable only for certain types of stochastic differential equations (e.g. methods based on characteristic functions or Hermite polynomial expansions of the transitional density).

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backward differences by computing central differences around intermediate grid points.

To concretize the finite difference approximation, consider a 'space' grid with distance h between adjacent knots: $x_j = x_0 + j \cdot h$; $j = 0, 1, ..., N_x$ and similarly equally spaced points along the time axis between $t = 0$ and the final time T: $t_i = i \cdot k$ with $i = 0, ..., N_t$ and $k = \frac{T}{N_t}$.

In a forward discretization, (7) would have to be replaced by

$$\frac{f_j^{i+1} - f_j^i}{k} = \frac{\mu_{j+1} f_{j+1}^i - \mu_j f_j^i}{h} + \frac{g_{j+1} f_{j+1}^i - 2 g_j f_j^i + g_{j-1} f_{j-1}^i}{h^2} \tag{8}$$

with $f_j^i := f(x_0 + j \cdot h, ik)$ and $\mu_j := \mu(x_0 + j \cdot h, \theta)$, $g_j := g(x_0 + j \cdot h, \theta)$. This forward approximation is also known as the *explicit* finite difference approximation as it provides a closed-form solution for the mesh points at time $i+1$. Replacing the forward difference on the left-hand side by the backward difference $f_j^i - f_j^{i-1}$, we obtain the *implicit* finite difference approximation. While the forward and backward approximations are of local accuracy (at the mesh points) $O(k) + O(h^2)$, higher accuracy can be obtained by taking the average of both the forward and backward difference approximation. This is known as the Crank-Nicolson method and can be shown to have local accuracy $O(k^2) + O(h^2)$. Note that the Crank-Nicolson approach effectively approximates the continuous-time diffusion at intermediate points $(i + \frac{1}{2})k$ rather than those on the grid itself.

Because of the necessity of restricting the approximation to a finite interval, boundary conditions have to be imposed in order to prevent transitions to inaccessible states. In the present application boundary conditions should prevent a leakage of probability mass to points outside the support of the transient density. The very natural condition to conserve mass within the support is, therefore:

$$f_{-\frac{1}{2}}^j = f(x_0 - \frac{1}{2}h, jk) = 0 \qquad \text{and} \qquad f_{N_x+\frac{1}{2}}^j = f(x_0 + (N_x + \frac{1}{2})h, jk) = 0. \tag{9}$$

While such simple Dirichlet boundary conditions preserve the local second order accuracy, more complex derivative boundary conditions in certain applications would require a careful analysis of the errors brought about by their discretization. In our setting, the no-flux boundary conditions guarantee conservation of probability mass within the underlying x-interval if (7) governs the dynamics of a transient density (i.e. if (7) is a Fokker-Planck equation).

The drift term of the Fokker-Planck equation for our process is given by:

$$A(x) = \frac{n_-}{2N} w_\uparrow(x) - \frac{n_+}{2N} w_\downarrow(x) = v(1-x)e^{\alpha_0 + \alpha_1 x} - v(1+x)e^{-\alpha_0 - \alpha_1 x} \tag{10}$$

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which, of course, coincides with the right-hand side of (6), while the diffusion term is:

$$D(x) = \frac{1}{N}(\frac{n_-}{2N}w_\uparrow(x)+\frac{n_+}{2N}w_\downarrow(x)) = \frac{1}{N}(v(1-x)e^{\alpha_0+\alpha_1 x}+v(1+x)e^{-\alpha_0-\alpha_1 x}). \tag{11}$$

This is certainly a case in which the conditional density can not be solved for explicitly due to the high degree of non-linearity of both the drift and diffusion components. For numerical integration, we can, however, resort to the Crank-Nicolson scheme as introduced above. Fig. 2 shows an example with a strongly peaked initial distribution which evolves into a bi-modal distribution over time. Underlying parameters are: $v = 3, \alpha_0 = 0, \alpha_1 = 1.2, N = 50$ for the parameters of the agent-based model, $h = 0.0025$ and $k = 0.01$ for the discretization in "space" and time, $T = 3$ for the time horizon of the numerical integration and a space grid extending from -1 to 1 in accordance with the support of the variable x has been used. The initial condition, $x_0 = 0$, has been approximated by a Normal distribution with density $\Phi^N(x_0 + A(x)k, D(x)k)$ evaluated at grid points $-1 + jh; j = 0, 1, \ldots, N_x$, in the x direction for the first time increment k. This avoids the problems of a Dirac δ-function as initial condition and can be interpreted as a first-order Euler approximation using the known drift and diffusion functions for the initialization of the approximation.

Fig. 2 about here

On the base of the Crank-Nicolson (or any other finite difference approximation), we can estimate the parameters of a diffusion process with discretely spaced observations via approximate maximum likelihood: The negative log-likelihood of a sample of observations $X_0, \ldots, X_T$ is

$$-logf_0(X_0 \mid \theta) - \sum_{s=0}^{T-1} logf(X_{s+1} \mid X_s, \theta) \tag{12}$$

where $f_0(X_0 \mid \theta)$ is the density of the initial state (which in practical applications will be skipped because of its negligible influence and the possible lack of a closed-form solution for the stationary density) and $f(X_{s+1} \mid X_s, \theta)$ is the value of the transitional density at $s+1$ conditioned on the previous observation at time s, X_s. This continuous density is approximated by our finite difference scheme. Poulsen (1999) shows that the pertinent estimator is consistent, asymptotically normal and can be asymptotically equivalent to full ML estimates, at least under the Crank-Nicolson approximation scheme. In his Theorem 3, he shows that the grid size has to behave like $k(T) = T^{-\delta}$ with $\delta > \frac{1}{4}$ which will be guaranteed in our applications.

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4 Monte Carlo Simulations of Approximate ML Estimation

We now turn to estimation of model parameters on the base of the numerical approximation to the Fokker-Planck equation. In order to study the performance of the method we conduct a small simulation experiment on the base of our canonical interaction model. Because of the time needed for approximate ML with numerical integration of the transient density we have to restrict this Monte Carlo study to a few selected parameter values. The following sets of parameters have been chosen:

- set I: $v = 3$, $\alpha_0 = 0$, $\alpha_1 = 0.8$,
- set II: $v = 3$, $\alpha_0 = 0.2$, $\alpha_1 = 0.8$,
- set III: $v = 3$, $\alpha_0 = 0$, $\alpha_1 = 1.2$,
- set IV: $v = 3$, $\alpha_0 = 0.2$, $\alpha_1 = 1.2$.

In all scenarios, $N = 50$, i.e. the population size is equal to 100 ($2N$). Our choice of parameters is governed by our interest to compare the performance in situations with uni-modal and bi-modal distributions, with and without a bias term $\alpha_0 \neq 0$.

Because of the computational demands of this method, the sample size has been restricted to $T = 200$ observations at discrete integer time intervals which have been extracted from a true multi-agent simulation with small time increments $\Delta t = 0.01$. The order of magnitude of this sample size is also in line with the number of available monthly observations of the ZEW index in our sample (which is 176). The time scaling parameter v has been fixed in order to have a certain number of switches between both modes in the bi-modal case as otherwise we would not expect the estimation procedure to detect a bi-modal distribution (whether this conjecture really holds, might be checked in subsequent Monte Carlo experiments). The Crank-Nicolson finite difference discretization is applied with widths $k = \frac{1}{8}$ ($k = \frac{1}{16}$)and $h = 0.02$ in the time and space direction, respectively (note that in the space direction $h = 0.02$ corresponds exactly to the discreteness of the index x for our setting with $N = 50$). In order to have a certain benchmark for comparison of accuracy of the parameter estimates, we compare the resulting estimates with those obtained under $k = 1$. The later can be interpreted as an Euler approximation since it approximates the transient density by a Normal distribution (with mean and standard deviation taken from the drift and diffusion functions of the Fokker-Planck equation) which in the Crank-Nicolson approach is used only for the initialization of the iterations. This Euler approximation does, of course, not yield consistent estimates and so we would expect it to be inferior to the Crank-Nicolson-ML approach. In order to get some insight into the dependence of the parameter estimates on the step size used in the Crank-Nicolson approximation, we also compare results

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obtained with time increments $k = \frac{1}{8}$ and $k = \frac{1}{16}$.

