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Abstract: Previous literature has identified oil and gas prices as being the main drivers of CO_2 prices in a univariate GARCH econometric framework (Alberola *et al.* (2008), Oberndorfer (2009)). By contrast, we argue in this article that the interrelationships between energy and emissions markets shall be modeled in a vector autoregressive and multivariate GARCH framework, so as to reflect the dynamics of the correlations between the oil, gas and CO_2 variables overtime. Using BEKK, CCC, and DCC-MGARCH models on daily data from April 2005 to December 2008, we highlight significant own-volatility, cross-volatility spillovers, and own persistent volatility effects for nearly all markets, indicating the presence of strong ARCH and GARCH effects. Besides, we provide strong empirical evidence of time-varying correlations in the range of [-0.3;0.3] between oil and gas, [-0.05;0.05] between oil and CO_2, and [-0.2;0.2] between gas and CO_2, that have not been considered by previous studies. These findings are of interest for traders and utilities in the energy sector, but also for a broader applied economics audience.

JEL Classification: Q48; Q57; Q58.

Keywords: Oil; Gas; CO_2; EU ETS; Vector Autoregression; Multivariate GARCH; Time-Varying Correlation; BEKK-MGARCH Model; CCC-MGARCH Model; DCC-MGARCH Model.

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1 Introduction

Statistical analyses of price determinants in the EU Emissions Trading Scheme (EU ETS) have so far relied on univariate GARCH[2] models to explain the interrelationships between energy and emissions markets. Based on a standard GARCH(1,1) model, Alberola *et al.* (2008) identify oil and gas as being the main CO_2 price drivers. High (low) energy prices contribute to an increase (decrease) of CO_2 prices[3]. Oberndorfer (2009) develops an empirical analysis of stock market effects in the EU ETS. Based also on a GARCH(1,1) model with oil, gas, and electricity volatility variables in the variance equation, the author identifies a positive relationship between CO_2 prices and stock returns in the electricity sector.

This article focuses on the contemporaneous interrelationships between energy (oil, gas) and emissions (CO_2) markets[4]. The directions of these links are in essence complex to capture. The EU ETS is a commodity market, which is influenced by other factors as well, notably fuel shifts and energy efficiency. The CO_2 price is determined by the demand and supply of CO_2 rights - a surplus induced by above-mentioned measures or reduced energy demand would reduce the CO_2 cost. There is a natural correlation however which comes to mind between oil, gas, and CO_2: this explaining factor is the economy. Macroeconomic conditions indeed influence all commodity markets (Caballero et al. (2008)). If declining oil prices do not reduce *per se* the CO_2 cost, a recession reduces both oil demand and industrial activities, and hence CO_2 emissions that reduce the demand for CO_2 rights, and ultimately the CO_2 price.

Of course, the full nature of the price and volatility interrelationships between the oil, gas, and CO_2 markets needs to be assessed with adequate econometric tools. It is fair to assume that gas or oil prices may be affected by the EU allowance market, which makes a multivariate approach necessary. As shown by Hsu Ku (2008) for the major equity and currency markets in the US, Japan, and the UK, transmission effects between markets and obvious time-varying correlations may be adequately captured by Multivariate GARCH models (MGARCH). This econometric technique has also been recently applied by Leeves (2008) to the flow rates of US workers between employment and unemployment to investigate links between flow-rates volatilities. Both studies reveal that substantial links and adjustment dynamics may be uncovered using a multivariate econometric analysis, that we propose to adapt in this article to energy and emissions markets.

Compared to previous literature, we adopt a *multivariate* econometric framework which al-

[2] A GARCH(p,q) model stands for a General Autoregressive Conditional Heteroskedasticity model of autoregressive order p and moving average q.

[3] In a somewhat different setting, Kanen (2006) identifies brent prices as the main driver of natural gas prices which, in turn, affect power prices and ultimately CO_2 prices. Bunn and Fezzi (2009) also identify econometrically that CO_2 prices react significantly to a shock on gas prices in the short term.

[4] A disclaimer is necessary: only the European emissions market can be analysed and is analysed here.

lows to analyze the *time-varying* correlations between the oil, gas and CO_2 price series. While previous literature relied only on univariate GARCH models, the class of MGARCH models allows to capture the *dynamics* of variance and covariance overtime. We are primarily interested in identifying cross innovations and volatility spillover effects between energy and emissions markets. We also investigate the persistence of shocks overtime, and whether this persistence is more marked for own market innovations, or for cross markets innovations. The results obtained will thus be of particular importance for traders, financial institutions, and regulated utilities that lack a precise identification of correlations between energy and emissions markets for hedging and risk-management purposes.

As in Hsu Ku (2008), we use both Vector Autoregressive VAR and MGARCH models as part of our estimation strategy. Our study period goes from April 22, 2005 to December 15, 2008[5]. Based on various specifications of multivariate GARCH models (BEKK, CCC, DCC)[6], we are able to highlight the transmission of price volatility among the three markets. Own volatility and cross volatility spillovers are significant for nearly all markets, indicating the presence of strong ARCH and GARCH effects. Strong own persistent volatility effects are also evident in all markets. Our main finding states that BEKK and constant-correlation MGARCH models are insufficient to assess the time-varying correlations between energy and emissions markets. The DCC MGARCH model provides the best results to examine the relationship between volatility and correlation. We find time-varying correlations in the order of [-0.3;0.3] between oil and gas, [-0.05;0.05] between oil and CO_2, and [-0.2;0.2] between gas and CO_2. To further elaborate on the meaning and the importance of the results, we can identify one major explaining factor behind such correlations between oil, gas, and CO_2 prices: the macroeconomy.

The remainder of the article is organized as follows. Section 2 briefly reviews VAR and MGARCH modeling. Section 3 summarizes the data used. Section 4 presents the estimation results. Section 5 concludes.

2 Review of Vector Autoregressive and Multivariate GARCH Models

When modeling the interrelationships between oil, gas and CO_2 prices, several choices arise in the empirical estimation strategy. The time-series may be studied independently as univariate time-series, each characterized by its own mean and autocovariance function. Alberola *et al.* (2008) and Oberndorfer (2009) have followed this approach, by including in the univariate

[5] *i.e.* from the start of CO_2 trading on exchanges until the expiration date of the CO_2 futures contract of maturity December 2008.

[6] As detailed below, BEKK stands for the Baba-Engle-Kraft-Kroner model, CCC for the constant conditional correlation model, and DCC for the dynamic conditional correlation model.

time-series of CO_2 prices exogenous regressors such as oil and gas prices.

Such an approach, however, fails to take into account the possible dependence *between* the time-series, which may be of great importance for understanding the observed values of the time-series. This perspective leads us to consider a vector of oil, gas and CO_2 prices whose conditional covariance matrix evolves through time. Let us start with a brief review of VAR and MGARCH models.

