HAL
open science

An MLPO algorithm for fast evaluation of the focal plane fields of reflector antennas

Christine Letrou, Amir Boag

To cite this version:

Christine Letrou, Amir Boag. An MLPO algorithm for fast evaluation of the focal plane fields of reflector antennas. EUCAP '12 : 6th European Conference on Antennas and Propagation, Mar 2012, Prague, Czech Republic. pp.3436-3437, 10.1109/EuCAP.2012.6206071 . hal-00718480

HAL Id: hal-00718480

https://hal.science/hal-00718480v1

Submitted on 17 Jul 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

An MLPO Algorithm for Fast Evaluation of the Focal Plane Fields of Reflector Antennas

Christine Letrou[1], Amir Boag[2]
[1] Lab. SAMOVAR (UMR CNRS 5157), Institut Telecom, TELECOM SudParis, Evry, France, christine.letrou@it-sudparis.eu
[2] School of Electrical Engineering, Tel Aviv University, Tel Aviv 69978, Israel, boag@eng.tau.ac.il

***Abstract*— A fast algorithm for the computation of focal fields of reflector antennas for a range of incidence angles and frequencies is proposed. The algorithm is based on the physical optics approximation. It uses a hierarchical domain decomposition of the reflector surface. For the smallest subdomains the received fields are computed over a coarse grid of angles, frequencies, and observation points. The fields for all incidence angles, frequencies, and focal plane points are obtained by a multilevel aggregation of the fields received by neighboring subdomains while employing interpolation and phase correction.**

Keywords- reflector antennas; physical optics; fast multilevel algorithms.

I. INTRODUCTION

Reflector antennas are widely used in a variety of communication, radar, and radio astronomical applications. Conventionally, the radiation pattern of a reflector antenna is computed for a given feed configuration in the transmit mode. On the other hand, design and optimization of complex feeds for reflector antennas operating in the receive mode often require evaluation of the focal fields for multiple directions of incidence and frequencies [1]. This issue is particularly important for large radio telescope antennas in the context of survey instruments requiring a wide field of view [2].

Radiation pattern evaluation in the transmit mode can be performed using a surface integral of equivalent currents, also referred to as a vector diffraction integral [3], assuming that these currents, related to the tangential electric and magnetic fields on the surface of the reflector, have already been computed. When the fields are computed by ray tracing in the local plane wave approximation this approach is often referred as the Physical Optics (PO). The PO contribution is often augmented by the Physical Theory of Diffraction (PTD) line integral, in order to improve the far sidelobe accuracy. In all cases, for problems with large electrical dimensions, the computation time is dominated by the (surface) PO integration time. Fast two-level and multilevel physical optics (MLPO) algorithms for radiation pattern evaluation have been developed in [4] and [5], respectively; and further extended to include the PTD contributions in [6]. In this work, we generalize the MLPO algorithm for the computation of the focal plane fields of reflectors operating in the receive mode.

II. PROBLEM FORMULATION

Consider a reflector antenna comprising a feed and a perfectly conducting reflector S depicted in Fig. 1. The antenna can be circumscribed by a sphere of radius R centered at the origin of the coordinate system. Harmonic time dependence $e^{j\omega t}$ is assumed and suppressed. We consider the task of computing the field over a relatively small region near the focal point F_0 for the antenna plane wave illumination with the incidence angles spanning a given angular sector and with various frequencies confined to a predefined band.

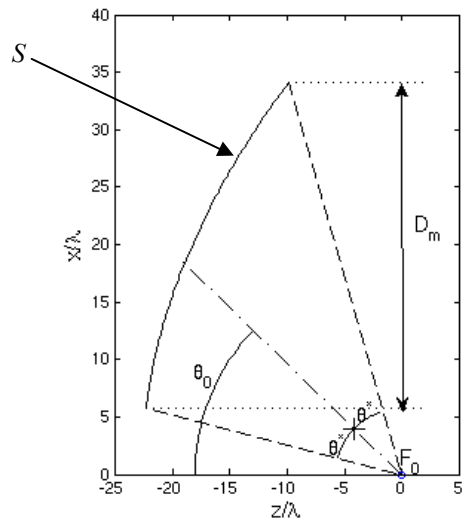

Fig. 1. Geometry of an offset reflector antenna (cut in the *xz*-plane).