Table 1 shows our results exhibiting the mean estimates, finite sample standard errors and root-mean squared errors for all underlying parameters. The main message is that we can estimate the parameters v, α_0 and α_1 quite accurately even for our relatively small sample of 200 observations. In all cases, the Crank-Nicolson estimates are by far better than those obtained on the base of the Euler approximation, in terms of bias and standard error. One also infers that estimated parameters become somewhat less reliable in the cases of parameter sets II and IV as compared to I and III, respectively. The reason is probably that a positive bias interferes with the effects of interaction so that the variability of estimated parameters across samples increases. Nevertheless, the overall bias and standard error still remain reasonable even in those cases with $\alpha_0 = 0.2$ (with the exception perhaps of the estimates of v for parameter set IV). In contrast, Euler estimates appear essentially useless in these cases. As concerns the influence of the density of the grid, we observe only minor differences between the Crank-Nicolson approximations with $k = \frac{1}{8}$ and $k = \frac{1}{16}$. In fact, results do not uniformly improve when reducing the time increments: while one obtains slight improvements for the parameters α_0 and α_1, the estimates of v seem to deteriorate. The near equivalance of both settings together with seemingly reasonable biases and standard errors suggests the conclusion that using finer grids would probably not improve significantly the quality of the parameter estimates. In an unpublished Appendix (available upon request), we also provide evidence for the alleged second-order accuracy of the Crank-Nicolson approximations which underscores its suitability for ML estimation.

Table 1 about here

Another set of Monte Carlo experiments is motivated by realizing that the number of agents (the system size) N appears as a variable in the diffusion part of the Fokker-Planck equation. Neglecting the issue of discreteness of N, we can, in principle, also use our approach to arrive at an estimate of the number of active agents instead of imposing a predetermined value of N. In our pertinent Monte Carlo experiments, we use again parameter sets I through IV, with $N = 25$, $N = 50$ or $N = 175$ in both cases. The results are exhibited in Table B1 in the Appendix. Given the small sample size, the behavior of the estimates seems also quite satisfactory. We comment on a few particular observations in the Appendix.

5 Empirical Application: Interaction Effects in a Business Climate Index

Since we have focused on a very simple interaction scheme, it is not obvious that its structural features should be easily applicable to economic data. Weidlich

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and Haag (1983, c. 5) and Kraft, Landes and Weise (1986) had proposed simple business cycle models with, for example, investment decisions being driven by an opinion process like the one outlined in Sec. 2. Such models could be estimated using the above methodology. We leave this more demanding multivariate application to future research and turn to a particular type of uni-variate time series in which interaction effects could arguably play some role. Various surveys of *business climate* or *sentiment* are regularly conducted in many countries that seem to receive much more attention by the public than by academic researchers. The leading examples are the Michigan Consumer Sentiment Index and the Conference Board Index for the U.S. economy, which have been reported monthly since the end of the 70ties (Ludvigson, 2004, Souleles, 2004). In Germany, similar surveys are conducted by the Ifo Institute (Ifo Business Climate Index) and the Center for European Research (ZEW) at the University of Mannheim (denoted the ZEW Index of Economic Sentiment). A broader range of confidence indices is compiled by the European Commission for the member states of the European union (European Commission, 2007). Many of these indices are close to the simple structure of our 'canonical' model in that they very literally ask for wether respondents are optimistic ("+") or pessimistic ("-") concerning the prospects of their economy. The only difference to our above model is that these indices mostly also allow for a neutral assessment. To accomodate this additional possibility we might assume that neutral subjects can be assigned half and half to the optimistic and pessimistic camp which, then, would allow us to apply our model directly to these data[5]. Here we focus on the ZEW index as one particularly interesting example. What makes it particularly suitable for our purpose is that in contrast to many other sentiment indices it represents the average of binary resp. tertiary responses in a very direct way, i.e. without any further aggregation involved, and that it has a rather constant number of participants (about 350 respondents) while other indices exhibit more fluctuations in their number of respondents over time. The group of respondents is furthermore more homogeneous than in most other surveys as it consists mainly of leading professionals from the finance and insurance industry. This selection of respondents implies, on the one hand, that there should be more communication within this group (directly and indirectly via targeted media) than in a more anonymous sample selected via randomized nation-wide telephone interviews. On the other hand, one could hypothesize that financial experts should be less prone to interaction effects which lends further interest to our results.

The index is, in fact, reported as the percentage of optimists minus pessimists so that it can be directly used as the opinion index x in Sec. 2. The available monthly record of the ZEW sentiment index (starting in December 1991 and

[5]As detailed in Weidlich and Haag (1983, c. 6) the above framework could easily be extended by allowing for a neutral valuation and various degrees of positive or negative sentiments by slight changes of individuals' transition rates. Adopting the formalization of transition rates proposed by Weidlich and Haag, the macroscopic dynamics of the index would indeed remain unchanged.

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running through July, 2006) had already been displayed in Fig. 1 above. What is striking is the very pronounced cyclical behavior of the ZEW index with very sudden movements upward and downward and a certain stagnation at times at a high or low plateau. One could, in fact, argue that the dynamics of the ZEW index is reminiscent of a bi-modal stochastic dynamics switching between a high positive and a moderately negative equilibrium. In the introduction, we had already compared this series with what it is designed to predict, the cyclical component in economic activity. This cyclical component appears in the lower panel of Fig. 1 in the form of residuals of monthly industrial production from the Hodrick-Prescott filter. Somewhat surprising, the *perception* of the business cycle dynamics as reflected in the survey allows a much more clear-cut categorization of its phases than the much more random appearance of filtered IP.

The ZEW surveys are based on about 350 respondents so that we might take this information as a parametric restriction on N (assuming N=175). We, then, have to estimate the parameters v, α_0 and α_1 in a baseline application of our interacting-agents framework. Results are shown in Table 2. Interestingly, the crucial parameter α_1 is significantly larger than unity indicating bi-modality of the limiting distribution. Despite the impression of a dominance of positive assessment over the whole sample period (quite in contrast to stereotypes of German *angst*) the bias term α_0 turns out to be not significantly different from 0. Unfortunately, simulations of the estimated model show, that it most likely would get stuck within one mode over a time horizon of the length of our sample (176 observations) and would on average at most switch only once from one mode to the other (cf. Figs. 3 and 5 below). This is due to the fact that, in our framework, transitions between modes are governed by chance fluctuations and become more and more unlikely the higher the number of agents. *Vice versa*, frequent switches would only occur for a relatively small size of the underlying population. In order to reconcile our observation of a relatively large number of apparent switches of the mood of the respondents with the 'official' system size of 350 respondents, we could argue that the 'effective' system size is smaller than the official number. This would happen if some respondents would actually move broadly synchronously and would, therefore, not act like independent agents (independent in performing their movements, not independent in the sense that their movements between "+" and "-" were not influenced by other agents). While we cannot check this assertion due to the anonymity of the data, we could let the index itself speak on the underlying effective system size by adding N to the list of parameters estimated via approximate ML. Table 2 shows that this added flexibility leads to a relatively large increase in the log likelihood and is preferred over the baseline model by both the AIC and BIC criteria. The 'effective' number of agents in our estimation is only about 40 ($2N$) compared to the much higher official sample size of about 350. As concerns the other parameters, α_0 still is insignificant, while the interaction coefficient falls marginally below 1 indicating uni-modality albeit with possibly large excursions into extreme configurations. Remarkably, the estimate of the parameter v decreases from 0.78 to 0.15 when proceeding from model 1 to model 2. The likely

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reason is that the first model would have to come up with a higher mobility of the population (higher propensity to change opinion) in order to compensate for the stagnatory tendency of the larger imposed population of model 1.

