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Error Analysis due to Laser Beams Misalignment of a Double Laser Self-Mixing Velocimeter

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Abstract. In this paper, we present a self-mixing double-head laser diode velocimeter. Analyzes are performed to evaluate the sensitivity to misalignment for this setup and calculate errors due to this misalignment. The analyses and calculations are verified by experimental results.

Keywords: Self-mixing, velocimetry, optical feedback interferometry, laser sensors.

I. INTRODUCTION

Velocity contactless measurements of moving targets like mechanical structures are often used in various industrial applications for nondestructive testing and quality control, like for example, for speed synchronization to stabilize the manufacturing process [1]. Moreover, velocity measurement processes may become crucial if it is one of the parameters governing the safety and performance of a system like in transportation.

Ultrasonic or microwave devices have a relatively poor spatial definition and optical sensors able to achieve this purpose are often too expensive, for example Laser Doppler Velocimetry (LDV) [2]. Optical feedback interferometry [3], commonly named self-mixing, is an attractive emerging solution enabling us to design low-cost laser sensors with good accuracy.

It has been proven in [4] that the robustness of a self-mixing velocimeter is increased by using a double laser diode (LD). This approach has proved to be useful when the angle between the target and the laser can not be controlled like for an on-board velocimeter for car safety [5]. However, the laser beams of the double LD velocimeter are assumed to be perfectly aligned. The purpose of this paper is to analyze errors due to laser beams misalignment of the double LD self-mixing velocimeter.

II. SELF-MIXING VELOCIMETRY

In optical feedback interferometry, the beam emitted by a laser diode is reflected by an external target in front of the LD back into the laser active cavity and interferes with the inner beam. This interference induces optical output power variations due notably to the Doppler effect [3]-[6]. Then, The target velocity $V_T$ is given by equation (1) where $\lambda$ is the emitted laser wavelength, $f_D$ is the Doppler frequency (DF) shift (fundamental frequency of the optical power signal) and $\gamma$ is the angle between the normal (N) to velocity vector and the optical propagation axis.

$$V_T = \frac{\lambda f_D}{2 \sin(\gamma)}$$  \hspace{1cm} (1)

However, with a single LD, the angle $\gamma$ must be known to determine the velocity of a target, limiting drastically the potential applications. It has been shown in [4] that better tolerance on variation $\Delta\gamma$ of angle $\gamma$ is obtained for superior values of $\gamma$ but at the usual price of a high bandwidth (high $f_D$). Therefore, a single LD velocimeter is limited for applications where $\Delta\gamma$ may vary widely. We are then proposing a double LD self-mixing velocimeter that permits to increase the potentialities of this sensing method.
III. DOUBLE LASER DIODE VELOCIMETER

The double-head LD prototype is presented in Fig. 1(a) where the two optical propagation axes are set orthogonal to improve the performance of the sensor [4]. B is the bisector of the angle (90°) between the two laser beams, N is the normal to the velocity vector \( V_T \) belonging to the plane of the two laser beams, \( \theta \) is the angle between the optical propagation axes and \( B \) (\( \theta=45^\circ \)), and \( \alpha \) is the angle between \( B \) and \( N \). Both LDs (named LD1 and LD2) are identical. \( V_T \) is given by

\[
V_T = \frac{\lambda}{2} f_{D1} \sin(\alpha - \theta) = \frac{\lambda}{2} f_{D2} \sin(\alpha + \theta)
\]  

(2)

where \( f_{D1} \) and \( f_{D2} \) are the Doppler frequency shifts of the optical power signals emitted by LD1 and LD2 respectively. Equation (2) gives

\[
\alpha = \tan^{-1}\left(\frac{f_{D2} + f_{D1}}{f_{D2} - f_{D1}} \tan \theta\right).
\]

(3)

By substituting \( \alpha \) given by (3) in (2) and with \( \theta=45^\circ \), we obtain

\[
|V_T| = \frac{\lambda}{2} \sqrt{f_{D1}^2 + f_{D2}^2} = \sqrt{V_x^2 + V_y^2}.
\]  

(4)

where \( V_x = \frac{\lambda}{2} f_{D1} \) and \( V_y = \frac{\lambda}{2} f_{D2} \) are the orthogonal projections of \( V_T \) onto the optical propagation axes \((Ax)\) and \((Ay)\) of LD1 and LD2 respectively (Fig. 1(b)).

