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Influence of bus partitioning on the reliability of transmissions

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Abstract:

This paper focuses on bus communication networks where every terminal is modeled as a multi-states component with a propagating failure state. An analytic expression of the reliability of the transmission between two terminals is first computed. From that, it is shown that partitioning the bus may lead to increase, decrease or let unchanged this reliability, according to the positions of the considered terminals in the different partitions. This analysis is aiming at providing an efficient help for the designers of communication networks of critical systems.

Keywords: communication networks, bus partition, reliability assessment, multi-states component, Markov chains

1. INTRODUCTION

Communications between the components of embedded systems are nowadays supported by communication networks which are very often based on bus architectures for obvious cost reasons; several commercial solutions (Ethernet-based industrial networks like Profinet, Powerlink or AFDX (Avionics Full Duplex), CAN (Controller Area Network), FlexRay) are currently available. Moreover, redundant solutions that use at least two buses have been developed to improve reliability.

Network reliability is a very large issue that can be decomposed in three classes: two-terminals problem, k-terminals problem and all-terminals problem [1]. This paper focuses only on the two-terminals problem, i.e. evaluation of the reliability of the transmission of data between two terminals, e.g. a computer and an actuator, to perform a specific function. This evaluation requires a model of the topology of the net as well as failure models of its components be defined. If the first issue is simple, a thorough analysis of the behavior of the network components is required to solve the second one. The usual binary model is indeed not appropriate for the network components that can fail in different ways, e.g. by failing silent, without modifying the communication abilities of the other components, or by preventing the adjacent components from communicating, even if they are themselves failure-free; a multi-states model must be introduced in that case. The states of this model represent the failure-free behavior and different modes of failure; degraded states, as proposed in [2], are not used because it is assumed that a component is either failure-free or failed in a given mode. A method to obtain the analytic expression of the reliability of a data transmission when this model is selected to describe the behavior of the terminals has been previously developed [3]; different topologies can then be compared more easily than with numerical approaches [4].

When large-sized systems are considered, bus partitioning is a common solution because it is generally not possible to connect all terminals with a unique cable. Partitioning a bus consists in dividing the network in several sub-networks, termed partitions, connected by data switches. Then, several questions arise for the designer of the network: How many partitions is the bus to be divided in? How must the terminals be gathered in these partitions? To answer correctly to these questions, the influence of bus partitioning on the reliability of the transmissions must be assessed, when critical systems are considered. The aim of this paper is to provide a help to the designer on the basis of the previous results. More precisely, analytic expressions of the reliability of data transmissions for different partitioning solutions will be constructed and compared; these comparisons will show how the reliability of data transmissions is modified by the partitioning according to the relative positions of the source and target terminals.

In the next section, the underlying models and the principles of the method to assess the reliability of a data transmission when multi-states models are considered are first reminded; this method is then exemplified on a case study based on a redundant bus. The impact of bus partitioning on reliability of data transmissions is addressed in section 3; on the basis of two case studies obtained from the previous one by defining different partitioning solutions, general conclusions are drawn.

2. TRANSMISSION RELIABILITY ASSESSMENT IN A NON-PARTITIONED BUS

The aim of this section is to describe briefly a method to obtain the analytic expression of the reliability of a transmission between two terminals connected by a redundant bus, assuming that all terminals of the network are modeled as multi-states components with a propagating failure. To avoid (or limit) combinatory explosion, an analysis of the network topology is performed prior to construction of the analytic expression. More details can be found in [3].

2.1. Network topology modeling

A communication network can be modeled as a non-directed graph $G = (\mathcal{N}, E)$ where \mathcal{N} is a set of nodes (a node representing a network component) and E is a set of non-directed edges between couples of elements of \mathcal{N} (an edge represents a physical link between two nodes).

In this graph, the set of paths which permit to ensure data transmission between two nodes i and j will be noted P_{ij} ; this set may contain one or several paths. An element of P_{ij} will be noted P_{ij}^k .

Two nodes are adjacent if there exists at least one edge between these nodes.