Table 2 about here

We have remarked in sec. 2 that our framework allows to incorporate exogenous effects on the opinion formation process. In order to do so we simply could expand the influence function U by introducing additional factors that could be of importance to the assessment of the business cycle by the respondents of the survey:

$$U_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_t. \tag{13}$$

Most naturally, y could be macroeconomic data of the same frequency itself (i.e. monthly), although our framework could also accommodate data of higher or lower frequency. Various such macro feedbacks have been investigated. As typical macroeconomic data at monthly frequency we tried interest rates, industrial production and changes of unemployment rates. In our model 3 we report the influence of industrial production (deseasonalized and HP filtered, as displayed in Fig. 1). Note that the direction of the feedback is not predetermined in our model, i.e. α_2 could turn out positive or negative. The outcome of the exercise shows that industrial production adds some explanatory power: we obtain a significantly negative coefficient together with lower values of the AIC and BIC criteria. For interest rates, in contrast (results are available upon request), the estimated coefficients α_2 are not significant and overall improvements compared to model 2 are smaller. Quite the same holds for various measures of unemployment (with the change over the past 12 months entering as regressor because of the non-stationarity of the raw data): parameter estimates oscillate between significant and insignificant depending on which measure is used, the AIC and BIC values are between those of models 2 and 3 and the parameter estimates of the interaction components are hardly affected. Remarkably, the coefficient for the influence of changes of unemployment is positive in all cases. Combining two or three macroeconomic factors leads to very modest improvements ($logL \approx 649$). Mostly, at most the coefficient for IP remains significant, while again the parameters for the interaction components are barely affected. However, even for model 3, the improvement compared to model 2 is much smaller than the increase in likelihood achieved by adding N as a free parameter (the step from model 1 to model 2). What is perhaps puzzling is the negative sign of the feedback effect from industrial production (similarly we obtained counterintuitive positive coefficients for unemployment and somewhat more plausible negative ones for interest rates) which is in contrast to a positive contemporaneous correlation of about 0.28 between both series. It appears to depict some type of 'contrarian' behavior: if the economic data is indicating a boom phase, our respondents already appear to forestall the overheating of

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the economy and the subsequent downturn and *vice versa*.[6] In the estimation exercise reported in Table 2, the realization of industrial production is that of the previous period (which, in fact, in the case of IP is not known at this time to survey participants since the first statistical estimates are only released somewhat later). We have also experimented with various leads and lags without much change of the results.

Models 4 and 5 in Table 2 depict another extension of our baseline model: here we include a kind of 'momentum' effect in the opinion dynamics. Eq. (13) is now modified to include the change of the climate index from the month $t-1$ to the last observation:

$$U_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_t + \alpha_3 (x_t - x_{t-1}). \tag{14}$$

One may interpret this as respondents reacting not only to the net influence of their environment but being particularly sensitive to changes of the business climate themselves.[7, 8] Note that a priori again both a positive as well as negative feedback (if any) could be imagined. In fact, the negative coefficient on industrial production might suggest a similar contrarian element for the perceived momentum. As it turns out (cf. Table 2), the momentum effect is significantly positive. It again leads to a remarkable improvement of the model, but does not affect previously estimated parameters by too much. Adding industrial production as an explanatory variable (model 5) again leads to a further increase of the likelihood which is, however, again much more modest compared to the gain obtained from model 4. Smaller gains would result from alternative macroeconomic factors. In summary it, therefore, appears that macroeconomic variables add only a very slight fraction of the explanatory power, while the major improvements are obtained via refinements of our social opinion formation

[6] One frequently finds press releases emphasizing an 'unexpected' decline or increase of the ZEW index in view of the tendencies of macroeconomic indicators at the same time.

[7] We have to be a bit careful about the interpretation of the momentum term in our stochastic process: in order to guarantee that the process has Markov properties, we assume that agents only become aware of the current 'momentum' $\Delta x_t = x_t - x_{t-1}$ at the time when the new survey is released (at time t). Respondents are, therefore, assumed to not update this variable between surveys. In this way, we can use it as an independent variable in the transition rates without having to modify the structure of the Fokker-Planck equation. If, in contrast, agents would update Δx_t between integer time steps, we would have to deal with a continuous time dynamics with delays for which finite difference approximations would become quite cumbersome.

[8] Following the recommendation of one referee, we have also estimated a more general variant of models IV and V where we allowed for an arbitrary time lag $x_t - x_{t-\delta}$ in the 'momentum' term. In order to estimate the new delay parameter δ, we have treated $x_{t-\delta}$ as a quasi-continuous quantity by taking the weighted average between observations $x_{t-int(\delta)}$ and $x_{t-int(\delta)-1}$ for non-integer δ (with $int(\cdot)$ denoting the integer part of its argument and weights being fixed in accordance with the fractional part of δ). Of course, the lower admissible bound for δ in this estimation exercise is 1 and it turned out, that the ML estimation converged to this lower bound. Given the rapid changes of the sentiment index, it seems plausible that the respondents could have a relatively short time horizon for assessing the 'momentum' of the business climate.

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process.[9]

6 Specification Tests

How closely do time series from the estimated models mimic the empirical behavior of the ZEW index? Fig. 3 exhibits three simulations over the same time horizon ($T = 176$ integer periods) of model 5 together with the empirical data. For these simulations, we have used time increments $\Delta t = 0.01$ for the ongoing opinion formation between integer time steps and have injected the knowledge of the current exogenous factor (HP-filtered industrial production) as well as the 'momentum' of the index itself at integer time steps. As it can be seen, the visual appearance of the three Monte Carlo runs is pretty similar to that of the index itself and the feedback from industrial production seems to direct the simulations towards a pattern that is broadly synchronous with the ups and downs of the empirical record. Model 2 to 4 are not too different in their appearance. In contrast, model 1 yields a very different pattern as shown in the lower right panel of Fig. 3 since with the higher 'official' number of respondents shifts between equilibria become less frequent than with $N \approx 20$. Fig. 4 shows the mean and 95 percent confidence bounds from the transient density computed for model 3 over the whole observation period given the first observation of the index as the initial condition and incorporating the feedback from industrial production. Since the empirical record stays within the 95 % bounds for practically the entire time horizon, we may conclude that we have no reason to reject the hypothesis that the empirical data could have emerged as one particular sample path from our stochastic model. We note that simulations of models 2, 4, and 5 would lead to very similar patterns. However, for models 2 and 4 the sample paths would not be synchronous to the empirical series simply because there is no exogenous factor.[10] As can be seen from Fig. 5, the 95 percent confidence interval from model 1 excludes the better part of the empirical record, so that this baseline model could be clearly rejected as a potential data-generating process. For model 5, we could not perform the same exercise since the discrete momentum effect is hard to capture in the Fokker-Planck equation. We can, however, resort to numerical simulations in this case which gave a 95 percent confidence interval (from 1000 repetitions) that improves slightly on the analytical results for model 3 in Fig. 4 (not shown here because it is almost undistinguishable from Fig. 3). Overall, our models 3 and 5, in fact, show how the fuzzy exogenous information in the lower panel of Fig. 1 could be translated into a much clearer image of the business cycle dynamics in the view of the respondents' sentiments (upper panel of Fig. 1) via the self-referential and

[9] This holds at all stages of our estimation exercise: if we add industrial production as an explanatory variable in model 1 (with fixed N=175), the likelihood only increases to -722.9 with virtually unchanging parameters for the social dynamics.

[10] While this synchronous behavior appears quite striking in simulated time series, the statistical improvement by models 3 and 5 compared to models 2 and 4 in terms of the 'distance' criterion in Table 3 is relatively modest.

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self-reinforcing dynamics of the opinion formation process.

