

A common signal detection model describes threshold and supra-threshold performance

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Instituts
thématiques



Inserm

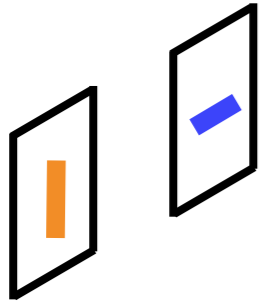


Institut national
de la santé et de la recherche médicale

Prototypical Psychophysical Experiment

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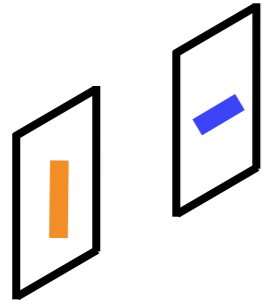
Trial



$$\phi_i \in \{\phi_1, \dots, \phi_n\}$$

Prototypical Psychophysical Experiment

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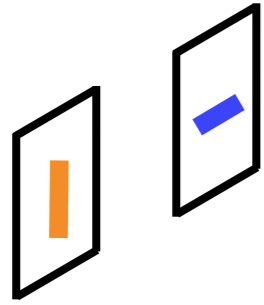
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Prototypical Psychophysical Experiment

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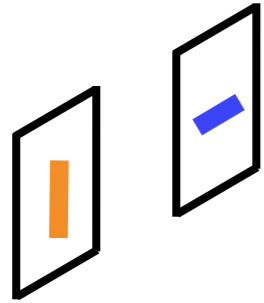
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+ ϵ

$\epsilon \sim \text{i.i.d.}$

Prototypical Psychophysical Experiment

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$$\psi_j \in \{\psi_1, \dots, \psi_m\} + \epsilon$$

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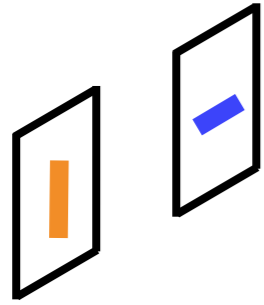
Response



$$R_{ij} \in \{1, \dots, p\}$$

Prototypical Psychophysical Experiment

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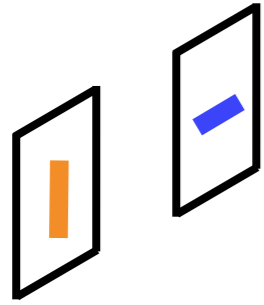
$$R_{ij} \in \{1, \dots, p\}$$

$$Pr(R_{ij}) = f(\{\phi_i\}; \Psi),$$

f is a Psychometric Function

Prototypical Psychophysical Experiment

Trial



$$\phi_i \in \{\phi_1, \dots, \phi_n\}$$

Observer



$$\psi_j \in \{\psi_1, \dots, \psi_m\} + \epsilon$$

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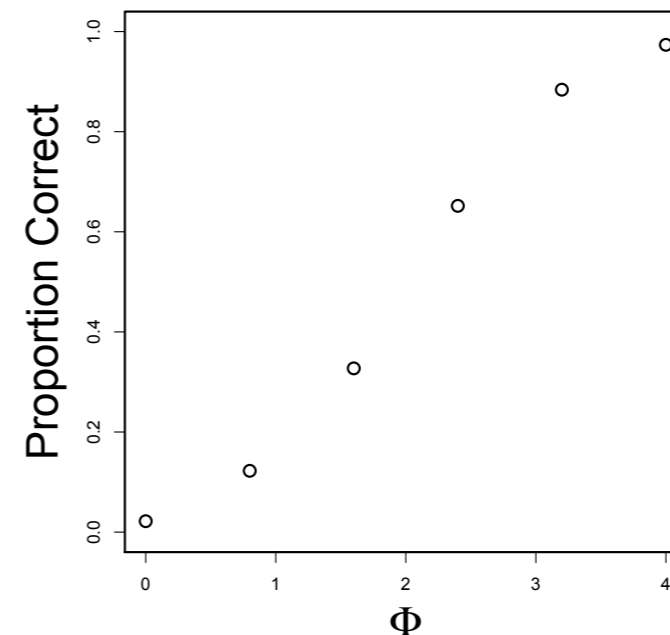
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$$Pr(R_{ij}) = f(\{\phi_i\}; \Psi),$$

f is a Psychometric Function



Modeling the Prototypical Psychophysical Experiment

How do the stimuli, $\{\phi_i\}$, and the perceptual response values, Ψ , combine to generate a *decision variable*?

$$\Delta = g(\Psi; \{\phi_i\}) \text{ and if linear} = \sum w_i \psi_i$$

Modeling the Prototypical Psychophysical Experiment

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If $\Delta > c$, choose $R = A$,
otherwise choose $R = B$,

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$$Pr(R_{ij}) = f(\{\phi_i\}; \Psi) = f(g(\Psi, \{\phi_i\})) = f(\Delta)$$

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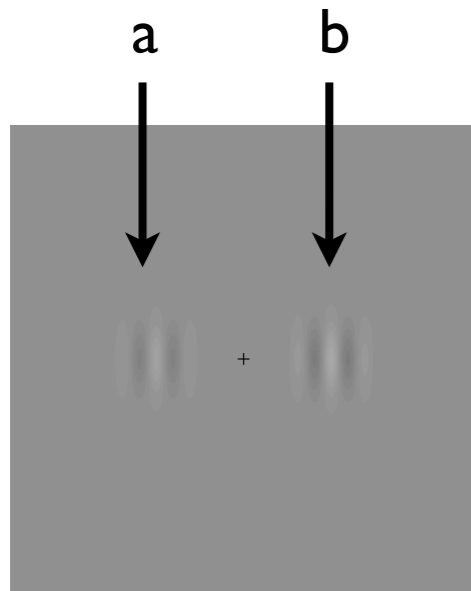
$$Pr(R_{ij}) = f(\{\phi_i\}; \Psi) = f(g(\Psi, \{\phi_i\})) = f(\Delta)$$

Given physical stimuli and behavioral responses,
estimate the internal responses by maximum likelihood:

$$L(\Psi; \{\phi_i\}, \{R_{ij}\}) = \prod_{i=1}^n Pr(\Delta_i)^{R_{ij}} (1 - Pr(\Delta_i))^{1-R_{ij}}$$

which simplifies to a generalized linear model (GLM) when the decision rule is linear!

Paired Comparisons



Find numbers, (ψ_a, ψ_b) , such that when
b is judged of higher contrast than a, $\psi_b > \psi_a$.

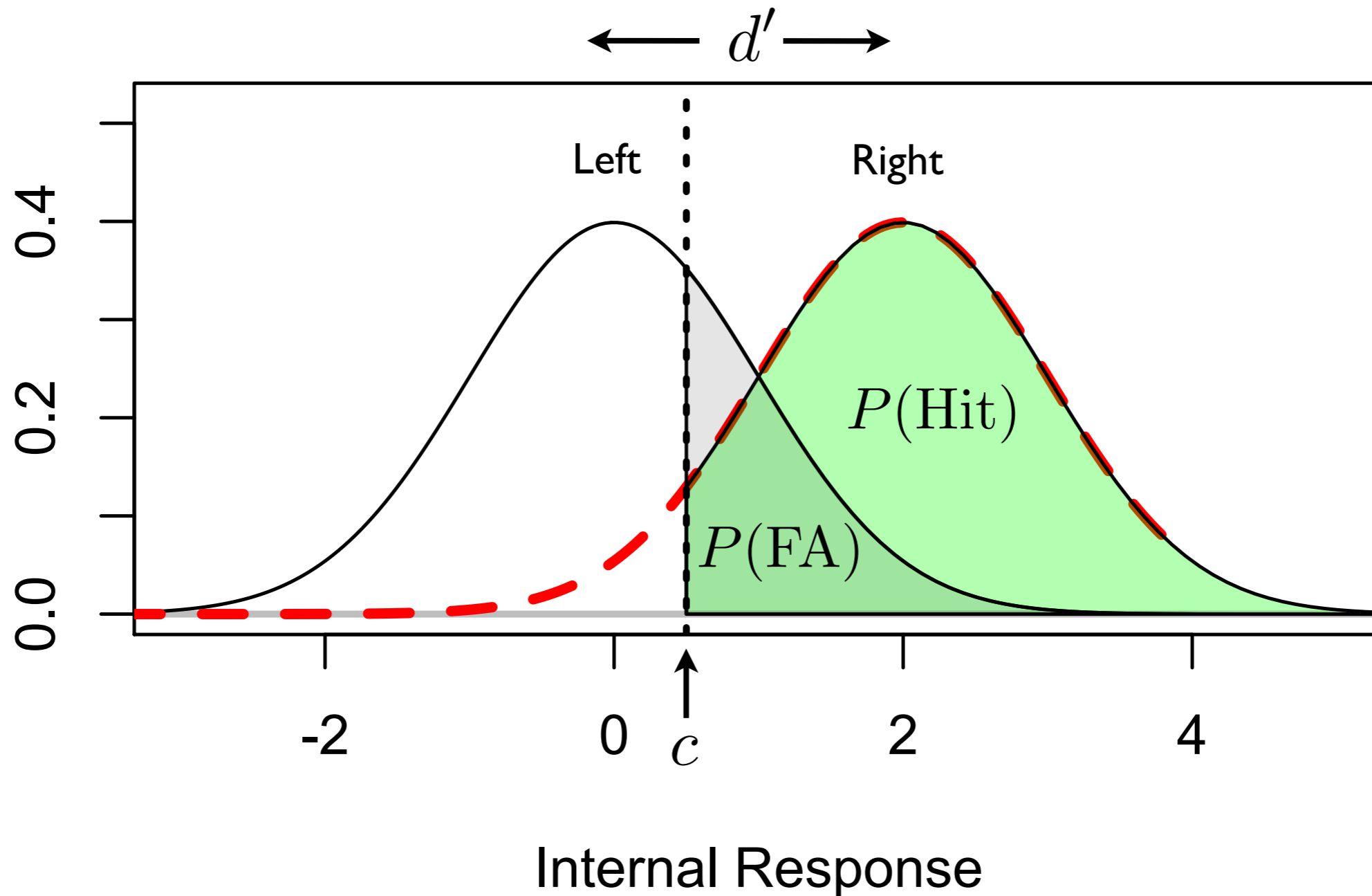
Decision variable:

$$\Delta = \psi_b - \psi_a + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \sigma^2)$$

$$(\psi_1, \psi_2, \dots, \psi_n)$$

An equal-variance Gaussian, signal detection model

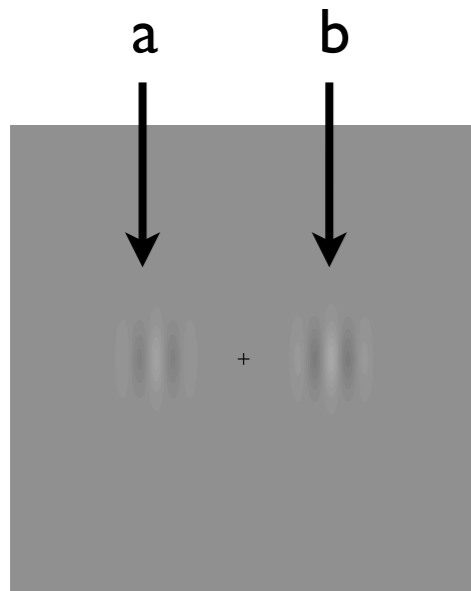
Signal Detection Theory Decision Space



```
glm(Response ~ Stimulus, binomial(probit))
```

Key idea is that the response, d' , is specified in units of the standard deviation of the noise.

Paired Comparisons



Find numbers, (ψ_a, ψ_b) , such that when
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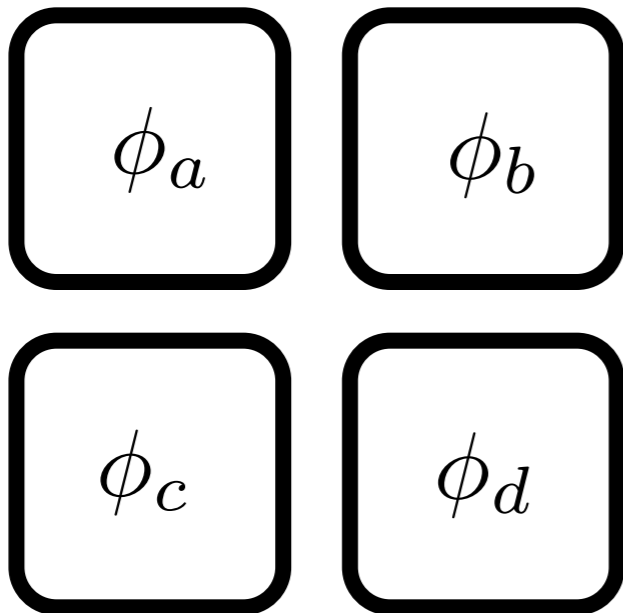
Any monotonic transformation of the above scale is equally valid!

Maximum Likelihood Difference Scaling (MLDS)

stimulus dimension ϕ_1 ϕ_2 \dots ϕ_p

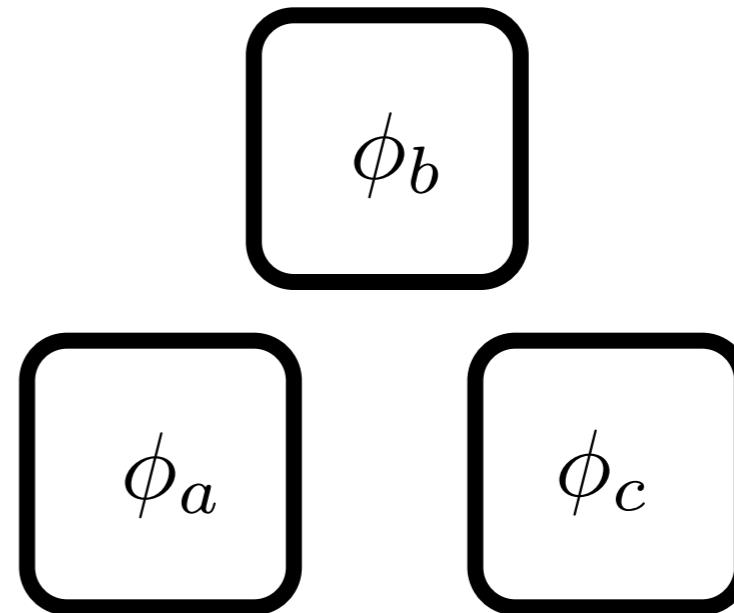


Quadruples



Between which pair (a, b) or (c, d)
is the difference greatest?

Triads



Between which pair (a, b) or (b, c)
is the difference greatest?

Estimate scale values, $\psi_1, \psi_2, \dots, \psi_p$, by maximum likelihood that best predict the observer's choices.

