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# $H_\infty$ controller design for variable speed wind turbines

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**Abstract:** In this paper, the application of a robust multivariable control system designed for a horizontal variable speed wind turbine is discussed. The control problem presented and the background of the design method is given. A  $H_\infty$  controller based of a linear model that reduces the mechanical fatigue of the turbine is designed. The alleviation of the mechanical loads could increase the lifespan and the quality of the produced energy. The  $H_\infty$  theory is used to synthesize controllers achieving robust performance and stabilization and it has the main advantage that it does not make any assumptions about the noise and external disturbances that affect the system and it minimizes the worst case estimation error. The performances obtained through this control method are discussed and presented by means of a set of simulations.

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## 1. INTRODUCTION

Wind energy has widely grown during the last decades and nowadays it is one of the most competitive forms of renewable energy. Wind turbines are large complex dynamically flexible structures that operate in turbulent and unpredictable environmental conditions where the efficiency and reliability highly depend upon a well designed control strategy.

The wind turbine studied in this paper is a horizontal variable speed wind turbine that functions in the above rated speed regime. This type of wind turbine is currently the most used technology, having the advantage that it can allow the rotor to operate at various speeds and therefore it can permit a more efficient capture of the wind energy with less stress in the turbine’s drive train during wind gusts [Bianchi et al. 2002]. The reader can find more wind turbine modelling techniques in [Cutululis et al. 2002] and also detailed explanations regarding the use and purpose of each type of model.

From a control point of view, the importance is not only on ensuring an optimal operation of the wind turbines, but also on load reduction and grid integration. In order to do this, the wind energy conversion systems, should be capable of providing good quality energy from a profoundly irregular primary source, which is the wind. The wind speed is modelled in the literature as a non-stationary random process, yielded by superimposing two components [Cutululis et al. 2002, Nichita et al. 2000, Nichita et al. 2002]

$$v(t) = v_s(t) + v_t(t) \quad (1)$$

with

$$\dot{v}_t(t) = -\frac{1}{T_v} \cdot v_t(t) + m_v(t). \quad (2)$$

Where  $v_s(t)$  is the low frequency component. It describes the behaviour of the wind currents on long term, and it comprises

low frequency variations of the wind. The second component,  $v_t(t)$  is a turbulent component, that corresponds to the fast, high frequency variations, and  $m_v$  is a white Gaussian noise. Normally  $v_t$  is described by von Karman or Kaimal spectra, but a good approximation of these spectra allows modelling  $v_t$  through a linear equation superimposed in its turn with the white noise (2). These factors make control to play an increasing role in wind turbine regulation, by a continuous essay to significantly improve all aspects of a wind energy conversion system.

The characteristics of the wind energy source are important in different aspects regarding wind energy exploitation. The energy available in the wind varies with the cube of the wind speed. Also the value of the wind speed determines the functioning regime of the turbine. Two major functioning regimes were identified. The first one corresponds to the low wind speed operation and here the control goal is to maximize the energy extraction. When the “rated speed” is reached, the turbine enters into the second regime. Typical values for the rated speed of the wind are 12-14m/s.

In the above rated region, the pitch angle and the electromagnetic torque are the control variables usually used to reduce the structural load and to maintain the output power around the nominal value imposed by the generator. The classical control scheme of the system can be seen in Fig.1.

We have focused our attention on this regime, where the system is multivariable and multi-objective. The pitch angle control has a direct impact on the aerodynamic forces developed on the rotor. Consequently, inappropriate controllers may induce tower bending and vibrations [Xing-Jia et al. 2008].

Various control synthesis options have been applied in response to wind turbine control problem such as PID controllers, LQG controllers or fuzzy logic controllers. The classical control structures proved to be simple and robust but

most of the times they require the implementation of multiple control loops in order to accomplish multiple control objectives. Nevertheless, the modern state space methods can be designed not only to optimize power or to regulate the turbine's speed but also to add damping to its flexible modes through state feedback [Wright et al. 2004]. This paper presents the characteristics of a horizontal variable speed wind turbine, the state space model of the system and the design of a (sub) optimal  $H_\infty$  controller.