FIGURE 3 ABOUT HERE

FIGURE 4 ABOUT HERE

Since the estimated interaction parameter, α_1, in models 2 through 4 is marginally below the bifurcation value of unity, the ups and downs of the sentiment index during the observation period would likely reflect shifts of unique equilibria that alternate between optimistic and pessimistic majorities. Note, however, that a standard, say 95 confidence interval for α_1 would not exclude the possibility $\alpha_1 > 1$ so that we could as well have an underlying bimodal process with switches between both modes triggered by exogenous forces together with the inherent volatility of the opinion dynamics.

As another specification test[11] we try to assess whether the abruptness of the up and down movements of the index is captured by our model. For this purpose we compute a series of one-period iterations of the transient density and extract the 95 percent confidence intervals conditional on the realization in the previous period. Fig. 6 shows the 95 % confidence bounds for the subsequent period's realization from model 5 which apparently is never left by the empirical record. Upon close investigation one might, however, find some of the downturns are getting close to the lower boundary while the ups are pretty much in the center of the 95 percent bound.

FIGURE 5 ABOUT HERE

FIGURE 6 ABOUT HERE

Table 3 provides a statistical analysis of 1000 Monte Carlo replications of models 1 through 5 on the base of the estimated parameters displayed in Table 2. In order to get an impression of how closely we match the statistical features of the data, we compare a selection of conditional and unconditional moments. The table shows the means and simulated 95 percent boundaries for the first four unconditional moments together with the relative deviation (the squared value of the mean divided by the variance) as defined in Chen (2002) and the mean absolute distance between the entries of each simulation and the 176 empirical observations. As we can see, for the first to third moments as well as the relative deviation, models 2 to 5 are all pretty close to the empirical numbers while model 1 (using the 'official' number of 350 active agents) is far off the mark in all cases. This confirms the visual impression reported above

[11] For a more direct test we could use the test for uniform residuals of out-of-sample density forecasts (Pedersen, 1994, Diebold et al., 1998). Using as in-sample data for parameter estimation the observations until the end of 2000 this relatively weak test does not reject models 2 and higher on the base of the remaining out-of-sample density forecasts.

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that the patterns of all models with an endogenous number of effective agents are relatively similar while model 1 stands out by its tendency of getting frozen in the lower mode due to the negative initial condition and the high level of persistence caused by the large number of 350 agents. For the remaining statistics, we first see that kurtosis is relatively poorly matched by all models, which might however be attributed to the volatility of this measure for small samples. The distance between the empirical observations and synthetic data again shows the greatest discrepancy for model 1 compared to all others while the feedback from industrial production in models 3 and 5 seems to have contributed to a better fit compared to models 2 and 4. Again, this provides a confirmation of our visual impression reported above.

Table 4 reports autocorrelations of the index for lags 1 to 10. A glance at smaller lags again indicates that ACFs from models 2 to 5 are all very close to their empirical counterpart while model 1 has a much lower degree of dependence. Interestingly, models 2 and 3 are only able to match about the first four lags while the autocorrelations remain much higher than the empirical ones for the longer lags. Inclusion of the 'momentum' effect leads to a better fit of the entire range of autocorrelations between 1 and 10 lags and also achieves a close agreement in the estimate of the parameter of fractional differentiation as given in the last row of Table 4. This statistics is the parameter for hypothesized hyperbolic decay of the autocovariances, $E[x_t x_{t-\tau}] \sim \tau^{2d-1}$ and it is estimated via the method proposed by Geweke and Porter-Hudak (1983). The motivation for inclusion of this statistics comes from the finding that various survey data in the political arena are characterized by long-term dependence in the sense of hyperbolic decay of their autocovariances and autocorrelation functions (Box-Steffensmeier and Smith, 1998).[12]

Table 3 about here

Table 4 about here

7 Conclusion

Given the immense public attention devoted to survey measures of business climate or economic sentiment, there has been surprisingly little work trying to model these data. Of course, under a rational expectations perspective, the most interesting aspect would be to test unbiasedness of such survey expectations and to find out whether they have predictive power beyond that of other macroeconomic data. However, not all economists firmly believe in the ubiquitous validity of the rational expectation hypothesis. If we go to the other extreme, business climate surveys might rather reflect Keynes' notorious animal spirits at work. In this paper we have adopted the latter viewpoint. However, rather

[12] Alfarano and Lux (2007) show that models with multi-modal distributions might lead to time series with apparent long memory.

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than taking the state of prevailing animal spirits as given, we have proposed a positive model to explain the fluctuations in respondents' confidence in the economic development. As it turned out, this model appears to have significant explanatory power for the ups and downs of the business climate index under investigation: the model's parameters for the conjectured social interaction are strongly significant, and apparently this social component of the opinion dynamics is much more important for the goodness-of-fit of various variants of our model than added macroeconomic variables. In the absence of alternative explanatory models, we conducted a series of specification tests that on the whole suggest that the empirical record could have been envisaged as a particular sample path from our model. Alternative 'rational' explanations of the development of the business climate would have to show that the pronounced swings could be explained by the release of important bits of information within the pertinent time intervals.[13] This is a problem similar to the identification of important news at the time of large changes of financial prices (cf. Cutler et al., 1989) and a casual search for such explanations did not reveal any plausible candidates for such information shocks. While we cannot exclude such explanations, we leave the burden of the proof to proponents of rational expectations and reiterate that the social contagion of animal spirits apparently provides us with a framework that explains the data well without having to rely on unobservable information shocks.

There are many directions into which research could fruitfully proceed from here: first, one should obviously study similar data sets from other countries to see whether interaction patterns are similar or not. We have already started such a comparative project and found quite similar results to those reported above in quite a number of cases. Second, if business cycles are, in fact, generated (at least partially) by animal spirits, the business climate measures would interact with objective economic quantities like industrial production.[14] It would, therefore, be worthwhile to include the opinion dynamics into a multi-variate setting of both objective measures of economic activity and more subjective survey indices. While conceptionally not too difficult to imagine, such a framework would be computationally extremely demanding and would require the development of more efficient numerical algorithms. Third, one would also like to identify animal spirits in cases where no survey data exists. This would pose the challenge of developing indirect methods of inference to identify hidden psychological states.

[13] One could certainly argue that the supposed interaction effects can be explained away by correlated information. In the absence of individual data, we cannot discriminate between unobserved exogenous factors and interaction effects (as shown by Lee, 2007, such an identification would require a sample with multiple groups of different sizes). In any case, an alternative explanation based on unobserved common shocks would be empirically void.

[14] Note that causality tests cannot distinguish between rational expectations and animal spirits: while positive causation between contemporaneous survey expectations and future realizations of economic activity are routinely interpreted as evidence for rational expectations (if unbiased), they could as well be the imprint of a true causality running from survey expectations to subsequent economic development.