2.1 Vector Autoregressive models

Following Sims (1980), a basic VAR model consists of a set of K endogenous variables $y_t = (y_{1t}, \ldots, y_{kt}, \ldots, y_{Kt})$ for $k = 1, \ldots, K$. A VAR(p) process may thus be defined as:

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t \tag{1}$$

where A_i are $(K \times K)$ coefficient matrices for $i = 1, \ldots, p$ and u_t is a K-dimensional process with $E(u_t) = 0$ and time-invariant positive definite covariance matrix $E(u_t u_t^T) = \sum_u$. For a given sample of $y_1, \ldots, y_T$ and sufficient pre-sample values $y_{-p+1}, \ldots, y_0$, the coefficients of a VAR(p) process can be estimated by OLS separately for each price series.

The main interest behind VAR modeling consists in generating stationary time-series with time invariant means, variances and covariance structure, given sufficient starting values. Besides, VAR models work quite well in many of the financial and econometric applications. Fitting a VAR model to energy and emissions markets thus appears as a natural extension of this methodology in line with our research question. Next, we recall various specifications of MGARCH.

2.2 Multivariate GARCH models

Consider k time-series of return innovations $\{X_{i,t}, i = 1, \ldots, k\}$. Stacking these innovations into a vector $\mathbf{X}_t$, we define $\sigma_{ii,t} = var(X_{i,t} \mid \Im_{t-1})$ and $\sigma_{ij,t} = cov(X_{i,t}, X_{j,t} \mid \Im_{t-1})$. We note $\Sigma_t = \sigma_{ij,t}$ the conditional variance-covariance matrix of all the time-series.

The main difficulty encountered with Multivariate GARCH modeling lies in finding a suitable system that describes the dynamics of Σ_t parsimoniously. Besides, the multiple GARCH equation needs to satisfy the positive definiteness of Σ_t, which is a numerically difficult problem. Finally, the number of parameters to be estimated increases very rapidly as the dimension of the time-series increases, which can take a very long time during the numerical implementation. To address these questions, we detail below three parametric formulations for the structure of the conditional covariance matrices.

2.2.1 BEKK MGARCH models

The first class of multivariate GARCH models that we study stems from the contributions of Bollerslev, Engle, and Wooldridge (1988), who provided with the VEC-GARCH model a straightforward extension of univariate GARCH models. We examine more particularly the following Baba-Engle-Kraft-Kroner (BEKK, Engle and Kroner (1995)) MGARCH model:

$$H_t = CC' + \sum_{j=1}^{q}\sum_{k=1}^{K} A'_{kj} r_{t-j} r'_{t-j} A_{kj} + \sum_{j=1}^{p}\sum_{k=1}^{K} B'_{kj} H_{t-j} B_{kj} \tag{2}$$

where A_{kj}, B_{kj}, and C are $N \times N$ parameter matrices, and C is lower triangular to ensure the positive definiteness of H_t. Note the BEKK model is covariance stationary if and only if the eigenvalues of $\sum_{j=1}^{q}\sum_{k=1}^{K} A_{kj} \otimes A_{kj} + \sum_{j=1}^{p}\sum_{k=1}^{K} B_{kj} \otimes B_{kj}$ are less than one in modulus, with $\otimes$ the notation for Kronecker products. Due to the computational burden involved by the estimation of a full BEKK model[7], we restrict the number of parameters by implementing the following "diagonal BEKK" MGARCH model:

$$H_t = CC' + A' r_{t-1} r'_{t-1} A + DE[A' r_{t-1} r'_{t-1} A \mid \Im_{t-2}]D \tag{3}$$

In eq(3), we now model the conditional variances and covariances of certain linear combinations of the vector of price returns r_t.

2.2.2 CCC MGARCH models

The second class of multivariate GARCH models examined is based on the decomposition of the conditional covariance matrix into conditional standard deviations and correlations. In such Constant Conditional Correlation (CCC) MGARCH models (Bollerslev (1990)), the conditional correlation matrix is time-invariant and the conditional covariance matrix may be written as follows:

$$H_t = D_t P D_t \tag{4}$$

where $D_t = diag(h_{1t}^{1/2}, ldots, h_{Nt}^{1/2})$ and $P = [\rho_{ij}]$ is positive definite with $\rho_{ii} = 1, i = 1, \ldots, N$. Off-diagonal elements of the conditional covariance matrix are computed as:

$$[H_t]_{ij} = h_{it}^{1/2} h_{jt}^{1/2} \rho_{ij}, i \neq j \tag{5}$$

[7] As the number of parameters $(p+q)KN^2 + N(N+1)/2$ increases, it may be difficult to obtain convergence during the estimation.

where $1 \leq i,j \leq N$. The conditional variances of r_{it} processes are similar to univariate GARCH(p,q) models:

$$h_t = \omega + \sum_{j=1}^{q} A_j r_{t-j}^2 + \sum_{j=1}^{p} B_j h_{t-j} \tag{6}$$

with ω a $N \times 1$ vector, A_j and B_j diagonal $N \times N$ matrices, and $r_t^2 = r_t \odot r_t$. When the conditional correlation matrix P is positive definite and the elements of ω and the diagonal elements of A_j and B_j positive, the conditional covariance H_t is positive definite.

2.2.3 DCC MGARCH models

Due to the possibly overly restrictive assumption of constant conditional correlations, we consider a third class of multivariate GARCH models which attempts at making the conditional correlation in eq(4) time-varying:

$$H_t = D_t P_t D_t \tag{7}$$

According to Engle's (2002) Dynamic Conditional Correlation (DCC) MGARCH model, we introduce the following dynamic matrix process:

$$Q_t = (1 - a - b)S + a\epsilon_{t-1}\epsilon'_{t-1} + bQ_{t-1} \tag{8}$$

with a and b respectively positive and non-negative scalar parameters such that $a + b < 1$, S the unconditional correlation matrix of the standardized errors ϵ_t, and Q_0 is positive definite. To produce valid correlation matrices, Q_t needs to be re-scaled as follows:

$$P_t = (I \odot Q_t)^{-1/2} Q_t (I \odot Q_t)^{-1/2} \tag{9}$$

Having detailed the VAR and MGARCH modeling on which our empirical estimation strategy hinges, we present in the next section the data used.

3 Data

We study three time-series of oil, gas, and CO_2 daily closing prices. Our study period goes from April 22, 2005 to December 15, 2008 which corresponds to a sample of 936 observations. The source of the data is the European Climate Exchange (ECX), Bloomberg and Reuters.

3.1 Oil and Gas Prices

We use the daily NYMEX Crude Oil Futures traded in \$/barrel, and the daily Zeebrugge Natural Gas Next Month contract traded in €/MWh. Price series are converted to € using the daily exchange rate provided by the European Central Bank.

Insert Table 1 about here

Descriptive statistics for oil and gas raw price series, log-returns, VAR and MGARCH residuals may be found in Table 1. The distributional properties of the oil and gas raw price series appear non-normal. The oil and gas markets are positively skewed and since the kurtosis (or degree of excess) in both of these energy markets exceeds three, a leptokurtic distribution is indicated.