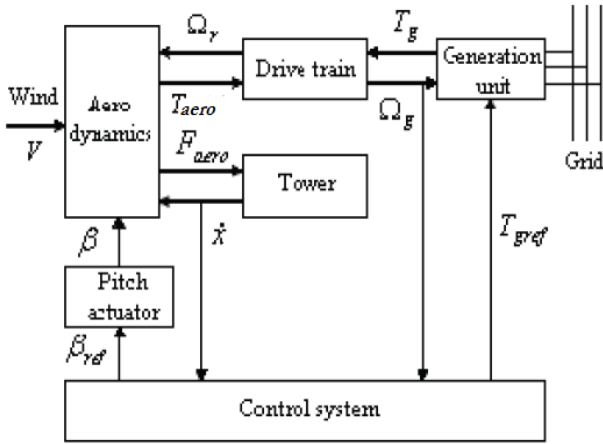


Fig.1 Wind energy conversion structure

In  $H_\infty$  control systems, the exact model is not required, and some uncertainty is allowed. The simulations show that the proposed control method is effective and highly robust.

## 2. WIND TURBINE MATHEMATICAL MODEL

Generally, the model of a wind energy conversion system can be structured as several interconnected subsystem models: an aerodynamic, a mechanical, electrical and actuator subsystems. But since the dominant dynamics lie in the mechanical subsystem, special attention will be paid in this direction.

The mechanical structure that we chose to study is seen as being arranged into several rigid bodies linked by flexible joints. The amount of these joints or degrees of freedom, determines the order of the model. In [Lupu et al. 2006] [Vanegas et al. 2008] and [Wright et al. 2004] one can observe the way in which the number of degrees of freedom of the system can increase the order of the non linear models of the turbine.

Therefore, it is important to consider on the model just those degrees of freedom that are directly coupled to the control [Bianchi et al. 2002]. By this reason, the model presented here, will include just the first mode of the drive train, the first mode of tower bending dynamics, and the first mode of the flapping of the blades. These degrees of freedom will suffice for the controller design that will be presented (Fig. 2). The drive train is modelled as a two rigid bodies linked by a flexible shaft (Fig. 3).

Also it was supposed that the two blades move in unison and support the same forces. In order to compute the model, we have started from a theory that states that a mechanical

system of arbitrary complexity can be described by the equation of motion

$$M \cdot \ddot{q} + C \cdot \dot{q} + K \cdot q = Q(q, \dot{q}, t, u) \quad (3)$$

where  $M$ ,  $C$  and  $K$ , are the mass, damping and the stiffness matrices,  $Q$  is the vector of forces acting on the system, and  $q_i$  is the generalized coordinate. For our model, the generalized coordinates are

$q = (\omega_T, \omega_G, \zeta_1, \zeta_2, y_T)$ , where  $\omega_T$  is the angular speed of the rotor,  $\omega_G$  stands for the angular speed of the generator,  $\zeta_1$  and  $\zeta_2$  are the flaps of the blades, while  $y_T$  represents the horizontal movement of the tower (Fig. 2).

