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4 **An integral** planning model that includes production, selling price,
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6 **cash flow management and flexible capacity**
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34 **Abstract**

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37 The integration of decisions regarding different areas into a single model is a
38 current trend in industrial planning that has been made possible thanks to
39 improvements in hardware and software capacity. In fact, many authors consider
40 that production and marketing decisions should be integrated. In this paper, we
41 discuss an aggregate planning problem that includes production, selling price,
42 **cash management and flexible capacity (by means of hiring and firing and with**
43 **the possibility of unlimited production subcontracting)**. The demand is considered
44 to be a nonlinear function of the product selling price. The problem, which is
45 modelled as a mixed integer linear program, can be solved using standard
46 optimization software. The results of a computational experiment and a numerical
47 example are shown to illustrate the performance of the proposed model and obtain
48 some managerial insights.
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59 **Keywords:** **integral planning**, aggregate planning, marketing, mixed integer linear
60 programming

1. Introduction

Planning is a required process when there is a significant time since decisions are taken until they are applied. Usually this process is taken under a hierarchic framework. This way, plans and schedules are included in successive levels and each level obtains, as an output, the input of the following level. In the Aggregate Planning (AP) level, which corresponds to a medium term (for example, one year) divided in periods (for example, months), products and resources are considered in an aggregate way (for example, in families). It is usually assumed that AP is focused in the operations (or production) area: AP receives from marketing a demand forecast and tries to adjust production at a minimum cost, considering as variables, normally, the production levels, the overtime and, sometimes, the staff size.

However, as it is stated by Singhal and Singhal (2007), AP could (and should) play a more relevant role, since it can be seen as a meeting point of main company areas activities planning. In fact, the integration of decisions regarding different areas into a single model is a current trend in planning. This integration is possible due to the improvements that have been made in hardware and software capacity over the last decades.

There are several examples of this line of work in the literature. For example, the model by Bhatnagar et al. (2007) that includes sub-processes, skills of workers, overtime and the number of contingent workers hired, the models by Lusa et al. (2008 and 2009) in which production and working time (under a flexible working time scheme) decisions in AP are integrated in a single model or production planning models that include cash management (Kirca and Murat, 1996; Guillén et al., 2007).

This paper is mainly focused on the integration of production and marketing (selling price) decisions in AP, including also flexible capacity (by means of hiring and firing workers and overtime) and cash management. Companies and the academic community began to consider the need to coordinate production and marketing in the late 1970s (Saphiro, 1977; Welam, 1977a-b; Freeland, 1980). Although few empirical studies quantify the improvement that can be obtained when production and marketing areas are integrated (O'Leary-Kelly et al., 2002), many authors consider that such integration is advantageous (Leitch, 1974; Sadjadi, 2005; Upasani, 2008). In fact, it seems clear

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2
3 that the inclusion of marketing strategy in production planning may reduce costs and
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5 increase benefits.
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8 Some authors have proposed a planning model that includes production and price
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10 decisions. Kim and Lee (1998a-b), Lee and Lee (1998a-b) and Sadjadi et al. (2005)
11
12 propose a price-production model for a short-term, single-period planning horizon.
13
14 Thomas (1970), Damon and Schramm (1972), Freeland (1980), Welam (1997b), Deng
15
16 and Yano (2006), Haugen et al. (2007a-b), Smith et al. (2009) and Gonzalez-Ramirez et
17
18 al. (2010) proposed a model for determining product prices and a production plan for a
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20 multi-period planning problem. Damon and Schramm (1972), Freeland (1980), Welam
21
22 (1997b), Lee and Lee (1998a-b), Sadjadi et al. (2005) and Deng and Yano (2006) also
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24 suggest that expenditures should be included as decision variables. Gilbert (1999, 2000)
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26 studied the case of companies that work with a catalogue of products for which the price
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28 should be constant throughout the whole planning period.

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30 With regard to capacity variables, only few authors, among those dealing with planning
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32 problems in which production and marketing (selling price) are integrated, consider the
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34 quantity of the workforce (e.g., Welam, 1977b; Kim and Lee, 1998b) or the number of
35
36 working hours (basically, overtime; e.g., Damon and Schramm, 1972) as decision
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38 variables. Furthermore, few authors consider limitations of capacity as a constraint in
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40 their models (e.g. Kim and Lee, 1998b; Gilbert, 2000; Deng and Yano, 2006; Haugen et
41
42 al. 2007a-b; Smith et al, 2009; Gonzalez-Ramirez et al., 2010).

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44 The aim of this paper is to help AP to become a suitable tool for medium-term
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46 forecasting of activities in the main company areas and to ensure their coordination. We
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48 propose an AP model that includes production, staff size, cash management and
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50 marketing decisions (selling price). Aggregate planning that fails to consider cash
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52 management can lead to cost unfavourable situations, and even to liquidity needs that
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54 cannot be attained by the company. The financial implications of decisions about
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56 production, inventory and hiring and firing are included in the proposed model. None of
57
58 the aforementioned papers include all of the decisions considered herein.