Insert Figure 1 about here

Figure 1 presents the price development for the Zeebrugge natural gas next month, and NYMEX crude oil futures price series from April 22, 2005 to December 15, 2008. In November 2005 and September 2008, natural gas prices soared to 90 €/MWh, and steadily decreased afterwards to 40 €/MWh in February 2008 and December 2008. The brent price series peaked over 80 €/barrel from May to August 2008.

3.2 CO_2 Price

For CO_2 prices, we use daily futures prices for the December 2008 contract traded in €/ton of CO_2 on ECX. In Figure 1, we observe that 2008 CO_2 futures prices convey a coherent price signal - around 20 €/ton of CO_2 - throughout the historical available data during Phase II (2008-2012) of the EU ETS. The futures price development features a lower bound around 15 €/ton of CO_2 in April 2007, and an upper bound around 35 €/ton of CO_2 in November 2008[8].

Descriptive statistics of the ECX futures contract of maturity December 2008 are presented in Table 1. We observe that the ECX December 2008 futures contract presents nonzero skewness and excess kurtosis[9]. These summary statistics also reveal a "fat tailed" leptokurtic distribution.

Insert Figure 2 about here

[8] Therefore, Phase II futures proved to be much more reliable than futures prices for delivery during Phase I (2005-2007) due to the banking restrictions enforced between 2007 and 2008 (Alberola and Chevallier (2009)).

[9] Note for a normally distributed random variable skewness is zero, and kurtosis is three.

To sum up, none of the raw time-series under consideration may be approximated by the normal distribution. Oil, gas and CO_2 log-returns are presented in Figure 2. In the next section, we present our estimation results.

4 Estimation Results

This section contains the estimation results for the VAR and MGARCH modeling of oil, gas, and CO_2 prices, denoted by $(OIL_t, GAS_t, CO2_t)'$.

4.1 VAR Results

In this section, we conduct a preliminary data analysis by applying a VAR(p) process to the returns of each price series (see Hsu Ku (2008) for a similar approach). The stability of VAR(p) processes appears indeed useful to ensure that the variables under consideration are stationary. First, the raw time-series plots are shown in Figure 1. A visual inspection indicates nonstationarity in each of the series, so we proceed by taking differenced natural logs, which allows convenient interpretation by means of approximate percentage change. The time-series plots of log-returns, given in Figure 2, appear stationary. Second, to confirm this diagnostic, we conduct unit root tests by applying the Augmented Dickey-Fuller (ADF) test regressions. We use the oil, gas, and CO_2 price series transformed into log-returns.

Insert Table 2 about here

The results of ADF tests are summarized in Table 2. These tests confirm that the original data are considered nonstationary. The oil, gas and CO_2 price series are stationary when taken in logarithmic first-difference transformation. It can be concluded that all time-series are integrated of order one (*I(1)*). After taking this nonstationarity into account, we need to determine the optimal length for an unrestricted VAR (with a maximal lag number of eight).

Insert Table 3 about here

Those results are reported in Table 3. All criteria unambiguously point out an optimal lag order $p = 1$. Thus, we choose the most parsimonious specification of a VAR(1) model. We can therefore proceed to fit a vector autoregressive model to the tri-variate log-returns of the time-series $(OIL_t, GAS_t, CO2_t)'$, which has a length of 936 observations. The VAR(p) model equation is:

$$\begin{bmatrix} OIL_t \\ GAS_t \\ CO2_t \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{bmatrix} \begin{bmatrix} OIL_{t-1} \\ GAS_{t-1} \\ CO2_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{bmatrix}$$

Insert Table 4 about here

The estimated parameters and robust standard errors are given in Table 4, along with diagnostic tests. The VAR(1) model has 12 parameters, of which 3 are significant. Note that the gas variable is negatively impacted by its own lag, while the CO_2 variable is positively impacted by the gas variable and its own lag at the 1% significance level. According to the diagnostic tests shown at the bottom of Table 4, there is no significant autocorrelation left in the residuals of this VAR(1) model[10]. However, there is significant autocorrelation in the three series of squared residuals, which indicates the necessity to use a Multivariate GARCH model for further analysis.

Insert Figure 3 about here

Besides, OLS-CUSUM tests (based on cumulated sums of OLS residuals against a single-shift alternative, see Kramer and Ploberger (1992)) for the presence of structural changes in the components of the VAR(1) model are shown in Figure 3. We reject the null that the process should have a peak around the breakpoint for all time-series. For all variables, the empirical fluctuation processes stay safely within their bounds.

In the next step of our empirical strategy, we proceed by fitting a suitable MGARCH model to the residuals $(\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t})'$ of the VAR(1) model for the oil, gas, and CO_2 variables.

4.2 MGARCH Results

In this section, we discuss first some issues concerning the estimation of the MGARCH models presented in Section 2.2, and second we present the results obtained for each class of model.

4.2.1 Estimation practicalities

In the unrestricted BEKK MGARCH model, too large values of K yield to an identification problem because several parameterizations yield the same representation of the model. To overcome these numerical difficulties, it is generally assumed that $p = q = K = 1$ in the application of eq(2). Such restrictions may be found in Kroner and Ng (1998), where $B = \delta A$ and $\delta > 0$ is a scalar. The estimation of the "diagonal BEKK" model for the three time-series is carried out maximizing the log-likelihood, assuming that residuals are Gaussian white noise[11].

[10] To conserve space, the autocorrelation function (ACF) for the residuals and squared residuals are not reproduced in the article, and may be obtained upon request to the authors.
[11] *i.e.* the log-likelihhod is computed on the basis of the normal distribution.

The estimation of CCC MGARCH models offers on the contrary a computationally attractive parameterization. Besides the univariate GARCH equations, the number of parameters needing to be estimated is equal to $N(N-1)/2$. Covariance stationarity is ensured if the roots of $det(I - \sum_{j=1}^{q} A_j\lambda^j - \sum_{j=1}^{p} B_j\lambda^j)$ lie outside the unit circle.

In the DCC MGARCH model, positive definiteness of H_t in eq(7) is ensured if the conditional correlation matrix P_t is positive definite at each point in time, in addition to having well-defined conditional variances $h_{it,i=1,\dots,N}$. This leads again to computationally demanding estimation procedures, as the correlation matrix has to be inverted for each t during every iteration.

In what follows, we set $p = 1$ and $q = 1$ for each class of MGARCH model. The BHHH algorithm (Berndt et al. (1974)) is used to produce quasi maximum likelihood parameter estimates and their corresponding asymptotic robust standard errors.

