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UNLOCKING THE STANDARD MODEL

II. 1 GENERATION OF QUARKS . MASSES AND COUPLINGS

B. Machet ^{1 2}

Abstract: We continue investigating the Standard Model with two Higgs doublets for one generation of fermions advocated for in [1], using the one-to-one correspondence, demonstrated there, between their components and bilinear quark operators. We show that all masses and couplings, in particular those of the two Higgs bosons, are determined by low energy considerations. They definitely stay in the perturbative regime. The mass of the lightest Higgs (scalar singlet), very weakly coupled to all particles except u and d quarks, is $m_s \approx gf_\pi \frac{m_\pi}{2\sqrt{2}m_W} \approx 33 \text{ KeV}$. The “quasi-standard” one has mass $m_\xi \approx \sqrt{2}m_\pi$. Their dependence on m_π , that is on quark masses, gives an indication concerning their evolution when more generations are added.

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1 Introduction

The extension of the Standard Model [2] for one generation of fermions advocated for in [1] is endowed with two Higgs doublets

$$K = \begin{pmatrix} \mathfrak{p}^1 - i\mathfrak{p}^2 \\ -(\mathfrak{s}^0 + \mathfrak{p}^3) \end{pmatrix}, \quad \langle \mathfrak{s}^0 \rangle = \frac{v}{\sqrt{2}}, \quad (1)$$

and

$$H = \begin{pmatrix} \mathfrak{s}^1 - i\mathfrak{s}^2 \\ -(\mathfrak{p}^0 + \mathfrak{s}^3) \end{pmatrix}, \quad \langle \mathfrak{s}^3 \rangle = \frac{\sigma}{\sqrt{2}}. \quad (2)$$

K and H are both isomorphic to the Higgs doublet of the Standard Model [2]. \mathfrak{s}^0 and \mathfrak{s}^3 have non-vanishing vacuum expectation values (VEV's) as written in (2). In there, \mathfrak{s} and \mathfrak{p} stand respectively for “scalar” and “pseudoscalar”, such that H and K are parity transformed of each other. Their components that we call generically h^0, h^1, h^2, h^3 transform respectively by $SU(2)_L$ and $SU(2)_R$ according to

$$\begin{aligned} T_L^i \cdot h^j &= -\frac{1}{2} (i \epsilon_{ijk} h^k + \delta_{ij} h^0), \\ T_L^i \cdot h^0 &= -\frac{1}{2} h^i, \end{aligned} \quad (3)$$

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and

$$\begin{aligned} T_R^i \cdot h^j &= -\frac{1}{2} (i \epsilon_{ijk} h^k - \delta_{ij} h^0), \\ T_R^i \cdot h^0 &= +\frac{1}{2} h^i. \end{aligned} \quad (4)$$

The main steps of this work are the following. In section 2 we give the general formula for the mass of the \vec{W} gauge bosons in terms of the two VEV's $\langle \mathfrak{s}^0 \rangle$ and $\langle \mathfrak{s}^3 \rangle$. In section 3 we introduce Yukawa couplings of quarks to both Higgs doublets K and H . It could have looked more natural to first introduce the scalar potential, but it turns out that the latter gets strongly constrained by the former. After giving their general expression, from which we extract the u and d quark masses in terms of $\langle \mathfrak{s}^0 \rangle$ and $\langle \mathfrak{s}^3 \rangle$, we investigate in section 4 their low energy limit by using the one-to-one correspondence demonstrated in [1] between K , H and 4-sets of bilinear quark operators. There, renormalizability is not a concern and Yukawa couplings can be rewritten in a very simple form in which, in particular, symmetries clearly show up. Using the Partially Conserved Axial Current hypothesis (PCAC) [3] [4] [5] and the Gell-Mann-Oakes-Renner (GMOR) [6] [5] relation enables to account for the mass of the pions and to determine the values of all but one Yukawa parameters. The last one is obtained by identifying the Goldstones of the spontaneously broken weak $SU(2)_L$ symmetry. A last constraint results from considering the $\pi^0 - \eta$ system and requesting that it be devoid of any tachyonic state. This determines the quantity $(m_u - m_d) \langle \bar{u}u - \bar{d}d \rangle$ ($(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle$) is determined by the GMOR relation). After gathering the values of the parameters in section 5, section 6 is devoted to the scalar potential. $V(K, H)$ is chosen to be invariant by the chiral group $U(2)_L \times U(2)_R$, which clearly identifies the Goldstone of chiral symmetry breaking. It only depends on two parameters, one quadratic and one quartic coupling. At low energy, it receives corrections from the bosonised (low energy) form of the Yukawa couplings, which yields an effective potential $V_{eff}(K, H)$. A last constraint comes from minimizing V_{eff} at the known VEV's of the two Higgs bosons, which reproduce the pion and W masses. It determines the value of the quartic coupling and the masses of the two Higgs bosons. In section 7, we determine their couplings to quarks, gauge bosons and leptons. Section 8 provides some additional considerations concerning symmetries, Goldstone and pseudo-Goldstone bosons. Several symmetries are at work and some fields play dual roles. We focus in particular on the custodial $SU(2)$ symmetry and on the respective roles of $\langle \bar{u}u + \bar{d}d \rangle$ and $\langle \bar{u}u - \bar{d}d \rangle$. Section 9 gives some remarks concerning more generations. Section 10 is a brief conclusion.