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60 After this introduction, the paper is organized as follows: Section 2 describes the
planning problem. Section 3 includes the optimization planning model, which is a
mixed integer linear program. Section 4 describes the characteristics and the solution of

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3 an example that illustrates the performance of the model. Section 5 contains the main
4 results of a computational experiment. Finally, Section 6 contains the conclusions and
5 the prospects of future research.
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12 2. Problem description

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15 The problem consists of determining for each period the production of the different
16 products (or family of products if they are aggregated), the overtime, the number of
17 workers to be hired/fired and the selling price of the products, which must be kept
18 within a lower and an upper bound.
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23 The demand for each product, which is assumed to be a function of its selling price, has
24 to be met in each period. To achieve this, production can be subcontracted **without**
25 **limitation** and new workers can be hired, provided that the number of workers does not
26 exceed an upper bound. Production capacity is assumed to be proportional to the
27 number of workers and the number of working hours, but there is a lower bound for the
28 number of workers (below which the system does not run) and an upper bound **(due to**
29 **space reasons or because with the existing equipments hiring more workers would not**
30 **lead to capacity increments)**. There is also an upper bound for overtime.
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39 Several demand functions have been proposed in the literature, depending on the
40 characteristics of the manufacturer and the reaction of the market to changes in the price
41 of products. Some authors have worked with a linear demand function (Thomas, 1970;
42 Kim and Lee, 1998a; Lee and Lee, 1998a; Gonzalez-Ramirez et al., 2010), while others
43 have used a **power** demand function (for example: Damon and Schramm, 1972; Welam,
44 1977a-b; Kim and Lee, 1998a-b; Sadjadi et al., 2005) or exponential demand function
45 (Salvietti and Smith, 2008; Smith et al, 2009). In addition, the case of seasonal demand
46 is addressed by Gilbert (1999, 2000) and Deng and Yano (2006). The studies by Gilbert
47 (1999, 2000) and Deng and Yano (2006) involve a price function that is the inverse of
48 the demand function; i.e. the price depends linearly on the demand.
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57 Here, we assume the case of a manufacturer who operates in a market in which there
58 may be other competitors. Of course, the demand might depend on the price of products
59 offered by the competitors. However, as justified in Deng and Yano (2006), a
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3 downward sloping demand curve can be considered representative of the market's broad
4 reaction to **increasing** price set by the manufacturer.
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8 It is also assumed that the manufacturer's customers do not hold inventory of any of the
9 products (this may occur with durable or semi-durable products like tools, equipment
10 and appliances). Otherwise, demand curves should be considered dependent across
11 periods, since an increase in demand in a given period, due to a price reduction, may
12 lead to a decrease in demand in the following periods, as customers may have held
13 inventory during the low price period.
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17 In reality, as stated by Deng and Yano (2006), there is some intertemporal substitution.
18 However, the degree depends upon the nature of the product, the willingness of
19 customers to search for longer and delay their purchases, and the extent of the price
20 fluctuations that may influence the perceived value of searching.
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24 The products can be stored and there is an upper bound for the inventory of each
25 product. In addition, the warehouse is assumed to have a limited capacity, so the volume
26 of stored products must be considered. The objective is to maximize the difference
27 between the income and the costs. The costs to be considered include production costs,
28 **subcontracting costs**, inventory holding costs, staff costs (wages), hiring costs, firing
29 costs, overtime costs and bank interests.
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33 Cash flows are managed by means of a credit account. Three different interest rates are
34 applied: to the borrowed amount, to the deposited amount and to the available credit
35 that is not used.
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39 40 41 42 43 44 45 46 47 48 **3. Model**

49 50 51 **3.1. Optimization model**

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53 A mixed integer linear program is proposed to solve the problem. The parameters,
54 variables and model equations are detailed below.
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57 58 **Parameters**

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60 *B* Maximum amount that can be borrowed from the credit account.

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- b_0 Initial amount of the balance of the credit account (which, of course, may be positive, null or negative, provided that it is $\geq -B$).
- BF_t Balance of cashing and payments, in period t ($t=1, \dots, T$), that are fixed (i.e. they do not depend on the decisions involved in the model). They may have positive (when cashing is greater than payments), null or negative values. BF_t may include past sales, fixed payments, etc.
- ce Cost of one hour of overtime.
- cd_t Firing cost in period t ($t=1, \dots, T$).
- ch_t Hiring cost in period t ($t=1, \dots, T$).
- cp_{qt} Production cost (exclusive labour cost) per unit for product q in period t ($q=1, \dots, Q$; $t=1, \dots, T$).
- cr_{qt} Subcontracting cost per unit for product q in period t ($q=1, \dots, Q$; $t=1, \dots, T$).
- cs_q Inventory holding cost per period for product q ($q=1, \dots, Q$).
- cw_t Cost per worker in period t ($t=1, \dots, T$).
- E_t Maximum overtime in period t ($t=1, \dots, T$), per worker.
- h_t Number of ordinary working hours available in period t ($t=1, \dots, T$).
- i_t^b, i_t^d, i_t^a ($< i_t^b$) Rates of interest, per period, that apply, respectively, to the borrowed amount (the interest is credited), to the deposited amount (balance of the account, when it is positive; in this case the company obtains an interest) and to the credit available and not used (B minus the borrowed amount). We assume that the interests are credited or debited in the account in the same periods that they are earned or due ($t=1, \dots, T$).
- LP_{qt}, UP_{qt} Lower and upper bound for the selling price of product q in period t ($q=1, \dots, Q$; $t=1, \dots, T$).
- LW, UW Lower and upper bound for the number of workers.