2.2. Component behavior models

The behavior of a network component will be then described by a continuous Markov chain where X_i is the set of states of this chain.

The common two-states (faultless or faulty) behavioral component model, also termed binary model, is used for the bus. However the behavioral model of a terminal is a multi-states model which includes three states:

- X_i^0 correct operation state,
- X_i^P propagating failure state. A component failure is termed "propagating" when its occurrence entails that every adjacent component becomes unable to ensure any communication, even if it is itself failure-free. According to [5] this failure is a propagating failure with a selected effect because only the components which are directly adjacent to the failed component become themselves unable to perform their service.
- X_i^F non-propagating failure state. A component failure is termed "non-propagating" when its occurrence does not impact the behavior of adjacent components.

The probability that a state of the chain is the active state at date t will be noted respectively $\pi_i^{X_i^0}(t)$, $\pi_i^{X_i^F}(t)$, $\pi_i^{X_i^P}(t)$.

In the sequel of this paper, some assumptions are introduced in this modeling:

- Failure occurrences are independent events.

- The components are not repairable; there is no transition starting from a failure state to the correct operation state.
- The failures are persistent; there is no transition between two failure states.

The second assumption has been introduced because no reparation is allowed during the mission if the network is part of an embedded system (in an aircraft, car, train, ...). The third assumption comes from industrial design rules that specify that a component becomes (and remains according to what precedes) faulty as soon as a fault occurs, whatever the duration of this fault.

With these assumptions, the behavior of terminals and buses can be described by the models of Figure 1(a) and 1(b).

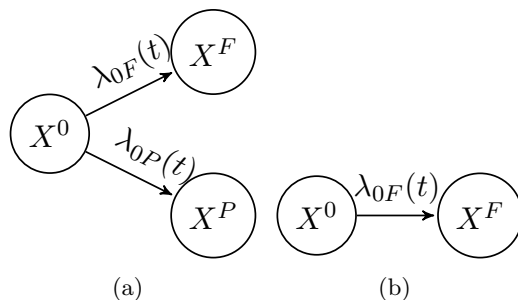


Figure 1: Terminal (a) and bus (b) behavior modeling ($\lambda_{XY}(t)$: failure rate)

2.3. Transmission reliability evaluation

From the models given previously, a method to obtain the analytic expression of the reliability of a transmission between two terminal nodes i and j , probability that there exists at least one path which permits to ensure data transmission between these two nodes, has been developed.

This method comprises two steps which are performed sequentially. The first step is aiming at determining for the whole set of paths between i to j , the components states combinations such that the transmission is possible. The analytic expression of the reliability is then constructed.

These two steps will be detailed in the following sub-sections and illustrated on the bus network where $6n$ identical terminals are connected by two identical buses B1 and B2 (Fig. 2(a)).

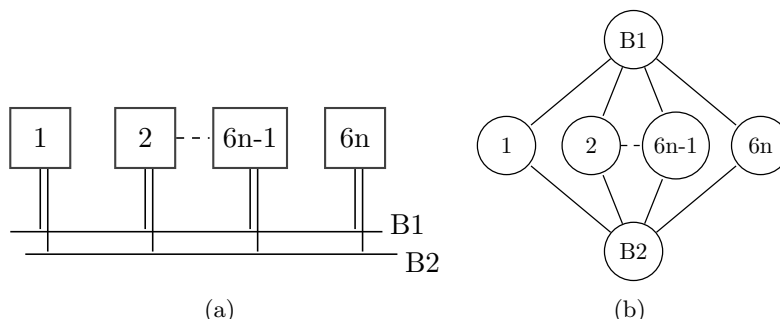


Figure 2: Redundant bus (a) and model of its topology (b)

This example uses $6n$ terminals because it is then possible to partition the bus into two or three sub-networks which comprise the same amount of terminals; hence, several partitioning solutions will be studied in section 3 to discuss the advantages and drawbacks of bus partitioning.