2 Kinetic terms for the Higgs doublets and gauge boson masses

The masses of gauge bosons arise from the kinetic terms

$$\frac{1}{2} \left(D_\mu K (D^\mu K)^\dagger + D_\mu H (D^\mu H)^\dagger \right) \quad (5)$$

for the two Higgs doublets K and H . D_μ is the covariant derivative with respect to the group $SU(2)_L$ of weak interactions. Owing to the laws of transformations (3), the VEV's of \mathfrak{s}^0 and \mathfrak{s}^3 generate a mass m_W for the \vec{W} gauge bosons

$$m_W^2 = \frac{g^2}{4} \left(\langle \mathfrak{s}^0 \rangle^2 + \langle \mathfrak{s}^3 \rangle^2 \right) = g^2 \frac{v^2 + \sigma^2}{8}, \quad (6)$$

in which g is the $SU(2)_L$ coupling constant

$$g \approx .61. \quad (7)$$

3 Yukawa couplings

We choose to first introduce Yukawa couplings because their low-energy limit (see section 4) will in particular constrain the effective scalar potential.

3.1 General expression

Quarks must be coupled to the two Higgs doublets K and H . Introducing the couplings ρ_u and ρ_d to K and λ_u and λ_d to H , the Yukawa Lagrangian writes ¹

$$\begin{aligned} \mathcal{L}_{Yukawa} = & + \rho_d \left(\frac{\bar{u}_L \bar{d}_L}{\sqrt{2}} \right) K d_R - \rho_u \left(\frac{\bar{u}_L \bar{d}_L}{\sqrt{2}} \right) (i\tau^2 K^*) u_R \\ & + \lambda_d \left(\frac{\bar{u}_L \bar{d}_L}{\sqrt{2}} \right) H d_R + \lambda_u \left(\frac{\bar{u}_L \bar{d}_L}{\sqrt{2}} \right) (i\tau^2 H^*) u_R \\ & + h.c., \end{aligned} \quad (8)$$

which gives, explicitly,

$$\begin{aligned} \mathcal{L}_{Yukawa} = & - \left(\delta_1 \frac{v}{\sqrt{2}\mu^3} (\bar{u}u + \bar{d}d) + \kappa_{12} \frac{\sigma}{\sqrt{2}\nu^3} (\bar{u}u - \bar{d}d) \right) \mathfrak{s}^0 - \left(\delta_{12} \frac{v}{\sqrt{2}\mu^3} (\bar{u}u + \bar{d}d) + \delta_2 \frac{\sigma}{\sqrt{2}\nu^3} (\bar{u}u - \bar{d}d) \right) \mathfrak{s}^3 \\ & + \left(\delta_1 \frac{v}{\sqrt{2}\mu^3} (\bar{u}\gamma_5 d \mathfrak{p}^- + \bar{d}\gamma_5 u \mathfrak{p}^+ + (\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) \mathfrak{p}^3) + \kappa_{12} \frac{\sigma}{\sqrt{2}\nu^3} (\bar{d}u \mathfrak{p}^+ - \bar{u}d \mathfrak{p}^- + (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) \mathfrak{p}^3) \right) \\ & - \left(\delta_{12} \frac{v}{\sqrt{2}\mu^3} (\bar{d}\gamma_5 u \mathfrak{s}^+ - \bar{u}\gamma_5 d \mathfrak{s}^- - (\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) \mathfrak{p}^0) + \delta_2 \frac{\sigma}{\sqrt{2}\nu^3} (\bar{d}u \mathfrak{s}^+ + \bar{u}d \mathfrak{s}^- - (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) \mathfrak{p}^0) \right). \end{aligned} \quad (9)$$

In (8) and (9) the signs have been set such that for positive $\langle \mathfrak{s}^0 \rangle$ and $\langle \mathfrak{s}^3 \rangle$, the fermion masses are positive for positive $\rho_{u,d}$ and $\lambda_{u,d}$ (given that a fermion mass term is of the form $-m\bar{\psi}\psi$). We introduced in (9) the parameters with dimension $[mass]^2$

$$\begin{aligned} \delta_1 &= \frac{\rho_u + \rho_d}{2} \frac{\sqrt{2}\mu^3}{v}, \\ \kappa_{12} &= \frac{\rho_u - \rho_d}{2} \frac{\sqrt{2}\nu^3}{\sigma}, \\ \delta_{12} &= \frac{\lambda_u + \lambda_d}{2} \frac{\sqrt{2}\mu^3}{v}, \\ \delta_2 &= \frac{\lambda_u - \lambda_d}{2} \frac{\sqrt{2}\nu^3}{\sigma}. \end{aligned} \quad (10)$$

3.2 Fermion masses

We define the two quantum Higgs fields s and ξ by

$$\mathfrak{s}^0 = \langle \mathfrak{s}^0 \rangle + s, \quad \mathfrak{s}^3 = \langle \mathfrak{s}^3 \rangle + \xi. \quad (11)$$

The two VEV's (given in (1) and (2)) contribute to the fermion masses according to

$$m_u = \rho_u \langle \mathfrak{s}^0 \rangle + \lambda_u \langle \mathfrak{s}^3 \rangle = \frac{v\rho_u + \sigma\lambda_u}{\sqrt{2}}, \quad m_d = \rho_d \langle \mathfrak{s}^0 \rangle + \lambda_d \langle \mathfrak{s}^3 \rangle = \frac{v\rho_d + \sigma\lambda_d}{\sqrt{2}}. \quad (12)$$

4 The low energy limit

At low energy we use the one-to-one correspondence between K, H and

¹ τ^2 is the Pauli matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. The doublets $\tilde{K} \equiv i\tau^2 K^*$ and $\tilde{H} \equiv i\tau^2 H^*$ are isomorphic to K and H .