In the PO approximation, the electric field at the observation point $\mathbf{r}^{\mathrm{f}}$ in the focal region due to a unit amplitude illuminating plane wave with a wavevector $\mathbf{k}$ and a unit polarization vector $\hat{\mathbf{p}}$ is given by

$$\mathbf{E}(\mathbf{r}^{\mathrm{f}} \mid \mathbf{k}, \hat{\mathbf{p}}) = \int_S \mathbf{A}(\mathbf{r}^{\mathrm{f}}, \mathbf{r} \mid \mathbf{k}, \hat{\mathbf{p}}) e^{-jk(|\mathbf{r}^{\mathrm{f}} - \mathbf{r}| + \hat{\mathbf{k}} \cdot \mathbf{r})} ds \qquad (1)$$

where $k = |\mathbf{k}|$ is the wavenumber, $\hat{\mathbf{k}} = \mathbf{k}/k$, and $\mathbf{r} \in S$ is a point on the reflector surface. Also in (1), $\mathbf{A}(\mathbf{r}^{\mathrm{f}}, \mathbf{r} \,|\, \mathbf{k}, \hat{\mathbf{p}})$ is a slowly varying vector function of the wavenumber (frequency and direction) and focal region coordinates. It comprises the effects of the polarization vector and the local normal to the surface.

We assume that the field in the focal plane needs to be evaluated over an $M \times M$ grid of points in the focal plane via the integral in (1). The numerical evaluation of this integral for a given direction of incidence and polarization requires summation of $O(N^2)$ terms, each of which involves computation of the integrand at a quadrature point. Here we consider $N = kR$ as a large parameter while M is relatively small. Complete characterization of an antenna requires the analysis of the receiving pattern for $O(N^2)$ directions covering the angular sector of interest, $O(N)$ frequencies spanning the relevant bandwidth, and two polarizations. Thus, straightforward evaluation of the antenna focal region fields leads to a staggering computational complexity of $c_1 M^2 N^5$, where c_1 is proportional to the computational cost of the integrand evaluation in (1). Such complexity makes the straightforward approach practically unfeasible, especially, if the design optimization requires repeated computations. To that end, in the next section we outline a fast algorithm designed to substantially reduce the computational complexity.

III. Multilevel Algorithm

Towards designing a fast algorithm, we can consider a domain decomposition of the reflector surface into N_s non-overlapping domains of roughly equal size, such that $S = \bigcup_{n=1}^{N_s} \bar{S}_n$. The exponential term in the integrand of (1) is the main reason for the rapid variation of the received fields that requires their dense sampling. Thus, we design a phase compensation factor for each subdomain by analyzing the phase behavior of the integrand, which comprises two terms: the near-field one involving the distance $r = |\mathbf{r}^{\mathrm{f}} - \mathbf{r}|$ and the far-field (plane wave) one - $\hat{\mathbf{k}} \cdot \mathbf{r}$. We define a phase compensated field reflected by the nth subdomain, $\bar{S}_n$, as

$$\tilde{\mathbf{E}}_n(\mathbf{r}^{\mathrm{f}} \,|\, \mathbf{k}, \hat{\mathbf{p}}) = \int_{\bar{S}_n} \mathbf{A}(\mathbf{r}^{\mathrm{f}}, \mathbf{r} \,|\, \mathbf{k}, \hat{\mathbf{p}}) e^{-jk[|\mathbf{r}^{\mathrm{f}} - \mathbf{r}| - \tilde{r} + \hat{\mathbf{k}} \cdot (\mathbf{r} - \bar{\mathbf{r}}_n)]} ds \qquad (2)$$