2.3.1. Research of the components states combinations for all paths

A transmission through the path P_{ij}^k between the nodes i and j is possible if:

- every component which is represented in the topological model by a node which belongs to the path P_{ij}^k is in the correct operation state;
- every component which is represented in the topological model by a node which is adjacent to a node which belongs to the path P_{ij}^k is in the correct operation state or in a non-propagating failure state;
- all other components are in any state.

If N_{ij}^k denotes the set of nodes which belong to the path P_{ij}^k and PN_{ij}^k the set of nodes which are adjacent to a node of the path P_{ij}^k , the set of allowed states, states that allow data be transmitted through the path P_{ij}^k , for a component represented by a node l ($l \in \mathcal{N}$), is $X_l^{P_{ij}^k}$, with:

$$X_l^{P_{ij}^k} = \begin{cases} X_l^0 & \text{if } l \in N_{ij}^k \\ X_l^0 \cup X_l^F & \text{if } l \in PN_{ij}^k \\ X_l & \text{else} \end{cases} \quad (1)$$

For the example, there are two paths to transmit data between the nodes 1 and $6n$, noted $P_{1 \leftrightarrow 6n}^1$ and $P_{1 \leftrightarrow 6n}^2$ (Fig. 3). Then, the allowed states of the components, defined by (1), are given in Table 1.

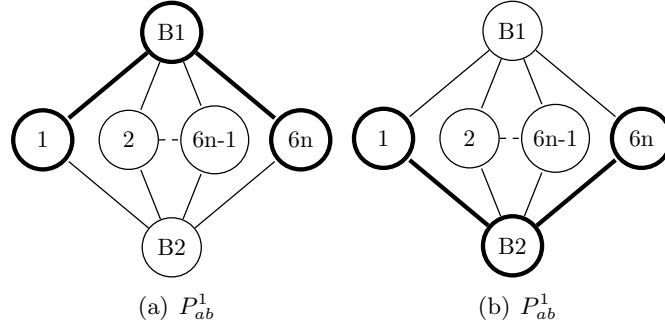


Figure 3: The two paths to transmit data between 1 and $6n$

Component	Path 1 \leftrightarrow 6n	
	$P_{1 \leftrightarrow 6n}^1$	$P_{1 \leftrightarrow 6n}^2$
1	X_1^0	X_1^0
for $l \in \{2, \dots, 6n - 1\}$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$
6n	X_{6n}^0	X_{6n}^0
B1	X_{B1}^0	$X_{B1}^0 \cup X_{B1}^F$
B2	$X_{B2}^0 \cup X_{B2}^F$	X_{B2}^0

Table 1: Set of allowed states for each component and each path

Both nodes 1 and $6n$ belong to the path, whatever it could be; hence, they must be in the correct operation state. At the opposite, the nodes 2 to $6n - 1$ do not belong to any path but are always adjacent to the used bus and therefore must not be in a propagating failure mode; this failure would prevent the nodes 1 and $6n$ from accessing the bus.

The active state of the network at a given date is the combination of the active states of the components at this date. The set $C_{ij}^{P_{ij}^k}$ of the allowed combinations for a transmission through a path is then obtained from the sets $X_l^{P_{ij}^k}$ of the allowed states of the components in the following way:

$$C_{ij}^{P_{ij}^k} = \prod_{l \in \mathcal{N}} X_l^{P_{ij}^k} \quad (2)$$

The set of allowed states of a component depends on the studied path; a component may for instance belong to a given path P_{ij}^m and be adjacent to a component of path P_{ij}^n , where P_{ij}^m and P_{ij}^n are two different paths through which data can be transmitted from/to i , to/from j . The set of the allowed components states combinations for all paths, noted C_{ij} , is then obtained by union of the sets of allowed combinations for each path:

$$C_{ij} = \bigcup_{P_{ij}^k \in \mathcal{P}_{ij}} C_{ij}^{P_{ij}^k} \quad (3)$$

For the example, it comes:

$$C_{1 \leftrightarrow 6n} = X_1^0 \times \left[\prod_{l=2}^{6n-1} (X_l^0 \cup X_l^F) \right] \times X_{6n}^0 \times [(X_{B1}^0 \times X_{B2}^0) \cup (X_{B1}^0 \times X_{B2}^F) \cup (X_{B1}^F \times X_{B2}^0)] \quad (4)$$