Q	Number of products.
T	Planning horizon.
US	Capacity of the warehouse (m^3).
US_q	Upper bound for the number of units of product q stored at any time ($q=1, \dots, Q$). This upper bound is not related to the warehouse capacity but, for example, to the desired stock turnover. Of course, $US_q \cdot v_q \leq US$.
v_q	Physical volume of one unit of product q ($q=1, \dots, Q$), in m^3 .
W_0	Number of workers at the beginning of the planning horizon.
ρ_{qt}	Number of units of product q produced per worker per hour in period t ($q=1, \dots, Q, t=1, \dots, T$).

Decision variables

$he_t \in \mathbb{R}^+$	Overtime done by all workers in period t ($t=1, \dots, T$).
$p_{qt} \in \mathbb{R}^+$	Selling price of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).
$wd_t \in \mathbb{R}^+$	Number of workers fired at the beginning of period t ($t=1, \dots, T$).
$wh_t \in \mathbb{R}^+$	Number of workers hired at the beginning of period t ($t=1, \dots, T$).
$x_{qt} \in \mathbb{R}^+$	Production of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).

Other variables

$b_t \in \mathbb{R}$	Balance of the credit account at the end of period t ($t=1, \dots, T$). Since these variables can be negative, null or positive, b_t is replaced with $b_t^+ - b_t^-$, being $b_t^- \in \mathbb{R}^+$ the credited amount and $b_t^+ \in \mathbb{R}^+$ the deposited amount (i.e., absolute values of the nonpositive and nonnegative balances, respectively), at the end of period t ($t=1, \dots, T$).
$D_{qt} \in \mathbb{R}^+$	Demand of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).

$I_{qt} \in \mathbb{R}^+$ Incomes due to sales of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).

$r_{qt} \in \mathbb{R}^+$ Units of product q to be subcontracted at period t ($q=1, \dots, Q; t=1, \dots, T$).

$s_{qt} \in \mathbb{R}^+$ Inventory of product q at the end of period t ($q=1, \dots, Q; t=1, \dots, T$).

$w_t \in \mathbb{Z}^+$ Number of workers available in period t ($t=1, \dots, T$).

Model

$$[MAX] z = \sum_{t=1}^T \sum_{q=1}^Q (I_{qt} - cp_{qt} \cdot x_{qt} - cs_q \cdot s_{qt} - cr_{qt} \cdot r_{qt}) + \sum_{t=1}^T (i_t^d \cdot b_t^+ - (i_t^b - i_t^a) \cdot b_t^- - cw_t \cdot w_t - ch_t \cdot wh_t - cd_t \cdot wd_t - ce \cdot he_t) \quad (1)$$

$$(b_t^+ - b_t^-) = (b_{t-1}^+ - b_{t-1}^-) + (BF_t + b_{t-1}^+ \cdot i_t^d + I_{qt}) - \left(\begin{array}{l} i_t^b \cdot b_{t-1}^- + i_t^a \cdot (B - b_{t-1}^-) + \\ \sum_{q=1}^Q (cp_{qt} \cdot x_{qt} + cs_q \cdot s_{qt} + cr_{qt} \cdot r_{qt}) + \\ cw_t \cdot w_t + ch_t \cdot wh_t + cd_t \cdot wd_t + ce \cdot he_t \end{array} \right) \quad t=1, \dots, T \quad (2)$$

$$I_{qt} = p_{qt} \cdot D_{qt}(p_{qt}) \quad q=1, \dots, Q; t=1, \dots, T \quad (3)$$

$$s_{q,t-1} + x_{qt} + r_{qt} = D_{qt}(p_{qt}) + s_{qt} \quad q=1, \dots, Q; t=1, \dots, T \quad (4)$$

$$\sum_{q=1}^Q \frac{x_{qt}}{\rho_{qt}} \leq W \cdot h_t + he_t \quad t=1, \dots, T \quad (5)$$

$$he_t \leq w_t \cdot E_t \quad t=1, \dots, T \quad (6)$$

$$w_t = w_{t-1} + wh_t - wd_t \quad t=1, \dots, T \quad (7)$$

$$\sum_{q=1}^Q v_q \cdot s_{qt} \leq US \quad t=1, \dots, T \quad (8)$$

$$s_{qt} \leq US_q \quad q=1, \dots, Q; t=1, \dots, T \quad (9)$$

$$LP_{qt} \leq p_{qt} \leq UP_{qt} \quad q=1, \dots, Q; t=1, \dots, T \quad (10)$$

$$b_t^- \leq B \quad (11)$$

$$\begin{aligned} x_{qt}, s_{qt}, wh_t, wd_t, he_t, p_{qt}, D_{qt}, I_{qt}, b_t^-, b_t^+ &\geq 0 \\ w_t &\in \mathbb{Z}^+ \end{aligned} \quad (12)$$