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A Appendix A: Diffusion Approximation of the Jump Process

Fokker-Planck equations for Poissonian jump processes of ensembles of particles and molecules are pervasively found in physics and chemistry (Risken, 1989; van Kampen, 2007). In the pertinent literature, the Fokker-Planck equation for such processes is typically obtained in a heuristic way via the so-called Kramers-Moyal expansion of the law of motion of the transition density. We provide details on this approximation below. As a starting point, note that conditional probabilities of our Markov process for the sentiment index x can be witten as:

$$P(x+\frac{1}{N},\, t+\tau \,|\, x,t) = \omega_{\uparrow}(x)\,\tau + o(\tau),$$

$$P(x-\frac{1}{N},\, t+\tau \,|\, x,t) = \omega_{\downarrow}(x)\,\tau + o(\tau), \tag{A1}$$

$$P(x',\, t+\tau \,|\, x,t) = o(\tau) \quad for \quad x' \notin \{x-\frac{1}{N},\, x,\, x+\frac{1}{N}\}$$

for sufficiently small τ. As a consequence, the change in time of the probability over all x is:

$$\begin{aligned} P(x,\, t+\tau) - P(x,\, t) \;=\; & (\omega_{\uparrow}(x-\frac{1}{N})\,\tau + o(\tau))\; P(x-\frac{1}{N},\, t) \\ & + (\omega_{\downarrow}(x+\frac{1}{N})\,\tau + o(\tau))\; P(x+\frac{1}{N},\, t) \\ & - (\omega_{\downarrow}(x)\,\tau + \omega_{\uparrow}(x)\,\tau + o(\tau))\; P(x,\, t) + o(\tau) \end{aligned} \tag{A2}$$

In the limit $\tau \to 0$ we obtain the so-called Master equation in continuous time:

$$\begin{aligned} \frac{dP(x,\, t)}{dt} \;=\; & \omega_{\uparrow}(x-\frac{1}{N})\; P(x-\frac{1}{N},\, t) \\ & + \omega_{\downarrow}(x+\frac{1}{N})\; P(x+\frac{1}{N},\, t) \;-\; (\omega_{\uparrow}(x)+\omega_{\downarrow}(x))\; P(x,\, t) \end{aligned} \tag{A3}$$

For large N, the right-hand side of (A3) can be approximated by a Taylor series:

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$$\begin{aligned}\frac{\partial P(x,\,t)}{\partial t} &= \omega_{\uparrow}(x)\;P(x,t) + \frac{\partial}{\partial x}[\omega_{\uparrow}(x)\;P(x,t)]\;(-\frac{1}{N}) \\ &+ \frac{1}{2}\frac{\partial^2}{\partial x^2}\;[\omega_{\uparrow}(x)\;P(x,t)](-\frac{1}{N})^2 + o(\frac{1}{N^2}) - \omega_{\uparrow}(x)\;P(x,t) \\ &+ \omega_{\downarrow}(x)\;P(x,t) + \frac{\partial}{\partial x}\;[\omega_{\downarrow}(x)\;P(x,t)]\;\frac{1}{N} \\ &+ \frac{1}{2}\frac{\partial^2}{\partial x^2}\;[\omega_{\downarrow}(x)\;P(x,t)]\;\frac{1}{N^2} + o(\frac{1}{N^2}) - \omega_{\downarrow}(x)\;P(x,t) \qquad \text{(A4)}\end{aligned}$$

We arrive at:

$$\begin{aligned}\frac{\partial P(x,t)}{\partial t} &= -\frac{1}{N}\frac{\partial}{\partial x}\;[(\omega_{\uparrow}(x) - \omega_{\downarrow}(x))\;P(x,t)] \\ &+ \frac{1}{2}\frac{1}{N^2}\frac{\partial^2}{\partial x^2}\;[(\omega_{\uparrow}(x) + \omega_{\downarrow}(x))\;P(x,t)] + o(\frac{1}{N^2}) \qquad \text{(A5)}\end{aligned}$$

As it turns out, the Taylor series expansion up to order two yields the standard format of a Fokker-Planck equation of a diffusion process with drift and diffusion terms depicted in eqs. (10) and (11) in the main text. It is worthwhile to note that convergence of the Taylor series expansion $\sum_{q=1}^{\infty}\frac{(-1^q)}{q!}\frac{\partial^q}{\partial x^q}\{a_q(x)\;P(x,t)\}$ is guaranteed in our example because of boundedness of $a_q(x)$. The so-called Kramers-Moyal expansion detailed above is equivalent to an approximation of the underlying jump Markov process by a continuous diffusion process whose infinitesimal generator has drift and diffusion terms coinciding with those of the second-order Taylor series expansion eq. (A5). The corresponding process $\tilde{x}_N$ can be described by the stochastic differential equation

$$d\tilde{x}_N = A(\tilde{x}_N)dt + \sqrt{D(\tilde{x}_N)}dB_t \qquad \text{(A6)}$$

with B_t a standard Brown motion and $A(\tilde{x}_t)$ and $D(\tilde{x}_t)$ the drift and diffusion terms (given by eqs. 10 and 11 in the main text in our particular case). The textbooks in statistical physics remain completely silent on the validity of this approximation. However, Theorem 11.3.1. in Ethier and Kurtz (1986, p. 460) shows that the heuristically derived diffusion process $\tilde{x}_N$ indeed approximates the original jump Markov process in a probabilistic sense. Since Ethier and Kurtz consider arbitrary density-dependent jump intensities, our model falls into their class of processes and their theorem is immediately applicable to our model. Skipping the involved details, Ethier and Kurtz's Theorem 11.3.1 shows that the heuristic diffusion approximation $\tilde{x}_N(t)$ of a population-based jump process with population size N (denoted by x_N) obeys for $N \geq 2$ and bounded time horizons $T > 0$:

$$\sup_{t \leq T} |x_N(t) - \tilde{x}_N(t)| \leq \Gamma_n^T\;\frac{logN}{N}$$

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with Γ_n^T a random variable whose distribution asymptotically decays as $\sim N^{-2}$. Since our empirical application is based on the iteration of the transient density over bounded intervals $[t_i, t_{i+1}]$ we might expect the probabilistic approximation to be sufficiently accurate. Indeed, our Monte Carlo experiments confirm that this approach works well both for fixed and endogenous N. Note that this 'naive' heuristic diffusion approximation is different from an approximate dynamics of the form:

$$\hat{x}_N(t) = \bar{x}_t + \frac{1}{\sqrt{N}}\xi_t \tag{A7}$$

with $\bar{x}_t$ the expectation of eq. (6) and ξ_t a diffusion process for the fluctuations around the deterministic mean value process. The later would coincide with what is called a small-noise approximation in statistical physics (van Kampen, c. X). Along similar lines as in the more involved model of Horst and Rothe (2008), a first-order approximation on the base of eq. (A7) would lead to the ordinary differential equation (6) in the main text (the deterministic limit for an infinite population). To second order, this concept would give rise to an Ornstein-Uhlenbeck process. The pertinent Fokker-Planck equation of this process would, however, be different from the one derived via the above heuristic approach (one would obtain a linear Fokker-Planck equation for ξ_t, cf. van Kampen, c. X, for details). However, this approximation would only be valid for the stable regime ($\alpha < 1$) or for the dynamics in the vicinity of one of the models in the unstable case ($\alpha > 1$). Starting out around the unstable mode $x = 0$ in the case $\alpha > 1$, the fluctuations would grow linearly and (A7) would not be an admissible approximation.

The standard small-noise approximation would, therefore, not be applicable in the presence of jumps between modes or arbitrary initial values possibly in the vicinity of an unstable mode. Nevertheless, it is worthwhile to note that asymptotically for $N \to \infty$ and for bounded time intervals, both approximations, $\tilde{x}_N$ and $\hat{x}_N$, are equivalent (cf. Ethier and Kurtz (1986), Theorem 11.3.2).

B Appendix B: Monte Carlo Runs with Endogenous N

Table B1 provides the results on our Monte Carlo runs with estimated parameters v, α_0, α_1, and N. The basic message is that even for the small sample sizes of our study, the extended sets of parameters can be efficiently estimated. The average biases across the 200 replications are small in most cases except for a few outliers. One particular outlier is the case $\alpha_0 = 0, \alpha_1 = 0.8, N = 175$ for which N is strongly biased upwards. However, as the medians show this bias might be due to some extreme realizations. Another interesting observation is that our algorithm has problems in disentangling the effects of a large bias and strong social interaction ($\alpha_0 = 0.2, \alpha_1 = 1.2$). This is perhaps not too surprising since with x fluctuating around a unique positive mode the factor $\alpha_1 x$ would

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exhibit only small fluctuations. Interestingly, however, this effect diminishes with increasing N.