2.3.2. Analytic expression of the reliability of the transmission

Let c be a components states combination and α_l^c the state of component l in this combination. The probability that the network be in this combination at date t is noted $\pi^c(t)$. If the probability that the component be in a state α_l^c at date t is noted $\pi_l^{\alpha_l^c}(t)$, then $\pi^c(t)$ is computed as follows:

$$\pi^c(t) = \prod_{l \in \mathcal{N}} \pi_l^{\alpha_l^c}(t) \quad (5)$$

because failures occurrences are independent events.

Therefore, the reliability of the transmission is the sum, for every path, of the probabilities of the allowed components states combinations:

$$\pi_{ij}(t) = \sum_{c \in C_{ij}} \pi^c(t) = \sum_{c \in C_{ij}} \prod_{l \in \mathcal{N}} \pi_l^{\alpha_l^c}(t) \quad (6)$$

For example, and assuming that every failure rate is constant, the analytic expression of the reliability of the transmission between the nodes 1 and $6n$ is then:

$$\pi_{1 \leftrightarrow 6n}^{1part}(t) = e^{-(\lambda_1^f + \lambda_1^p) \cdot t} \cdot \left[\prod_{l=2}^{6n-1} \left(\frac{\lambda_l^f + \lambda_l^p \cdot e^{-(\lambda_l^f + \lambda_l^p) \cdot t}}{\lambda_l^f + \lambda_l^p} \right) \right] \cdot e^{-(\lambda_{6n}^f + \lambda_{6n}^p) \cdot t} \left[(e^{-\lambda_B \cdot t})^2 + 2 \cdot (e^{-\lambda_B \cdot t}) \cdot (1 - e^{-\lambda_B \cdot t}) \right] \quad (7)$$

where λ_l^f and λ_l^p are the non-propagative and propagative failure rate of the component l , and λ_B the failure rate of buses.

This method will be applied in the next section to a bus network with 2 and 3 partitions.

3. TRANSMISSION RELIABILITY IN A PARTITIONED BUS

The influence of bus partitioning on the reliability of transmissions is addressed in this section. Partitioning a bus consists in dividing the initial network into several subnetworks which are separated by data switches whose role is to avoid fault propagation from a subnet to another one.

This new component will be considered as a binary component, hence modeled as a Markov chain with two states (correct operation state and non-propagating failure state). The method described in the previous section can be then used to obtain the expression of the reliability of a transmission between two terminal nodes located in two different subnets or not, by considering not only the allowed states of the buses and terminals but also those of the switches, if a communication path crosses these components.

This will be illustrated below on the basis of two case studies built from Figure 2. From the results obtained in these studies, a general discussion will be pursued to point out the effects of bus partitioning on the reliability of transmissions.

3.1. Case of two partitions

This section considers a network obtained from the example of Figure 2 by dividing the bus in two partitions which include the same number ¹ ($3n$) of terminals. These partitions are interconnected by a data switch. A first analysis will focus on the reliability of the transmission between two terminals which belong to two different partitions (nodes 1 and $6n$); then focus will be put on transmission between two terminals which belong to the same partition (nodes 1 and $3n$).