Equation (1) is the objective function to be maximized and corresponds to the income (sales plus bank interests) minus the costs due to production, inventories, bank interests, wages, hiring, firing and overtime; (2) corresponds to the credit account balance: the balance of the credit account in period t (b_t , which has been replaced with $b_t^- - b_t^+$) is equal to the balance in period $t-1$ ($b_{t-1}^- - b_{t-1}^+$) plus the balance of cashing and payments that are fixed before the beginning of the planning horizon (BF_t , which can be positive, null or negative) plus inputs (interest of the deposited amount of the credit account and incomes due to sales) minus the outputs (interest of the borrowed amount from the credit account, interest of the unused available credit and payments due to production, inventory, subcontracting, personnel, hiring and firing and overtime); (3) is the income function. Equation (4) is the production, inventory and demand balance; (5) imposes the upper bound for production, depending on the number of working hours; (6) imposes the upper bound for the total overtime in a period. Equation (7) corresponds to the staff balance. For the sake of simplicity, hiring is supposed to be instantaneous; a different situation could be modelled, for example, by including a lag between the period in which a worker is hired and the period in which the worker is available to work. Equations (8) and (9) impose an upper bound to the inventory. Equation (10) imposes the lower and the upper bound for the selling price; (11) imposes the upper bound for the negative balance of the credit account. Finally, (12) expresses the nonnegative, integer and binary character of some variables.

In the solutions of the computational experiments, either b_t^+ or b_t^- was always equal to zero. This happens because the interests that are paid for credited amount are greater than those gained for the deposited amount (if the balance of the credit account is equal to, for example, 600, it is preferable to assign 600 to b_t^+ and 0 to b_t^- than any other incorrect possibility in which both variables are positive as, for example, assigning 700 to b_t^+ and 100 to b_t^-). In case that the values of the interests do not guarantee that this happens, new variables and constraints should be added to the model, as follows:

$y_t \in \{0,1\}$ Auxiliary binary variable to guarantee that either b_t^+ or b_t^- are equal to zero.

If $b_t^+ > 0$ then y_t takes value 1, and if $b_t^- > 0$, y_t takes value 0 ($t=1, \dots, T$).

$$\left. \begin{array}{l} b_t^+ \leq M_t \cdot y_t \\ b_t^- \leq B \cdot (1 - y_t) \end{array} \right\} \quad t = 1, \dots, T \quad (13)$$

where M_t is a large enough number (an upper bound for b_t^+).

3.2. Demand function, $D_{qt}(p_{qt})$

On the basis of the assumptions stated at section 2, the following demand function is considered:

$$D_{qt}(p_{qt}) = \alpha_{qt} - \beta_{qt} \cdot p_{qt}^{\gamma_{qt}} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (14)$$

Where $\alpha_{qt}, \beta_{qt}, \gamma_{qt}$ ($q=1, \dots, Q; t=1, \dots, T$) are the parameters of the demand function, with $\alpha_{qt} > 0$, $\beta_{qt} > 0$, $\gamma_{qt} \geq 0$. It is assumed that, as normally occurs, the demand decreases according to the price (the higher the price, the lower the demand).

Note that the potential variability in demand due to exogenous factors can be considered by means of the function parameters, whose value may vary over time.

Of course, demand cannot take negative values; this means that $p_{qt} \leq \left(\frac{\alpha_{qt}}{\beta_{qt}} \right)^{1/\gamma_{qt}}$ and,

$$\text{hence, that } UP_{qt} \leq \left(\frac{\alpha_{qt}}{\beta_{qt}} \right)^{1/\gamma_{qt}}.$$

The parameter α corresponds to the maximum demand, which would be obtained with a selling price equal to 0 (i.e., if the company gave the product for free). The parameter β corresponds to the demand scaling factor and represents the fact that the company does not operate in a monopolistic market. Finally, the parameter γ is the price elasticity of demand and represents the sensitivity of the market to changes in the selling price.

Using the demand function displayed in equation (14) and considering that there is no lost or delayed demand, the income function, which is also nonlinear, is detailed in Equation (15).

$$I_{qt} = p_{qt} \cdot D_{qt}(p_{qt}) = p_{qt} \cdot \alpha_{qt} - \beta_{qt} \cdot p_{qt}^{\gamma_{qt}+1} \quad (15)$$

In next section the linearisation of demand and income functions is detailed.

3.3. Linearising demand and income functions

The proposed demand function (equation (14)) is convex for $\gamma_{qt} \leq 1$ (for $\gamma_{qt} = 1$ the function would be linear) and concave otherwise, while the income function is concave for all parameter values. When the demand function is convex, it can be linearised by approximating it to a stepwise function. However, since generally speaking its convexity cannot be assumed, the linearisation of demand and income has been done by considering them as discrete functions.