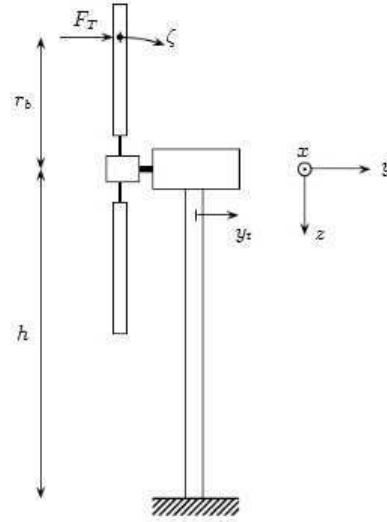


Fig. 2 The mechanical structure of the wind turbine

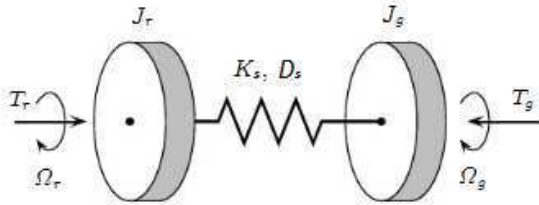


Fig. 3 The two mass model representation of the drive train

We considered that the thrust forces acting on each blade are equal  $F_{aero1} = F_{aero2} = F_{aero}$ , and that they deform the blades in an identical way, therefore we can write  $\zeta_1 = \zeta_2 = \zeta$ , which transformed  $q$  into  $q = (\omega_T, \omega_G, \zeta, y_T)$ . In the same time, one can find  $Q$  as being:

$$Q = (C_{aero}, -C_{em}, F_{aero}, 2 \cdot F_{aero}) \quad (4)$$

The considered forces that are acting on the system are:  $C_{aero}$ , the aerodynamic torque,  $C_{em}$ , the electromagnetic torque, and  $F_{aero}$ , representing the thrust. The thrust force acting on the entire rotor and the power extracted are expressed in terms of non-dimensional thrust and power coefficients,  $C_P$  and  $C_T$  respectively, as follows

$$P_{aero} = \frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot C_P(\lambda, \beta) \cdot v^3 \quad (5)$$

$$F_{aero} = \frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot C_T(\lambda, \beta) \cdot v^2$$

where  $\rho$  represents the air density,  $R$  is the blade radius, and  $v$  is the average speed of the wind. In the simulations that we did, we have considered the speed of the wind as a state variable in which the slow variant component gives the operating points while the turbulent component is seen as a disturbance that can determine the system to have variations around the operating points.

The power coefficient is one of the most important parameters of the wind turbine because it offers information upon the efficiency of the turbine, it helps defining the control objectives in the below rated regime and also it characterizes the aerodynamic torque that moves the turbine's rotor. The power coefficient and also the thrust coefficient can be expressed in a polynomial form, and depend on two parameters, namely the tip speed ratio  $\lambda$  and the pitch angle  $\beta$  of the blades.

In order to derive the mathematical model, one has used the Lagrange equation that offers a systematic procedure to calculate such models.

$$\frac{d}{dt} \left( \frac{\partial E_c}{\partial \dot{q}_i} \right) - \frac{\partial E_c}{\partial q_i} + \frac{\partial E_d}{\partial \dot{q}_i} + \frac{\partial E_p}{\partial q_i} = Q \quad (6)$$

Here,  $E_c$ ,  $E_d$ , and  $E_p$  denote the kinetic, dissipated and potential energies.

After a few calculations, applied for our system, one obtains:

$$\begin{aligned} E_c &= \frac{J_T}{2} \cdot \dot{\omega}_T^2 + \frac{J_G}{2} \cdot \dot{\omega}_G^2 + \frac{M_T}{2} \cdot \dot{y}_T^2 + M_P \cdot (\dot{y}_T + r_P \cdot \dot{\zeta})^2 \\ E_D &= \frac{d_A}{2} \cdot (\omega_T - \omega_G)^2 + d_P \cdot (r_P \cdot \dot{\zeta})^2 + \frac{d_T}{2} \cdot \dot{y}_T^2 \\ E_P &= \frac{k_A}{2} \cdot (\theta_T - \theta_G)^2 + k_P \cdot (r_P \cdot \zeta)^2 + \frac{k_T}{2} \cdot y_T^2 \end{aligned} \quad (7)$$

These energies were calculated under the supposition that the generalized force that acts on the rotor is applied on a point situated at the distance  $r_P$  on each blade from the hub of the rotor (Fig. 2).