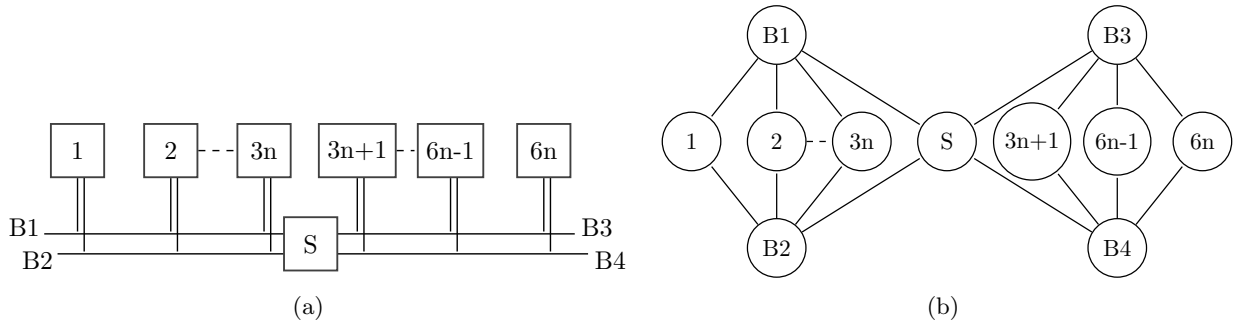


Figure 4: Two partitions bus (a) and model of its topology (b)

3.1.1. Reliability of the transmission between two terminals located in different partitions

There are four paths for communication between nodes 1 and $6n$ which are:

- $P_{1 \leftrightarrow 6n}^1$ using B1 and B3
- $P_{1 \leftrightarrow 6n}^2$ using B1 and B4
- $P_{1 \leftrightarrow 6n}^3$ using B2 and B3
- $P_{1 \leftrightarrow 6n}^4$ using B2 and B4

¹A similar analysis can be performed for other distribution of components in two subnets

Both nodes 1 and $6n$ as well as the switch belong to the path, whatever it could be; hence, they must be in the correct operation state. At the opposite, the nodes 2 to $6n - 1$ do not belong to any path but are always adjacent to the selected bus and therefore must not be in a propagating failure mode; Last, at least one bus must be in the correct operation state in each partition. Table 2 gives the sets of allowed states for each component and each path.

Component	Path 1 \leftrightarrow $6n$			
	$P_{1\leftrightarrow 6n}^1$	$P_{1\leftrightarrow 6n}^2$	$P_{1\leftrightarrow 6n}^3$	$P_{1\leftrightarrow 6n}^4$
1	X_1^0	X_1^0	X_1^0	X_1^0
for $l \in \{2, \dots, 6n - 1\}$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$
$6n$	X_{6n}^0	X_{6n}^0	X_{6n}^0	X_{6n}^0
B1	X_{B1}^0	X_{B1}^0	$X_{B1}^0 \cup X_{B1}^F$	$X_{B1}^0 \cup X_{B1}^F$
B2	$X_{B2}^0 \cup X_{B2}^F$	$X_{B2}^0 \cup X_{B2}^F$	X_{B2}^0	X_{B2}^0
B3	X_{B3}^0	$X_{B3}^0 \cup X_{B3}^F$	X_{B3}^0	$X_{B3}^0 \cup X_{B3}^F$
B4	$X_{B4}^0 \cup X_{B4}^F$	X_{B4}^0	$X_{B4}^0 \cup X_{B4}^F$	X_{B4}^0
S	X_S^0	X_S^0	X_S^0	X_S^0

Table 2: Set of allowed states for each component and each path

From this table and equation (3), the set of states combinations allowed for the transmission between nodes 1 and $6n$ is:

$$\begin{aligned}
C_{1\leftrightarrow 6n}^{2parts} = & X_1^0 \times \left[\prod_{l=2}^{6n-1} (X_l^0 \cup X_l^F) \right] \times X_{6n}^0 \times X_S^0 \times [(X_{B1}^0 \times X_{B2}^0 \times X_{B3}^0 \times X_{B4}^0) \cup (X_{B1}^0 \times X_{B2}^0 \times X_{B3}^0 \times X_{B4}^F) \cup \\
& (X_{B1}^0 \times X_{B2}^F \times X_{B3}^0 \times X_{B4}^0) \cup (X_{B1}^0 \times X_{B2}^F \times X_{B3}^0 \times X_{B4}^F) \cup \\
& (X_{B1}^0 \times X_{B2}^F \times X_{B3}^F \times X_{B4}^0) \cup (X_{B1}^F \times X_{B2}^0 \times X_{B3}^0 \times X_{B4}^0) \cup (X_{B1}^F \times X_{B2}^0 \times X_{B3}^0 \times X_{B4}^F) \cup \\
& (X_{B1}^F \times X_{B2}^0 \times X_{B3}^F \times X_{B4}^0)] \quad (8)
\end{aligned}$$