For doing so, a finite set of values has been considered for the selling price values. Actually, this can be interesting for many companies, since allow considering other features that may be quite difficult to model (psychological criteria), such as considering prices close to a value (the usual X.99 price), prices that may look appealing for customers or prices that are multiple of a certain value (even though this particular condition can be modelled in different ways). Of course, it is possible to consider a wide range of selling prices by designing a wide enough set of admissible values.

For linearising the demand and income functions, the following additional data and variables are defined:

Data:

P_{qt} Set of admissible values for the price of product q in period t ($q=1, \dots, Q$; $t=1, \dots, T$).

Variables:

$y_{nqt} \in \{0,1\}$ Takes value 1 if the price of product q in period t (p_{qt}) is equal to n ($q=1,\dots,Q; t=1,\dots,T; n \in P_{qt}$).

The demand and the income nonlinear functions are modelled as follows:

$$\left. \begin{aligned} p_{qt} &= \sum_{n \in P_{qt}} y_{nqt} \cdot n \\ \sum_{n \in P_{qt}} y_{nqt} &= 1 \end{aligned} \right\} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (16), (17)$$

$$D_{qt} = \alpha_{qt} - \beta_{qt} \cdot \sum_{n \in P_{qt}} y_{nqt} \cdot n^{\gamma_{qt}} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (18)$$

$$I_{qt} = p_{qt} \cdot \alpha_{qt} - \beta_{qt} \cdot \sum_{n \in P_{qt}} y_{nqt} \cdot n^{\gamma_{qt}+1} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (19)$$

Equations (16) and (17) force the binary variable y_{nqt} be equal to 1 when the price of the product q in period t is equal to n , and 0 otherwise. Equations (18) and (19) assign the demand and income values, respectively, according to the binary variables (the price is replaced by n when the binary variable that indicates whether the price is equal to n or not takes value 1).

Now, the optimization model can be formulated as a mixed integer linear model. For doing so, the following steps should be done:

- Add P_{qt} as a parameter and y_{nqt} as variables
- Add equations (16) and (17)
- Replace equation (3) by equation (19)
- Replace $D_{qt}(p_{qt})$ in equation (4) by the expression on the right of equation (18)
- Delete equation (10) (obviously, the first value in P_{qt} will be equal to LP_{qt} and the last one to UP_{qt}).
- In equation (12), add $y_{nqt} \in \{0,1\}$

4. Numerical examples and managerial insights

In this section, we provide the results of a numerical example. The most relevant parameters used for the experiment are given in Table 1 and Figure 1 (as it can be observed in this figure, a seasonal demand has been considered). Since real data was not available, the different parameters were designed with the aim of obtaining realistic scenarios. The model was solved by means of CPLEX 11.0 in less than three seconds using a standard PC.

[INSERT TABLE 1. PARAMETER VALUES USED IN THE EXAMPLE]