Maloney & Yang (2003) *J. Vision*, **3**, 573-585

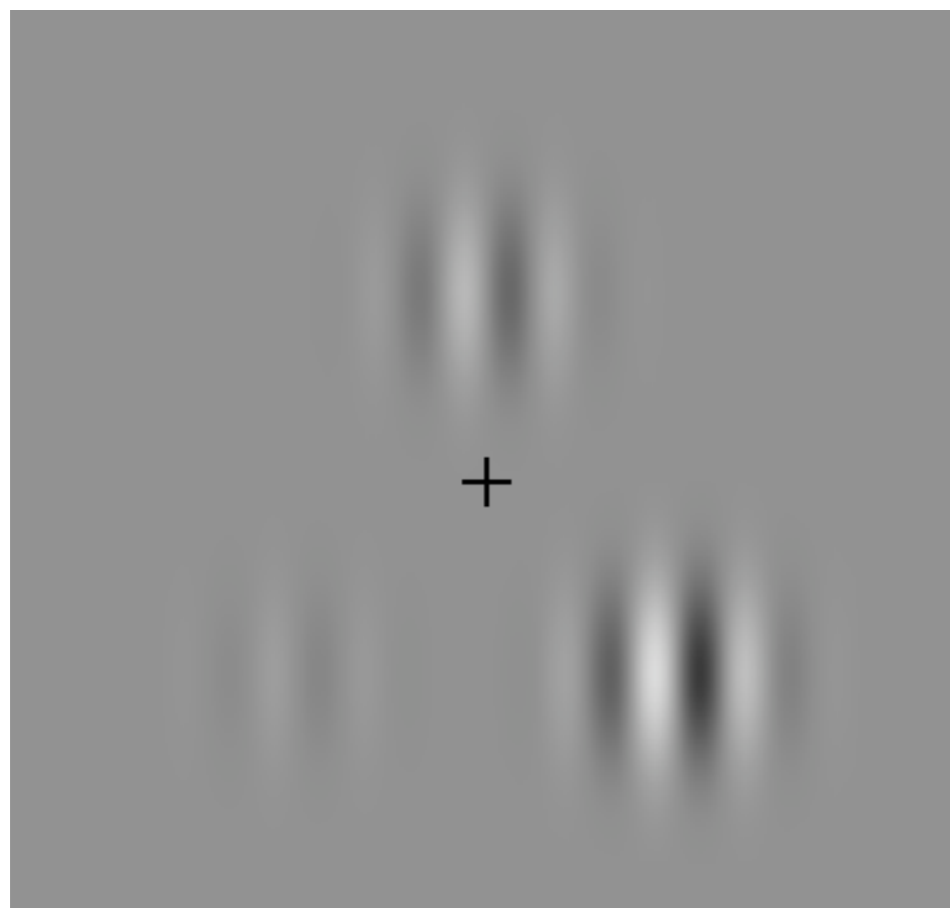
Knoblauch & Maloney (2010) *J. Stat. Software*, **25**, 1-25

Knoblauch & Maloney (Sept., 2012) **Modeling Psychophysical Data in R**, Springer

Maximum Likelihood Difference Scaling (MLDS)

Given a physical scale, $\{\phi_1, \dots, \phi_n\}$, choose an ordered triad;

judge whether the *interval* is greater between the (1, 2) or (2, 3).
For example,



The upper stimulus is more similar to the one on the left or the right?

Decision Rules and MLE

Triads:

$$\begin{aligned}\Delta(a, b, c) + \epsilon &= (\psi_c - \psi_b) - (\psi_b - \psi_a) + \epsilon = \\ &= \psi_c - 2\psi_b + \psi_a + \epsilon > 0 \\ \epsilon &\sim \mathcal{N}(0, \sigma^2)\end{aligned}$$

Likelihood:

$$\mathcal{L}(\Psi, \sigma; R_i) = \prod_{i=1}^n \Phi\left(\frac{\Delta_i}{2\sigma}\right)^{R_i} \left(1 - \Phi\left(\frac{\Delta_i}{2\sigma}\right)\right)^{1-R_i},$$

an *equal-variance, Gaussian model*, scaled so that ψ_i are in units of \mathbf{d}' (in theory!).

$$g(\mathbf{E}[Pr(R = 1)]) = \beta_2\psi_2 + \cdots + \beta_p\psi_p$$



Knoblauch, K. & Maloney, L. T. (2008) MLDS : Maximum likelihood difference scaling in R, Journal of Statistical Software, 25, 1-26, <http://www.jstatsoft.org/v25/i02/>.

```
data(kktriad, package = "MLDS")  
head(kktriad)
```

```
  resp S1 S2 S3  
1     1  8  7  2  
2     1  8  6  4  
3     0 10  5  2  
4     1  1  2 11  
5     0  8  5  3  
6     1  1  3 11  
      .  
      .  
      .
```

```

mx <- max(kktriad[, -1]) ## get max stimulus level
d <- within(kktriad, { ## make factors w/ same levs
  S1 <- factor(S1, levels = seq_len(mx))
  S2 <- factor(S2, levels = seq_len(mx))
  S3 <- factor(S3, levels = seq_len(mx))
})
m.lst <- lapply(names(d[, -1]), function(nm) {
  f <- as.formula(paste("~", nm))
  m <- model.matrix(f, d) ## sub-model matrices
  if (nm == "S2")
    m <- -2 * m
  m
})
m <- Reduce("+", m.lst) ## combine

```

```
head(m)
```

	(Intercept)	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	...	resp	S1	S2	S3	
1	0	1	0	0	0	0	-2	1	0	0	0	...	1	1	8	7	2
2	0	0	0	1	0	-2	0	1	0	0	0	...	2	1	8	6	4
3	0	1	0	0	-2	0	0	0	0	1	0	...	3	0	10	5	2
4	0	-2	0	0	0	0	0	0	0	0	1	...	4	1	1	2	11
5	0	0	1	0	-2	0	0	1	0	0	0	...	5	0	8	5	3
6	0	0	-2	0	0	0	0	0	0	0	1	...	6	1	1	3	11

```
kkdat.df <- data.frame(kktriad[, 1], m[, -1])
```

```
names(kkdat.df) <- c("resp", paste0("S", 2:mx))
```

```
glm(resp ~ . - 1, binomial(link = probit), data = dsInc.df)
```

```
Call: glm(formula = resp ~ . - 1, family = binomial(link = probit),  
data = kkdat.df)
```

Coefficients:

	S2	S3	S4	S5	S6	S7	S8	S9
S10		S11						
	-0.3925	-0.2270	-0.2098	-0.1781	-0.1663	-0.3945	-0.3860	-0.4710
	-0.3589	-0.1264						

Degrees of Freedom: 165 Total (i.e. Null); 155 Residual

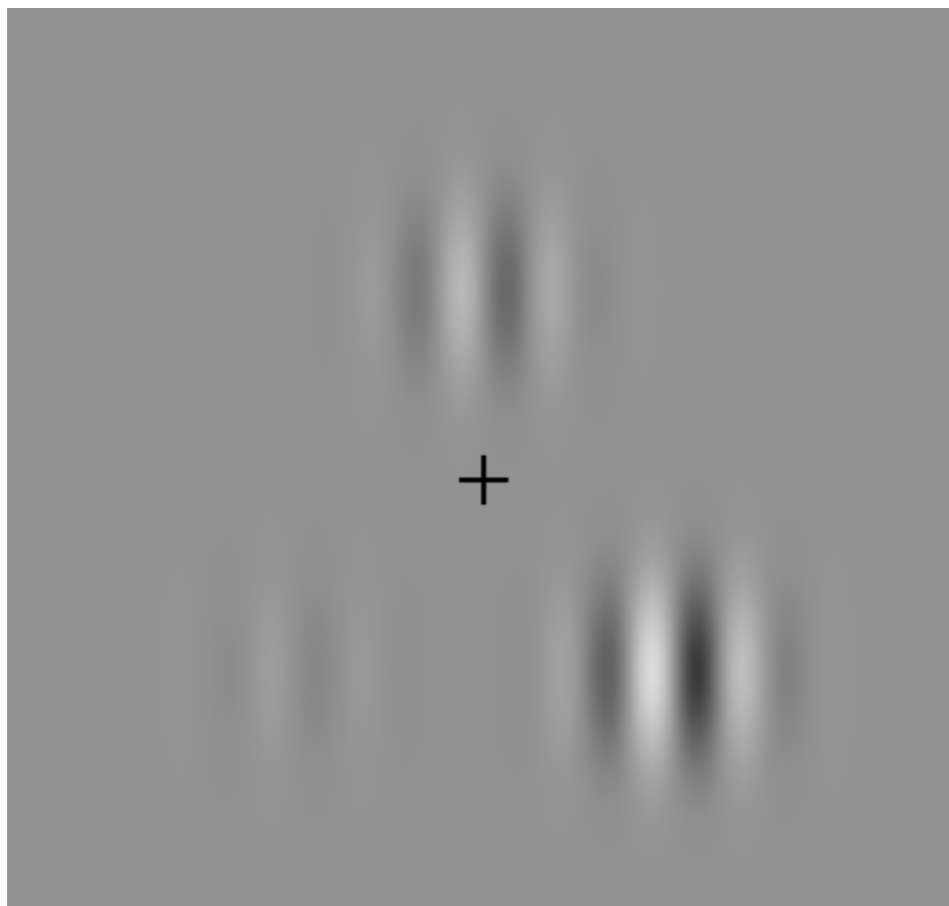
Null Deviance: 228.7

Residual Deviance: 218.6 AIC: 238.6

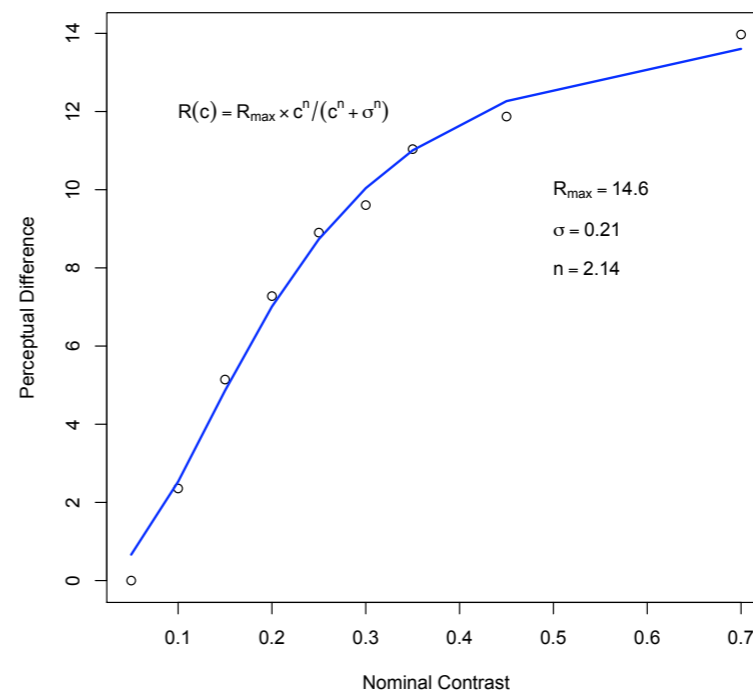
```
mlds(kktriad) ## wrapper function around above from MLDS package
```

Maximum Likelihood Difference Scaling (MLDS)