From (6), the analytic expression of the reliability is then:

$$\begin{aligned}
\pi_{1\leftrightarrow 6n}^{2parts}(t) = & e^{-(\lambda_1^f + \lambda_1^p).t} \cdot \left[\prod_{l=2}^{6n-1} \left(\frac{\lambda_l^f + \lambda_l^p \cdot e^{-(\lambda_l^f + \lambda_l^p).t}}{\lambda_l^f + \lambda_l^p} \right) \right] \cdot e^{-(\lambda_{6n}^f + \lambda_{6n}^p).t} \cdot e^{-\lambda_S.t} \cdot \\
& \left[(e^{-\lambda_B.t})^4 + 4 \cdot (e^{-\lambda_B.t})^3 \cdot (1 - e^{-\lambda_B.t}) + 4 \cdot (e^{-\lambda_B.t})^2 \cdot (1 - e^{-\lambda_B.t})^2 \right] \quad (9)
\end{aligned}$$

where λ_S is the failure rate of the switch.

Comparison of (9) and (7) permits to quantify the effect of bus partitioning on the reliability of transmission. This can be achieved by computing the following ratio:

$$\frac{\pi_{1\leftrightarrow 6n}^{2parts}(t)}{\pi_{1\leftrightarrow 6n}^{1part}(t)} = e^{-\lambda_S.t} \cdot e^{-\lambda_B.t} \cdot (2 - e^{-\lambda_B.t}) \quad (10)$$

The function $e^{-\lambda_S.t} \cdot e^{-\lambda_B.t} \cdot (2 - e^{-\lambda_B.t})$ is decreasing with a maximum value 1 for $t = 0$. It is therefore possible to assert that at every moment : $\pi_{1\leftrightarrow 6n}^{2parts}(t) < \pi_{1\leftrightarrow 6n}^{1part}(t)$. Partitioning the bus decreases the reliability of the transmission between two terminals located in different partitions.

3.1.2. Reliability of the transmission between two terminals located in the same partition

This issue will be exemplified with a transmission between the terminals 1 and 3n. As no failure can be propagated from one partition to the other one owing to the switch, only the 3n terminals of the partition are to be considered. From (6), the analytic expression of the reliability is:

$$\pi_{1 \leftrightarrow 3n}^{2parts}(t) = e^{-(\lambda_1^f + \lambda_1^p).t} \cdot \left[\prod_{l=2}^{3n-1} \left(\frac{\lambda_l^f + \lambda_l^p \cdot e^{-(\lambda_l^f + \lambda_l^p).t}}{\lambda_l^f + \lambda_l^p} \right) \right] \cdot e^{-(\lambda_{3n}^f + \lambda_{3n}^p).t} \left[(e^{-\lambda_B.t})^2 + 2 \cdot (e^{-\lambda_B.t}) \cdot (1 - e^{-\lambda_B.t}) \right] \quad (11)$$

The effect of bus partitioning can be quantified by computing the ratio below, where $\pi_{1 \leftrightarrow 3n}^{1part}$ represents the reliability of the transmission when the bus is not partitioned (expression easily obtainable from (7) by replacing 6n by 3n):

$$\frac{\pi_{1 \leftrightarrow 3n}^{2parts}(t)}{\pi_{1 \leftrightarrow 3n}^{1parts}(t)} = \left(\prod_{l=3n+1}^{6n} \left(\frac{\lambda_l^f + \lambda_l^p \cdot e^{-(\lambda_l^f + \lambda_l^p).t}}{\lambda_l^f + \lambda_l^p} \right) \right)^{-1} \quad (12)$$

As the product located at the denominator of this ratio is comprised between 0 and 1, it is possible to say that at every moment : $\pi_{1 \leftrightarrow 3n}^{2parts}(t) \geq \pi_{1 \leftrightarrow 3n}^{1part}(t)$. The reliability of the transmission is improved by the partitioning.