$$\begin{aligned}
\mathfrak{K} &= \frac{1}{\sqrt{2}} \frac{v}{\mu^3} \begin{pmatrix} \phi^1 - i\phi^2 \\ -(\phi^0 + \phi^3) \end{pmatrix} = \frac{v\sqrt{2}}{\mu^3} \begin{pmatrix} \bar{d}\gamma_5 u \\ -\frac{1}{2}(\bar{u}u + \bar{d}d) - \frac{1}{2}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) \end{pmatrix} \equiv \begin{pmatrix} \mathfrak{k}^1 - i\mathfrak{k}^2 \\ -(\mathfrak{k}^0 + \mathfrak{k}^3) \end{pmatrix}, \\
\langle \bar{u}u + \bar{d}d \rangle &= \mu^3, \\
\mathfrak{H} &= \frac{1}{\sqrt{2}} \frac{\sigma}{\nu^3} \begin{pmatrix} \xi^1 - i\xi^2 \\ -(\xi^0 + \xi^3) \end{pmatrix} = \frac{\sigma\sqrt{2}}{\nu^3} \begin{pmatrix} \bar{d}u \\ -\frac{1}{2}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) - \frac{1}{2}(\bar{u}u - \bar{d}d) \end{pmatrix} \equiv \begin{pmatrix} \mathfrak{h}^1 - i\mathfrak{h}^2 \\ -(\mathfrak{h}^0 + \mathfrak{h}^3) \end{pmatrix}, \\
\langle \bar{u}u - \bar{d}d \rangle &= \nu^3,
\end{aligned} \tag{13}$$

that has been established in [1] and identify accordingly

$$(\mathfrak{s}^0, \mathfrak{p}^1, \mathfrak{p}^2, \mathfrak{p}^3) \simeq (\mathfrak{k}^0, \mathfrak{k}^1, \mathfrak{k}^2, \mathfrak{k}^3), \quad (\mathfrak{p}^0, \mathfrak{p}^1, \mathfrak{p}^2, \mathfrak{p}^3) \simeq (\mathfrak{h}^0, \mathfrak{h}^1, \mathfrak{h}^2, \mathfrak{h}^3). \tag{14}$$

4.1 Rewriting Yukawa couplings

The first consequence of this correspondence is that, defining

$$m_{12}^2 = \kappa_{12} + \delta_{12}, \tag{15}$$

the Yukawa couplings can be rewritten as

$$\mathcal{L}_{Yukawa} = -\delta_1 K^\dagger K + \frac{1}{2} m_{12}^2 (K^\dagger H + H^\dagger K) - \delta_2 H^\dagger H, \tag{16}$$

or, indifferently, since renormalizability is not an issue at low energy, as a sum of 4-fermion interactions

$$\mathcal{L}_{Yukawa} = -\delta_1 \mathfrak{K}^\dagger \mathfrak{K} + \frac{1}{2} m_{12}^2 (\mathfrak{K}^\dagger \mathfrak{H} + \mathfrak{H}^\dagger \mathfrak{K}) - \delta_2 \mathfrak{H}^\dagger \mathfrak{H}. \tag{17}$$

4.2 PCAC and the Gell-Mann-Oakes-Renner relation

Owing to the Partially Conserved Axial Current (PCAC) hypothesis [3][4]

$$i(m_u + m_d) \bar{u}\gamma_5 d = \sqrt{2} f_\pi m_\pi^2 \pi^+, \tag{18}$$

which identifies the interpolating pion field with a bilinear quark operator and to the Gell-Mann-Oakes-Renner relation [6]

$$(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle = 2f_\pi^2 m_\pi^2, \tag{19}$$

one gets

$$\mathfrak{p}^\pm \simeq -\frac{i\pi^\pm}{\sqrt{2}}, \tag{20}$$

such that the term proportional to δ_1 in (17) triggers in particular adequate pion mass terms if

$$v = \frac{f_\pi}{\sqrt{2}} \tag{21}$$

and

$$\rho_u + \rho_d = \frac{m_u + m_d}{4f_\pi}. \tag{22}$$

Going back to the definition of δ_1 in (10), one gets

$$\delta_1 = \frac{1}{2} m_\pi^2. \tag{23}$$

Since $f_\pi \ll m_W$, (21) plugged into (6) entails

$$\sigma \approx \frac{2\sqrt{2}m_W}{g}, \quad (24)$$

which shows that the \vec{W} 's get their mass essentially from the VEV of \mathfrak{s}^3 . The ratio of the VEV's of the two Higgs doublets comes out accordingly as

$$\tan \theta = \frac{\langle \mathfrak{s}^3 \rangle}{\langle \mathfrak{s}^0 \rangle} = \frac{\sigma/\sqrt{2}}{v/\sqrt{2}} \approx \frac{4m_W}{gf_\pi} \approx 5711. \quad (25)$$

Eqs. (12) and (24) then determine λ_u and λ_d

$$(\lambda_u + \lambda_d) = \frac{7g}{16m_W}(m_u + m_d), \quad (\lambda_u - \lambda_d) = \frac{g}{2m_W} \left((m_u - m_d) - \frac{f_\pi}{2}(\rho_u - \rho_d) \right), \quad (26)$$

that is

$$\lambda_u = g \frac{15m_u - m_d - 4f_\pi(\rho_u - \rho_d)}{32m_W}, \quad \lambda_d = g \frac{15m_d - m_u + 4f_\pi(\rho_u - \rho_d)}{32m_W}, \quad (27)$$

in terms of $\rho_u - \rho_d$ which is, at this point, still undetermined.

4.3 Goldstones and pseudo-Goldstones

4.3.1 The charged Goldstones of the broken $SU(2)_L$

Since $\langle \mathfrak{s}^3 \rangle$ provides most of the mass of the \vec{W} 's, the charged Goldstone bosons of the broken $SU(2)_L$ weak symmetry are, to a very good approximation, the excitations of \mathfrak{s}^3 by the generators T_L^+ and T_L^- , that is \mathfrak{s}^+ and $\mathfrak{s}^- \in H$.