where $\bar{\mathbf{r}}_n$ denotes the center of the smallest sphere of radius $\bar{R}_n$ circumscribing $\bar{S}_n$. Also in (2), the near-field distance compensation is given by $\tilde{r} = \sqrt{r^2 + \bar{R}_n^2 / 2}$ as proposed in [7]. The phase compensated field is a slowly varying function of direction, frequency, and the focal region coordinates. Thus, it can be evaluated for each subdomain on a coarse grid of directions, frequencies and focal region points where the numbers of points depend on $\bar{N} = k\bar{R}$ with $\bar{R} = \max \bar{R}_n$.

The total field is then evaluated by summing the subdomain contributions

$$\mathbf{E}(\mathbf{r}^{\mathrm{f}} \,|\, \mathbf{k}, \hat{\mathbf{p}}) = \sum_{n=1}^{N_s} e^{-jk[\tilde{r} + \hat{\mathbf{k}} \cdot \bar{\mathbf{r}}_n]} \tilde{\mathbf{E}}_n(\mathbf{r}^{\mathrm{f}} \,|\, \mathbf{k}, \hat{\mathbf{p}}) \qquad (3)$$

where, prior to the aggregation, $\tilde{\mathbf{E}}_n$ is interpolated to the final dense grid of directions, frequencies, and points, and subsequently multiplied by the phase restoration factor. Such a two-level scheme leads to very substantial computational savings, however even higher numerical efficiency can be achieved by using a multilevel approach [4, 5].

The domain decomposition scheme described above can now be cast into a multilevel algorithm by considering a hierarchical domain decomposition of S. Like in [5], phase compensated fields are first computed for the smallest subdomains. The field aggregation is performed in a multilevel fashion where the fields of each four neighboring subdomains are combined at each level transition via interpolation and phase compensation. The multilevel scheme reduces the asymptotic complexity to roughly $c_1 N^2 + c_2 M^2 N^3$ where c_2 represents the computational cost of a single interpolation. It should be noted that computationally the numerical quadrature is significantly more expensive than the interpolation, i.e., $c_1 \gg c_2$. This observation makes the first term in the above complexity estimate dominant for moderately sized antennas and further emphasizes the computational savings produced by the fast algorithm.

References

[1] D. B. Hayman, T. S. Bird, K. P. Esselle, P. Hall, "Encircled power study of focal plane field for estimating focal plane array size," *Proc. IEEE Int. Antennas Propag. Soc. Symp.,* July 3–8, 2005, vol. 3A, pp. 371- 374.

[2] M. V. Ivashina, M. Kehn, P.-S. Kildal, R. Maaskant, "Decoupling Efficiency of a Wideband Vivaldi Focal Plane Array Feeding a Reflector Antenna," *IEEE Trans. Antennas Propagat.*, vol. 57, no. 2, pp. 373 - 382, Feb. 2009.

[3] W. L. Stutzman and G. A. Thiele, *Antenna Theory and Design*, 2nd ed. New York: Wiley, 1998.

[4] A. Boag and C. Letrou, "Fast Radiation Pattern Evaluation for Lens and Reflector Antennas," *IEEE Trans. Antennas and Propagation*, vol. 51, no. 5, pp. 1063-1068, May 2003.

[5] A. Boag and C. Letrou, "Multilevel Fast Physical Optics algorithm for radiation from non-planar apertures," *IEEE Trans. Antennas Propagat.*, vol. 53, no. 6, pp. 2064-2072, June 2005.

[6] C. Letrou and A. Boag, "Generalized Multilevel Physical Optics (MLPO) for Comprehensive Analysis of Reflector Antennas," *IEEE Trans. Antennas and Propagation,* accepted for publication.

[7] A. Boag, E. Michielssen, and A. Brandt, "Non-uniform Polar Grid Algorithm for Fast Field Evaluation," *IEEE Antennas and Wireless Propagation Letters*, vol. 1, no. 7, pp. 142-145, 2002.