To sum up, dividing the bus in two partitions decreases the reliability of inter-partitions transmissions (transmissions between terminals located in different partitions) and increases that of intra-partition transmission.

3.2. Case of three partitions

A similar analysis can be performed for a network partitioned in three identical subnetworks (Figure 5).

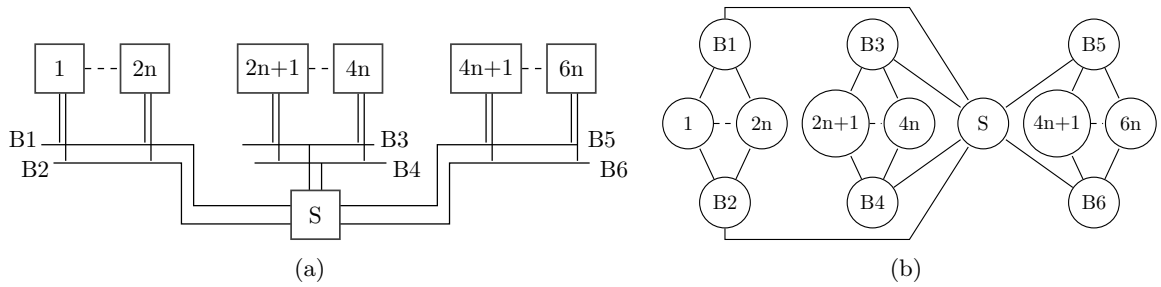


Figure 5: Three partitions bus (a) and model of its topology (b)

3.2.1. Reliability of the transmission between two terminals located in different partitions

Focusing on the transmission between 1 and 6n, four paths are to be considered. The sets of the allowed components states for each one of these paths is given at Table 3. It matters to underline that only the components of two partitions are involved in this analysis. The active states of the components of the third partition (terminals 2n to 4n-1 and buses B3 and B4) do not influence the transmission.

Component	Path 1 ↔ 6n			
	$P_{1↔6n}^1$	$P_{1↔6n}^2$	$P_{1↔6n}^3$	$P_{1↔6n}^4$
1	X_1^0	X_1^0	X_1^0	X_1^0
for $l \in \{2, \dots, 2n\}$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$
for $l \in \{4n+1, \dots, 6n-1\}$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$	$X_l^0 \cup X_l^F$
6n	X_{6n}^0	X_{6n}^0	X_{6n}^0	X_{6n}^0
B1	X_{B1}^0	X_{B1}^0	$X_{B1}^0 \cup X_{B1}^F$	$X_{B1}^0 \cup X_{B1}^F$
B2	$X_{B2}^0 \cup X_{B2}^F$	$X_{B2}^0 \cup X_{B2}^F$	X_{B2}^0	X_{B2}^0
B5	X_{B5}^0	$X_{B5}^0 \cup X_{B5}^F$	X_{B5}^0	$X_{B5}^0 \cup X_{B5}^F$
B6	$X_{B6}^0 \cup X_{B6}^F$	X_{B6}^0	$X_{B6}^0 \cup X_{B6}^F$	X_{B6}^0
S	X_S^0	X_S^0	X_S^0	X_S^0