However, the $SU(2)_L$ invariant Yukawa couplings that need to be introduced to provide fermions with ‘‘soft’’ masses also give, at low energy, among other couplings, a ‘‘soft’’ mass to \mathfrak{s}^+ and \mathfrak{s}^- through the term proportional to δ_2 . The situation for \mathfrak{s}^+ and \mathfrak{s}^- is different from that of the pions which can become pseudo-Goldstone bosons and stay as physical particles. The spontaneously broken $SU(2)_L$ symmetry requires true Goldstones, which can only go along with

$$\delta_2 = 0 \Leftrightarrow \lambda_u = \lambda_d = \frac{7g}{32m_W}(m_u + m_d). \quad (28)$$

Looking at (10), one could think that $\nu^3 \equiv \langle \bar{u}u - \bar{d}d \rangle = 0$ could be a solution to $\delta_2 = 0$. However, we shall see later in subsection 8.1 that $\langle \bar{u}u \rangle$ must be different from $\langle \bar{d}d \rangle$ as a trigger of both weak and custodial symmetry breaking.

By (26), (28) determines

$$\rho_u - \rho_d = \frac{2(m_u - m_d)}{f_\pi}, \quad (29)$$

and, combined with (22),

$$\rho_u = \frac{9m_u - 7m_d}{8f_\pi}, \quad \rho_d = \frac{9m_d - 7m_u}{8f_\pi}. \quad (30)$$

4.3.2 The $\mathfrak{p}^3 - \mathfrak{p}^0$ system

The $(\mathfrak{p}^3, \mathfrak{p}^0)$ or $(\mathfrak{k}^3, \mathfrak{h}^0)$ or, equivalently (π^0, η) system gets endowed by the Yukawa couplings with a mass matrix

$$\begin{pmatrix} 2\delta_1 & m_{12}^2 \\ m_{12}^2 & 2\delta_2 \end{pmatrix}. \quad (31)$$

However, since δ_2 has been fixed to zero in subsection 4.3.1, this system now exhibits a tachyonic state unless

$$m_{12}^2 = 0 \Leftrightarrow (\rho_u - \rho_d) \frac{\nu^3}{\sigma} = -(\lambda_u + \lambda_d) \frac{\mu^3}{v} \Leftrightarrow \frac{m_u - m_d}{m_u + m_d} = -\frac{7}{8} \frac{\mu^3}{\nu^3} \equiv -\frac{7}{8} \frac{\langle \bar{u}u + \bar{d}d \rangle}{\langle \bar{u}u - \bar{d}d \rangle}, \quad (32)$$

in which we have used (15).(10), (28), (30) and the definitions of μ^3 and ν^3 that were introduced in (13). Then, \mathbf{p}^0 is a true Goldstone and \mathbf{p}^3 keeps its mass m_π^2 . They do not mix. This fits the picture of \mathbf{p}^0 being the third Goldstone boson of the broken $SU(2)_L$ symmetry, and \mathbf{p}^3 being the neutral member of the triplet of pseudo-Goldstone bosons of the broken chiral symmetry $SU(2)_L \times SU(2)_R$ down to the diagonal $SU(2)$. Other considerations concerning symmetries will be given in section 8.

4.3.3 The unitary gauge. Leptonic decays of pions

In the unitary gauge the crossed couplings between the \vec{W} gauge bosons and the (derivative of the) $SU(2)_L$ Goldstone bosons $\mathbf{p}^0, \mathbf{s}^+, \mathbf{s}^-$ are canceled, which leaves untouched the similar couplings between \vec{W} and the three pions. Their proportionality to $v \propto f_\pi$ yield in particular leptonic decays of pions in agreement with the standard PCAC calculation.

5 Summary of the parameters

By low energy considerations, we have determined the following parameters, introduced in particular in (8) and (10):

$$\begin{aligned} \rho_u &= \frac{9m_u - 7m_d}{8f_\pi}, & \rho_d &= \frac{9m_d - 7m_u}{8f_\pi}, & \lambda_u &= \lambda_d = \frac{7g(m_u + m_d)}{32m_W}, \\ \delta_1 &= \frac{1}{2}m_\pi^2, & \kappa_{12} &= -\delta_{12} = -\frac{7gf_\pi m_\pi^2}{8m_W}, & \delta_2 &= 0, \\ (m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle &\stackrel{(19)}{=} 2f_\pi^2 m_\pi^2, & (m_u - m_d) \langle \bar{u}u - \bar{d}d \rangle &\stackrel{(32)}{=} -\frac{7}{4}f_\pi^2 m_\pi^2. \end{aligned} \quad (33)$$

These should be plugged into the renormalizable form (9) of the Yukawa Lagrangian. Note that, unlike its low energy avatar (16), it depends on κ_{12} and $\delta_{12} = -\kappa_{12}$, and not on $m_{12}^2 = 0$.

6 The scalar potential

6.1 A $U(2)_L \times U(2)_R$ invariant potential

We shall consider a quartic $U(2)_L \times U(2)_R$ invariant potential

$$V(K, H) = -\frac{m_H^2}{2} (K^\dagger K + H^\dagger H) + \frac{\lambda_H}{4} \left((K^\dagger K)^2 + (H^\dagger H)^2 \right), \quad (34)$$

which thus decomposes into two independent potentials, one for K and one for H .

This is possible because (see [1]) K and H are stable by both $SU(2)_L$ and $SU(2)_R$ and transform into each other by $U(1)_L$ and $U(1)_R$ (with the appropriate signs). This last symmetry dictates in particular the equality of the couplings (quadratic and quartic) for the two doublets.