4.2.2 BEKK MGARCH results

Now we proceed to identifying a tri-variate BEKK(1,1) MGARCH model to the residuals of the VAR(1) model. The model follows the equations:

$$\epsilon_t = \mathtt{H}_t^{1/2}\,\nu_t, \quad \mathtt{H}_t = \mathtt{C}'\mathtt{C} + \sum_{i=1}^{q} \mathtt{A}_i'\epsilon_{t-i}\epsilon_{t-i}'\mathtt{A}_i + \sum_{j=1}^{p} \mathtt{B}_j'\mathtt{H}_{t-j}\mathtt{B}_j \tag{10}$$

with $(Z_t) = (W_t, X_t, Y_t)'$ the time-series of oil, gas, and CO_2 variables, $\mathtt{H_t}$ the conditional covariance matrix of Z_t, $cov(Z_t \mid \Im_{t-1}) = \mathtt{H_t}$, $\{\mathtt{C}, \mathtt{A_i}, \mathtt{B_j}\}$ the parameter matrices, ν_t a three-dimensional white noise with covariance matrix $cov(\nu) = \mathtt{I}_n$, and $\mathtt{I}_n$ the unity matrix of order n.

Insert Table 5 about here

The estimated parameters, together with their robust standard errors in parenthesis, are shown in Table 5. The BEKK(1,1) MGARCH model has 27 parameters, of which 16 are significant. The maximum eigenvalue is 0.86901. The coefficients for the variance-covariance equations are generally significant for own- and cross-innovations, and significant for own- and cross-volatility spillovers in the individual price series of oil, gas and CO_2, indicating the presence of strong ARCH and GARCH effects. In evidence, 89% (8 out of 9) of the estimated ARCH coefficients, and 44% (4 out of 9) of the estimated GARCH coefficients are significant at the 1% level.

Own-innovation spillovers in the energy and emissions markets are large and significant, indicating the presence of strong ARCH effects. The own-innovation spillover effects range from -0.55 for gas to 0.58 for CO_2. In terms of cross-innovation effects, past innovations in most markets exert an influence on the other energy and/or emissions markets. For example, in the case of the CO_2 market cross-innovations, the oil market is significant with a coefficient of -0.15. The exception to the presence of cross-innovation effects is gas on the CO_2 market.

In the GARCH set of parameters, 44% of the estimated coefficients are significant. The lagged volatility spillover effects for oil, gas, and CO_2 are equal to, respectively, -0.84, -0.29, and -0.64. This means that past volatility shocks in each individual market have a greater effect on their own future volatility than the past volatility shocks in the other energy/emissions markets. In terms of cross-volatility for the GARCH parameters, the only significant parameter appears to be oil on the CO_2 market. That is, past volatility shocks in the oil market have the greatest effect on the future volatility of the CO_2 market. The latter result is in line with previous literature, which highlighted the predominant role of oil price changes in driving CO_2 price changes. We are able to confirm this effect in a multivariate and dynamic econometric framework.

The sum of the ARCH and GARCH coefficients measures the overall persistence in each market's own volatility, *i.e.* -0.45, 0.84, and -0.07 for the oil, gas and CO_2 markets respectively. The cross-volatility spillover effect of the oil market on the CO_2 market is equal to 0.22. As a diagnostic check of the fitted model, the range of residuals is now closer to what we expect from a standard normal distribution, as the ACF plots for each time-series are contained within the critical values[12]. However, the ACF of the squared residuals between lags 9 and 14 clearly exhibit some autocorrelation. Thus, the diagnostic checks of the BEKK(1,1) MGARCH model suggest that the model may be misspecified with respect to the necessary white noise residuals properties[13]. We need to try and refine the trivariate GARCH model in order to remove the small number of remaining significant correlations in the ACFs of the standardized residuals[14].

To this purpose, we detail in the next section the results obtained with constant conditional correlation and dynamic conditional correlation models.

[12] These plots are not reproduced in the article to conserve space, and may be obtained to the authors upon request.

[13] To make further visual assessment of the goodness of fit of the model, the interested reader may ask to the authors a graph of the standardized residuals. Note that while for an univariate GARCH standardized residuals are simply the model residuals divided by the conditional standard deviation, the standardized residuals for a multivariate GARCH are obtained by the *whitening matrix transformation* $\Sigma_t^{-1/2}\mathbf{Z}_t$. For a well-fitted multivariate GARCH model, this transformation produces standardized residual vectors that have an approximately diagonal conditional covariance matrix.

[14] Since the BEKK(1,1) MGARCH model appears misspecified, we do not display the plots of the conditional standard deviations and correlations to conserve space.

4.2.3 CCC MGARCH results

We estimate below Bollerslev's (1990) constant-correlation model in which the conditional variances of $\{y_t\}, t = 1, \ldots, T$ time-series with K elements each so that $y_t = (y_{1t}, \ldots, y_{Kt})'$ follow a GARCH process, while the correlations are constant:

$$\begin{aligned} \sigma_{it}^2 =& \omega_i + \alpha_i \sigma_{i,t-1}^2 + \beta_i y_{i,t-1}^2, \quad i = 1, \ldots, K \\ \sigma_{ijt} =& \rho_{ij} \sigma_{it} \sigma_{jt}, \quad 1 \leq i < j \leq K \end{aligned} \tag{11}$$

with ω_i, α_i, and β_i nonnegative, $\alpha_i + \beta_i < 1$ for $i = 1, \ldots, K$, σ_{ijt} the covariance elements, and $\Gamma = \{\rho_{ij}\}$ the correlation matrix positive definite. A tri-variate GARCH(1,1) model with constant-correlation is fitted to the VAR(1) residuals of oil, gas and CO_2 variables. Compared to the BEKK(1,1) MGARCH, the CCC(1,1) MGARCH does not exhibit autocorrelation, since the squared residuals remain in the range of the critical values[15]. Besides, the fitted model has resulted in smaller ACF values for the standardized residuals relative to the series observed. To provide further guidance in assessing the fit of the CCC(1,1) MGARCH model, the standardized residuals show that most, but not all, of the autocorrelation structure has been removed by the fitted CCC(1,1) MGARCH model[16]. Nevertheless, the residuals of the CCC(1,1) MGARCH model satisfy the required white noise properties, and useful interpretations may be derived from our estimates.

Insert Figure 4 about here

The time-varying standard deviations estimated from the CCC(1,1) MGARCH model for the oil, gas and CO_2 variables are shown in Figure 4.

Insert Table 6 about here

The results are summarized in Table 6. The CCC(1,1) MGARCH model has 12 parameters, of which 9 are significant. The last three rows of Table 6 show estimates of the correlation parameters. The other rows show parameter estimates of univariate GARCH(1,1) models for each time-series. As can be seen, most of the estimated univariate GARCH parameters, except the constant terms, are statistically significant and positive. The level of the ARCH coefficient, which represents the reaction to new information, is quite low. The value of $\alpha + \beta$ is close to one for each time-series, which suggests that the variance process is not integrated (Engle and Bollerslev (1986)). As for the correlation parameters, we observe that

[15] ACF of residuals and squared residuals are not reproduced in the article to conserve space, and may be asked upon request to the authors.

[16] This figure is not reproduced in the article to conserve space, and may be asked upon request to the authors.

the correlations across the three energy and emissions markets are quite low, all below 0.1. There is strong evidence (significance at the 1% level) of time-varying correlations between the tri-variate time-series. Not surprisingly, we find evidence against constant correlations.