				Crank-Nicolson ($k = 1/8$)				Crank-Nicolson ($k = 1/16$)			
				v	α_0	α_1	N	v	α_0	α_1	N
a_0	=	0.000	means	3.390	0.001	0.777	32.400	4.024	0.001	0.774	32.618
a_1	=	0.800	medians	3.268	0.000	0.807	26.011	3.401	0.000	0.812	26.726
N	=	25	FSSE	2.471	0.007	0.172	19.703	2.565	0.007	0.180	20.063
			RMSE	2.634	0.007	0.173	21.000	2.756	0.007	0.181	21.414
a_0	=	0.000	means	4.766	0.002	0.736	80.047	4.840	0.002	0.713	80.058
a_1	=	0.800	medians	3.595	0.000	0.847	61.461	3.876	0.000	0.856	64.865
N	=	50	FSSE	3.763	0.007	0.277	62.459	3.950	0.007	0.303	64.302
			RMSE	4.149	0.007	0.284	69.170	4.349	0.008	0.314	70.834
a_0	=	0.000	means	6.589	0.001	0.673	392.688	8.252	0.002	0.673	479.833
a_1	=	0.800	medians	3.385	0.000	0.824	191.185	4.362	0.000	0.863	243.992
N	=	175	FSSE	7.136	0.005	0.359	433.211	10.354	0.009	0.475	588.831
			RMSE	7.972	0.005	0.380	483.861	11.587	0.009	0.490	661.749
a_0	=	0.200	means	1.696	0.220	0.759	28.370	27.620	0.221	0.758	28.118
a_1	=	0.800	medians	3.718	0.191	0.813	26.637	4.215	0.190	0.813	26.711
N	=	25	FSSE	20.696	0.123	0.229	9.451	38.345	0.120	0.222	8.707
			RMSE	23.250	0.125	0.232	10.012	45.488	0.122	0.225	9.228
a_0	=	0.200	means	6.965	0.249	0.711	52.319	7.048	0.259	0.694	50.922
a_1	=	0.800	medians	3.216	0.208	0.790	50.401	3.410	0.208	0.789	48.795
N	=	50	FSSE	16.054	0.158	0.281	19.450	14.627	0.168	0.300	18.396
			RMSE	16.497	0.165	0.294	19.539	15.141	0.178	0.318	18.379
a_0	=	0.200	means	4.035	0.279	0.662	185.145	4.985	0.294	0.637	184.206
a_1	=	0.800	medians	3.196	0.209	0.784	175.450	3.334	0.219	0.770	164.994
N	=	175	FSSE	2.992	0.229	0.399	86.822	5.742	0.240	0.418	92.877
			RMSE	3.159	0.242	0.422	87.197	6.062	0.258	0.448	93.101
a_0	=	0.000	means	3.161	0.000	1.197	26.372	3.110	0.000	1.196	26.050
a_1	=	1.200	medians	3.001	0.000	1.197	26.168	3.065	0.000	1.197	26.021
N	=	25	FSSE	0.746	0.009	0.015	3.975	0.598	0.009	0.015	3.563
			RMSE	0.762	0.009	0.015	4.196	0.606	0.009	0.015	3.705
a_0	=	0.000	means	3.097	0.004	1.187	51.587	3.099	0.004	1.187	51.468
a_1	=	1.200	medians	3.053	0.000	1.197	51.526	3.072	0.000	1.196	51.356
N	=	50	FSSE	0.761	0.031	0.050	6.593	0.686	0.031	0.050	6.516
			RMSE	0.765	0.032	0.052	6.766	0.691	0.031	0.052	6.664
a_0	=	0.000	means	3.825	0.026	1.127	176.176	3.781	0.055	1.138	175.935
a_1	=	1.200	medians	2.967	0.011	1.172	159.542	2.999	0.011	1.174	158.175
N	=	175	FSSE	7.201	0.356	0.513	242.764	3.902	0.723	0.666	242.663
			RMSE	7.230	0.357	0.517	242.159	3.970	0.724	0.667	242.057
a_0	=	0.200	means	3.410	0.669	0.619	19.130	4.556	0.656	0.635	19.296
a_1	=	1.200	medians	1.501	0.596	0.715	18.465	1.705	0.588	0.736	18.976
N	=	25	FSSE	9.547	0.390	0.488	5.205	14.034	0.391	0.489	5.183
			RMSE	9.531	0.609	0.758	7.837	14.086	0.600	0.747	7.699
a_0	=	0.200	means	22.016	0.256	1.134	54.254	39.919	0.342	1.042	52.693
a_1	=	1.200	medians	10.118	0.173	1.244	54.290	20.864	0.210	1.186	53.523
N	=	50	FSSE	30.421	0.338	0.414	12.290	40.372	0.367	0.449	12.193
			RMSE	35.811	0.341	0.419	12.976	54.633	0.393	0.481	12.457
a_0	=	0.200	means	5.686	0.318	1.059	171.394	5.994	0.341	1.031	170.926
a_1	=	1.200	medians	2.834	0.195	1.209	174.389	3.027	0.280	1.102	167.431
N	=	175	FSSE	15.959	0.520	0.628	48.890	15.518	0.544	0.658	53.062
			RMSE	16.144	0.532	0.642	48.901	15.766	0.561	0.677	53.086

Table B1: Note: The table reports the statistics of 200 Monte Carlo runs of each parameter set with a sample size of $T = 200$.

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		Euler ($k = 1$)			Crank-Nicolson ($k = 1/8$)			Crank-Nicolson ($k = 1/16$)		
		v	α_0	α_1	v	α_0	α_1	v	α_0	α_1
set I	mean	0.999	-0.001	0.642	2.980	-0.000	0.793	3.023	-0.000	0.794
	FSSE	0.091	0.007	0.052	0.567	0.005	0.028	0.585	0.005	0.028
	RMSE	2.003	0.007	0.166	0.564	0.005	0.028	0.583	0.005	0.028
set II	mean	0.439	0.578	0.123	2.992	0.216	0.772	3.547	0.211	0.782
	FSSE	0.048	0.097	0.171	1.046	0.057	0.105	1.422	0.038	0.069
	RMSE	2.561	0.390	0.698	1.041	0.059	0.108	1.517	0.039	0.071
set III	mean	1.019	0.000	1.173	2.884	0.000	1.196	2.952	0.000	1.196
	FSSE	0.126	0.024	0.034	0.457	0.009	0.015	0.499	0.009	0.015
	RMSE	1.913	0.024	0.043	0.469	0.009	0.016	0.499	0.009	0.015
set IV	mean	0.232	1.741	-0.698	1.369	0.262	1.123	1.748	0.245	1.144
	FSSE	0.026	0.350	0.426	0.245	0.127	0.159	0.439	0.127	0.159
	RMSE	2.768	1.580	1.945	1.326	0.141	0.175	1.326	0.135	0.168

Table 1: Approximate ML Estimates: The table displays the mean parameter estimates over 200 Monte Carlo replications together with their finite sample standard errors (FSSE) and root mean squared errors (RSME).

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	ν	α_0	α_1	α_2	α_3	N	logL	AIC	BIC
Model 1 (baseline)	0.78 (0.06)	0.01 (0.01)	1.19 (0.01)				-726.9	1459.8	1464.1
Model 2 (end. N)	0.15 (0.07)	0.09 (0.06)	0.99 (0.14)			21.21 (9.87)	-655.9	1319.7	1322.0
Model 3 (feedback from IP)	0.13 (0.06)	0.09 (0.07)	0.93 (0.16)	-4.55 (2.53)		19.23 (8.78)	-650.4	1310.9	1311.1
Model 4 (momentum effect)	0.14 (0.05)	0.10 (0.06)	0.91 (0.14)		2.11 (0.76)	27.24 (9.63)	-627.5	1265.1	1265.4
Model 5 (momentum + IP)	0.12 (0.05)	0.11 (0.06)	0.86 (0.16)	-2.82 (1.65)	2.23 (0.81)	25.12 (8.95)	624.9	1261.9	1260.1

Table 2: Parameter Estimates for Stochastic Models of Interacting Agents. Note: Details on the underlying models appear in the main text. The numbers in brackets are standard errors of parameter estimates.