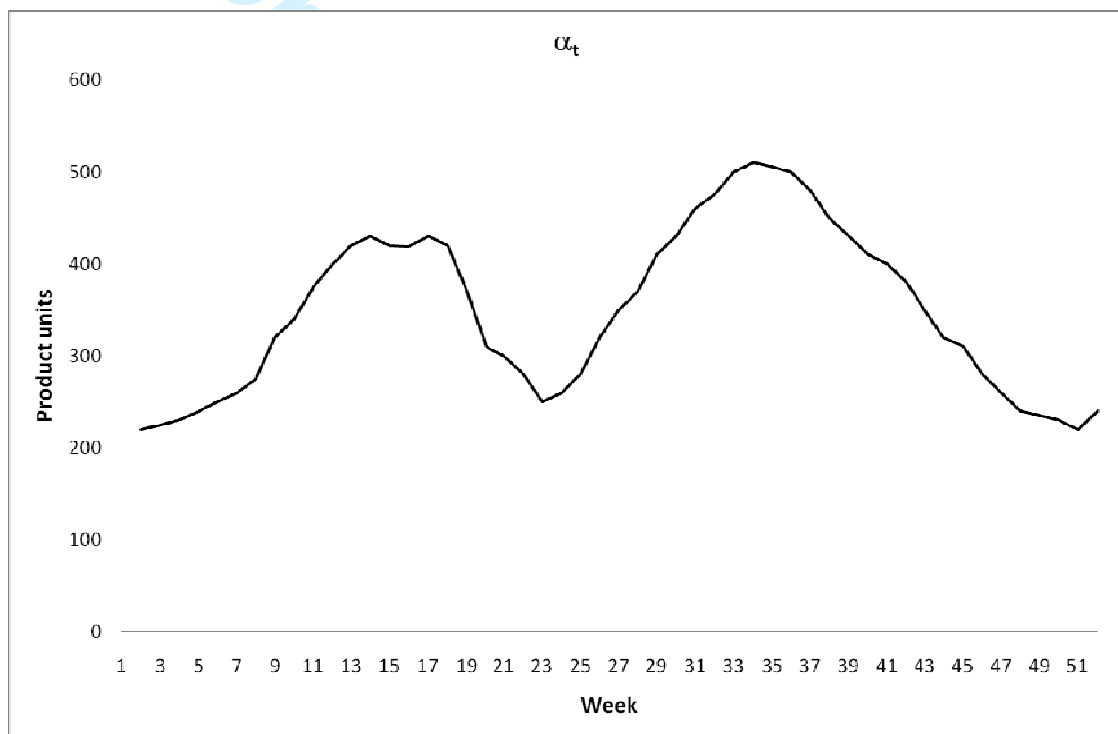


Figure 1. Values of α_t in the example

Figures 2, 3 and 4 present the optimal solution that was obtained. For each week, the selling price is shown in Figure 2, the number of workers is shown in Figure 3 and the demand, production capacity, production (both in normal hours and in overtime), subcontracting and inventory are shown in Figure 4.

Figure 2 also shows that there are some periods in which it could be considered that the change in the selling price is large. Smaller changes can be ensured in order to avoid unforeseen market behaviour by including the following constraint:

$$-k_{qt} \leq p_{qt} - p_{q,t-1} \leq k_{qt} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (20)$$

where k_{qt} is the maximum change allowed in the price of product q in period t (note that in this case p_{q0} should also be defined).