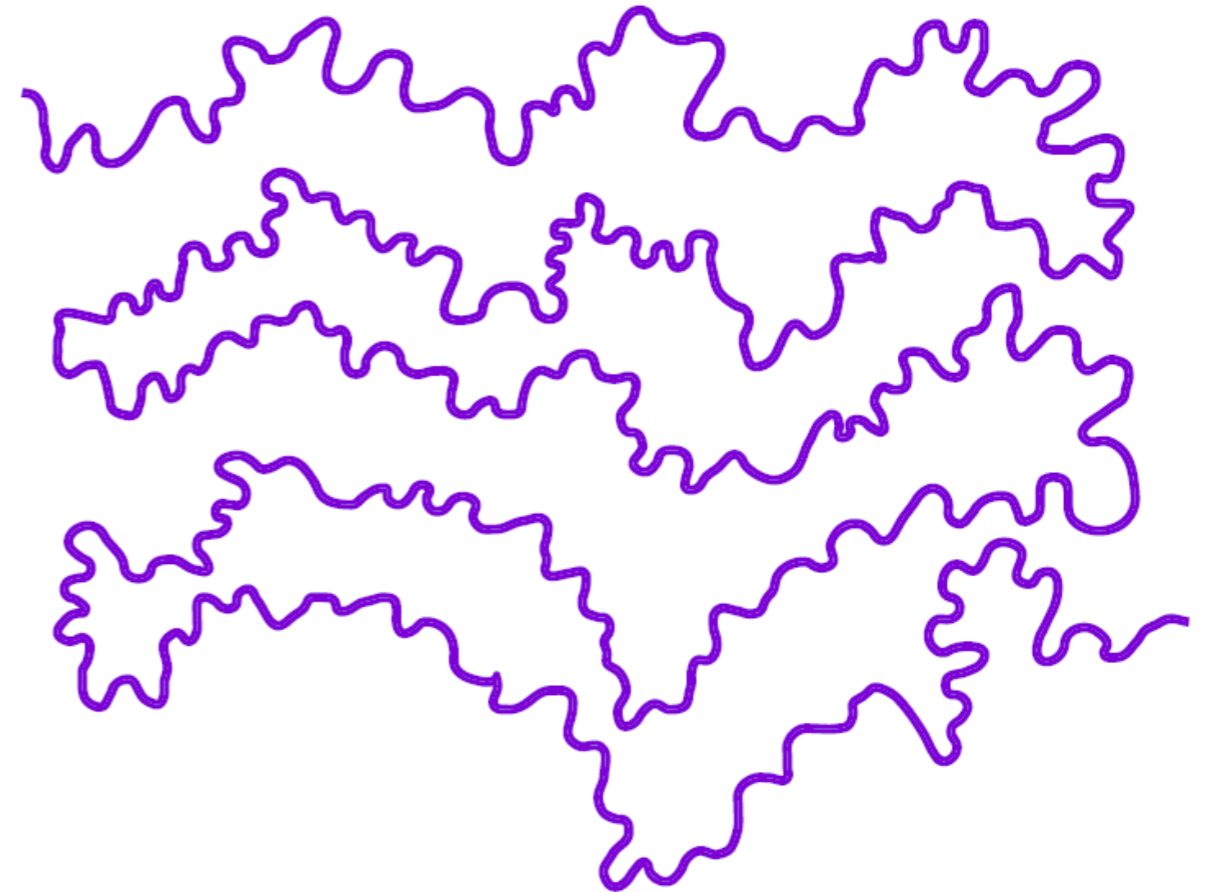
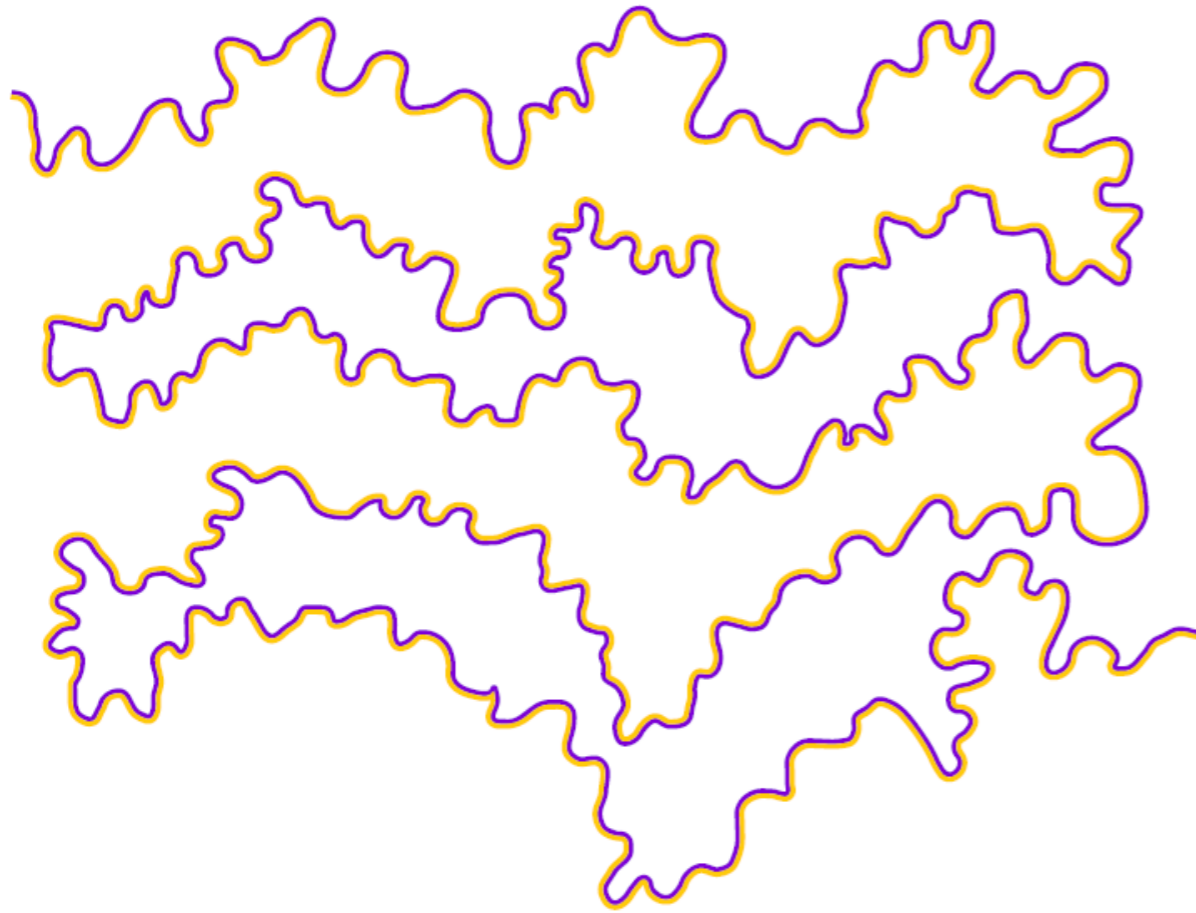
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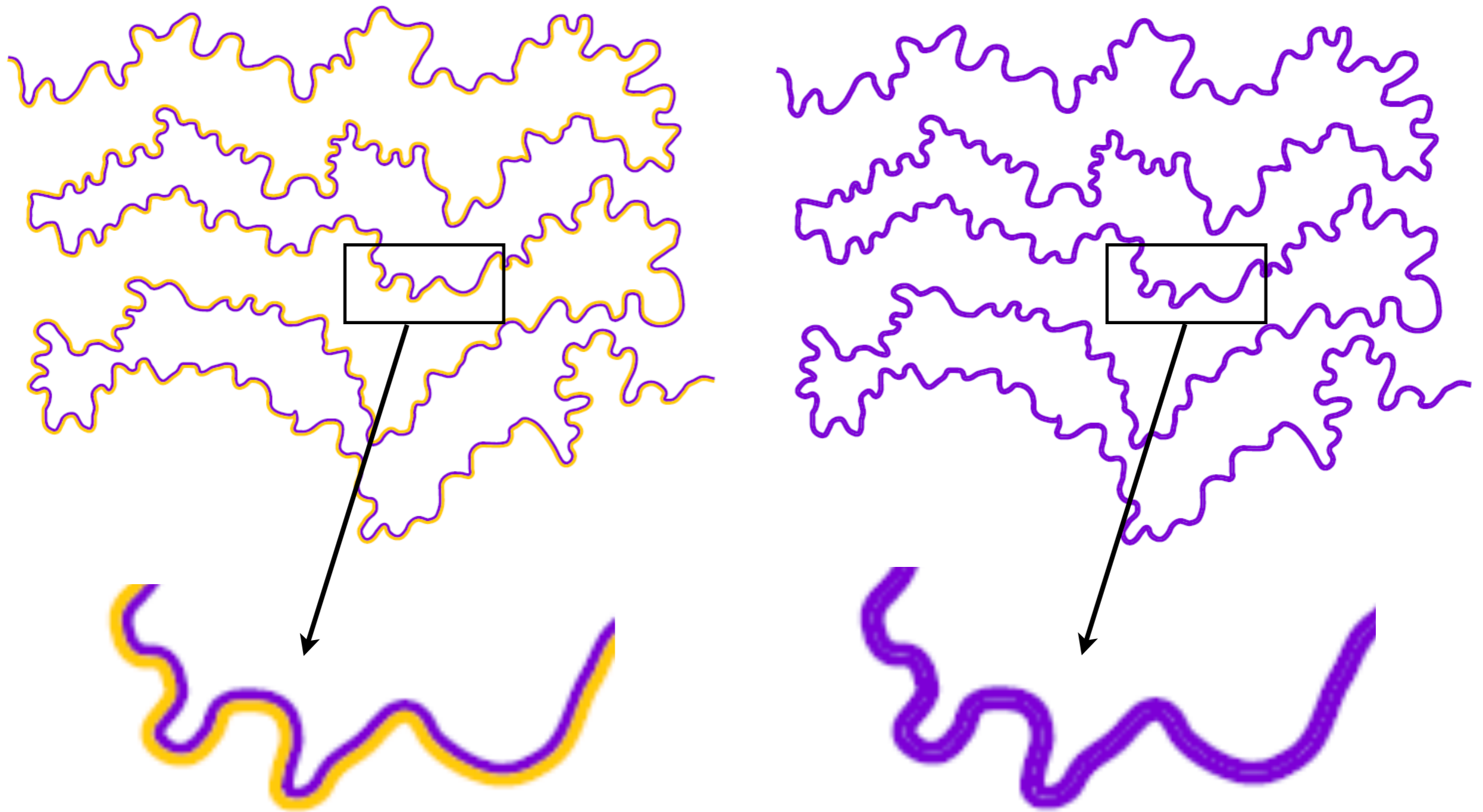
The Watercolor Effect (WCE)



Pinna, B. (1987). *XXI Congresso degli Psicologi Italiani*, 158.

Pinna et al. (2001). *Vision Research*, **41**, 2669–2676.

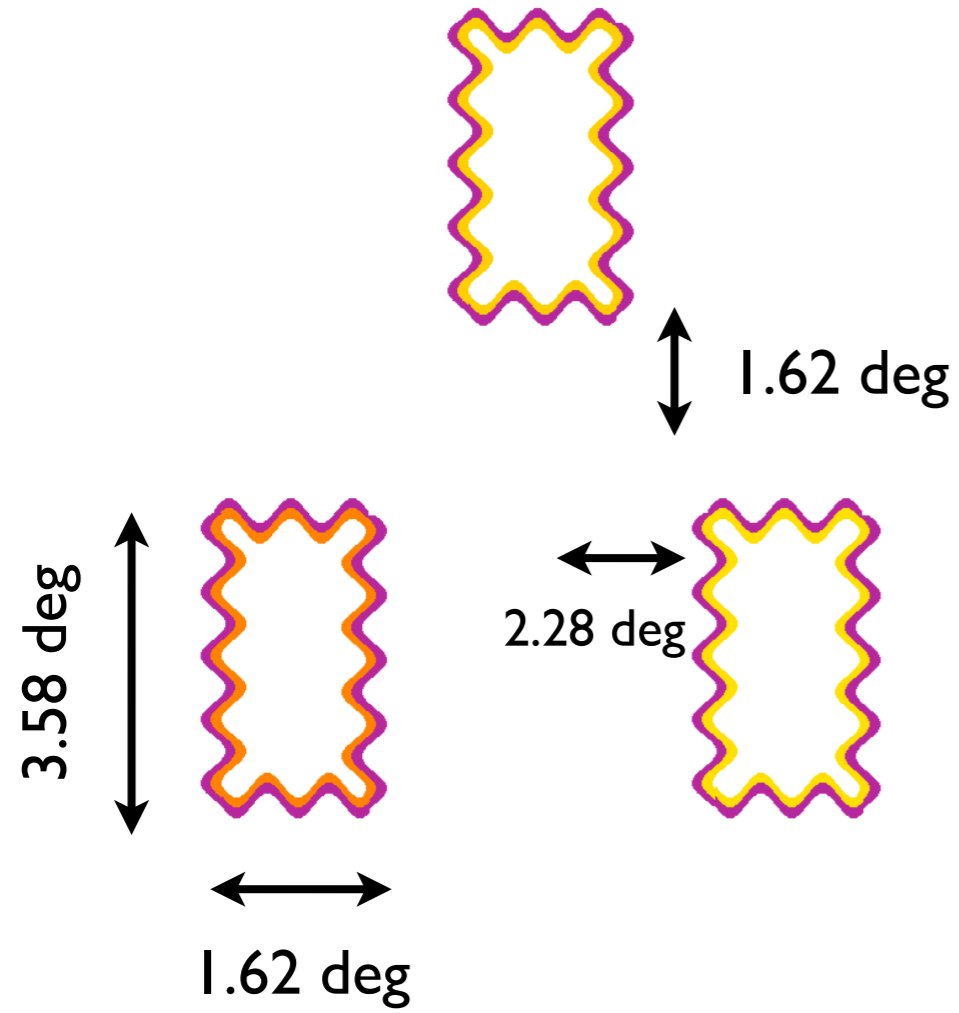
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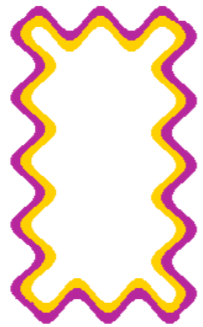
Pinna et al. (2001). *Vision Research*, **41**, 2669–2676.

Test Condition



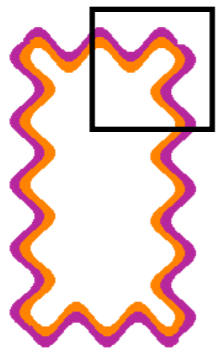
Frequency: 1.5 c/deg

Test Condition

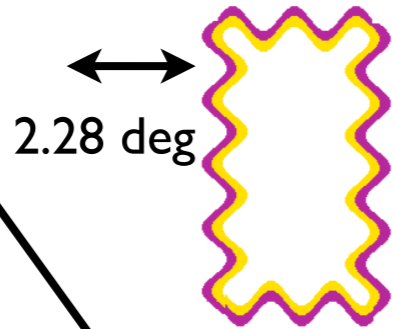


1.62 deg

3.58 deg



1.62 deg



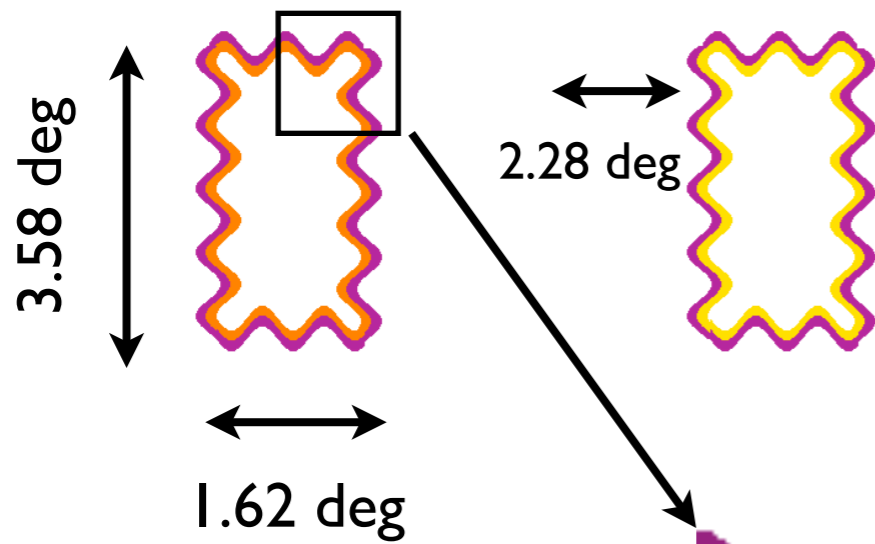
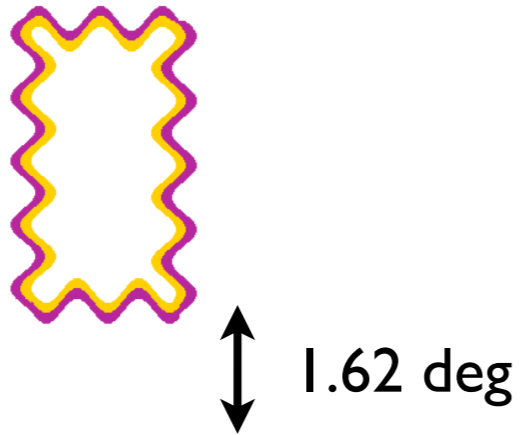
2.28 deg



16 arc min

Frequency: 1.5 c/deg

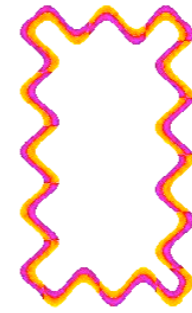
Test Condition



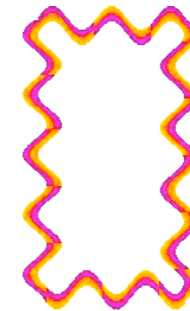
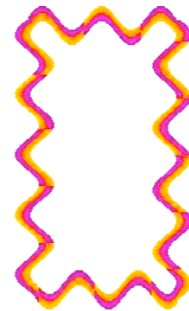
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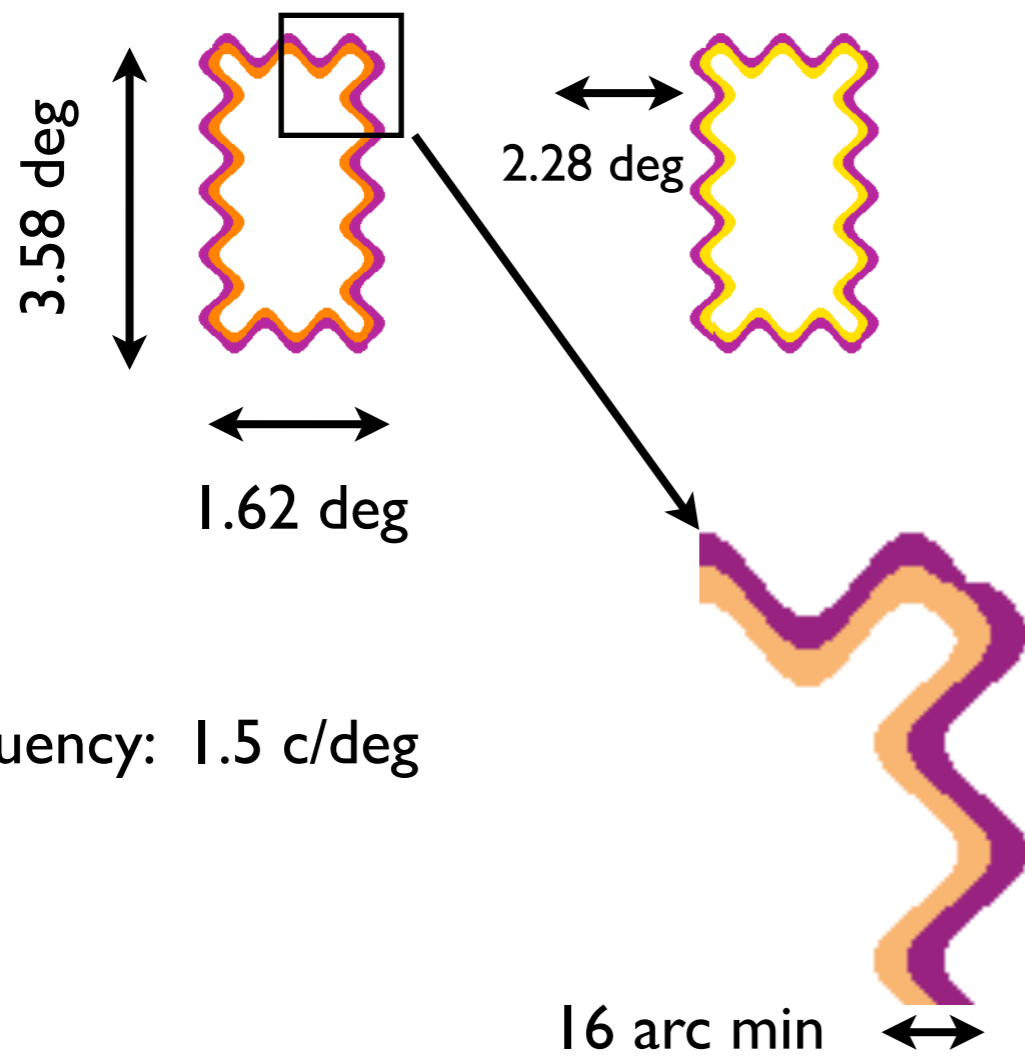
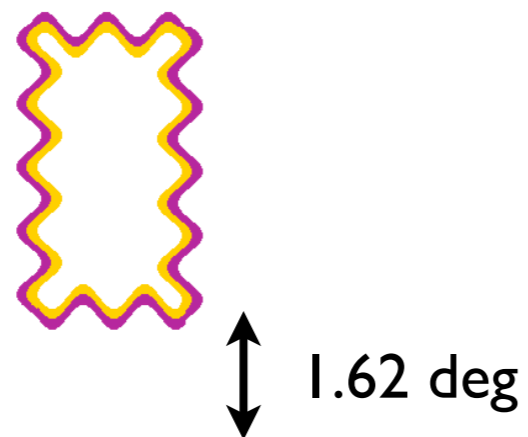
Braided Control