Table 3: Set of allowed states for each component and each path

The analytic expression of the reliability is then easily obtained from (9) by modifying the bounds of the product (only the nodes 2 to 2n and 4n+1 to 6n-1 can impact the transmission):

$$\pi_{1↔6n}^{3parts}(t) = e^{-(\lambda_1^f + \lambda_1^p).t} \cdot \left[\prod_{l=2}^{2n} \left(\frac{\lambda_l^f + \lambda_l^p \cdot e^{-(\lambda_l^f + \lambda_l^p).t}}{\lambda_l^f + \lambda_l^p} \right) \right] \cdot \left[\prod_{l=4n+1}^{6n-1} \left(\frac{\lambda_l^f + \lambda_l^p \cdot e^{-(\lambda_l^f + \lambda_l^p).t}}{\lambda_l^f + \lambda_l^p} \right) \right] \cdot e^{-(\lambda_{6n}^f + \lambda_{6n}^p).t} \cdot e^{-\lambda_S.t} \cdot \left[(e^{-\lambda_B.t})^4 + 4 \cdot (e^{-\lambda_B.t})^3 \cdot (1 - e^{-\lambda_B.t}) + 4 \cdot (e^{-\lambda_B.t})^2 \cdot (1 - e^{-\lambda_B.t})^2 \right] \quad (13)$$

This expression can be compared to (7) and (9) on the basis of the following ratios:

$$\frac{\pi_{1↔6n}^{3parts}(t)}{\pi_{1↔6n}^{1parts}(t)} = e^{-\lambda_S.t} \cdot e^{-\lambda_B.t} \cdot (2 - e^{-\lambda_B.t}) \left[\prod_{l=2n+1}^{4n} \left(\frac{\lambda_l^f + \lambda_l^p \cdot e^{-(\lambda_l^f + \lambda_l^p).t}}{\lambda_l^f + \lambda_l^p} \right) \right]^{-1} \quad (14)$$

$$\frac{\pi_{1↔6n}^{3parts}(t)}{\pi_{1↔6n}^{2parts}(t)} = \left(\prod_{l=2n+1}^{4n} \left(\frac{\lambda_l^f + \lambda_l^p \cdot e^{-(\lambda_l^f + \lambda_l^p).t}}{\lambda_l^f + \lambda_l^p} \right) \right)^{-1} \quad (15)$$

These results show that the reliability of the transmission is improved by introducing an additional partition ($\pi_{1↔6n}^{3parts}(t) > \pi_{1↔6n}^{2parts}(t)$); the number of terminals that can fail in a propagating mode and are to be considered is indeed smaller. However, no definite conclusion can be drawn for the comparison with a non-partitioned bus because depends on the relative values of the failure rates of the switch, buses and terminals.

3.2.2. Reliability of the transmission between two terminals located in the same partition

The analysis and conclusion are similar to those of section 3.1.2. The reliability of intra-partitions transmission is improved.

3.3. Generalising results

From the previous two studies, it is possible to state that, when partitioning a bus into k partitions:

- the reliability of transmissions between terminals located in the same partition (intra-partition transmissions) is increased;
- the reliability of transmissions between terminals located in different partitions (inter-partitions transmissions) is increased, unchanged or decreased, depending on the failure rates, but this attribute increases when the number of partitions grows.

These results provides an efficient help for the designer of the communication network. The terminals that exchange very critical data, e.g. a safety-related sensor and the corresponding actuator, will have to be located in the same partition. The number k of partitions as well as the size of each partition are to be defined to find a good trade-off between complexity of the design and reliability of inter-partitions transmissions. The method that has been presented yields the expressions to compute this reliability.

4. CONCLUSION

This paper has proposed a method to solve a two-terminals problem: evaluation of the reliability of the data transmission between two terminals of a communication network where terminals are described as multi-states components with a propagating failure state. This method provides the analytic expression of this reliability whatever the topology of the network.

Application to non-partitioned and partitioned buses shown that partitioning always increases reliability of intra-partition transmissions but may decrease that of inter-partitions transmissions; however, this attribute is increased when the number of partitions grows.

Even if only constant failure rates have been considered, the principle of the analysis (determination of the allowed states combinations) does not rely on any assumption on the failure rates. Then, a first perspective for further work is the extension to networks whose components own other failure distribution functions.

Another promising perspective is the development of more sophisticated multi-states components models that encompass failure detection and prevention mechanisms that can be found in some network components, like bus guardians. This will extend the applicability of the approach.

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