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# Performance of time-marching techniques dedicated to nonsmooth systems: a nonlinear modal analysis approach

Mathias Legrand Denis Laxalde

Laboratory of Structural Dynamics and Vibration

Department of Mechanical Engineering

McGill University, Montreal, QC, Canada

(+1) 514-398-5321

mathias.legrand@mcgill.ca denis.laxalde@mcgill.ca

## Context

Nonlinear modal analysis is slowly becoming a mature tool for improved investigations of flexible structures subject to vibration. Nonlinear modes [4] (NLM) may be defined as non-trivial periodic solutions of autonomous dynamical systems. They are invariant curved subspaces embedded in the original configuration space and share similar properties with their linear and flat counterparts such as eigenfrequencies.

The present study takes advantage of their properties to assess the performances of time-stepping techniques dedicated to flexible mechanical systems featuring unilateral contact conditions. In [3], an original numerical technique dedicated to such solutions was proposed; the system of equations being solved in the frequency-domain within an optimization algorithm while contact constraints are accounted for in the time-domain.

It is thought that the performances of time-marching techniques could be analyzed through their capabilities to find such periodic orbits, as solutions to an initial value problem. Quantities such that numerical damping per cycle introduced by the solutions methods could be found. The proposed approach can be regarded as an extension of the usual techniques relying on the conservation of energies, which are invariants as well.

## Nonlinear modal analysis of unilateral contact problems

In the framework of infinitesimal perturbations, consider an elastic structure occupying an open bounded domain  $\Omega \in \mathbb{R}^n$  ( $n = 2, 3$ ) with Dirichlet and Signorini boundary conditions on  $\Gamma_d$  and  $\Gamma_c$ . Denoting by  $\mathbf{u} \in \mathbb{R}^n$ , the displacement field of a material point  $\mathbf{x} \in \Omega$ , the gap function  $\mathbf{g}(\mathbf{u}) = \mathbf{u}(\mathbf{x}) \cdot \mathbf{n} - g_0(\mathbf{x})$  is defined with respect to the outward normal  $\mathbf{n}$  to  $\Gamma_c$  and the contact pressure is  $\tau_n = \boldsymbol{\sigma} \cdot \mathbf{n}$ . The motion of the autonomous system is sought in the form of a Fourier series:

$$\mathbf{u}(t) = \sum_{n \in \mathbb{Z}} \hat{\mathbf{u}}_n e^{jn\omega t} \quad \text{with} \quad \hat{\mathbf{u}}_n = \frac{1}{T} \int_T \mathbf{u}(t) e^{-jn\omega t} dt \quad (1)$$

The nonlinear eigenvalue problem is then expressed in a strong form as:

$$-\operatorname{div} \boldsymbol{\sigma}(\hat{\mathbf{u}}_n) = (n\omega)^2 \rho \hat{\mathbf{u}}_n \quad \text{on} \quad \Omega \times \mathbb{Z} \quad (2a)$$

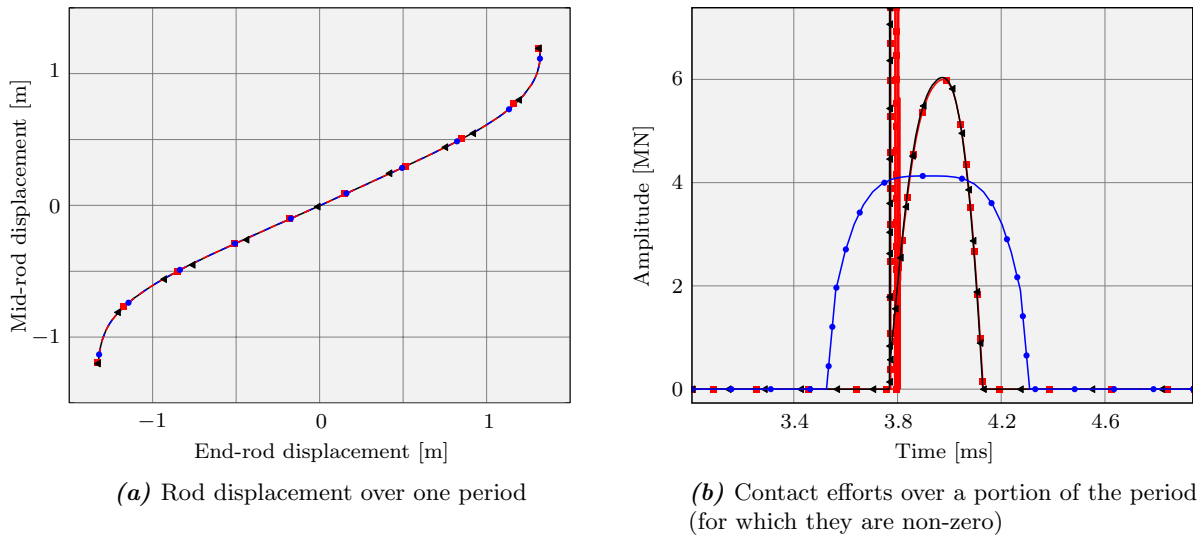
$$\hat{\mathbf{u}}_n = 0 \quad \text{on} \quad \Gamma_d \times \mathbb{Z} \quad (2b)$$

$$\mathbf{g}(\mathbf{u}) \leq 0, \quad \tau_n \geq 0 \quad \text{and} \quad \mathbf{g}(\mathbf{u}) \cdot \tau_n = 0 \quad \text{on} \quad \Gamma_c \times [0, T] \quad (2c)$$

A weak formulation of (2) can be derived as a constrained minimization problem. The respective equations are then solved through an augmented Lagrangian strategy and result in the construction of the previously mentioned modal invariants [3].

## Applications

A purely elastic rod potentially contacting a rigid foundation is considered. The calculated first nonlinear mode provides the proper initial conditions from which the system should remain on the respective invariant manifold. These initial conditions are used within two opposite time-stepping methods, namely a second order explicit technique [1] and a first order implicit procedure [2]. Results are displayed in Fig. 1. The three methods agree very well for the displacement unknowns. The sensitivity to contact is reflected on the curved relation between relative displacements within the rod. The respective predicted contact forces feature very similar amplitudes even though the starting and ending instants are quite distinct between time-marching and frequency-domain approaches.



**Figure 1.** Comparison of the three numerical approaches: implicit [■—■], explicit [◄—◄] and NLM [●—●]

## Conclusions

The present contribution is limited to the introduction of a new methodology targeting the numerical analysis of time-marching approaches dedicated to nonsmooth systems. First results are promising. More challenging mechanical configurations involving several contacting nodes will be investigated. Focus will also be given in the development of modern time driven and event driven strategies. Such techniques could then be exploited to explore the dynamics of the contacting structures around invariant manifolds.

## References

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