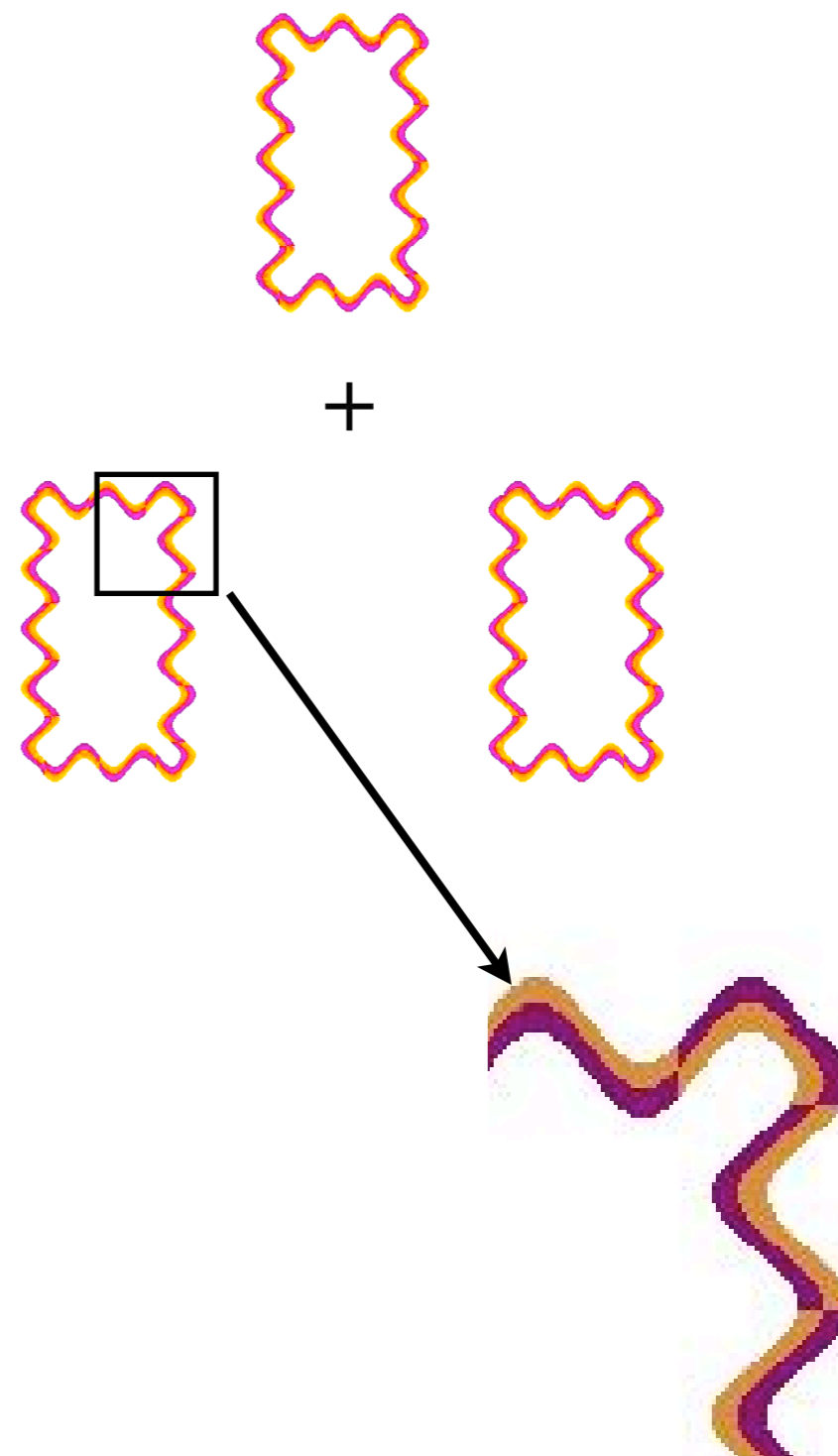
+



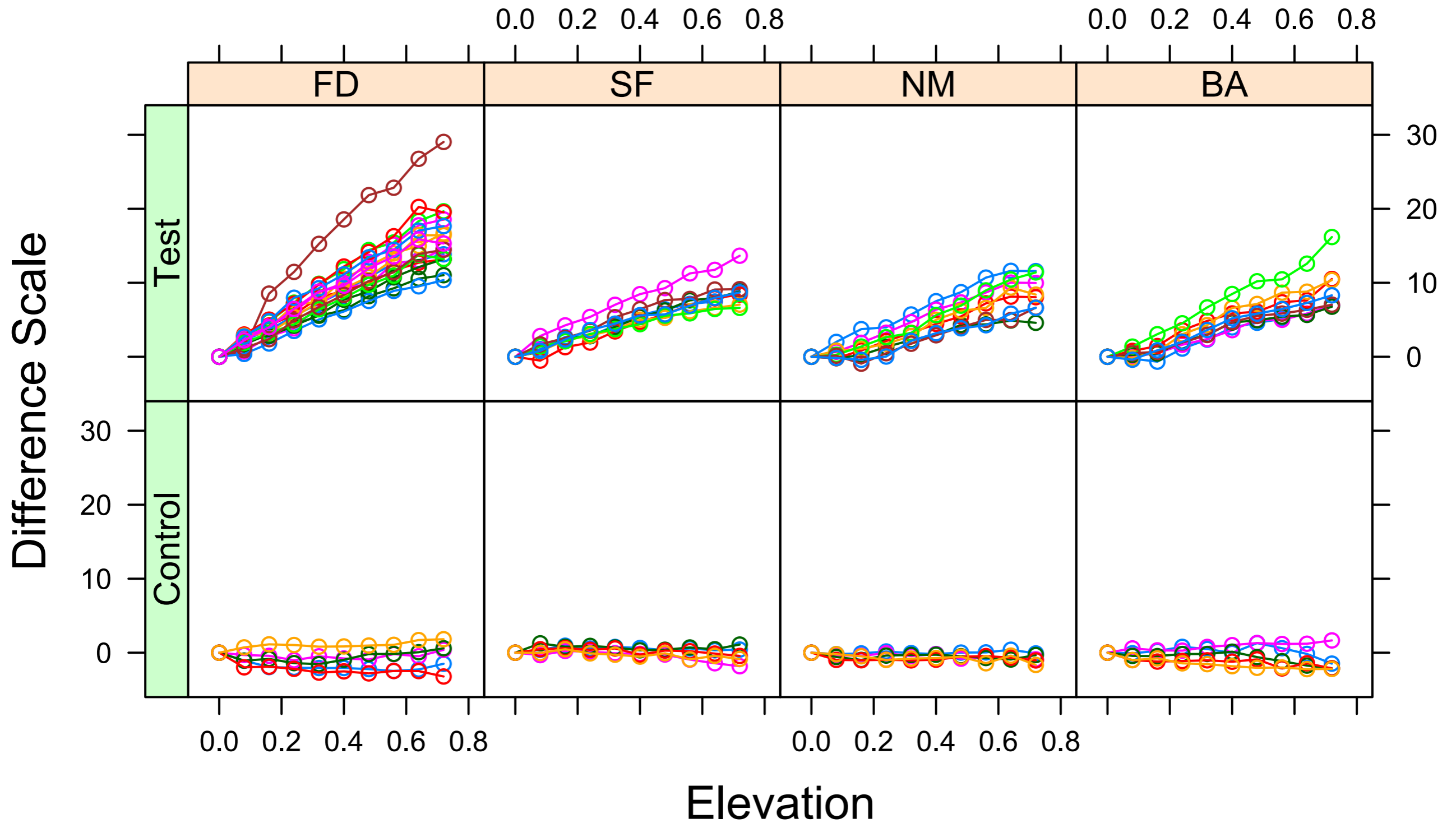
Test Condition



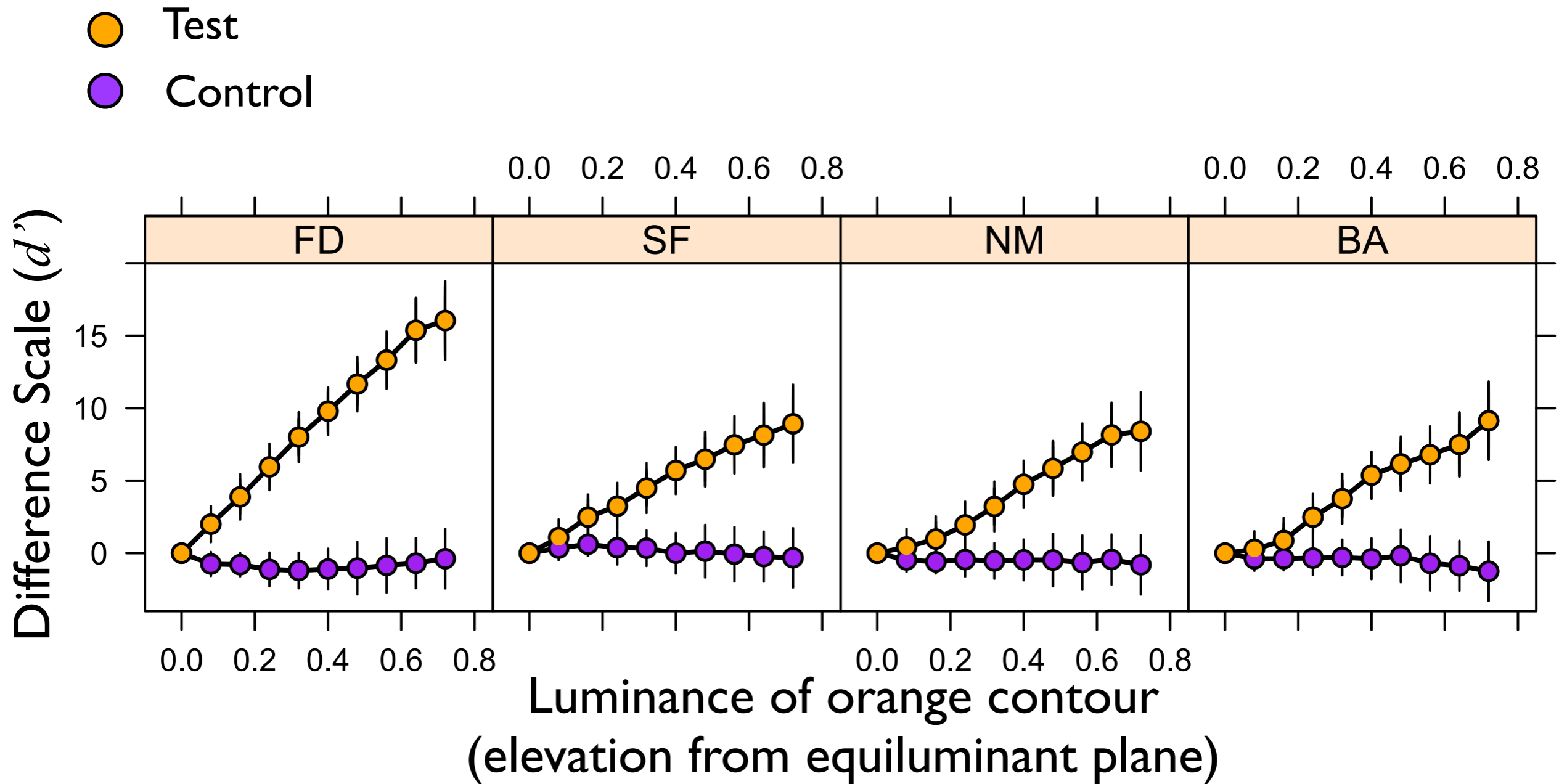
Braided Control



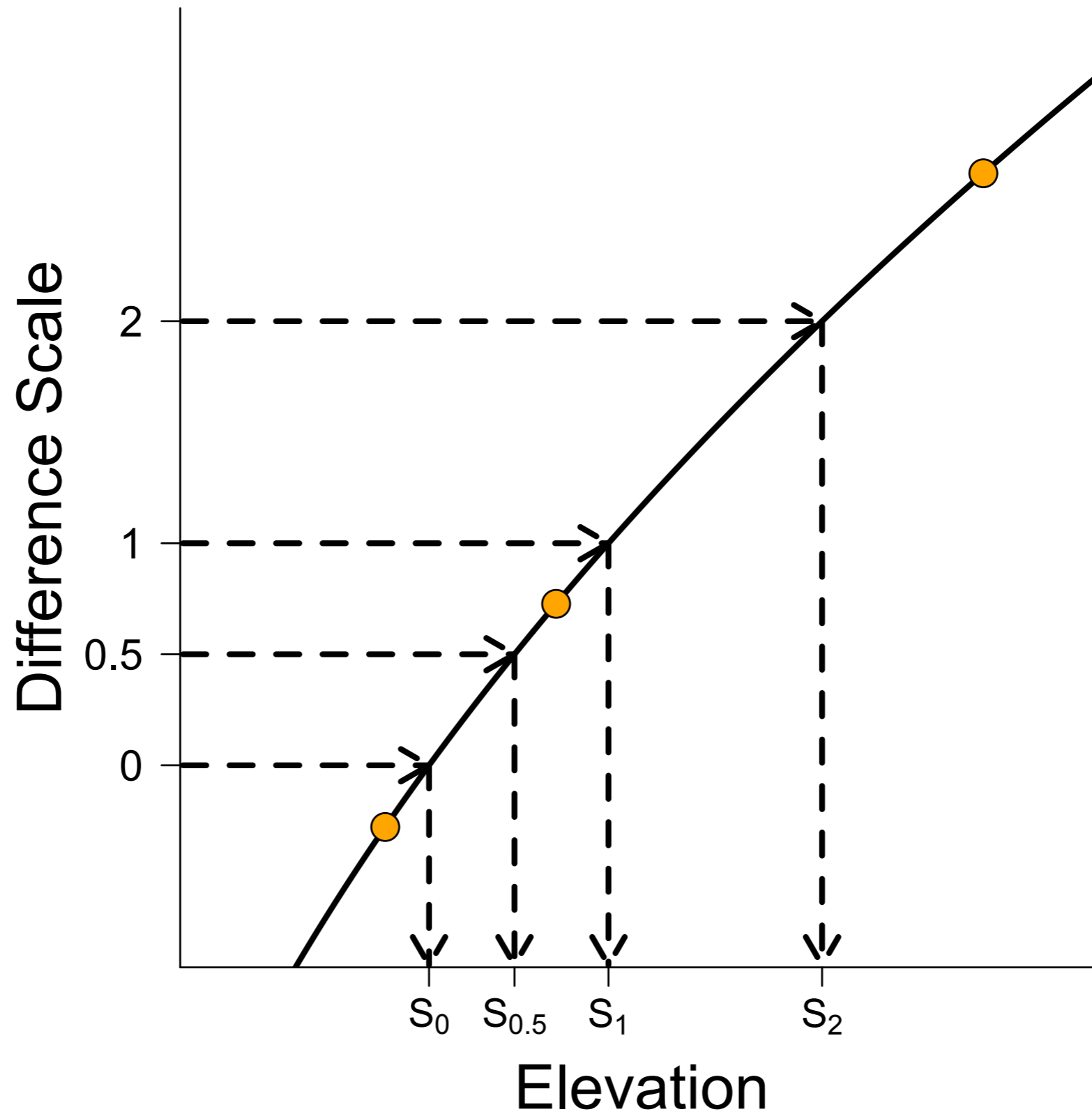
Positive Elevations: Repeatability



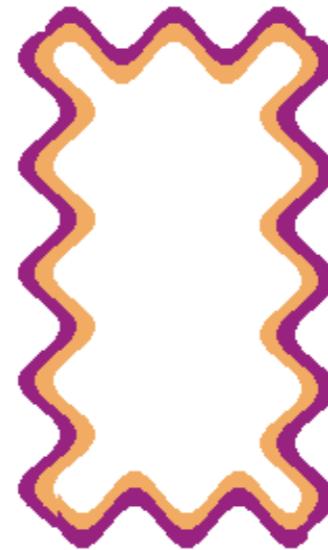
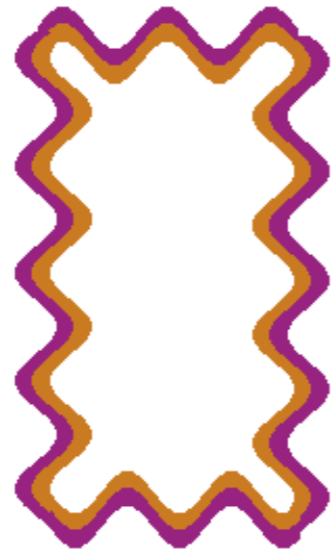
Positive Luminance Elevations



Do Differences Scales predict discrimination performance?



Elevation Discrimination Experiment



Rating whether left or right interior is more orange