$SU(2)_L$ breaking by $v \neq 0$ and $\sigma \neq 0$ generates three Goldstone bosons in each Higgs multiplet: $\vec{\mathbf{p}} \in K$, the pseudoscalar singlet \mathbf{p}^0 and the two charged scalars $\mathbf{s}^\pm \in H$. This also fits the scheme according to which $v \neq 0$ and $\sigma \neq 0$ spontaneously break the chiral $U(2)_L \times U(2)_R$ down to $U(1) \times U(1)_{em}$ (see [1]); there, too, six Goldstones are generated. The pseudoscalar triplet $\vec{\mathbf{p}} \in K$ gets a small mass from the $SU(2)_L$ invariant

Yukawa couplings while the pseudoscalar singlet p^0 and the two charged scalars $\mathfrak{s}^\pm \in H$ must be protected from this since they are also the three Goldstones to be eaten by the \vec{W} gauge bosons (see section 4). p^0 plays a double role in that is also the Goldstone of the breaking of $U(1)_L \times U(1)_R$ down to the diagonal $U(1)$, which, at the level of the algebra, is related to parity breaking.

$v \neq 0$ is associated with $\langle \bar{u}u + \bar{d}d \rangle \neq 0$, responsible for the breaking of $SU(2)_L \times SU(2)_R$ down to $SU(2)$ with the three pions as (pseudo)-Goldstone bosons, while $\sigma \neq 0$ is associated with $\langle \bar{u}u - \bar{d}d \rangle \neq 0$ which is also responsible for the breaking of the custodial $SU(2)$ into $U(1)_{em}$ and of the \vec{W} mass.

Our choice for the potential amounts to requesting that, in the absence of Yukawa couplings, all fields are Goldstones but for the two Higgs bosons.

In the most general potential for two Higgs doublets the following terms have accordingly been discarded:

- $(m^2 K^\dagger H + h.c.)$, with $m \in \mathbb{C}$ would mediate in particular transitions between scalars and pseudoscalars that should not occur classically;
- $\lambda_4(K^\dagger K)(K^\dagger H) + h.c.$, $\lambda_5(H^\dagger H)(K^\dagger H_2) + h.c.$ with $\lambda_4, \lambda_5 \in \mathbb{C}$ would also mediate unwanted classical transitions between scalars and pseudoscalars;
- $\lambda_3(K^\dagger H)^2 + h.c.$ with $\lambda_3 \in \mathbb{C}$ would in particular contribute to the mass of the neutral pion and not to that of the charged pions. Such a classical $\pi^+ - \pi^0$ mass difference which is not electromagnetic nor due to $m_u \neq m_d$ is unwelcome;
- $\lambda_1(K^\dagger K)(H^\dagger H)$, $\lambda_2(K^\dagger H)(H^\dagger K)$, with $\lambda_1, \lambda_2 \in \mathbb{R}$ would also spoil the Goldstone nature of the pions and η .

6.2 The low energy effective potential

At low energy, the renormalizable $V(K, H)$ is supplemented by $(-1) \times$ the bosonised form of the Yukawa Lagrangian (16). This yields the effective potential

$$\begin{aligned} V_{eff}(K, H) &= V(K, H) + \delta_1 K^\dagger K - \frac{1}{2} m_{12}^2 (K^\dagger H + H^\dagger K) + \delta_2 H^\dagger H \\ &= -\frac{m_H^2}{2} (K^\dagger K + H^\dagger H) + \frac{\lambda_H}{4} \left((K^\dagger K)^2 + (H^\dagger H)^2 \right) + \delta_1 K^\dagger K - \frac{1}{2} m_{12}^2 (K^\dagger H + H^\dagger K) + \delta_2 H^\dagger H. \end{aligned} \quad (35)$$

It is further simplified since we could show that $\delta_2 = 0$ and $m_{12}^2 = 0$ (see section 4) and V_{eff} accordingly reduces to

$$V_{eff}(K, H) = -\frac{m_H^2 - m_\pi^2}{2} K^\dagger K - \frac{m_H^2}{2} H^\dagger H + \frac{\lambda_H}{4} \left((K^\dagger K)^2 + (H^\dagger H)^2 \right). \quad (36)$$

Last, to suitably reproduce the $\vec{\pi}$ and \vec{W} masses, we know that it should have a minimum at values of v and σ given by (21) and (24). The two equations $\left. \frac{\partial V_{eff}}{\partial s^0} \right|_{\langle s^0 \rangle = \frac{f_\pi}{2}} = 0$ and $\left. \frac{\partial V_{eff}}{\partial s^3} \right|_{\langle s^3 \rangle = \frac{2m_W}{g}} = 0$ yield respectively $m_H^2 = \lambda_H \langle s^0 \rangle^2 + m_\pi^2$ and $m_H^2 = \lambda_H \langle s^3 \rangle^2$ such that

$$\lambda_H \approx \frac{g^2 m_\pi^2}{4m_W^2} \left(1 + \frac{g^2 f_\pi^2}{16m_W^2} \right), \quad (37)$$

which puts it definitely in the perturbative regime. It is because of the presence of m_π^2 that λ_H is different from zero. $m_\pi \neq 0$ keeps accordingly the theory away from instability.

6.3 The masses of the two Higgs bosons s and ξ

Since the effective scalar potential is now fully determined, one can calculate the masses m_s and m_ξ of the Higgs bosons and one gets

$$\begin{aligned} m_s &= v\sqrt{\lambda_H} \approx gf_\pi \frac{m_\pi}{2\sqrt{2}m_W}, \\ m_\xi &= \sigma\sqrt{\lambda_H} \approx \sqrt{2}m_\pi. \end{aligned} \quad (38)$$

The masses are small and justify *a posteriori* our low energy treatment of the scalar effective potential. The two Higgs bosons s and ξ do not mix.

6.3.1 The roles of m_W and m_π

In our rebuilding of Standard Model with only one generation, we find that the masses of the two Higgs bosons are both proportional to m_π and small. But they are not small in the same way. If m_π is replaced by the mass of some heavier bound state $m \leq 2\sqrt{2}m_W \equiv \sigma \approx 230 \text{ GeV}$, m_s will stay very small $m_s \leq gf_\pi \approx 56 \text{ MeV}$ while m_ξ will grow like m_{quark} . The presence of a very light Higgs boson looks in particular a robust feature. As far as the other one(s) is (are) concerned, larger masses can be expected.