In the above equations,  $J_T$  and  $J_G$  represent the rotor and the generator moments of inertia,  $M_T$  and  $M_P$  are the masses of the tower and of the blade,  $d_P$ ,  $d_A$  and  $d_T$  represent the damping coefficients for the blade, drive shaft and tower. Similarly,  $k_P$ ,  $k_A$  and  $k_T$  stand for the spring coefficients of the blade, drive shaft and tower.  $\theta_T$  and  $\theta_G$  are the angular positions of the rotor and generator.

The interconnection of the models of different plant subsystems, leads to a global highly non linear system, mainly because of the expressions of the aerodynamic torque  $C_{aero}$ , whose mathematical expression is given by:

$$C_{aero} = \frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot C_p(\lambda, \beta) \cdot \frac{v^3}{\omega_T} \quad (8)$$

and also due to the nonlinearity of the thrust force  $F_{aero}$ .

For control design purposes, we linearized the model around a operating point  $S_{op}$ :

$$C_{aero} = D_{c\omega} \cdot \omega_T + D_{c\beta} \cdot \beta + D_{cv} \cdot v \quad (9)$$

$$F_{aero} = D_{f\omega} \cdot \omega_T + D_{f\beta} \cdot \beta + D_{fv} \cdot v$$

Here

$$D_{c\omega} = \left( \frac{\partial C_{aero}}{\partial \omega_T} \right)_{Sop}, D_{c\beta} = \left( \frac{\partial C_{aero}}{\partial \beta} \right)_{Sop}, D_{cv} = \left( \frac{\partial C_{aero}}{\partial v} \right)_{Sop}$$

and

$$D_{f\omega} = \left( \frac{\partial F_{aero}}{\partial \omega_T} \right)_{Sop}, D_{f\beta} = \left( \frac{\partial F_{aero}}{\partial \beta} \right)_{Sop}, D_{fv} = \left( \frac{\partial F_{aero}}{\partial v} \right)_{Sop}.$$

Besides these equations, in order to interconnect the models of the individual subsystems, one must include into the model, the pitch controller. This was modelled here as a first degree order system [Lescher et al. 2005]

$$\frac{\beta}{\beta_{ref}} = \frac{1}{1 + T_\beta \cdot s}, \text{ where } \beta_{ref} \text{ is the desired pitch angle and } \beta$$

is the actual pitch angle of the blades.

We have taken into consideration the fact that the pitch servomotor has some physical limitations, and we have modelled them by including into our model one saturation in the position and one in the speed. For this study we have supposed that the saturation values in position are  $-45^\circ$  and  $45^\circ$ , and that the servomotor does not exceed the speed of  $10^\circ/s$ . In Fig. 4 one can observe the way the pitch servomotor's dynamics were modelled.

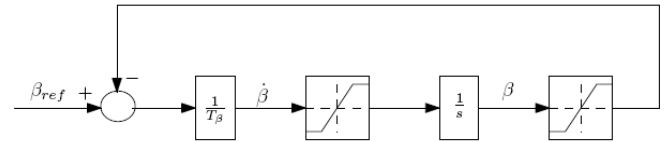


Fig. 4 The pitch servomotor dynamics modelling

After combining all these equations, one can put (6) and (1) into the classical state-space representation:

$$\dot{x}(t) = A \cdot x(t) + B \cdot u(t) + E \cdot m_v \quad (10)$$

$$y(t) = C \cdot x(t) + D \cdot u(t)$$

In which

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -51.4 & 0 & 0 & -0.69 & 0.28 & 0 & 0 & -2 & 0.04 \\ 268.29 & 0 & 0 & 1.46 & -1.46 & 0 & 0 & 0 & 0 \\ 0 & -390.47 & 30.35 & 0.15 & 0 & -3.9 & 0.17 & -0.13 & 0.04 \\ 0 & 457.14 & -242.85 & -0.21 & 0 & 4.57 & -1.42 & -0.21 & 0.07 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5.55 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.14 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -2.44 \cdot 10^{-5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.55 & 0 \end{pmatrix}^T$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, D = \begin{pmatrix} 0 & 0 & 10^{-5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$E = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