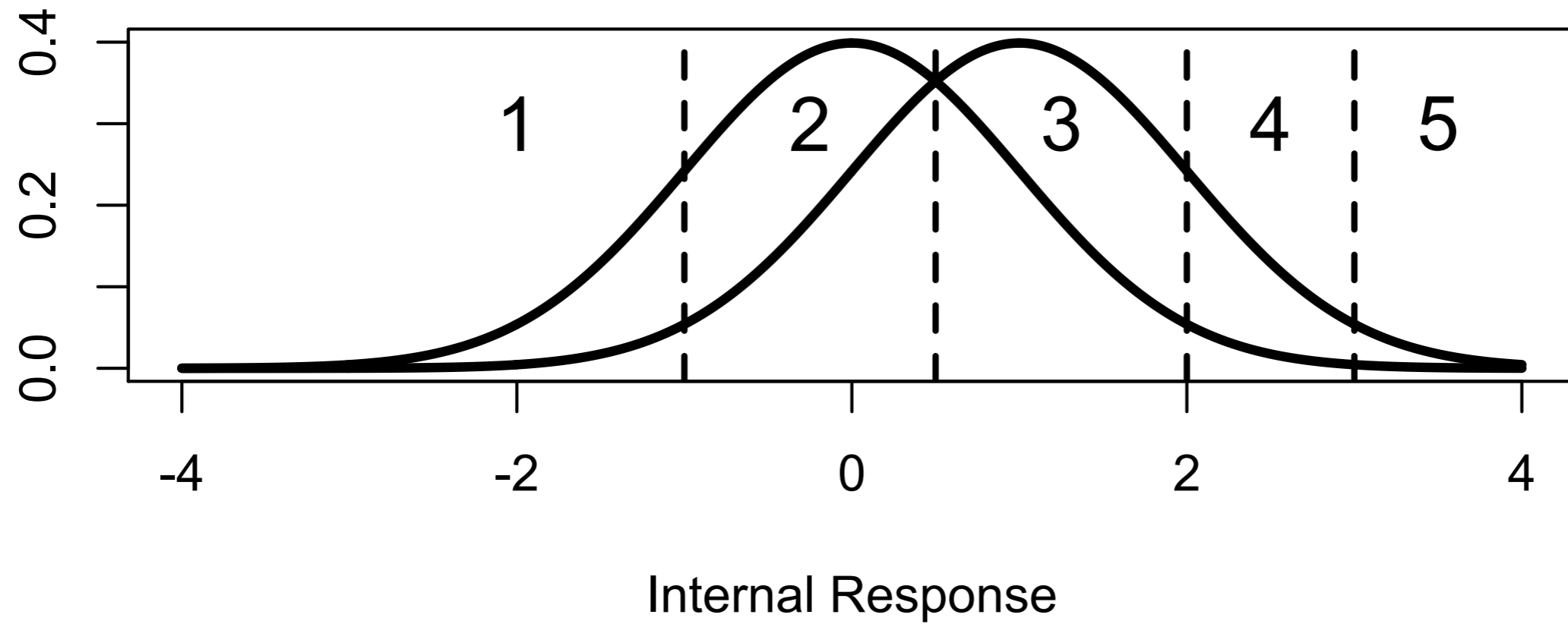
Five point rating scale: 1 - definitely left, 5 definitely right

Two Conditions: Test, Braided Control

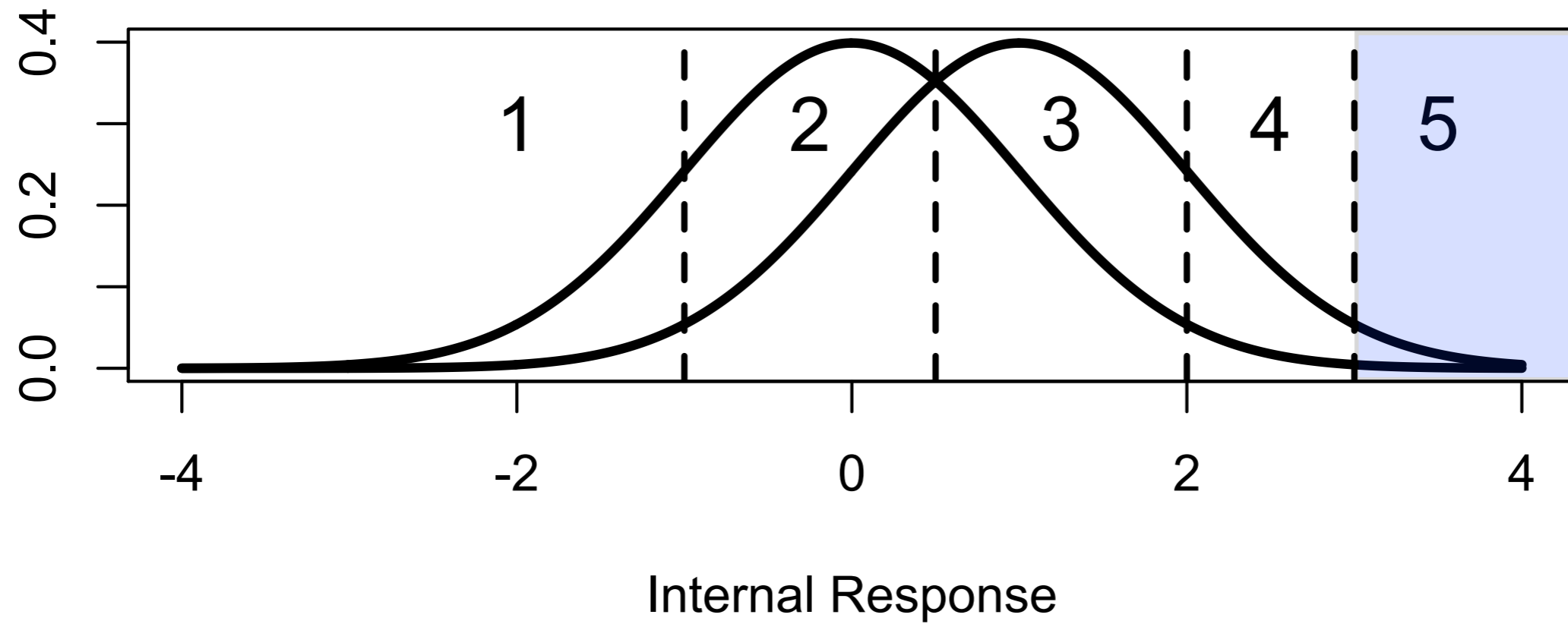
Three elevations: nominally $d' = 0.5, 1.0, 2.0$ (as predicted from individual MLDS curves)

Rating scales analyzed with Proportional Odds Ordinal Regression Model

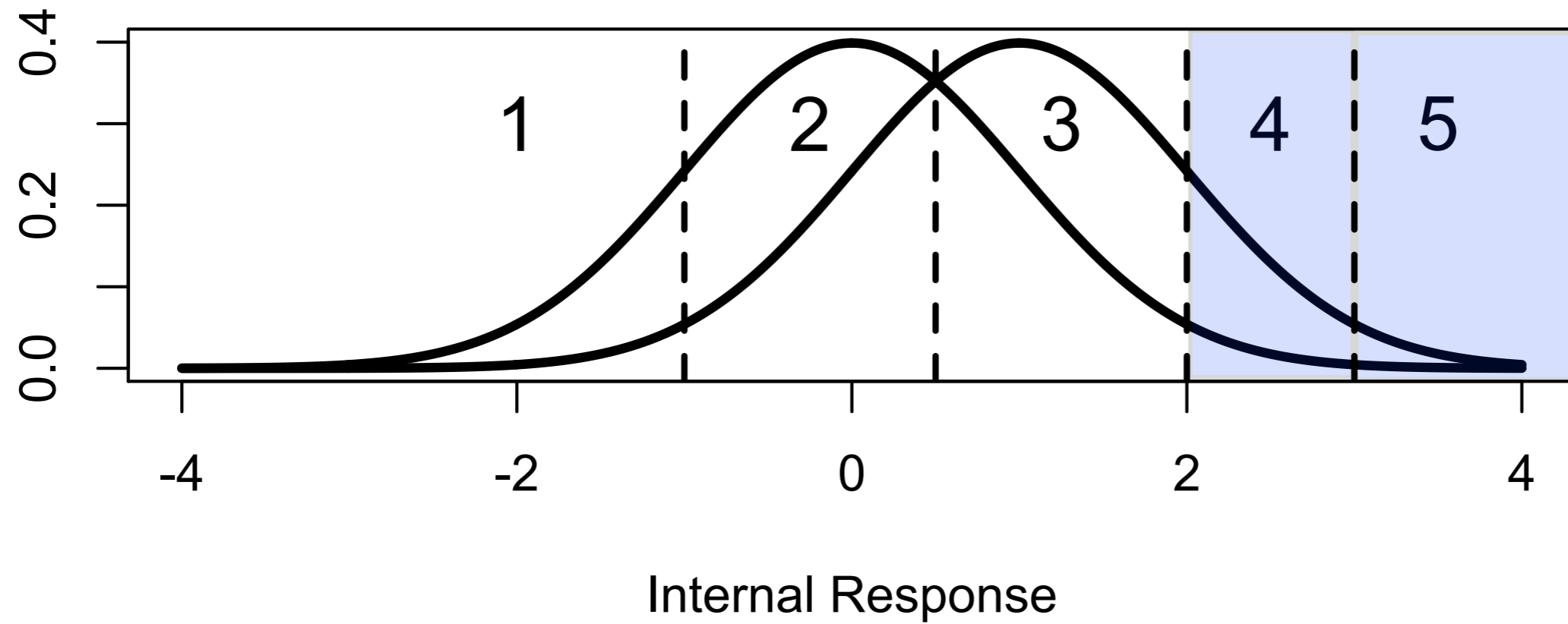
Analysis of rating scales: Signal Detection Theory



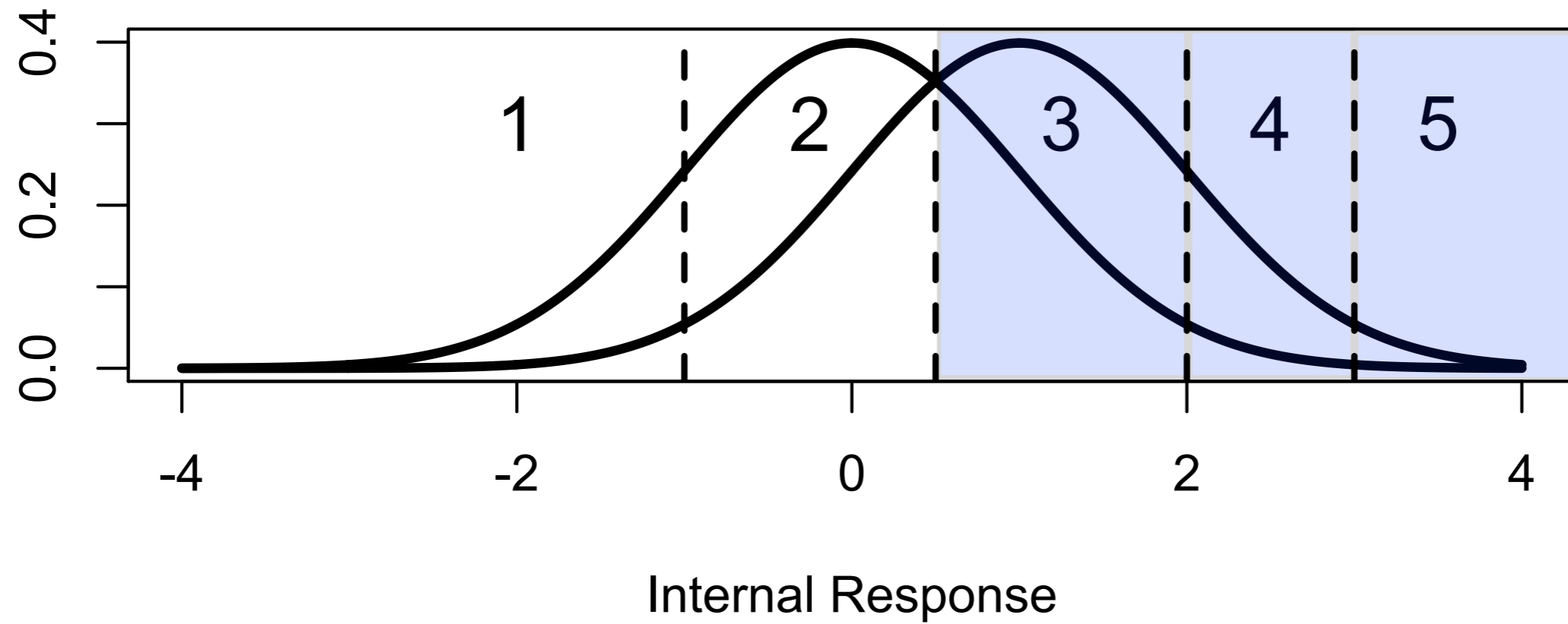
Analysis of rating scales: Signal Detection Theory