As highlighted in previous literature (Alberola *et al.* (2008), Oberndorfer (2009), Bunn and Fezzi (2009)), CO_2 price changes are dependent, to a large extent, on price changes of other energy markets such as oil and gas. If these markets have low correlations, and/or their relationships are not stable overtime[17], we would expect the correlations between the three time-series to be time-varying. This comment brings us to investigate in the next section the interrelationships between oil, gas and CO_2 prices from a dynamic correlation perspective.

4.2.4 DCC MGARCH results

For the ease of presentation, we re-state the DCC(m,n) MGARCH model estimated (Engle (2002)):

$$h_{i,t} = \omega_i + \sum_{p=1}^{P_i} \alpha_{i_p} r_{i,t-p}^2 + \sum_{q=1}^{Q_i} \beta_{i_q} h_{i,t-p} \quad i = 1, \ldots, k \tag{12}$$

$$\begin{aligned} Q_t &= \left(1 - \sum_{m=1}^{M} \alpha_m^* - \sum_{n=1}^{N} \beta_n^*\right) \overline{Q} + \sum_{m=1}^{M} \alpha_m^* (\epsilon_{t-m}\epsilon_{t-m}') + \sum_{n=1}^{N} \beta_n^* Q_{t-n} \\ R_t &= \tilde{Q}_t^{-1} Q_t \tilde{Q}_t^{-1} \end{aligned} \tag{13}$$

with $\epsilon_t = D_t^{-1} r_t$, $\epsilon_t \sim N(0, R_t)$, $\tilde{Q}_t$ a diagonal matrix containing the square root of the diagonal entries of Q_t, and $\overline{Q}_t$ the matrix of unconditional covariances. Eq(12) is a standard univariate GARCH model, and eq(13) is referred to as a DCC(m,n) model. We fit eq(12) and (13) to the VAR(1) residuals of oil, gas, and CO_2 variables. As explained in Section 4.2.1, we choose to adopt the most parsimonious specification with $m = 1$ and $n = 1$. Similarly to the constant-correlation model, the ACF plots of residuals and squared residuals do not exhibit autocorrelation[18]. It may be concluded that the residuals of the DCC(1,1) MGARCH model satisfy the necessary white-noise properties.

Insert Figure 5 about here

In Figure 5, we can also look at the normal $Q - Q$ plots of the standardized residuals for all estimated models. We observe that the deviation from the normal distribution is highest in

[17] Alberola *et al.* (2008) highlight that the influence of other energy markets (such as oil and gas) on CO_2 price changes varies depending on institutional events.

[18] These plots are not reproduced in the article to conserve space, and may be asked upon request to the authors.

the VAR model. The $Q-Q$ plots for oil almost lie on a straight line in all MGARCH models, while gas seems to differ from the normal distribution. Compared to gas, the mismatch appears less pronounced for the CO_2 variable in all MGARCH models.

Insert Table 7 about here

DCC(1,1) MGARCH estimates are reported in Table 7. The DCC(1,1) MGARCH model has 11 parameters, of which 7 are significant. As sensitivity tests, we notice the remarkable stability of the coefficients and robust standard error estimates between the CCC(1,1) and DCC(1,1) MGARCH models. The correlation structure of the DCC(1,1) MGARCH model has a clear interpretation: there is a non-constant interaction of the three time-series with respect to conditional correlation, and this correlation impacts current correlation with a lag of 1. This interaction effect would be neglected if the three time-series of VAR residuals were modeled in isolation, each with a univariate GARCH model. Next, we reproduce Engle and Sheppard's (2001) test for the presence of dynamic correlation in the residuals of the DCC(1,1) MGARCH model:

$$\begin{aligned} &H_0 : R_t = \overline{R} \quad \forall t \in T \\ &H_a : vech(R_t) = vech(\overline{R}) + \beta_1 vech(R_{t-1} + \beta_2 vech(R_t - 1) + \ldots + \beta_p vech(R_{t-1})) \end{aligned} \tag{14}$$

Engle and Sheppard's (2001) p value and χ^2 statistic testing for the dynamic correlation between the oil, gas and CO_2 residuals are presented in the last two rows of Table 7. Under the null the constant and all of the lagged parameters in the model should be zero. Thus, we reject the null of a constant correlation in favor of a dynamic structure.

Insert Figure 6 about here

In Figure 6, we provide a visual representation of the dynamic correlations between the oil, gas and CO_2 variables estimated from the DCC(1,1) MGARCH model[19]. To allow direct comparison, the dashed lines in Figure 6 represent the estimated constant conditional correlations from the CCC(1,1) MGARCH model. The constant-correlation model tends to bias (upward in the case of $\rho_{GAS,CO2}$, downward in the case of $\rho_{OIL,CO2}$) the actual correlations observed in the dynamic correlation model. The DCC MGARCH model thus provides a more accurate description of the dynamics of the correlations between the oil, gas, and CO_2 variables overtime.

[19] To conserve space, we do not reproduce the plots of the time-varying standard deviations which are very similar to the CCC(1,1) MGARCH model, due to the stability of the coefficients estimated with the DCC(1,1) MGARCH model.

The values observed for $\rho_{OIL,GAS}$ are comprised between -0.3 and 0.3, which represents a significantly higher bandwith than the value of -0.02 estimated in the CCC(1,1) MGARCH. The main reason behind this sharp difference between the constant and dynamic correlations models lie in the presence of two peaks in the dynamics of $\rho_{OIL,GAS}$ observed in August and September 2005. These peaks have also been observed in the raw price series, as commented in Section 3.1. Otherwise, $\rho_{OIL,GAS}$ oscillates around the value found in the constant correlation model.

$\rho_{OIL,CO2}$ spans the range of values comprised between -0.05 and 0.05, with a few peaks in the observed correlations. The constant correlation of -0.04 estimated with the CCC(1,1) MGARCH model lies outside of the interval where both series oscillates (around 0.01). This plot gives us a clear picture of the dynamic correlations behind the evolution of the oil and CO_2 price series overtime.

As for $\rho_{GAS,CO2}$, the correlation structure spans the range from nearly -0.2 to over 0.1, which differs from the value of 0.018 found in the CCC(1,1) MGARCH model. Significant peaks may be found during the period going from April to October 2006, which corresponds to institutional developments during Phase I of the EU ETS[20]. The visual inspection of Figure 6 has overall confirmed the superiority of the DCC MGARCH model over the CCC MGARCH model to examine the contemporaneous relationships between volatility and correlation on energy and emissions markets. We may conclude that there is strong evidence of time-varying correlations among the selected oil, gas, and CO_2 variables.

5 Conclusion

Previous studies on the interrelationships between energy (oil, gas) and emissions (CO_2) markets have focused on univariate GARCH models (Alberola *et al.* (2008), Oberndorfer (2009)). These articles identified oil and gas (among energy prices) as being the main drivers of the CO_2 price (see also Bunn and Fezzi (2009) for structural interactions results). To take this analysis one step further, the goal of our article is to further analyze the co-movements of the oil, gas, and CO_2 price series. Multivariate GARCH models appear as an adequate tool to fulfill such an objective (see Hsu Ku (2008) and Leeves (2008) for recent applications).