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	data	models				
		1	2	3	4	5
mean:	0.352	-0.588	0.349	0.324	0.391	0.343
(95%)		(-0.631, -0.405)	(0.045, 0.552)	(0.035, 0.528)	(0.214, 0.521)	(0.173, 0.479)
std. dev:	0.370	0.098	0.355	0.384	0.314	0.356
(95%)		(0.068, 0.185)	(0.251, 0.484)	(0.293, 0.506)	(0.245, 0.391)	(0.284, 0.424)
skewness:	-0.620	0.615	-0.908	-0.991	-0.947	-0.857
(95%)		(0.029 ,1.685)	(-1.880, 0.097)	(-1.878, 0.053)	(-1.620, 0.247)	(-1.571, -0.181)
kurtosis:	-0.428	0.575	0.591	0.540	0.804	0.207
(95%)		(-0.652, 3.717)	(-1.322, 4.347)	(-1.283, 3.296)	(-0.911, 3.585)	(-1.000, 2.315)
rel. deviation:	0.905	49.705	1.368	1.030	1.766	1.049
(95%)		(8.706, 82.360)	(0.024, 4.022)	(0.022, 2.792)	(0.342, 3.956)	(0.209, 2.439)
distance:		0.952	0.335	0.297	0.233	0.223
(95%)		(0.899, 0.986)	(0.255, 0.493)	(0.220, 0.455)	(0.171, 0.318)	(0.165, 0.311)

Table 3: Unconditional moments from 1.000 Monte Carlo simulations of models 1 through 5 (95 percent confidence intervals from the simulations are given in brackets)

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		models				
ACF	data	1	2	3	4	5
1	0.935	0.630	0.923	0.939	0.904	0.930
(95 %)		(0.456 0.963)	(0.845 0.967)	(0.908 0.968)	(0.844 0.944)	(0.890 0.955)
2	0.830	0.404	0.853	0.880	0.796	0.848
		(0.162 0.929)	(0.715 0.936)	(0.820 0.934)	(0.674 0.879)	(0.769 0.901)
3	0.709	0.266	0.789	0.819	0.691	0.762
		(0.013 0.890)	(0.595 0.907)	(0.732 0.900)	(0.523 0.811)	(0.653 0.845)
4	0.584	0.175	0.729	0.758	0.592	0.675
		(-0.080 0.857)	(0.496 0.883)	(0.652 0.866)	(0.393 0.751)	(0.541 0.784)
5	0.465	0.116	0.673	0.696	0.499	0.589
		(-0.133 0.820)	(0.398 0.860)	(0.566 0.833)	(0.266 0.699)	(0.432 0.723)
6	0.363	0.075	0.620	0.633	0.419	0.508
		(-0.171 0.784)	(0.319 0.840)	(0.478 0.797)	(0.169 0.638)	(0.335 0.662)
7	0.272	0.048	0.571	0.571	0.355	0.434
		(-0.188 0.747)	(0.241 0.813)	(0.392 0.759)	(0.092 0.594)	(0.250 0.616)
8	0.186	0.032	0.525	0.512	0.302	0.366
		(-0.197 0.703)	(0.184 0.793)	(0.317 0.722)	(0.049 0.553)	(0.171 0.565)
9	0.094	0.022	0.482	0.454	0.251	0.298
		(-0.213 0.668)	(0.121 0.774)	(0.239 0.678)	(-0.013 0.516)	(0.088 0.514)
10	0.017	0.014	0.442	0.398	0.196	0.228
		(-0.220 0.631)	(0.078 0.752)	(0.167 0.640)	(-0.084 0.468)	(0.002 0.463)
d	0.553	0.194	0.826	0.923	0.551	0.668
		(-0.343 0.978)	(0.338 1.261)	(0.455 1.346)	(0.027 0.992)	(0.202 1.051)

Table 4: Autocorrelations and estimated parameter of fractional differentiation d from 1.000 Monte Carlo simulations (95 percent confidence intervals from the simulations are given in brackets).

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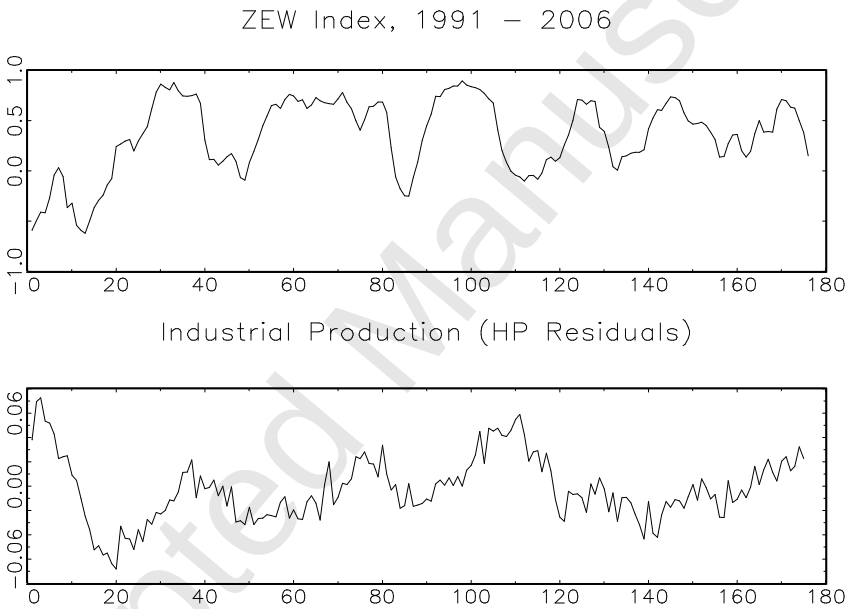

Figure 1: ZEW Sentiment Index and Industrial Production.

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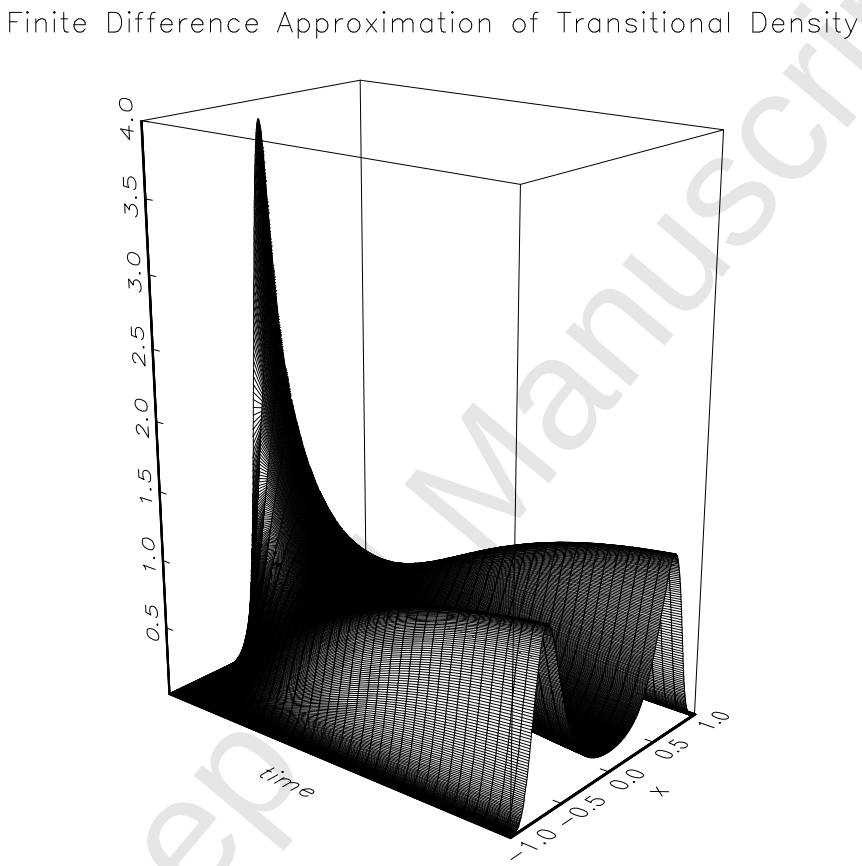

Figure 2: An illustration of the development of the transient density in the bimodal case. The initial state x_0 has been approximated by a Normal distribution with small standard deviation and mean x_0.