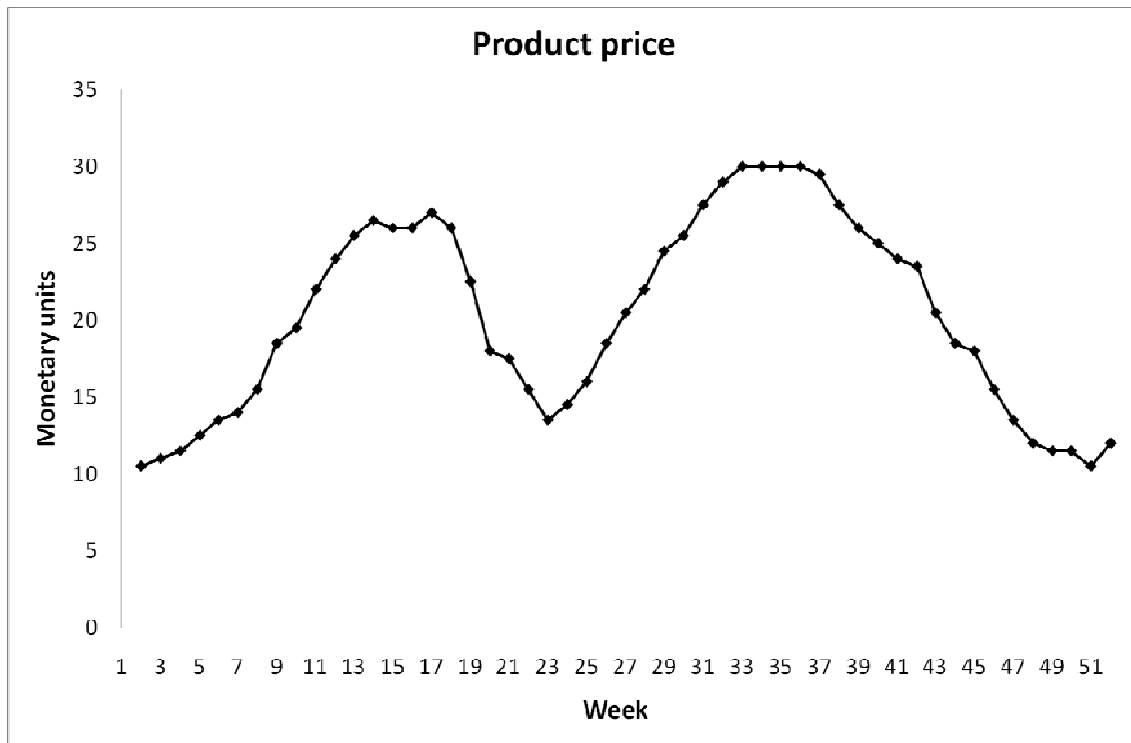


Figure 2. Optimal selling price

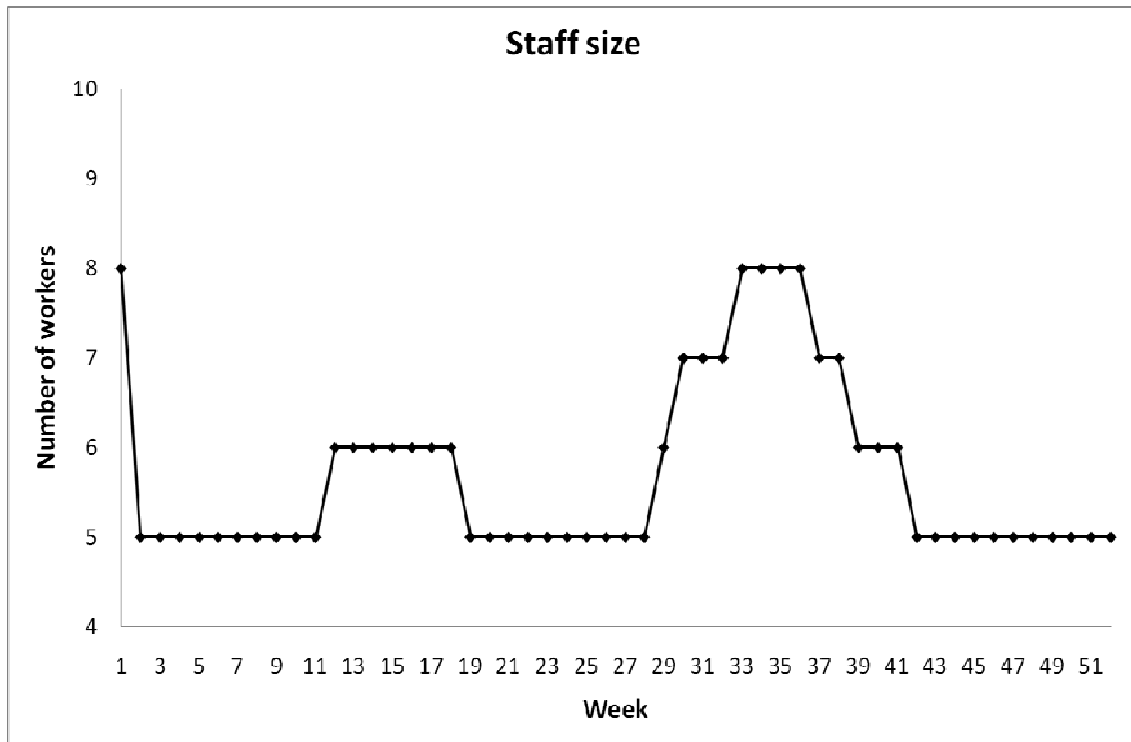


Figure 3. Staff size for the optimal solution

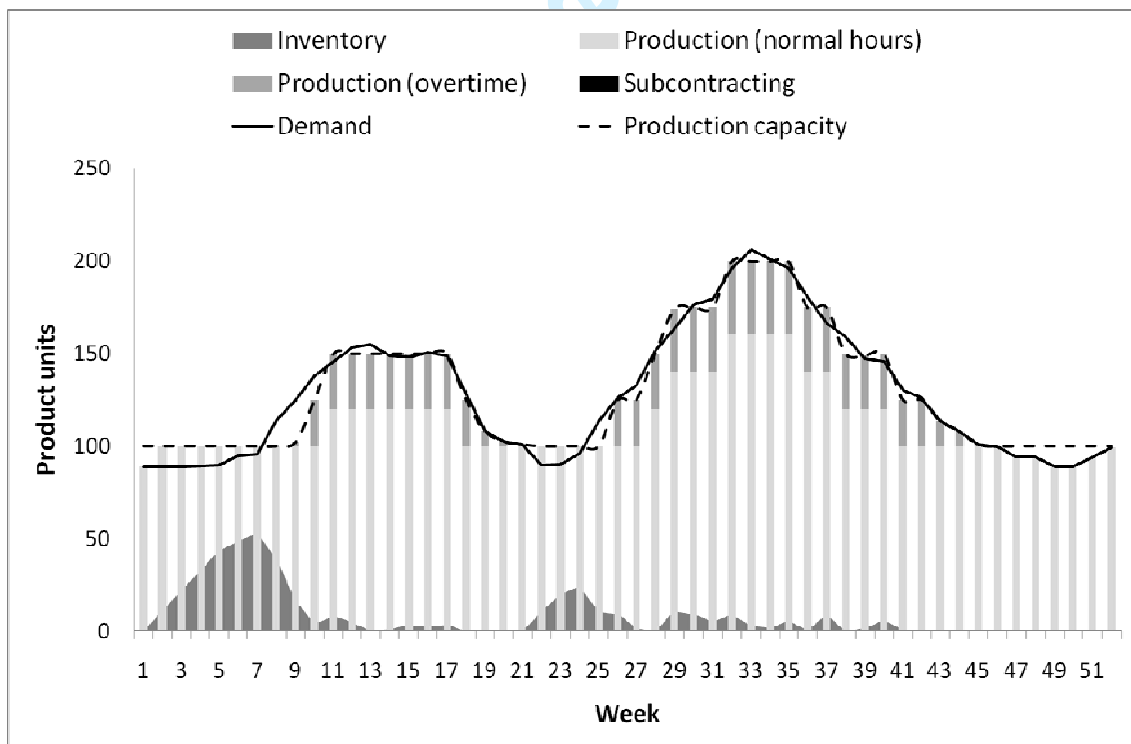


Figure 4. Demand, production capacity, production (normal hours and overtime), subcontracting and inventory for the optimal solution

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Figure 1 show a seasonal profile of the demand (α data). Analysing Figures 1 and 2 it can be clearly seen that when α goes up, also the price goes up so the demand does not increase too much; similarly, when α goes down, the price also goes down to prevent a large decrease in the demand. Changing the selling price helps having demand under (a relative) control and, hence, avoiding large subcontracting costs (which would occur if demand increased too much) or large reductions in the income (if demand decreases too much).