The system is multivariable; there have been identified three inputs and four outputs (Fig. 5). In the simulations, the turbulent component of the wind was considered as a white noise passed through a Nichita filter [Nichita et al. 2002] which has the transfer function

$$H_F(s) = K_F \cdot \frac{m_1 \cdot T_F \cdot s + 1}{(T_F \cdot s + 1) \cdot (m_2 \cdot T_F \cdot s + 1)}, \quad \text{with } T_F = 11/36,$$

$K_F = 4.11$ ,  $m_1 = 0.4$ , and  $m_2 = 0.25$ . Further details about the wind model can be found in [Munteanu et al. 2007].

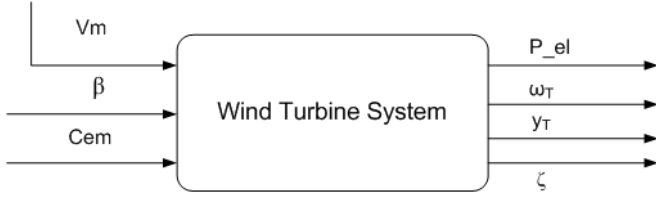


Fig. 5 The block scheme of the controlled system

As shown in Fig. 5, the input variables of the system are considered:  $v_m$  the average value of the wind speed, and the two control variables: the pitch angle,  $\beta$ , and the electromagnetic torque  $C_{em}$ .

Here, we have considered the state vector  $x^T = (\theta_T - \theta_G, \zeta, y_T, \omega_T, \omega_G, \dot{\zeta}, y_T, \beta, v)^T$ , the output of the system  $y = (P_{el}, \omega_T, \zeta, y_T)$ , and the command signal  $u = (\beta, C_{em})$ . The first component of the output vector represents the electrical power generated by the turbine. It can be computed as  $P_{el} = \omega_G \cdot C_{em}$  but in this paper, its normalized value was used.

The other output variables that we are interested in are:  $\omega_T$  because the goal is to try to maintain it constant to its nominal value, no matter the changes that appear in the environment, the flap mode of the blades and of the tower  $\zeta$  and  $y_T$  respectively, because, it is desired that these variables be as small as possible.

The two available control variables are the pitch angle and the electromagnetic torque. The numeric values of the wind turbine's parameters can be found in the APPENDIX, at the end of the paper.

### 3. DESIGN OF THE $H_\infty$ CONTROLLER

As previously said, many control techniques were proposed along the time for different models of wind turbines, depending on the objectives set.

The idea of conveniently sizing a trade-off between energy efficiency and increasing the lifetime of the wind turbines by alleviating fatigue loads is continuously being paid special attention, even when employing controllers like PI or PID. However, these approaches do not allow a rigorous control design in order to perform a fine tuning of the trade-off between the energy performance and the reliability demands [Munteanu et al. 2007].

The LQG regulators proved superior performances in ensuring this objective.

Nevertheless, the Kalman theory supposes that the plant has a linear description, that the exogenous noises and disturbances impinging on the feedback system are stochastic in nature and have known statistical properties. Reality shows that accurate system models are not readily available for industrial problems and in addition, engineers rarely understand the statistical nature of the noise processes that act on the industrial processes [Simon 2006].

Therefore, the  $H_\infty$  theory comes to solve this problem by offering a simple synthesis procedure that explicitly addresses questions of modelling errors. The  $H_\infty$  controller can work with deterministic systems and it is robust with changes in the model parameters. It minimizes the  $H_\infty$  norm of the closed-loop system transfer function and not the mean square error like in the case of an LQG control technique.