If m_W is to control the mass of (one of) the Higgs boson(s), it can only be indirectly. For one generation, the Higgs masses are more under the control of chiral symmetry than of its weak subgroup: indeed, m_π and not m_W sets the scale of the Higgs boson masses: m_W only occurs in the denominator to damp one of the two. In particular, the masses of the two Higgs bosons vanish at the limit $m_\pi \rightarrow 0$, that is, by the GMOR relation (19), either when $\langle \bar{u}u + \bar{d}d \rangle \rightarrow 0$ or when $(m_u + m_d) \rightarrow 0$. Since we have also determined (see (33)) that $(m_u - m_d) \langle \bar{u}u - \bar{d}d \rangle$ vanishes with m_π , this limit corresponds either to $\langle \bar{u}u \rangle = 0 = \langle \bar{d}d \rangle$ or to $m_u = 0 = m_d$.

7 Couplings of the Higgs bosons

7.1 Couplings of Higgs bosons to quarks

Plugging the definition (11) into (9) yields the following couplings of the Higgs bosons s and ξ to quarks

$$\begin{aligned} & -s(\rho_u \bar{u}u + \rho_d \bar{d}d) - \xi(\lambda_u \bar{u}u + \lambda_d \bar{d}d) \\ = & -s \left(\delta_1 \frac{v}{\sqrt{2}\mu^3} (\bar{u}u + \bar{d}d) + \kappa_{12} \frac{\sigma}{\sqrt{2}\nu^3} (\bar{u}u - \bar{d}d) \right) - \xi \left(\delta_{12} \frac{v}{\sqrt{2}\mu^3} (\bar{u}u + \bar{d}d) + \delta_2 \frac{\sigma}{\sqrt{2}\nu^3} (\bar{u}u - \bar{d}d) \right) \end{aligned} \quad (39)$$

which, using the values (33) of the parameters, yields

$$\mathcal{L}_{Higgs-quarks} = -s \left(\frac{9m_u - 7m_d}{8f_\pi} \bar{u}u + \frac{9m_d - 7m_u}{8f_\pi} \bar{d}d \right) - \xi \frac{7g(m_u + m_d)}{32m_W} (\bar{u}u + \bar{d}d). \quad (40)$$

While the ξ Higgs boson couples to quarks in a quasi-standard way ($\propto gm_{quark}/m_W$), the s Higgs boson is more strongly coupled, though still ‘‘perturbatively’’ to light quarks u and d .

7.2 Couplings of Higgs bosons to gauge bosons

They arise from the kinetic terms (5). Using (21) and (24), one gets

$$\mathcal{L}_{HiggsWW} = \frac{gm_W}{2} W_\mu^2 \xi + \frac{g^2 f_\pi}{8} W_\mu^2 s. \quad (41)$$

ξ couples accordingly in a ‘‘standard’’ way $\simeq gm_W$ to two W 's while the coupling of s , $\mathcal{O}(g^2 f_\pi)$ is much smaller by a factor $\mathcal{O}(10^{-3})$.

7.3 Couplings of Higgs bosons to leptons

Yukawa couplings to leptons need introducing four parameters, ρ_e and ρ_ν for \mathfrak{s}^0 and the quantum Higgs s , λ_e and λ_ν for \mathfrak{s}^3 and the quantum Higgs ξ

$$\begin{aligned} \mathcal{L}_{Yuk-lept} &= \left((\rho_\nu \bar{\nu}\nu + \rho_e \bar{e}e) \mathfrak{s}^0 - (\lambda_\nu \bar{\nu}\nu + \lambda_e \bar{e}e) \mathfrak{s}^3 \right) \\ &+ \left(\frac{\rho_\nu + \rho_e}{2} (\bar{\nu}\gamma_5 e \mathfrak{p}^- + \bar{e}\gamma_5 \nu \mathfrak{p}^+ + (\bar{\nu}\gamma_5 \nu - \bar{e}\gamma_5 e) \mathfrak{p}^3) \right) + \frac{\rho_\nu - \rho_e}{2} \left((\bar{e}\nu \mathfrak{p}^+ - \bar{\nu}e \mathfrak{p}^- + (\bar{\nu}\gamma_5 \nu + \bar{e}\gamma_5 e) \mathfrak{p}^3) \right) \\ &- \left(\frac{\lambda_\nu + \lambda_e}{2} (\bar{e}\gamma_5 \nu \mathfrak{s}^+ - \bar{\nu}\gamma_5 e \mathfrak{s}^- - (\bar{\nu}\gamma_5 \nu - \bar{e}\gamma_5 e) \mathfrak{p}^0) \right) + \frac{\lambda_\nu - \lambda_e}{2} \left((\bar{e}\nu \mathfrak{s}^+ + \bar{\nu}e \mathfrak{s}^- - (\bar{\nu}\gamma_5 \nu + \bar{e}\gamma_5 e) \mathfrak{p}^0) \right). \end{aligned} \quad (42)$$

Using again (21) and (24) provides the lepton masses

$$m_e = \rho_e \frac{f_\pi}{2} + \lambda_e \frac{2m_W}{g}, \quad m_\nu = \rho_\nu \frac{f_\pi}{2} + \lambda_\nu \frac{2m_W}{g}. \quad (43)$$

7.3.1 The low energy limit

Let us use again the one-to-one correspondence between the components of the Higgs multiplets and bilinear quark operators (13). Using PCAC (18) and the Gell-Mann-Oakes-Renner relation (19), we could relate the charged pion fields π^\pm and the charged pseudoscalar components \mathfrak{p}^\pm of the Higgs doublet K by (20). Yukawa couplings (42) are then seen to trigger, among others, leptonic decays of charged pions. These come in addition to the ‘‘standard ones’’ obtained from the $W_\mu \partial^\mu \pi$ crossed couplings that originate from the kinetic terms (5) at low energy (see subsection 4.3.3) and which agree with PCAC usual calculations.