Compared to previous literature, we investigate the time-varying correlations of the energy and emissions markets in a *multivariate* modeling framework. Our econometric methodology consists in fitting various specifications of multivariate GARCH to the residuals of a vector autoregressive model for the three price series. Following a brief review of VAR and MGARCH

[20] Namely, the first verification of emissions occurred in April 2006 (Alberola *et al.* (2008)), while the European Commission announced banking restrictions between 2007 and 2008 in October 2006 (Alberola and Chevallier (2009))

models, we have detailed the main interests for investigating dynamic correlations between energy and emissions markets. Then, we have detailed the daily data used for oil, gas, and CO_2 prices from April 2005 to December 2008. After fitting a VAR model to the log-returns of the time-series, the observed squared residuals revealed significant autocorrelations in all time-series, indicating a further need for tri-variate MGARCH modeling. Hence, we investigated the interrelationships between the oil, gas, and CO_2 markets by using three classes of MGARCH: BEKK models, constant conditional correlation (CCC) and dynamic conditional correlation (DCC) models.

The BEKK MGARCH model allows to identify own and cross volatility spillovers between energy and emissions markets. We also establish that past volatility shocks have a stronger effect on their own future volatility rather than on the other energy/emissions markets. However, we noticed that the squared residuals of the BEKK MGARCH models exhibit some autocorrelation, which suggests model misspecification. As for constant-correlation models, we find strong evidence of time-varying correlations between the oil, gas and CO_2 price series. For this reason, we conclude that the CCC MGARCH model does not capture adequately the dependence between the conditional correlations of the oil, gas, and CO_2 price series.

The class of DCC MGARCH model features white noise residuals properties, while reflecting the dynamic correlations overtime. It also uncovers clear interactions between the VAR residuals with respect to conditional correlation. These interactions impact the current correlation structure with a lag of one, and would have been neglected if the three time-series were modeled in isolation using univariate GARCH models. The estimates of time-varying correlations typically features values of [-0.3;0.3] between oil and gas, [-0.05;0.05] between oil and CO_2, and [-0.2;0.2] between gas and CO_2. The DCC MGARCH model may therefore be identified as being the most satisfactory.

Through this empirical analysis, we have been able to identify strong connections between energy and emissions markets. These results convey interesting applied economics insights, as they inform us about the dynamic correlations between oil, gas, and CO_2 prices modeled jointly overtime. They may be used directly in the banking and finance industry, as well as by brokers for companies regulated by the EU ETS, to make informed hedging decisions. In extension of this work, an interesting area for future research lies in the investigation of the transmission of shocks on the term structure of the energy and emissions markets, as is usual for other financial markets (Christiansen (2000)).

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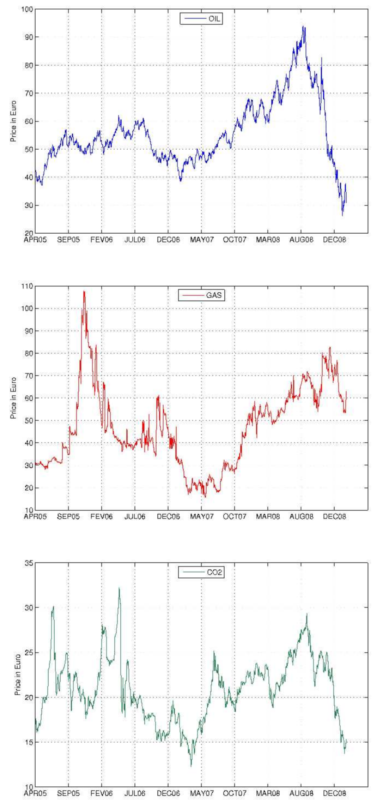

Figure 1: Raw Price Series of NYMEX Crude Oil Futures, Zeebrugge Natural Gas Next Month, and ECX December 2008 CO_2 Futures from April 22, 2005 to December 15, 2008
Source: Reuters, Bloomberg and ECX

Figure 1
215x279mm (600 x 600 DPI)

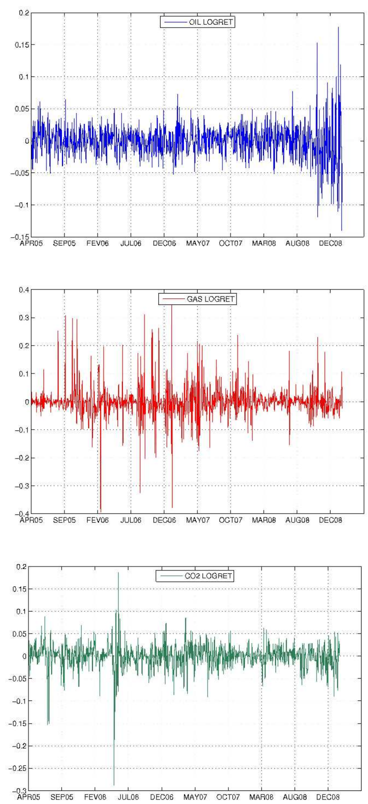

Figure 2: Log-returns of Oil, Gas, and CO_2 Variables

Figure 2
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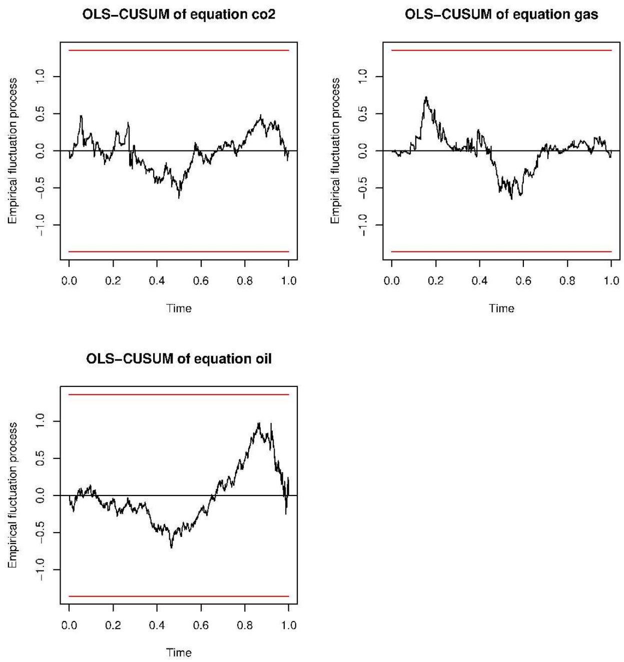