The idea of the  $H_\infty$  is the minimisation of the effect of the noises and disturbances acting on the system, without making any assumption on them.

The solution proposed in this paper comprises a  $H_\infty$  state regulator and a  $H_\infty$  filter. The role of the filter is to minimize the worst state estimation error. Thus we obtain a  $H_\infty$  output feedback controller combining a state feedback and a filter in a similar way with the well known LQG controller. We will consider first the standard  $H_\infty$  control problem [Doyle et al. 1989]. The closed loop system is given in the picture below (Fig.6).

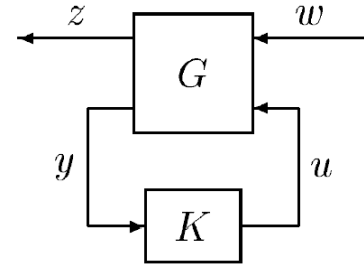


Fig. 6 The general representation of the closed loop system

There,  $G$  is the transfer function of the process and  $K$  is the  $H_\infty$  controller one tries to find. In its general form  $G$  can be written as

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \quad (11)$$

which corresponds to the following state space process model

$$\begin{aligned} \dot{x} &= A \cdot x + B_1 \cdot w + B_2 \cdot u \\ z &= C_1 \cdot x + D_{11} \cdot w + D_{12} \cdot u \\ y &= C_2 \cdot x + D_{21} \cdot w + D_{22} \cdot u \end{aligned} \quad (12)$$

By making an equivalence between (12) and (10), one can observe the fact that the variable  $m_v$  from (10) corresponds to the variable  $w$  from (12). The triple  $(A, B_2, C_2)$  has to be controllable and observable in order to guarantee the existence of a controller  $K$  that internally stabilizes the system. The feedback controller  $K(s)$  that is searched, has to make the system internally stable and minimize the  $H_\infty$  norm of the transfer function from the perturbation  $w$  to the error signal  $z$

$$\min_{K \text{ stabilizing}} \|T_{zw}(s)\|_\infty = \gamma_0$$

where  $T_{zw}$  is the closed loop transfer function from  $w$  to  $z$  given by

$$T_{zw} = G_{11} + G_{12} \cdot K \cdot (I - G_{22} \cdot K)^{-1} \cdot G_{21} \quad (13)$$

Therefore, the  $H_\infty$  controller will have to minimize as much as possible the influence of the disturbances and noises on the output of the system.

In practice, usually one can only get a sub-optimal solution that makes  $\min_{K_{stabilizing}} \|T_{zw}(s)\|_\infty < \gamma$  [Chen et al. 2002].

In [K. Zhou et al. 1995] it has been shown that the standard  $H_\infty$  controller can be obtained by connecting a specific state estimator and a state feedback under the worst system disturbances. However, the state estimate used here is not the  $H_\infty$  optimal state estimate of the process.

In this paper we apply an alternative approach for  $H_\infty$  output feedback control using a serial connection of  $H_\infty$  filter and  $H_\infty$  state regulator, which makes possible to obtain a  $H_\infty$  optimal estimate of the process state.

For  $z = Cx$ , the  $H_\infty$  state regulator has the form  $u = -K_c \cdot x$ ,  $K_c = B^T X$ , where  $X$  is the nonnegative definite solution or the Riccati equation

$$A^T \cdot X + X \cdot A + X \cdot (E \cdot E^T / \gamma^2 - B \cdot B^T) \cdot X + C^T \cdot C = 0. \quad (14)$$

According to the bounded real lemma, such a nonnegative definite solution  $X$  exists if the condition  $\min_{K_{stabilizing}} \|T_{zw}(s)\|_\infty < \gamma$  is satisfied.