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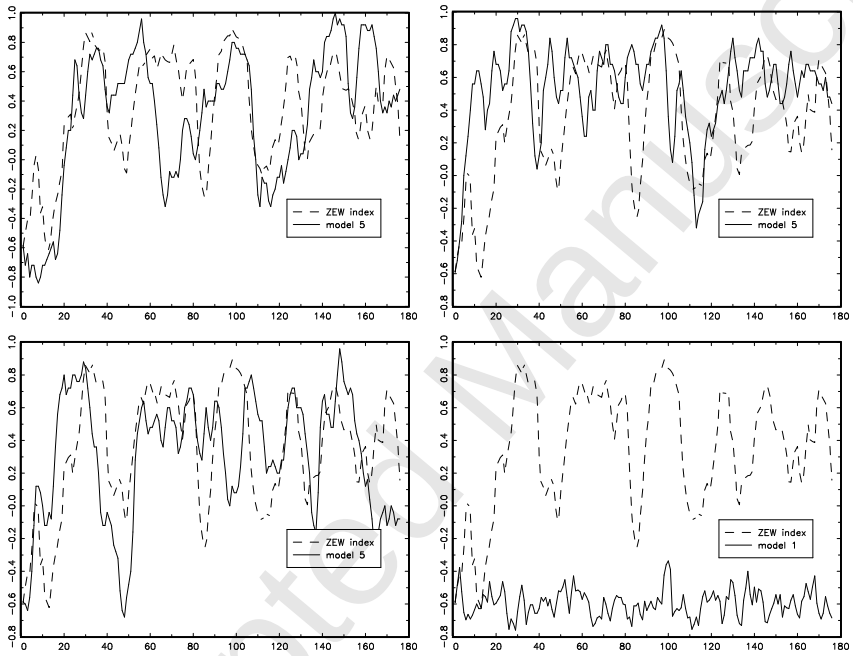

Figure 3: Simulated trajectories from models 5 and 1 (lower right-hand panel). The broken lines show the empirical data

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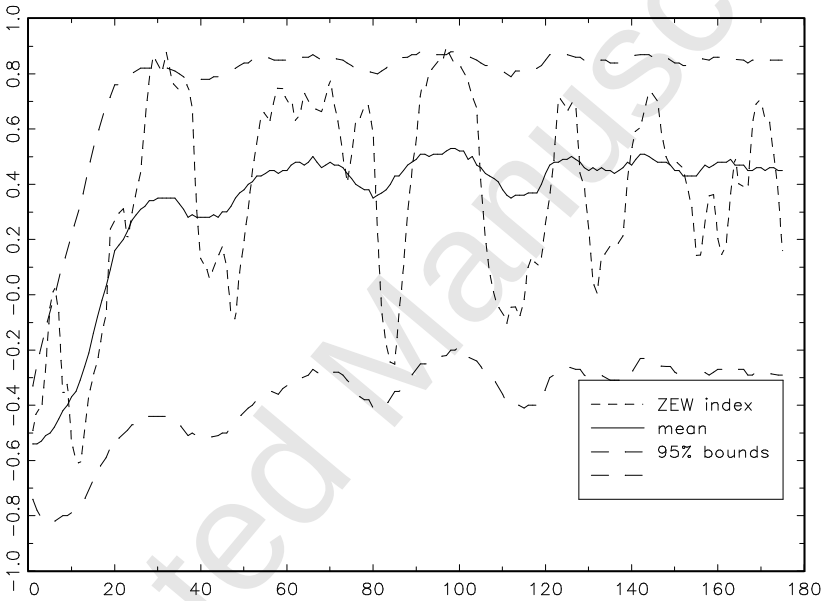

Figure 4: Mean and 95 percent confidence interval for Model 3 (from Fokker-Planck Equation conditional on initial observations and external macroeconomic information)

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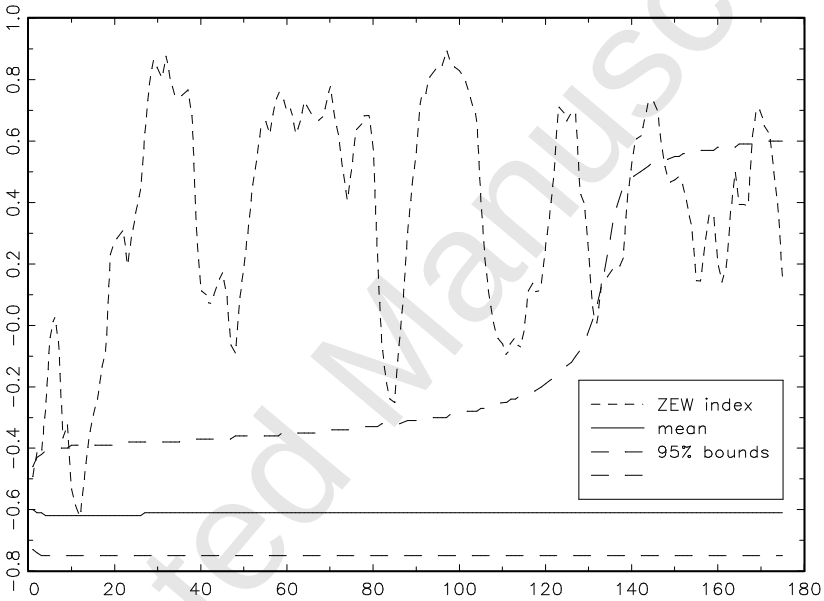

Figure 5: Mean and 95 percent confidence interval from Model 1 (from Fokker-Planck Equation conditional on initial observations and external macroeconomic information.)

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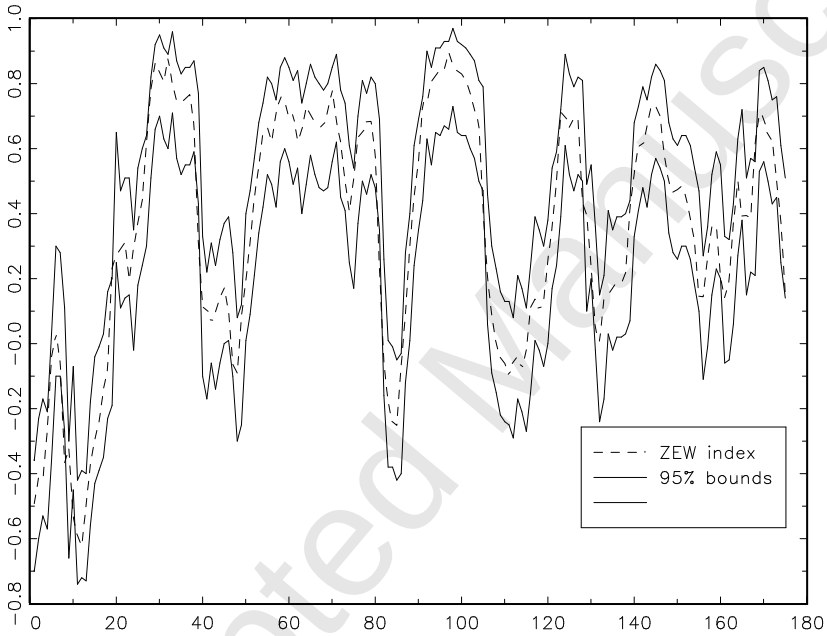

Figure 6: 95 Percent interval for one-step iterations of transient density of Model 5.

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