By measuring both DFs, the direction and the module of the velocity vector \( V_T \) are estimated using (3) and (4) respectively. Note that the velocity \( V_T \) measured by the double LD velocimeter is the orthogonal projection of the velocity vector \( V \) onto the plane \((x,y)\) containing the two LDs (Fig. 1(b)).

\[
\begin{align*}
V_x & = \frac{\lambda}{2} f_{D1} \\
V_y & = \frac{\lambda}{2} f_{D2}
\end{align*}
\]

\[
\begin{align*}
V_T & = \sqrt{V_x^2 + V_y^2} \\
\theta & = \tan^{-1}\left(\frac{f_{D2} + f_{D1}}{f_{D2} - f_{D1}} \tan \theta\right)
\end{align*}
\]

Several analyses and experiments have permitted to prove the performance and robustness of the double LD velocimeter in terms of accuracy and insensitivity to angle variations of the target [4]. However, the laser beams are assumed to be coplanar and illuminating the same point on the target. Though, the laser beams are possibly not perfectly illuminating the same point and even not coplanar. In the next sections, velocity estimation errors of the double-head LD velocimeter due to laser beams misalignment are quantified for flat surface solid target in translational and rotational motion respectively.
IV. QUANTIFICATION OF ERRORS DUE TO LASER BEAMS MISALIGNMENT FOR A TARGET IN TRANSLATIONAL MOTION

When a solid target is moving with translational motion, all its points have the same velocity vector $V_T$. Thus, if the two laser beams are illuminating two different points A and B on the target (Fig. 2) then A and B have the same velocity vector $V_T$:

$$V_A = V_B = V_T$$  \hspace{1cm} (5)

Therefore, the equations (2), (3) and (4) are still valid. In this case, the measurement is not influenced by the misalignment of the laser beams. And there are no velocity estimation errors due to this misalignment.

![FIGURE 2](image)

**FIGURE 2.** Laser beams emitted by LD1 and LD2 illuminate 2 different points A and B on the target.

V. QUANTIFICATION OF ERRORS DUE TO LASER BEAMS MISALIGNMENT FOR A TARGET IN ROTATIONAL MOTION AROUND A FIXED AXIS

When a solid target is moving with a rotational motion around a fixed axis perpendicular to the target (Fig. 3), the velocity $V_A$ of a point A on the target is related to the angular velocity $w$ of the target via $V_A = r_A w$ where $r_A$ is the distance from the axis of rotation to point A, i.e. $r_A = OA$. Thus, in a rigidly rotating target, the velocity increases linearly with (perpendicular) distance from the axis of rotation. The direction of $V_A$ is perpendicular to the radius $OA$.

Therefore, velocity estimation errors may occur when the two laser beams illuminate 2 different points A and B on the rotating target. This is due to the fact that equation (2) is not valid anymore. These errors are quantified for two cases. In the first one, the laser beams are considered to be coplanar but illuminating two different points on the target. In the second one, they are not coplanar and the second one has a parallel direction to its original direction, i.e. the optical propagation axis of LD2 is translated in parallel to z-axis (Fig. 7(a)). Note that in these two cases the target is considered in a plane perpendicular to the (x,y) plane (Fig. 7(a)).

A. Coplanar Laser Beams

A. Theoretical Analysis

Laser beam emitted by LD1 illuminate a point A on the rotating target. A is situated on the z-axis which belongs to the plane of the target orthogonal to the (x,y) plane. LD2 is translated normally to its optical propagation axis in parallel to x-axis (Fig. 3(a)). The second laser beam illuminates now an alternative point noted B on the target. The 2 points A and B belong to the intersection $\langle AB \rangle$ of the target plane and the (x,y) plane containing the 2 laser beams (Fig. 3(a)). The velocities at point A and B are respectively:
\[ V_A = r_A w \quad \text{and} \quad V_B = r_B w \quad (6) \]

where \( r_A \) and \( r_B \) are the distances from the axis of rotation to point A and B respectively. Note that \( V_A \) is in the plane of the 2 laser beams contrary to \( V_B \) which is out of the (x,y) plane. The orthogonal projection \( V_{Bx} \) of \( V_B \) onto the (x,y) plane is given by
\[ V_{Bx} = V_B \sin \delta \quad (7) \]

where \( \delta \) is the angle between (OB) and (BX) (Fig. 3(b)). \( \delta \) is given by
\[ \sin \delta = \frac{r_A}{r_B} \quad (8) \]

Equations (6), (7) and (8) give
\[ V_{Bx} = r_A w = V_A. \quad (9) \]

Therefore, equation (2) is valid in this case. Hence there are no velocity estimation errors when the two laser beams are still coplanar and the target is in a perpendicular plane to the laser beams plane.