Figure 3: OLS CUSUM tests for the VAR(1) model with Oil, Gas and CO_2 Variables

Figure 3
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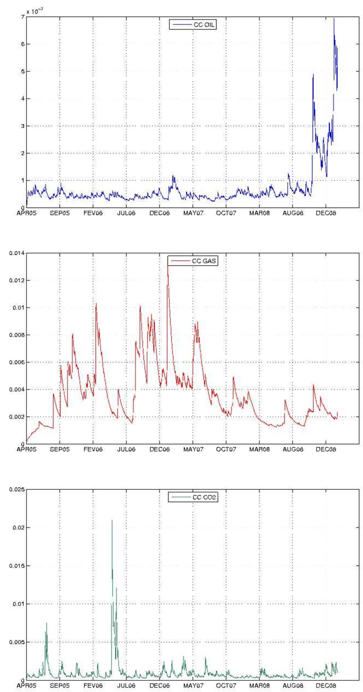

Figure 4: Time-varying standard deviations estimated with the CCC(1,1) MGARCH for the Oil, Gas, and CO_2 Variables

Figure 4
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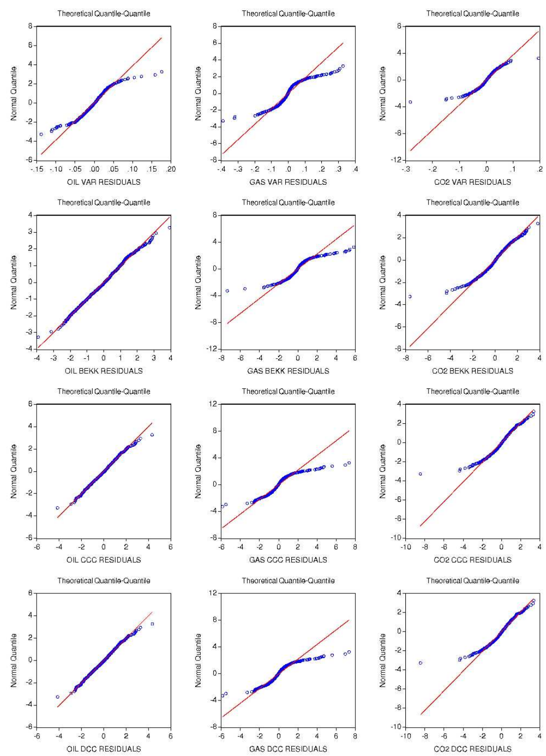

Figure 5: Normal $Q-Q$ plots of the standardized residuals for the VAR(1) (first row), BEKK(1,1) MGARCH (second row), CCC(1,1) MGARCH (third row), DCC(1,1) MGARCH (fourth row) with the Oil (left panel), Gas (middle panel), and CO_2 (right panel) Variables

Note: Normal $Q-Q$ plot stands for the quantiles of the standardized residuals plotted against the quantiles of the normal distribution.

Figure 5
215x279mm (600 x 600 DPI)

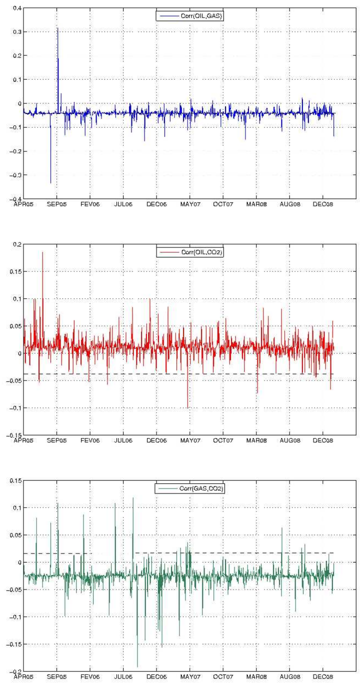

Figure 6: Dynamic Conditional Correlations between Oil and Gas (top panel), Oil and CO_2 (middle panel), Gas and CO_2 (bottom panel) estimated with the DCC(1,1) MGARCH model

Note: In each panel, the dashed line is the estimated constant conditional correlation from the CCC(1,1) MGARCH model.

Figure 6
215x279mm (600 x 600 DPI)

Variable	Mean	Median	Max	Min	Std. Dev.	Skew.	Kurt.
Raw Price Series							
Oil	55.7279	53.1892	93.9841	26.2558	11.9011	0.8585	3.6833
Gas	46.6878	44.3250	107.7500	15.7200	17.8437	0.5299	3.0022
CO_2	20.9298	20.9000	32.2500	12.2500	3.5804	0.2084	2.7831
Log-Returns							
Oil	-0.0003	0.0004	0.1776	-0.1401	0.0268	0.0988	8.4889
Gas	0.0008	-0.0014	0.3590	-0.3949	0.0603	0.5472	13.2876
CO_2	-0.0001	0.0012	0.1865	-0.2882	0.0287	-1.3195	17.7714

Table 1: Summary Statistics

Note: *Oil* refers to NYMEX crude oil, *Gas* to Zeebrugge Natural Gas Next Month, and CO_2 to ECX December 2008 futures price series. *Std.Dev.* stands for Standard Deviation, *Skew.* for Skewness, and *Kurt.* for Kurtosis. The number of observations is 936.

Variable	Deterministic Terms	Lags	Test Value	Critical Values		
				1%	5%	10%
Oil	constant, trend	2	-18.0672	-3.96	-3.41	-3.12
ΔOil	constant	1	-23.4714	-3.43	-2.86	-2.57
Gas	constant, trend	2	-20.0859	-3.96	-3.41	-3.12
ΔGas	constant	1	-24.4663	-3.43	-2.86	-2.57
CO_2	constant, trend	2	-16.406	-3.96	-3.41	-3.12
ΔCO_2	constant	1	-19.9418	-3.43	-2.86	-2.57

Table 2: ADF Tests for Oil, Gas and CO_2 Variables

Note: *Oil* refers to NYMEX crude oil, *Gas* to Zeebrugge Natural Gas Next Month, and CO_2 to ECX December 2008 futures price series. The critical values reported are from Dickey and Fuller (1981) and Hamilton (1994). The number of observations is 936.

Lag	1	2	3	4	5	6	7	8
$AIC(n)$	-1.997933	-1.997209	-1.996408	-1.994872	-1.995287	-1.994975	-1.993828	-1.992055
$HQ(n)$	-1.99450	-1.992437	-1.989847	-1.986521	-1.985147	-1.983045	-1.980109	-1.976545
$SC(n)$	-1.990114	-1.984699	-1.979207	-1.972980	-1.968704	-1.963700	-1.957862	-1.951397
$FPE(n)$	0.000210	0.000212	0.000214	0.000217	0.000216	0.000217	0.000219	0.000223

Diagnostic Tests						
Lag	Q_16	p value	JB_4	p value	$MARCH_5$	p value
$p = 1$	165.1727	0.03965	13208.33	0.00001	481.9319	0.00001

Table 3: VAR Optimal Lag Length Determination for Oil, Gas and CO_2 Variables

Note: *Oil* refers to NYMEX crude oil, *Gas* to Zeebrugge Natural Gas Next Month, and CO_2 to ECX December 2008 futures price series. $AIC(n)$ refers to the Akaike Information Criterion for a lag of order n, $HQ(n)$ refers to the Hannan-Quinn Criterion for a lag of order n, $SC(n)$ refers to the Schwarz Criterion for a lag of order n, and $FPE(n)$ refers to the Final Prediction Criterion for a lag of order n. The number of observations is 936. Diagnostic tests are provided for the optimal lag length $p = 1$. Q_{16} refers to the Ljung-Box-Pierce Portmanteau Test Q Statistic with a maximal lag of order 16, JB_4 is the Jarque-Berra Normality Tests Statistic for a maximal lag of order 4, and $MARCH_5$ is the Multivariate ARCH Test Statistic for a maximal lag of order 5.