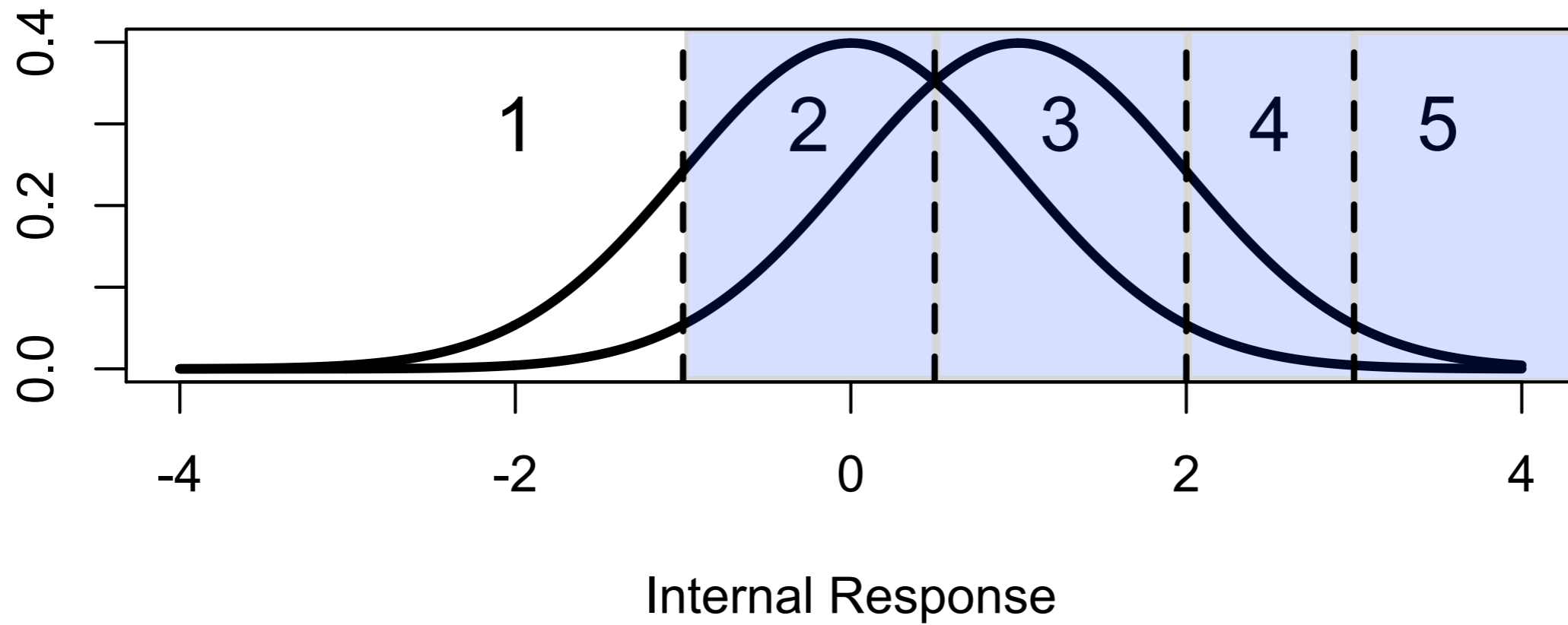
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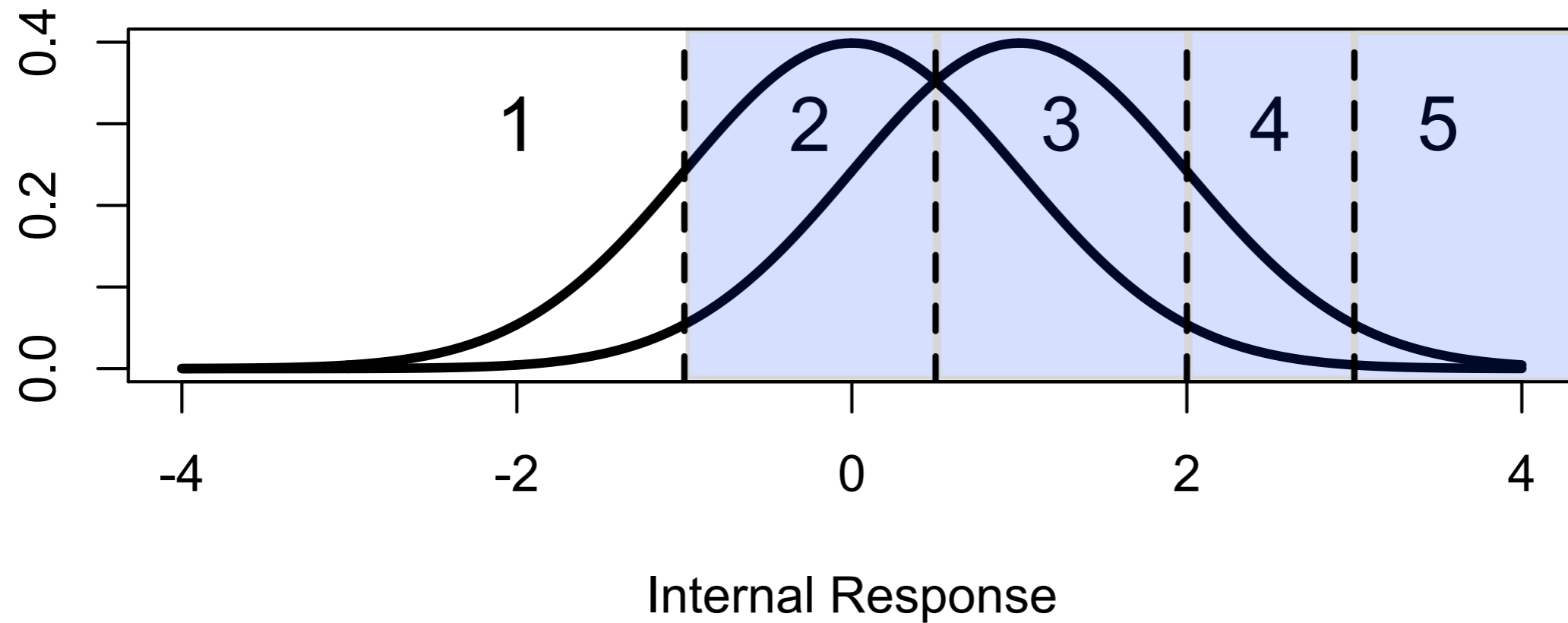
Analysis of rating scales: Signal Detection Theory



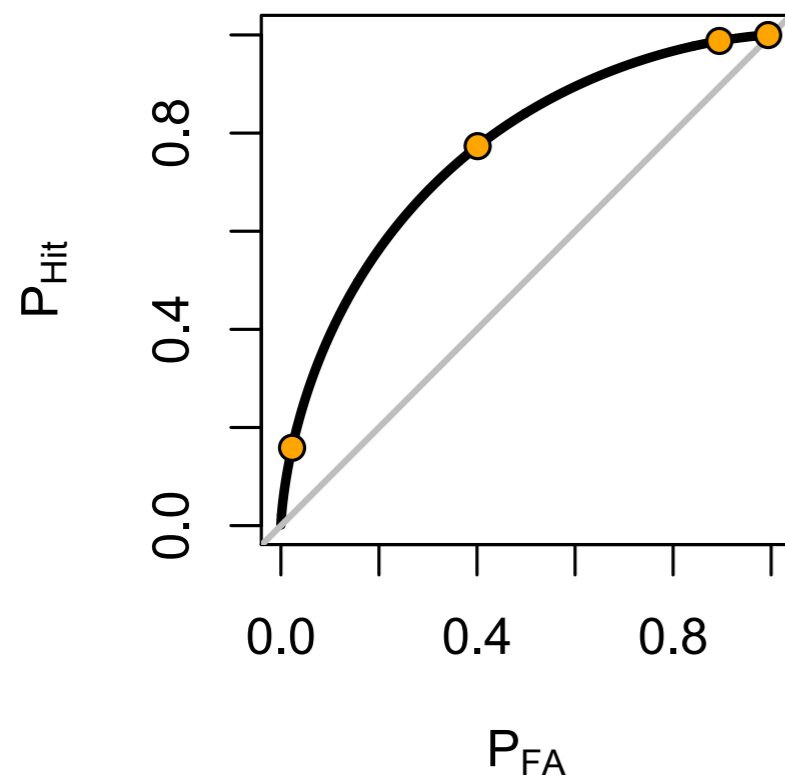
Analysis of rating scales: Signal Detection Theory



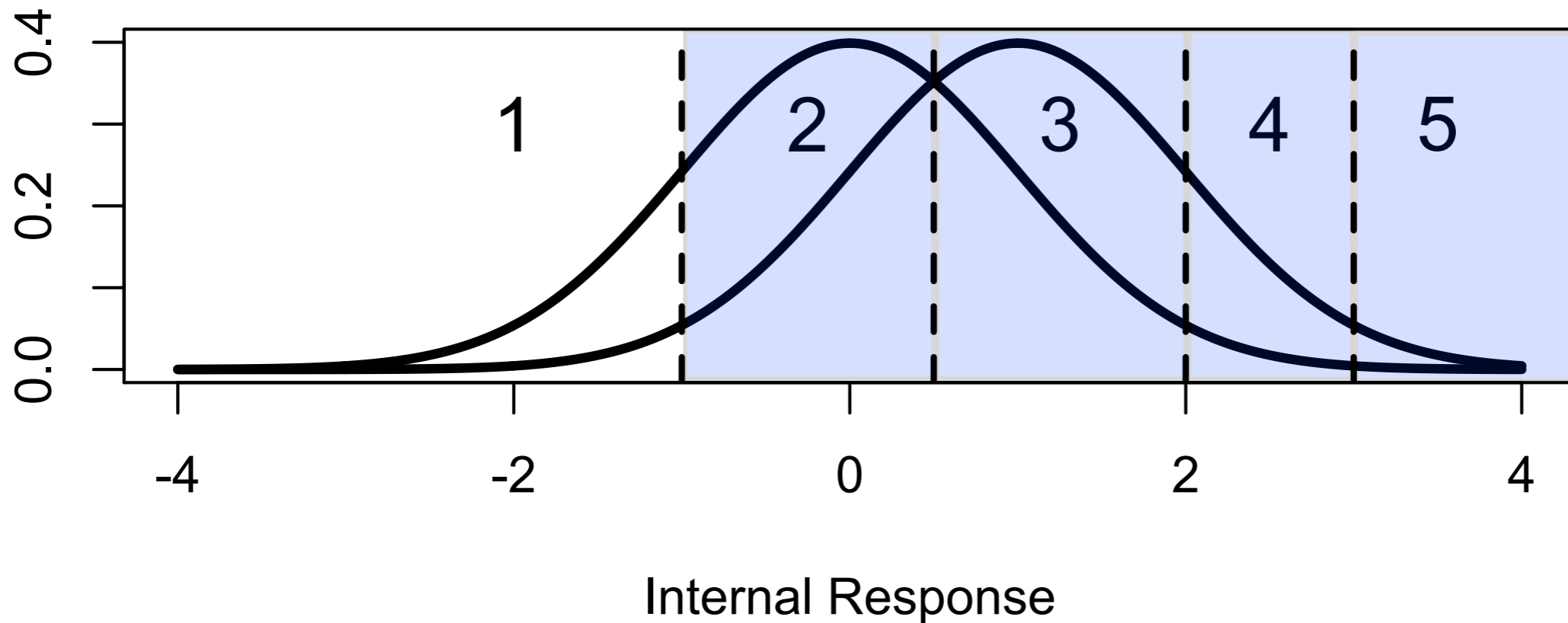
Analysis of rating scales: Signal Detection Theory



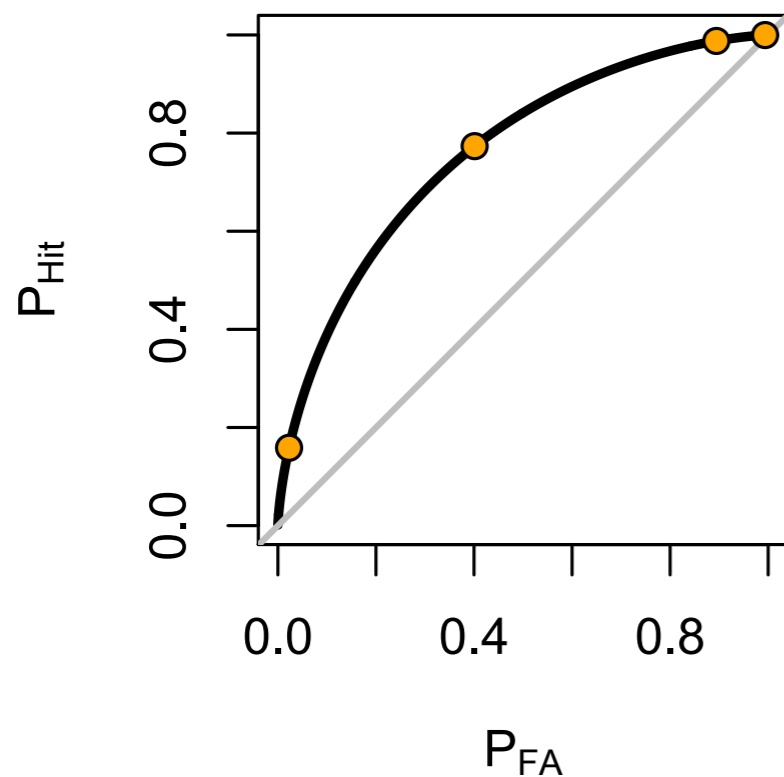
ROC curve analysis



Analysis of rating scales: Signal Detection Theory



ROC curve analysis



Cumulative link models
(e.g., polr from MASS package or
clm from ordinal package)

$$g(\mathbb{E}[\Pr(Y < \theta_k | x_k)]) = \theta_k - X\beta$$

```
head(dis.df, 10)
```

```
  Cond Stim Inten Rating Ele
1     C    L     2     4    1
2     T    L     2     4    1
3     T    L     2     4    1
5     T    L     2     5    1
6     C    L     2     5    1
7     C    L     2     4    1
10    T    L     2     3    1
11    T    R     2     4    1
13    T    R     2     4    1
14    C    L     2     4    1
```

```
str(dis.df)
```

```
'data.frame': 1920 obs. of  5 variables:
 $ Cond   : Factor w/ 2 levels "C","T": 1 2 2 2 1 1 2 2 2 1 ...
 $ Stim   : Factor w/ 2 levels "L","R": 1 1 1 1 1 1 1 2 2 1 ...
 $ Inten  : num  2 2 2 2 2 2 2 2 2 2 ...
 $ Rating: Ord.factor w/ 5 levels "1"<"2"<"3"<"4"<..: 4 4 4 5 5 4 3 4 4
4 ...
 $ Ele    : num  1 1 1 1 1 1 1 1 1 1 ...
```



```
library(ordinal) ## by R. H. B. Christensen (CRAN)
```

```
mod[[Cnd]] <- clm(Rating ~ Stim, data = dis.df, link = "probit",  
  subset = Cond == Cx & Inten == Ix) ## = var model
```

```
mod2[[Cnd]] <- clm(Rating ~ Stim, scale = ~ Stim, data = dis.df,  
  link = "probit", subset = Cond == Cx & Inten == Ix) ## != var model
```

```
summary(mod[[2]])
```

```
formula: Rating ~ Stim
```

```
data: dis.df
```

```
subset: Cond == Cx & Inten == Ix
```

link	threshold	nobs	logLik	AIC	niter	max.grad	cond.H
probit	flexible	360	-327.35	664.69	7(1)	2.36e-08	1.4e+02

```
Coefficients:
```