FIGURE 3. Coplanar laser beams illuminating two points A and B on a rotating target whose plane is perpendicular to the (x,y) plane of the 2 laser beams. (a) view in 3D and (b) view 2D in the plane of the target.

B. Experimental Results

The experimental set-up is composed of 2 LDs from Hitachi (HL7851G) emitting at a wavelength \( \lambda = 785 \) nm, with a maximum power of 50 mW. They are fixed on 2 orthogonal rails (i.e. \( \theta = 45^\circ \)) corresponding to x-axis and y-axis. The target is a disc rotating at a constant velocity and the angle \( \alpha \) is equal to \( 10^\circ \).

Both LDs have 2 degrees of freedom: translation in the direction of the rails and normally to the rails. Translation in the direction of the rails is used to adjust the distance (around 30 cm) between the LDs and the target. Translation normally to the rails is used to misalign the 2\textsuperscript{nd} laser beam from the original position A on the z-axis to an alternative position B (Fig. 3(a)). This translation maintains the laser beams in the same plane ((x,y) plane). The target has 2 degrees of freedom: 2 orthogonal translations in the plane of the rails used to align in the beginning the z-axis with the laser beams.

The acquisition of the self-mixing signal was performed with a 1 million-point memory scope at 2-MHz sampling frequency. Three acquisitions were taken at each position of B for \( 0 < AB < 4.5 \) cm. Target velocity is estimated using the method described in the 3\textsuperscript{rd} section.

The experimental results are showed in Fig. 4, Fig. 5 and Fig. 6. Figure 4 presents the variation of \( f_{D1} \) and \( f_{D2} \) regarding the initial measurement where both laser beams illuminate the original position A. The variations in \( f_{D1} \) are due to measurements errors. Note that the variations \( \Delta f_{D2} \) are a little bit higher than \( \Delta f_{D1} \) due to the sweeping of the
rotating disc, i.e. the disc is not perfectly rotating around its fixed axis. However, $\Delta f_{D2}$ remains lower than 1.5% of original $f_{D2}$. Figure 5 shows the original and estimated velocities function of misalignment distance $AB$. Estimated velocities vary roughly around the original one. The standard deviations and errors of these estimations are presented in Fig. 6. One can note that Mean Percentage Error (MPE), i.e. relative error in percentage between the estimated velocity and the original one, remains lower than 1%.

It can then be observed from the results showed above that the double-head LD velocimeter remains accurate for coplanar misaligned laser beams illuminating a rotating disc orthogonal to laser beams plane.

FIGURE 4. Variation of Doppler frequency measurements when the laser beams are coplanar but not illuminating the same point on the target.
FIGURE 5. Variation of velocity measurements when the laser beams are coplanar but not illuminating the same point on the target.

FIGURE 6. (a) Standard deviations, (b) mean squared error and (c) mean percentage error of velocity estimations for coplanar laser beams not illuminating the same point on the target.

B. Non Coplanar Laser Beams

A. Theoretical Analysis

Now, LD2 is translated normally to its optical propagation axis in parallel to z-axis (Fig. 7(a)). The 2 points A and B are on the z-axis. In this case, the velocity vectors $V_A$ and $V_B$ are in the (x,y) plane. We define $\delta r$ as the difference between the distances from the axis of rotation to point B and A:
And the variation of $V_B$ regarding $V_A$ is given by

$$\delta V = V_B - V_A = \delta r \omega.$$  \hspace{1cm} (11)

$\delta V$ induces a variation $\delta f_{D2}$ in $f_{D2}$ estimation:

$$\delta f_{D2} = \frac{2 \delta V \sin(\alpha + \theta)}{\lambda}$$  \hspace{1cm} (12)

Then the measured $f_{D2}$ is shifted by $\delta f_{D2}$ versus the actual one corresponding to velocity $V_A$. The velocity measured by the double LD velocimeter is in this case:

$$V_T = \frac{\lambda}{2} \sqrt{f_{D1}^2 + (f_{D2} + \delta f_{D2})^2}$$  \hspace{1cm} (13)

The relative error of the measured velocity $V_T$ regarding the actual velocity $V_A$ is given by

$$\frac{V_T - V_A}{V_A} = \sqrt{1 + \frac{\delta f_{D2}^2 + 2 f_{D2} \delta f_{D2}}{f_{D1}^2 + f_{D2}^2}} - 1$$  \hspace{1cm} (14)

Then, if the second laser beam illuminates a point $B$, the velocity estimation error is the same than in $B$. Therefore the velocity estimation error depends only of $\delta r$ according to $z$-axis (Fig. 7(b)).