Parameter	w_1	w_1	w_1	$\gamma_{1,1}$	$\gamma_{1,2}$	$\gamma_{1,3}$	$\gamma_{2,1}$	$\gamma_{2,2}$	$\gamma_{2,3}$	$\gamma_{3,1}$	$\gamma_{3,2}$	$\gamma_{3,3}$
Estimate	0.00240	0.00069	0.00113	-0.04430	-0.00305	-0.00134	-0.01268	-0.16630***	0.006472	0.03261	0.11620***	0.12060***
Standard Error	0.00176	0.00391	0.00187	0.03278	0.00730	0.03494	0.01451	0.03231	0.01547	0.03056	0.06803	0.03257

Diagnostic Tests	OIL_t	GAS_t	$CO2_t$
$R - Squ.$	0.00720	0.03099	0.01555
$Adj.R - Squ.$	0.002926	0.02682	0.01131
SE	0.02674	0.05953	0.02850
$Log - Lik.$	5372.948	5372.948	5372.948
$F - Stat.$	0.1514	0.00001	0.00566

Table 4: VAR(1) Estimation Results for Oil, Gas and CO_2 Variables

Note: *Oil* refers to NYMEX crude oil, *Gas* to Zeebrugge Natural Gas Next Month, and CO_2 to ECX December 2008 futures price series. The optimal lag order for the VAR is $p = 1$. *** denotes 1% significance, ** 5% significance, and * 10% significance levels. The number of observations is 936. $R - Squ.$ stands for the R-Squared, $Adj.R - Squ.$ for the Adjusted R-Squared, SE for the standard error, $Log - Lik.$ for the log-likelihood, and $F - Stat.$ for the F-Statistic. The value of the $F - Stat.$ is the p-value. The VAR(p) model estimated is shown in Section 4.1.

Parameter	Estimate		
	Oil	*Gas*	CO_2
$\mathtt{C}_{OIL}$	-0.00910***	0.01089***	0.00001
	(0.00138)	(0.00613)	(0.00001)
$\mathtt{C}_{GAS}$	0.00001	-0.04645***	0.00001
	(0.00001)	(0.00367)	(0.00001)
$\mathtt{C}_{CO_2}$	0.00083	0.01069***	-0.01481***
	(0.00141)	(0.00551)	(0.00144)
$\mathtt{A}_{OIL}$	0.38894***	0.18887**	-0.14831***
	(0.03606)	(0.10386)	(0.04072)
$\mathtt{A}_{GAS}$	0.04024***	-0.54739***	0.03205
	(0.01798)	(0.06579)	(0.02130)
$\mathtt{A}_{CO_2}$	0.01986***	0.23559***	0.57486***
	(0.03490)	(0.09673)	(0.04516)
$\mathtt{B}_{OIL}$	-0.83827***	-0.00119	-0.08685***
	(0.02893)	(0.09296)	(0.03915)
$\mathtt{B}_{GAS}$	-0.04919	-0.29449***	-0.06569
	(0.03600)	(0.13907)	(0.04264)
$\mathtt{B}_{CO_2}$	0.02625	-0.06593	-0.64418***
	(0.03283)	(0.111374)	(0.04679)

Diagnostic Tests	
AIC	-6399.865
Eig.	0.86901

Table 5: BEKK(1,1) MGARCH Estimates for Oil, Gas and CO_2 Variables

Note: *Oil* refers to NYMEX crude oil, *Gas* to Zeebrugge Natural Gas Next Month, and CO_2 to ECX December 2008 futures price series. Robust standard errors in parentheses. *** indicates 1% significance, ** 5% significance, and * 10% significance levels. The number of observations is 936. *AIC* is the Akaike Information Criterion. *Eig.* is the maximum eigenvalue. The BEKK(p,q) MGARCH model estimated is shown in Section 4.2.2.

Parameter	Estimate
GARCH parameters	
ω_{OIL}	0.0001
	(0.0001)
α_{OIL}	0.1144***
	(0.0002)
β_{OIL}	0.8557***
	(0.0004)
ω_{GAS}	0.0001
	(0.0001)
α_{GAS}	0.0404***
	(0.0001)
β_{GAS}	0.9458***
	(0.0002)
ω_{CO_2}	0.0001
	(0.0001)
α_{CO_2}	0.2507***
	(0.0081)
β_{CO_2}	0.6942***
	(0.0059)
Correlation Parameters	
$\rho_{OIL,GAS}$	-0.0243***
	(0.0001)
ρ_{OIL,CO_2}	-0.0398***
	(0.0001)
ρ_{GAS,CO_2}	0.0176***
	(0.0003)
$Log-Lik.$	5671.3939

Table 6: CCC(1,1) MGARCH Estimates for Oil, Gas and CO_2 Variables

Note: *Oil* refers to NYMEX crude oil, *Gas* to Zeebrugge Natural Gas Next Month, and CO_2 to ECX December 2008 futures price series. Robust standard errors in parentheses. *** indicates 1% significance, ** 5% significance, and * 10% significance levels. The number of observations is 936. The CCC(p,q) MGARCH model estimated is shown in Section 4.2.3.

Parameter	Estimate
GARCH parameters	
ω_{OIL}	0.0001
	(0.0001)
α_{OIL}	0.1144***
	(0.0002)
β_{OIL}	0.8557***
	(0.0004)
ω_{GAS}	0.0001
	(0.0001)
α_{GAS}	0.0404***
	(0.0001)
β_{GAS}	0.9458***
	(0.0002)
ω_{CO_2}	0.0001
	(0.0001)
α_{CO_2}	0.2507***
	(0.0081)
β_{CO_2}	0.6942***
	(0.0059)
Correlation Parameters	
α_1^*	0.0190***
	(0.0003)
β_1^*	0.0001
	(0.0096)
$Log - Lik.$	5671.6345
$ES\ p\ value$	0.0333
$ES\ \chi^2\ stat$	0.9835

Table 7: DCC(1,1) MGARCH Estimates for Oil, Gas and CO_2 Variables

Note: *Oil* refers to NYMEX crude oil, *Gas* to Zeebrugge Natural Gas Next Month, and CO_2 to ECX December 2008 futures price series. Robust standard errors in parentheses. *** indicates 1% significance, ** 5% significance, and * 10% significance levels. The number of observations is 936. $ES\ p\ value$ and $ES\ \chi^2\ stat$ are Engle and Sheppard's (2001) dynamic correlation tests statistics for a maximum lag of order 1. The DCC(m,n) MGARCH model estimated is shown in Section 4.2.4.