Assuming that  $y(t) = C \cdot x(t) + D \cdot u(t) + \eta$  where  $\eta$  is the measurement noise, the  $H_\infty$  filter is defined as

$$\dot{\hat{x}} = A \cdot \hat{x} + B \cdot u + K_f \cdot (y - C \cdot \hat{x} - D \cdot u), \quad K_f = Y \cdot C$$

Where  $Y$  is the nonnegative definite solution of the Riccati equation

$$A \cdot Y + Y \cdot A^T + Y \cdot (I / \gamma^2 - C^T \cdot C) \cdot Y + E \cdot E^T = 0. \quad (15)$$

#### 4. RESULTS

The simulations were done using MATLAB/SIMULINK software and the results proved good performances. The chosen operating point for the linearization of the system corresponds to the average value of the wind speed of 18 m/s.

In Fig. 7 one can see the control scheme that was used for the simulation.

The simulation was done considering that the system is perturbed by wind gusts represented by the variable  $w$  and also that the measured variable  $y$  is affected by some measurement noise  $\eta$ . First a  $H_\infty$  state regulator  $u = K_c \cdot x$  was computed for  $\gamma = 1$ . The system was put in the form given by equation (12) with  $z = Cx$ .

As the full state is not available, the feedback command is realized as  $u = K_c \cdot \hat{x}$ , where  $\hat{x}$  is the  $H_\infty$  estimate of  $x$ . The value chosen here for  $\gamma$  was 0.8. In turn, the weighting matrix

$Kr$  was chosen as  $Kr = -1.05 I$ , where  $I$  is the unity matrix of order 2.

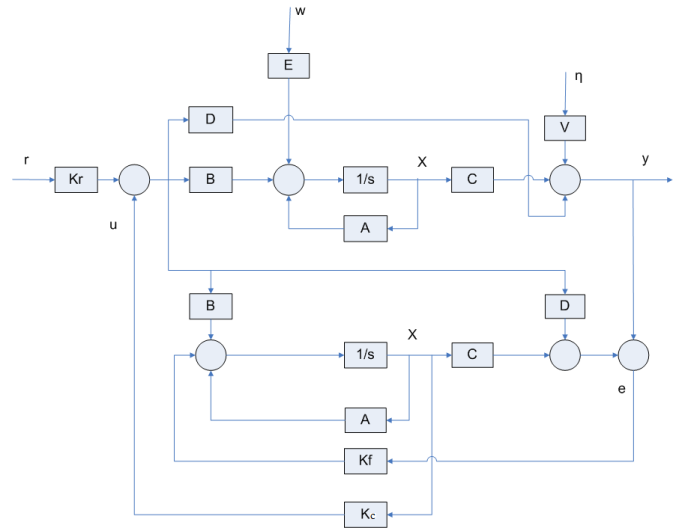


Fig. 7 The system controlled by  $H_\infty$  state regulator and filter. The simulation results are presented in the Fig 8-11 below.