This means that, in a first approximation (and it is not the goal of this work to go beyond), we should take

$$\rho_\nu \approx 0 \approx \rho_e. \quad (44)$$

In case observed leptonic pion decay turn out to differ from PCAC estimates, the issue could be raised whether (44) should be revisited.

In relation with (43) the choice (44) leads to a standard coupling of the Higgs boson ξ to leptons, proportional to gm_{lepton}/m_W , while the ones of s vanish (or are extremely close to).

8 Symmetries again

8.1 The roles of $\langle \bar{u}u + \bar{d}d \rangle$ and $\langle \bar{u}u - \bar{d}d \rangle$

$\langle \bar{u}u + \bar{d}d \rangle \neq 0$ is the signal for what is commonly called ‘‘chiral symmetry breaking’’, the breaking of $SU(2)_L \times SU(2)_R$ down to the diagonal $SU(2)$. $\langle \bar{u}u - \bar{d}d \rangle \neq 0$ breaks $SU(2)_L$, and the custodial $SU(2)$ down to $U(1)_{em}$. Let us show that $\langle \bar{u}u \rangle$ cannot be equal to $\langle \bar{d}d \rangle$. Indeed, for $\nu^3 = 0$ one gets from the last line of (10) $\delta_2 = 0 = \kappa_{12}$. Then

$$m_{12}^2 = \delta_{12} = \frac{7gf_\pi m_\pi^2}{8m_W}, \quad (45)$$

in which we used the definition of δ_{12} in (10), the GMOR relation (19) and (26).

Performing the minimization of the effective potential $V_{eff}(K, H)$ while still supposing that $V(K, H)$ is $U(2)_L \times U(2)_R$ invariant gives the two equations

$$m_H^2 = \lambda_H \langle \mathfrak{s}^0 \rangle^2 + 2\delta_1 + \delta_{12} \frac{\langle \mathfrak{s}^3 \rangle}{\langle \mathfrak{s}^0 \rangle}, \quad m_H^2 = \lambda_H \langle \mathfrak{s}^3 \rangle^2 + \delta_{12} \frac{\langle \mathfrak{s}^0 \rangle}{\langle \mathfrak{s}^3 \rangle}, \quad (46)$$

which yield, since $\langle \mathfrak{s}^3 \rangle \gg \langle \mathfrak{s}^0 \rangle$ (see (21) and (24))

$$\lambda_H \approx \frac{2\delta_{12}}{\langle \mathfrak{s}^3 \rangle^2} + \frac{\delta_{12}}{\langle \mathfrak{s}^0 \rangle \langle \mathfrak{s}^3 \rangle} = \frac{9g^2 m_\pi^2}{8m_W^2}. \quad (47)$$

The mass matrix of the $\mathfrak{s}^0 - \mathfrak{s}^3$ system becomes then

$$\begin{pmatrix} \frac{\partial^2 V_{eff}}{(\partial \mathfrak{s}^0)^2} \equiv \frac{\lambda_H}{2} f_\pi^2 - \frac{4m_W}{g f_\pi} \delta_{12} & \frac{1}{2} \frac{\partial^2 V_{eff}}{\partial \mathfrak{s}^0 \partial \mathfrak{s}^3} \equiv \frac{1}{2} \delta_{12} \\ \frac{1}{2} \frac{\partial^2 V_{eff}}{\partial \mathfrak{s}^0 \partial \mathfrak{s}^3} \equiv \frac{1}{2} \delta_{12} & \frac{\partial^2 V_{eff}}{(\partial \mathfrak{s}^3)^2} \equiv 8\lambda_H \frac{m_W^2}{g^2} - \frac{g f_\pi}{4m_W} \delta_{12} \end{pmatrix} \approx \begin{pmatrix} -\frac{7}{2} m_\pi^2 & \frac{7g f_\pi m_\pi^2}{16m_W} \\ \frac{7g f_\pi m_\pi^2}{16m_W} & 9m_\pi^2 \end{pmatrix} \quad (48)$$

which exhibits a tachyonic s Higgs boson $m_s^2 \approx -\frac{7}{2} m_\pi^2$. The theory with $\langle \bar{u}u \rangle \neq \langle \bar{d}d \rangle$ is thus unstable.

Since we have everywhere supposed that the minimum of the effective potential fits the \vec{W} and $\vec{\pi}$ masses, we conclude that chiral and weak symmetry breakings as they are observed are only possible for $\langle \bar{u}u \rangle \neq \langle \bar{d}d \rangle$.

Unlike for the pions the masses of which are related to $\langle \bar{u}u + \bar{d}d \rangle$ by the GMOR relation (19), there is no such relation between m_W and $\langle \bar{u}u - \bar{d}d \rangle$ (see the last line of (33)). Moreover, even when $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ (that is, $\langle \nu^3 \rangle = 0$) $\langle \mathfrak{s}^3 \rangle$ can be equal to $\sigma/\sqrt{2}$ because, in its expression (13), ν^3 cancels between the numerator and the denominator. This is why it looks opportune to rather speak of $\langle \bar{u}u \rangle \neq \langle \bar{d}d \rangle$ as the *catalyst* of weak (and custodial) symmetry breaking.