Figure 8 presents the relative error of velocity estimations function of relative misalignment $\delta r/r_A$ of the second laser beam and angle $\alpha$. $\delta r/r_A = -1$ corresponds to when the second laser beam illuminates the centre of rotation $O$, i.e. $B$ is coincident with $O$, $\delta r/r_A = 0$ when the 2 laser beams are ideally aligned in $A$, i.e. $B$ is coincident with $A$ and $\delta r/r_A = 1$ when $r_B = 2 r_A$. For $\delta r/r_A$ around 0, i.e. when the two laser beams are almost aligned, velocity estimations error is low. Then it increases with the absolute value of $\delta r/r_A$ to attend roughly 90% for $\delta r/r_A = \pm 1$ and $\alpha = 40^\circ$.  

\[ \text{FIGURE 7. Non coplanar laser beams illuminating two points A and B on a rotating target whose plane is perpendicular to the (x,y) plane of the 2 laser beams. (a) view in 3D and (b) view 2D in the plane of the target.} \]
Note that higher errors are obtained for higher values of $\alpha$. This is due to that velocity estimation is more correlated to $f_{D2}$ than to $f_{D1}$ for high values of $\alpha$ and contrary for low values of $\alpha$.

### B. Experimental Results

The same experimental set-up as the previous section is used. But now, the LDs second degree of freedom turn into translation up/down normally to the rails, i.e. in parallel to z-axis. It is used to misalign the 2nd laser beam from its original position $A$ ($r_A = OA \approx 1.4 \text{ cm}$) on the z-axis to an alternative position $B$ on the z-axis too (Fig. 7(a)). In this case, the laser beams are not coplanar anymore.

The acquisition of the self-mixing signal was performed with a 1 million-point memory scope at 2-MHz sampling frequency. Three acquisitions were taken at each position of $B$ for $-0.5 < AB < 3 \text{ cm}$.

The experimental results are showed in Fig. 9 and Fig. 10. Figure 9 presents the variation of measured and calculated $f_{D1}$ and $f_{D2}$ (using equation (12)), i.e. $f_{D2} = \text{original } f_{D2} + \delta f_{D2}$, regarding the initial measurement where both laser beams illuminate the original position $A$ ($AB = 0$). The variations in $f_{D1}$ are due to measurement errors. Note that (a) calculated $f_{D1}$ are equal to original $f_{D1}$ because, theoretically, $f_{D1}$ remains equal to its original value and (b) calculated and measured $\Delta f_{D2}$ increase linearly with the misalignment distance $AB$ ($AB = \delta r$). Figure 10(a) shows the original, estimated and calculated velocities (using equation (13)) function of misalignment distance $AB$. Mean percentage error of estimated velocities and MPE calculated using equation (14) are presented in Fig. 10(b). They increase almost linearly with $AB$. The results presented in Fig. 9 and Fig. 10 show the accordance of the measurements with calculations.

It can then be observed from the results showed above that the double-head LD velocimeter is not accurate for non coplanar laser beams illuminating a rotating disc orthogonal to laser beams plane. Errors due to this misalignment have been theoretically calculated and approved with experimental measurements. They depend of the misalignment distance and the angle $\alpha$.

![FIGURE 8](image_url) Relative error in percentage between the measured velocity and the actual velocity.(a) for $-1 < \delta r/r_A < 0$ and (b) for $0 < \delta r/r_A < 1$.

### VI. Conclusion

In this paper, a robust double-head LD self-mixing sensor was presented. Errors due to laser beams misalignment were analyzed for flat surface solid target in translational and rotational motion respectively. For a target in translational motion, analyzes demonstrated that the double LD velocimeter is insensitive to laser beams misalignment. Moreover, the velocimeter is insensitive to coplanar laser beams misalignment for a rotating target orthogonal to laser beams plane. However, when the laser beams are not coplanar, measurements errors result from this misalignment. These errors have been quantified.

Experimental results validated the theoretical analyses and calculations. Additional analysis and experiments may be performed to quantify the errors due to laser beams misalignment for alternative orientations and motions of the target.
FIGURE 9. Variation of Doppler frequency measurements when the laser beams are not coplanar.

FIGURE 10. (a) Original, calculated and estimated velocities and (b) mean percentage error of velocity estimations for non coplanar laser beams.

REFERENCES