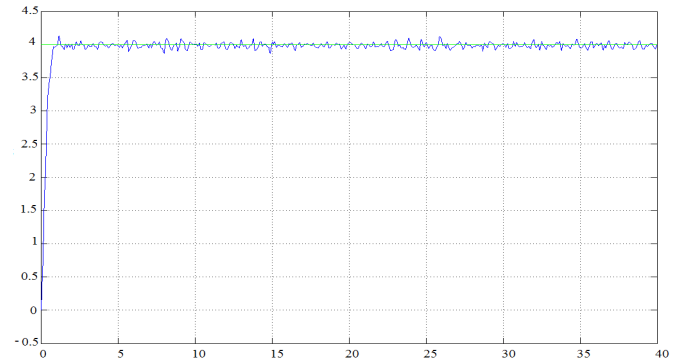


Fig. 8 The variation of the angular speed of the rotor

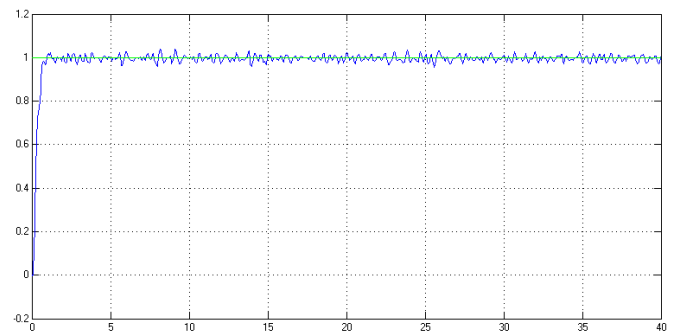


Fig. 9 The variation of the normalized electrical power

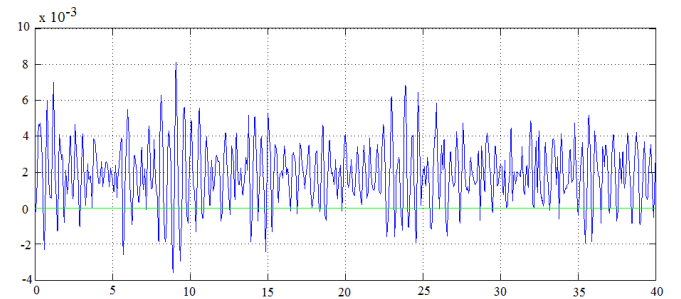


Fig. 10 The variation of the first flap mode of the blades

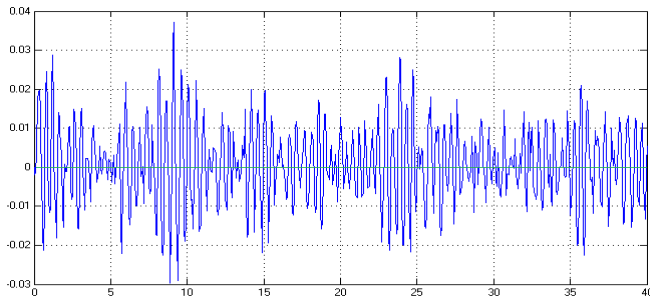


Fig. 11 The horizontal movement of the tower

One can observe from the pictures above that the angular speed of the rotor and the electrical power produced by the turbine follow their reference values, while the variables that we wanted to minimize have also very small values.

Namely, the blades suffer a deflection of maximum 8 mm which compared to the dimensions of the blades can be considered insignificant.

Also the tower's displacement does not exceed 4cm, this being again an acceptable value if one thinks at the dimensions of the tower.

The robustness of the system results from the  $H_\infty$  controller features, this control guaranteeing the asymptotic stability of the closed loop system as well as the attenuation of the exogenous disturbances.

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#### Appendix The values of the wind turbine's parameters

THE NUMERICAL VALUES OF THE WIND TURBINE PARAMETERS		
Symbol	Physical measure	Value
$J_t$	Turbine inertia	214 000 Kg * m <sup>2</sup>
$J_g$	Generator inertia	41 Kg * m <sup>2</sup>
$M_T$	Tower and nacelle mass	35000 kg
$M_p$	Blade mass	3000 kg
$k_p$	Blade Stiffness Coefficient	1000 Kg* m <sup>2</sup> /s <sup>2</sup>
$k_T$	Tower Stiffness Coefficient	8500 Kg* m/s <sup>2</sup>
$k_A$	Drive Shaft Stiffness Coefficient	11000 Kg* m <sup>2</sup> /s <sup>2</sup>
$d_p$	Blade Damping coefficient	10 000 Kg* m <sup>2</sup> /s
$d_T$	Tower Damping coefficient	50 000 Kg* m/s
$d_A$	Drive shaft damping coefficient	60 000 Kg* m <sup>2</sup> /s
$r_p$	Distance from the rotor hub	8 m
N	Number of blades	2
D	The rotor diameter	34 m
$P_n$	Nominal Power	400 kW
$\Omega_{nom}$	Nominal rotor speed	4 rad/s
h	Tower height	47 m