	Estimate	Std. Error	z value	Pr(> z)	
StimR	-0.01913	0.12590	-0.152	0.879	## d' estimate

```
Threshold coefficients:
```

	Estimate	Std. Error	z value	
1 2	-2.54793	0.25311	-10.067	## criterion boundaries
2 3	-0.36517	0.09288	-3.932	
3 4	-0.21297	0.09202	-2.314	
4 5	2.52980	0.25422	9.951	

```
summary(mod2[[2]])
```

```
formula: Rating ~ Stim
```

```
scale: ~Stim
```

```
data: dis.df
```

```
subset: Cond == Cx & Inten == Ix
```

```
link threshold nobs logLik AIC niter max.grad cond.H  
probit flexible 360 -326.79 665.59 10(1) 4.11e-13 2.1e+02
```

```
Coefficients:
```

```
Estimate Std. Error z value Pr(>|z|)  
StimR -0.04645 0.12074 -0.385 0.7
```

```
log-scale coefficients:
```

```
Estimate Std. Error z value Pr(>|z|)  
StimR -0.1426 0.1382 -1.032 0.302 ## log(slope ratio)
```

```
Threshold coefficients:
```

```
Estimate Std. Error z value  
1|2 -2.43387 0.26738 -9.103  
2|3 -0.35406 0.09325 -3.797  
3|4 -0.21272 0.09182 -2.317  
4|5 2.36110 0.27796 8.494
```

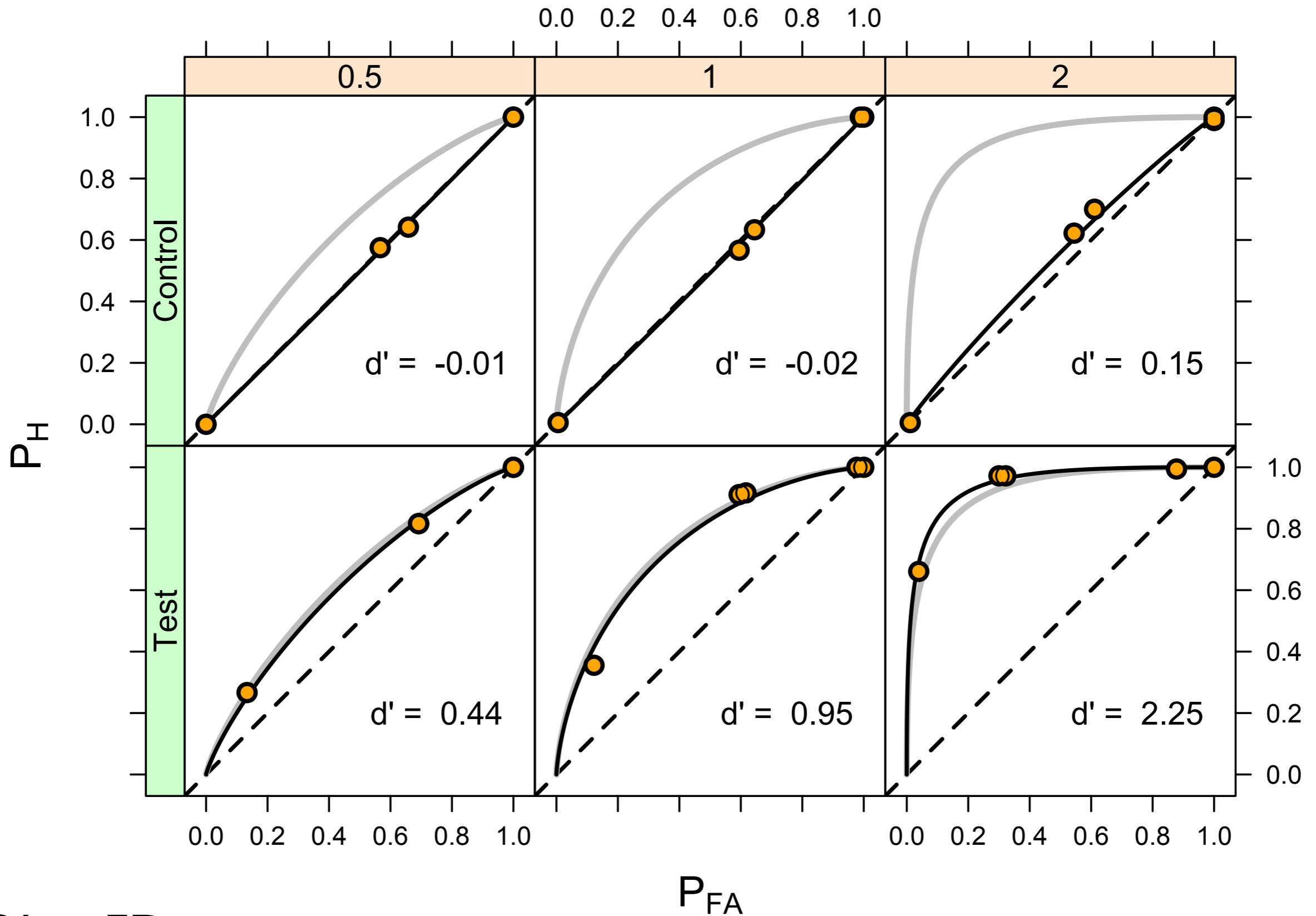
Comparison of equal and unequal variance models

```
anova(mod[[2]], mod2[[2]])
```

Likelihood ratio tests of cumulative link models:

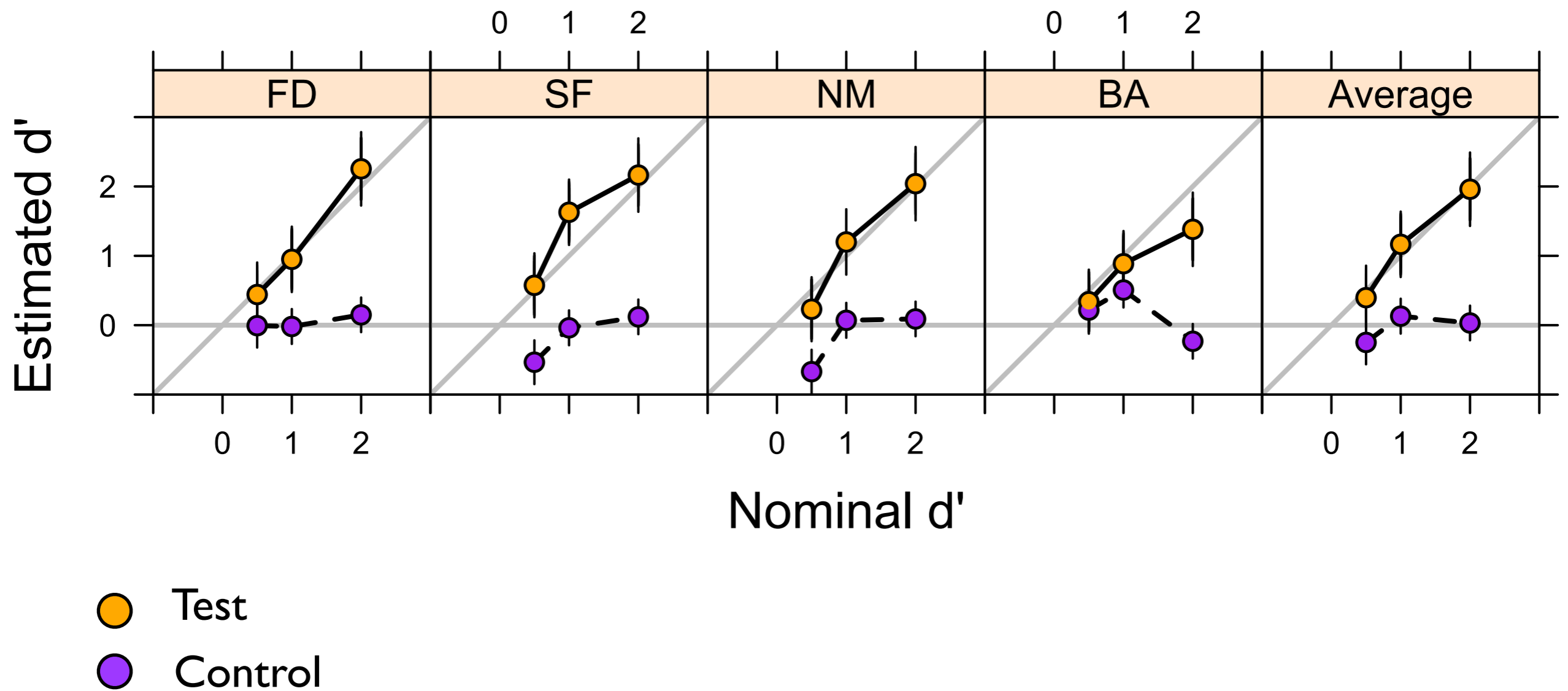
	formula:	scale:	link:	threshold:					
mod[[2]]	Rating ~ Stim	~1	probit	flexible					
mod2[[2]]	Rating ~ Stim	~Stim	probit	flexible					
	no.par	AIC	logLik	LR.stat	df	Pr(>Chisq)			
mod[[2]]	5	664.69	-327.35						
mod2[[2]]	6	665.59	-326.79	1.1047	1	0.2932			

Elevation Discrimination: ROC curves



Obs: FD

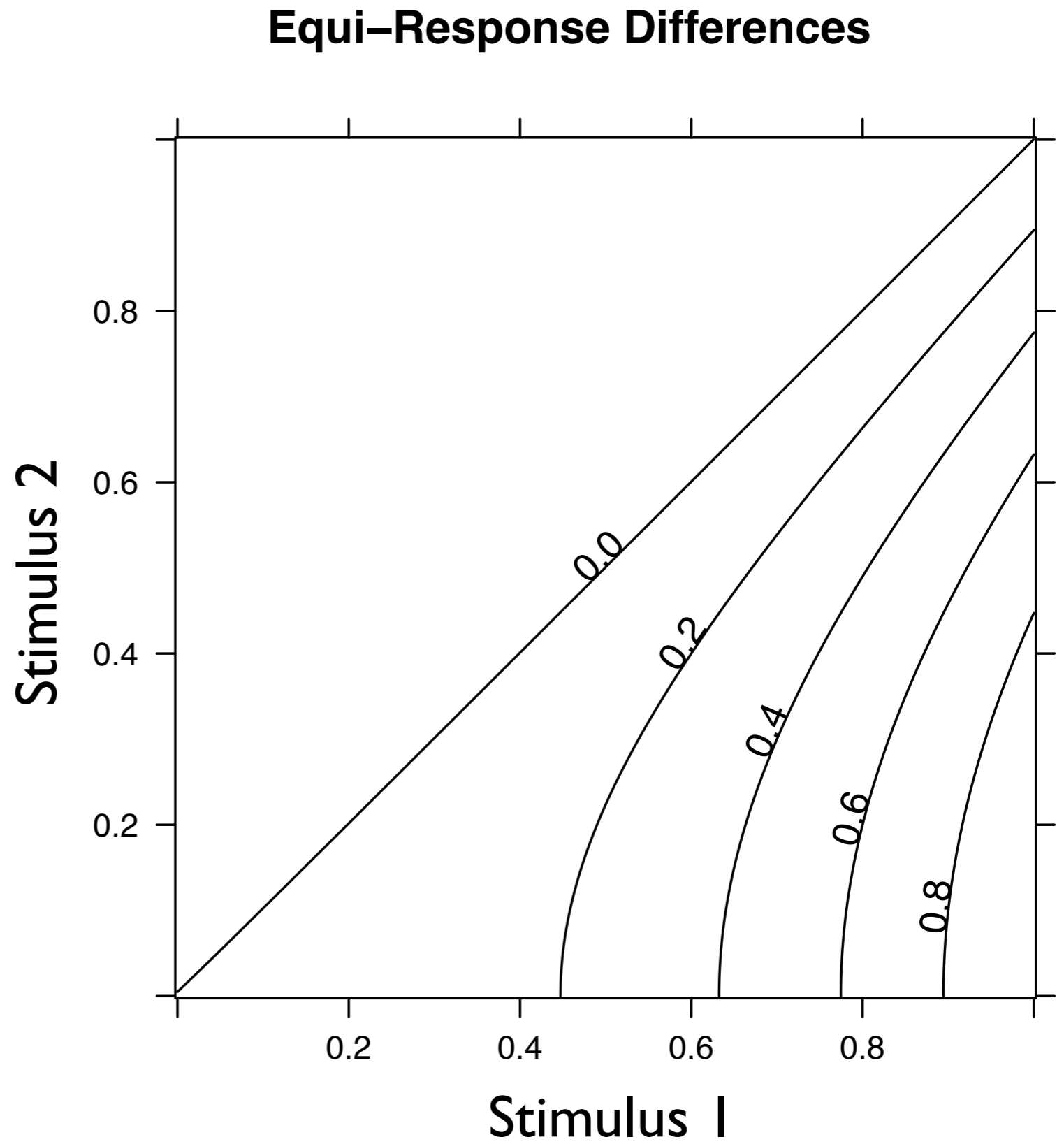
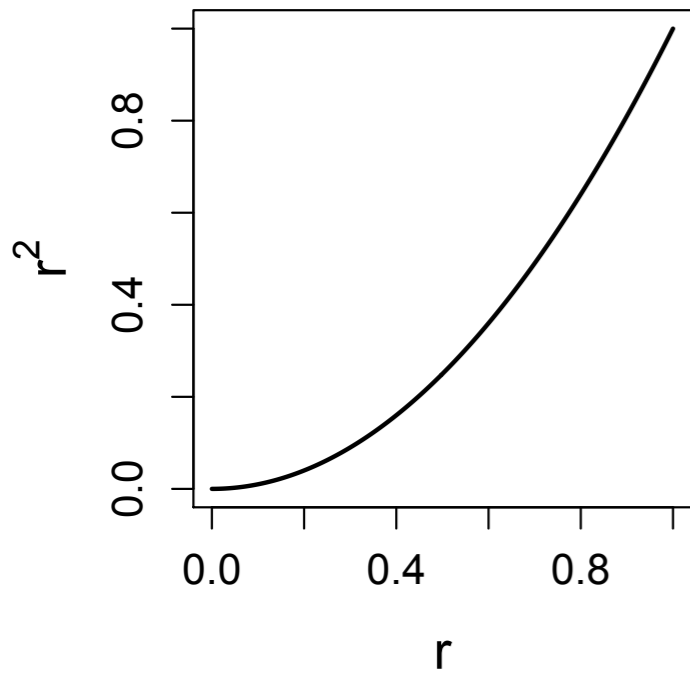
Estimated vs. Predicted d'