8.2 The custodial $SU(2)$

While $(\bar{u}u + \bar{d}d)$ gets annihilated by all generators of $SU(2)$, $(\bar{u}u - \bar{d}d)$ does not, it only gets annihilated by $T^3 = Q$ (see [1]). So, $\langle \bar{u}u \rangle \neq \langle \bar{d}d \rangle$ spontaneously breaks the custodial $SU(2)$ down to $U(1)_{em}$. In this breaking one expects two Goldstones. They are the excitations by T^+ and T^- of the vacuum \mathfrak{s}^3 , that is the two scalars \mathfrak{s}^+ and \mathfrak{s}^- eaten by W^\pm , and which coincide with the two charged Goldstones of the spontaneously broken weak $SU(2)_L$.

The electroweak Lagrangian is invariant by the custodial $SU(2)$ as soon as the \vec{W} 's form an $SU(2)$ vector. But, in the broken phase, the W^3 can only eat \mathfrak{s}^0 which is a $SU(2)$ singlet. This is how the generation of the \vec{W} mass breaks the custodial symmetry.

8.3 Goldstones and pseudo-Goldstones

Three true Goldstones are eaten by the \vec{W} 's to get massive: they are \mathfrak{p}^0 , \mathfrak{s}^+ and \mathfrak{s}^- , belonging to the doublet H . \mathfrak{p}^0 is also the Goldstone of the $U(1)_L \times U(1)_R$ spontaneous breaking down to the diagonal $U(1)$. The three $\vec{\pi}$ (the three pions) are the pseudo-Goldstones of the broken $SU(2)_L \times SU(2)_R$ down to $SU(2)$.

The only non-Goldstones are ξ and s in the sense that, though their masses also vanish with m_π , they do not seem connected with the breaking of any continuous symmetry. The first could only be excited by acting either on \mathfrak{p}^0 with T_L^3 or T_R^3 , or on \mathfrak{p}^3 with \mathbb{I}_L or \mathbb{I}_R . However, in a first approximation, neither \mathfrak{p}^0 nor \mathfrak{p}^3 , being a pseudoscalar, has a non-vanishing VEV. Likewise, H could only be excited either by acting on \mathfrak{p}^3 with T_L^3 or T_R^3 , or on \mathfrak{p}^0 with \mathbb{I}_L or \mathbb{I}_R . The same argumentation rejects thus both as Goldstone bosons, unless some additional spontaneously broken continuous symmetry is at work, which is to be uncovered.

9 A few hints for more generations

Before concluding, it is worth pointing at a few features concerning the case of a larger number N of generations (some information can also be found in [7]). A more detailed study is postponed to [9].

There are features of this work which only belong to the case of one generation. For example the fact that the η pseudoscalar meson (pseudoscalar singlet) becomes the longitudinal neutral W^3 . In the case of more generations, it may happen that this role is still held by the singlet $\propto \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{c}\gamma_5 c + \bar{d}\gamma_5 s + \dots$, but it is no longer the η , or by another neutral combination. Though this can only be known by a precise study, it is likely that the η can then live again its life as a physical pseudoscalar meson.

Other features are certainly, at the opposite, robust, like the fact that there is a very light Higgs boson with mass $\leq gf_\pi \approx 56 \text{ MeV}$. Likewise, from the expression (37) for the quartic Higgs coupling λ_H , it seems reasonable to believe that, even if the mass of the pion gets replaced by the mass of a much heavier bound state, λ_H will stay smaller than 1 and thus “perturbative”. It can only get equal to 1 if m_π is replaced by $2m_W/g \approx 265 \text{ GeV}$, such that one should only be careful when the “top” generation is concerned, for which “non-perturbative” phenomena could appear.

The logic of the present work and of [1] is that all (pseudo)scalar doublets isomorphic to the one of the Standard Model of Glashow, Salam and Weinberg [2] should be incorporated. This would stay an empty or meaningless statement without noticing that the standard Higgs doublet has transformations by the chiral group (3) (4) that are identical to those of bilinear quark operators. For one generation, this doubled the number of possible doublets, with parity distinguishing the two of them. In the case of N generations, it was shown in [7] that there exists $2N^2$ such doublets, divided, by parity again, in two sets. Their $8N^2$ real components can be put in one-to-one relationship with the same number of scalar and pseudoscalar $J = 0$ mesons that occur for $2N$ flavors of quarks. The same logic as the one followed here requires accordingly that the Standard Model be then endowed with $2N^2$ complex Higgs doublets. Among these, one expects in particular as many Higgs fields as there exist quark-antiquark $\langle \bar{q}_i q_i \rangle$ condensates, that is, $2N$. Owing to the large number of parameters involved, it looks of course too optimistic to think that one can easily calculate all masses and couplings as we did here. This path stays nevertheless in our opinion the most natural to follow, the underlying guess being that the mystery of Higgs boson(s) simply lies inside the one of scalar (and eventually pseudoscalar) $J = 0$ mesons.

10 Conclusion and prospects

As we re-built it, the Standard Model for one generation of fermions is complete in the sense that all masses and couplings of all fields present in the Lagrangian and of all $J = 0$ pseudoscalar mesons are determined. Pions are accounted for with the correct decays and, of the four expected scalar mesons, two (the charged ones) become the longitudinal charged W^\pm while the last two are the Higgs bosons s and ξ . Both have small masses and are perturbatively coupled and self-coupled. While ξ is close to standard, s is extremely light and has peculiar properties that deserve a specific investigation concerning the role that it can hold in nature [8]. As far as we can see, this minimal extension of the Standard Model is different from what other authors have been considering; it is different as a 2-Higgs doublet model [10] [11] [12], and it is different in that, for a larger number of generations $N > 1$, it cannot stay as a 2-Higgs doublet model and should be endowed with $2N^2$ Higgs doublets. At this stage, no physics “beyond the Standard Model” is needed² but, since the one generation case can only be considered as a “toy Standard Model”, this is one among the features that should be carefully scrutinized for more generations of fermions [9].

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²The only hint in favor of it may be the vanishing of the masses of the two Higgs bosons at the chiral limit, which makes them look “like pseudo-Goldstone bosons” (see subsection 8.3).

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