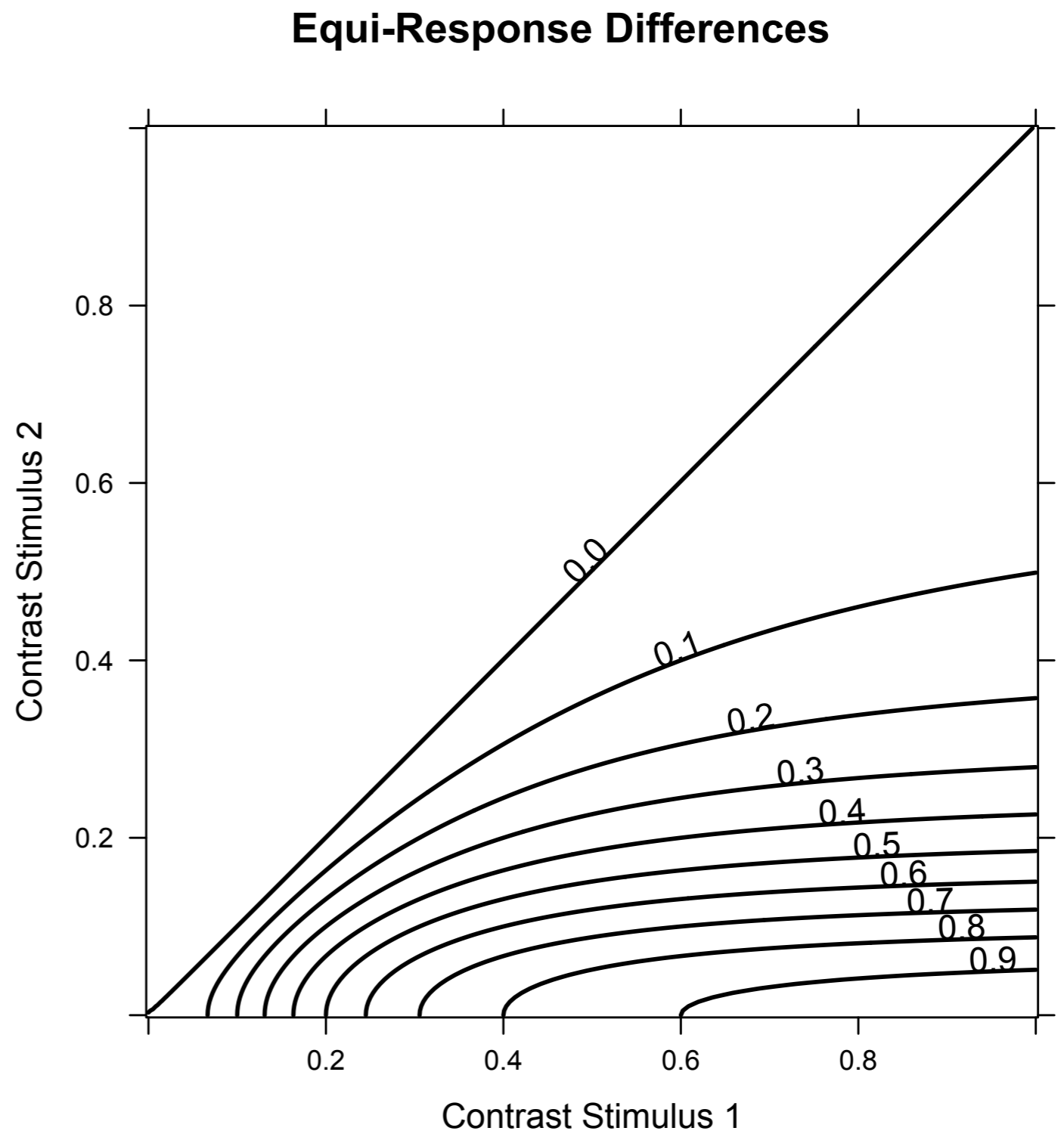
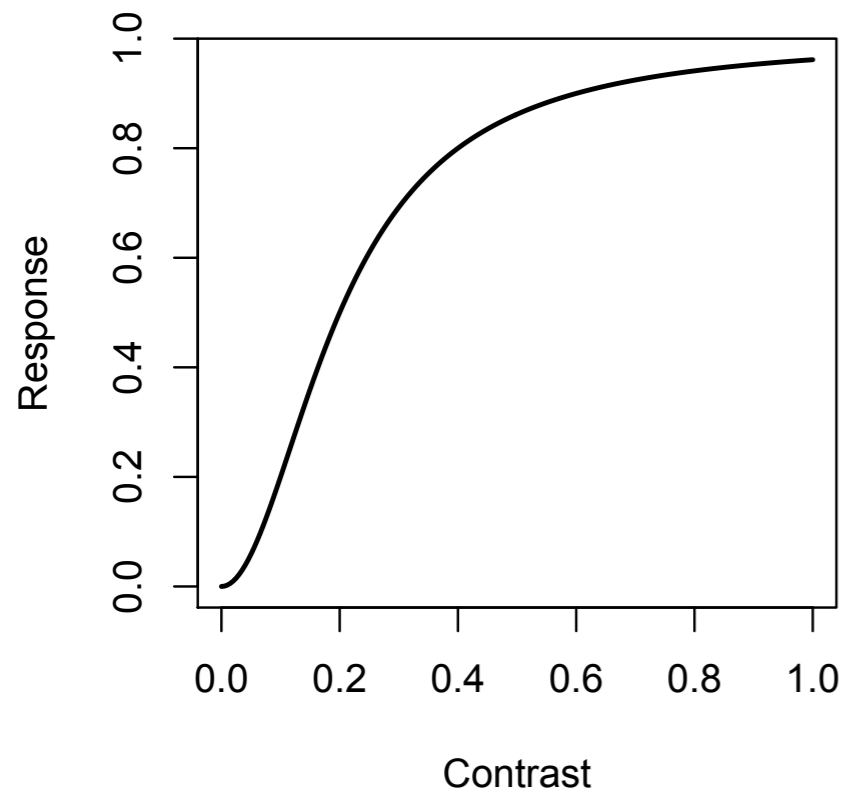
Conclusions

- 1) The signal detection model used to model modern psychophysical experiments is simply formalized as a generalized model and a GLM when the decision rule is linear.
- 2) The equal-variance, Gaussian signal detection model can be applied to measuring small stimulus differences (discrimination) as well as large ones (difference scaling, conjoint measurement, etc.).
- 3) MLDS, implemented in the package MLDS on CRAN, provides an efficient method for obtaining an interval scale describing the appearance of stimuli along a physical continuum.
- 4) We demonstrate that for a long-range, filling-in phenomena, the Watercolor Effect, the perceptual scale obtained by MLDS predicts discrimination performance for small differences, thus demonstrating that the equal-variance, Gaussian model is valid from threshold to perception.

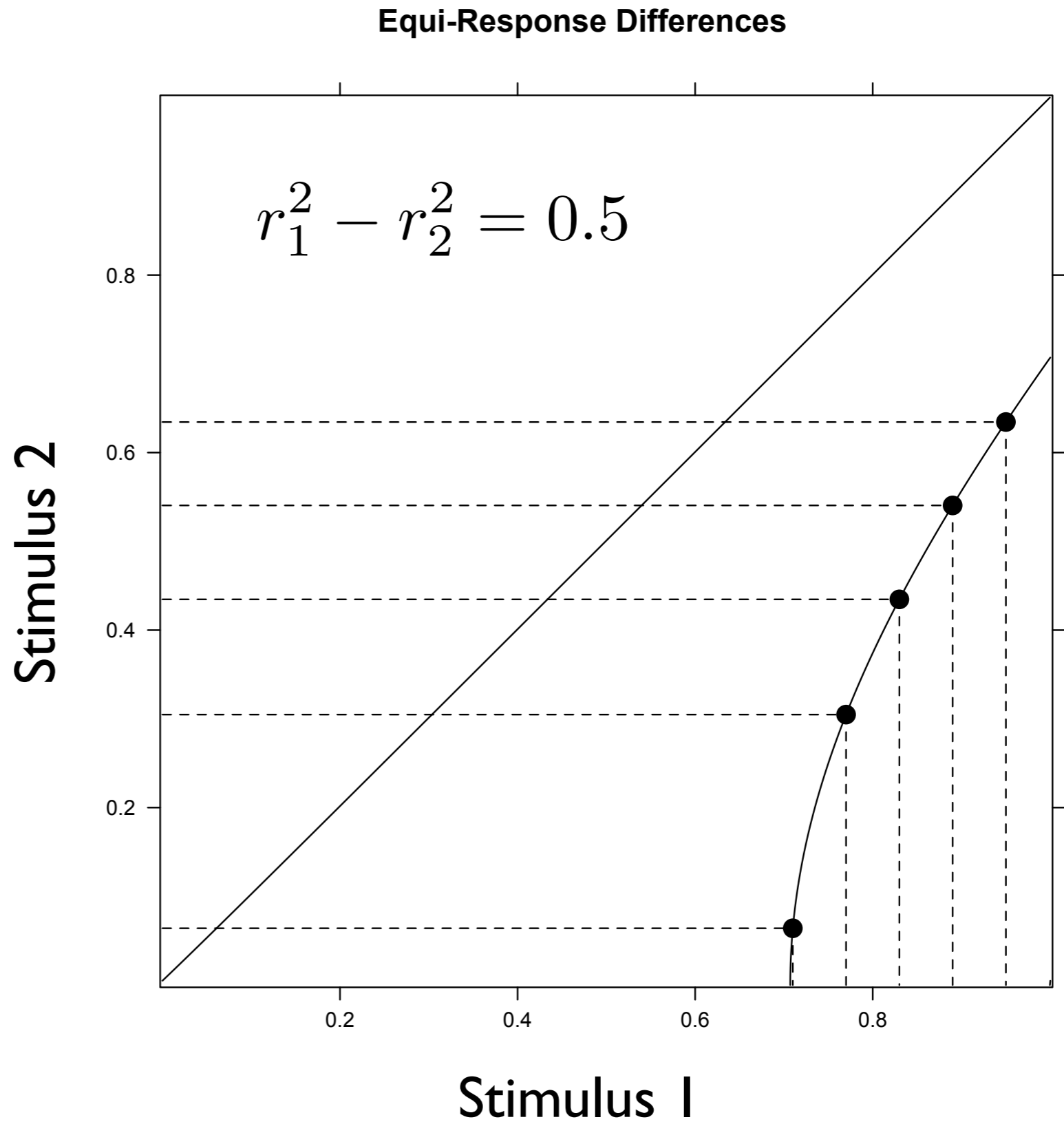
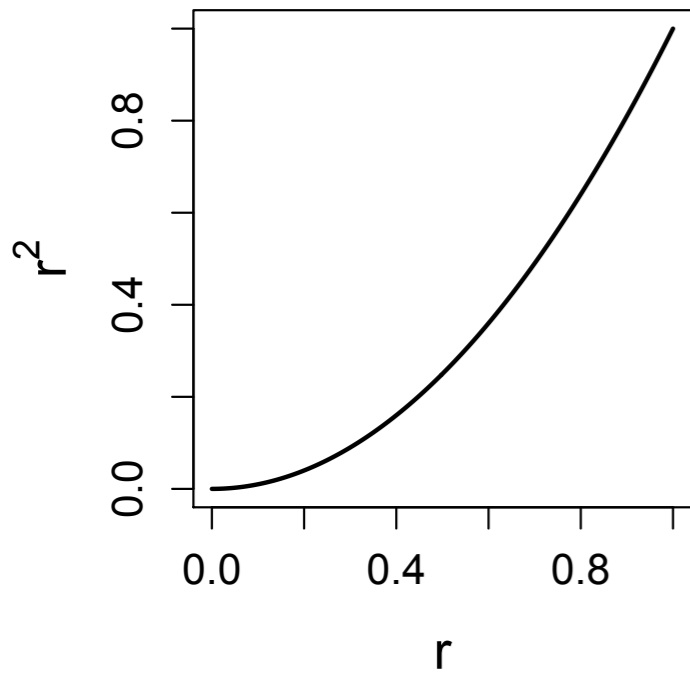
$$\psi(r) \approx r^2$$



$$\psi(c) = R_{\max} \frac{c^n}{c^n + \sigma^n}$$

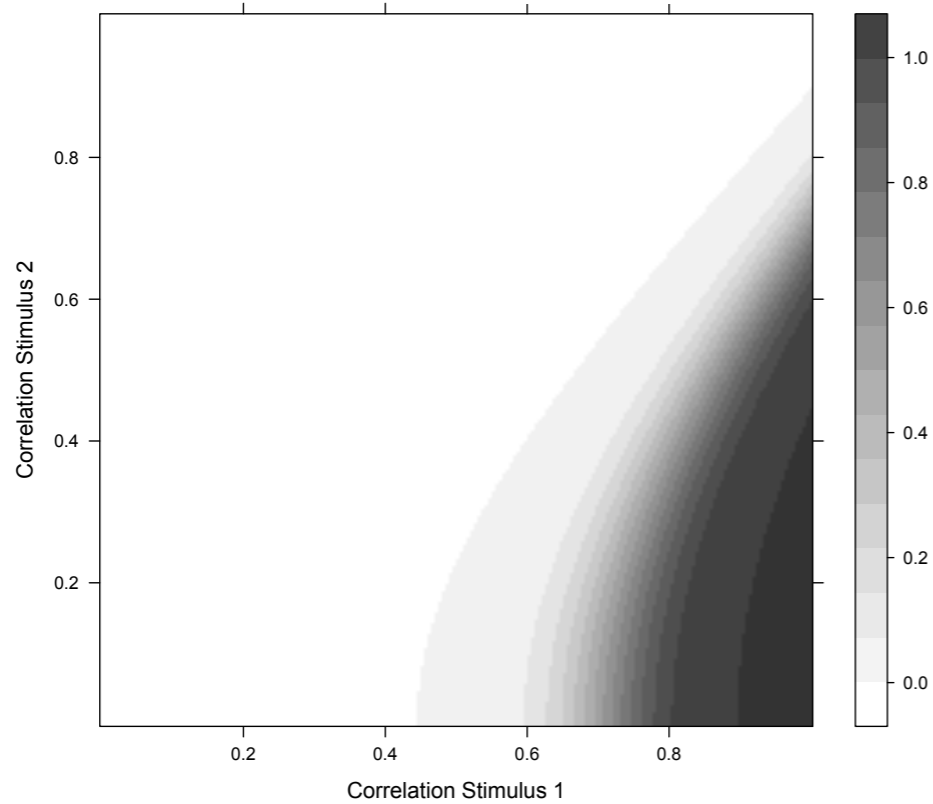


$$\psi(r) \approx r^2$$



$$r_1^2 - r_2^2 = 0.5$$

$\sigma = 0.1$



$\sigma = 0.5$