In Figures 3 and 4 it can be observed that, as expected, when the demand goes up it is desirable to hire workers, while when the demand goes down the number of workers decreases. It can be seen, also, that when there is a small change in the demand (as the one in week 14) the number of workers does not change; this is reasonable if we take into account that hiring and firing workers has a significant cost. The fact that the capacity (number of workers and overtime) is adapted to the demand can be clearly observed comparing demand and capacity lines in Figure 4.

Finally, Figure 5 shows the evolution of the cash inflows, cash outflows and credit account balance (this has been divided by 10 so that changes in inflows and outflows are clearer). Both inflows and outflows follow the activity level of the company, and analysing the outflows it can be seen that in some periods there is a peak, which corresponds to hiring or firing workers.

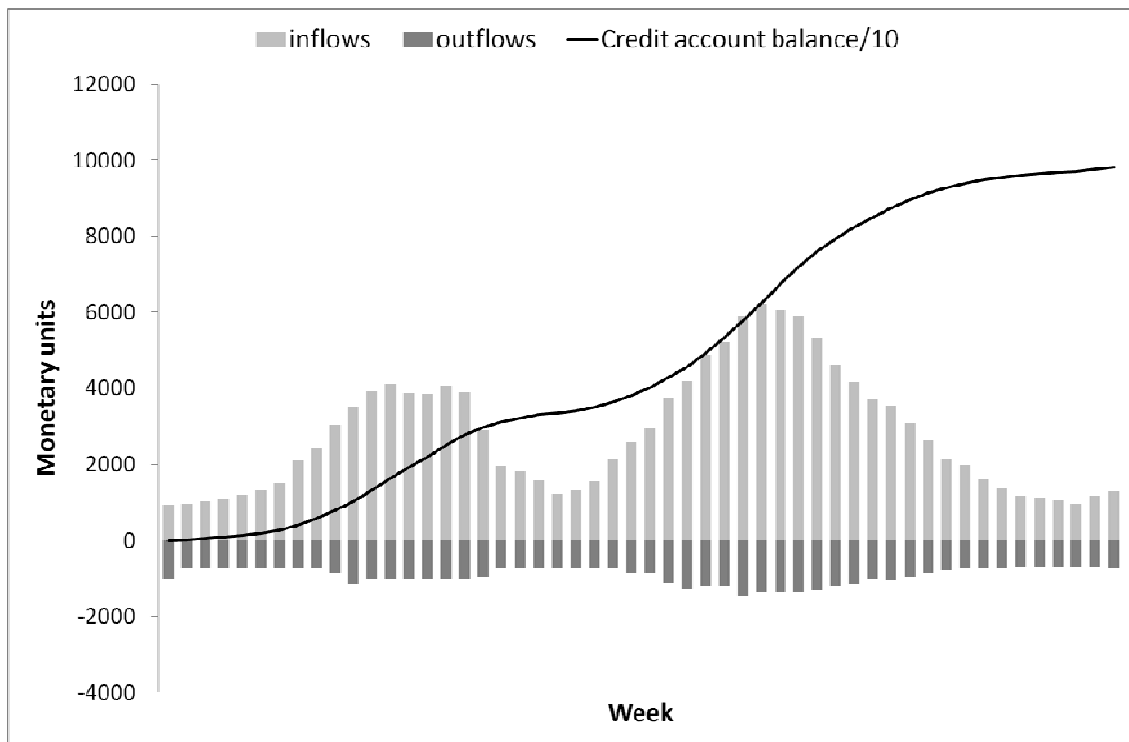


Figure 5. Cash inflows, cash outflows and credit account balance for the optimal solution

The convenience (or benefit) of using a model such as the one proposed here, in which production, hiring and firing and product price decisions are integrated in Aggregate Planning (AP), may depend on the company parameters. To analyse this issue, different models and scenarios were considered in a new experiment:

Models:

- M: Number of workers and selling price is considered to be fixed (i.e., production planning)
- M-w: The number of workers can be modified; the selling price is fixed (staff decisions are included)
- M-p: The number of workers is fixed; the selling price can be modified (marketing decisions are included)
- M-wp: Both number of workers and selling price can be modified (i.e., coincides with the model proposed in this paper).

In models M, M-w and M-p the decisions that are supposed to be fixed are considered to be constant during the planning horizon and, among all the possible values, those that optimize the objective function have been set (i.e., in the M model, the number of

workers and the selling price for the whole horizon are included as two decision variables; for models M-w and M-p an analogue thing is done with the selling price and the number of workers, respectively).

Scenarios:

- α : highly seasonal, medium seasonal, low seasonal (see Figure 6)
- γ : high (0.8; the market is highly sensible to changes in selling price), low (0.5; the sensitivity to changes in price is low)
- Labour costs: high ($cw_t=200$), low ($cw_t=100$)

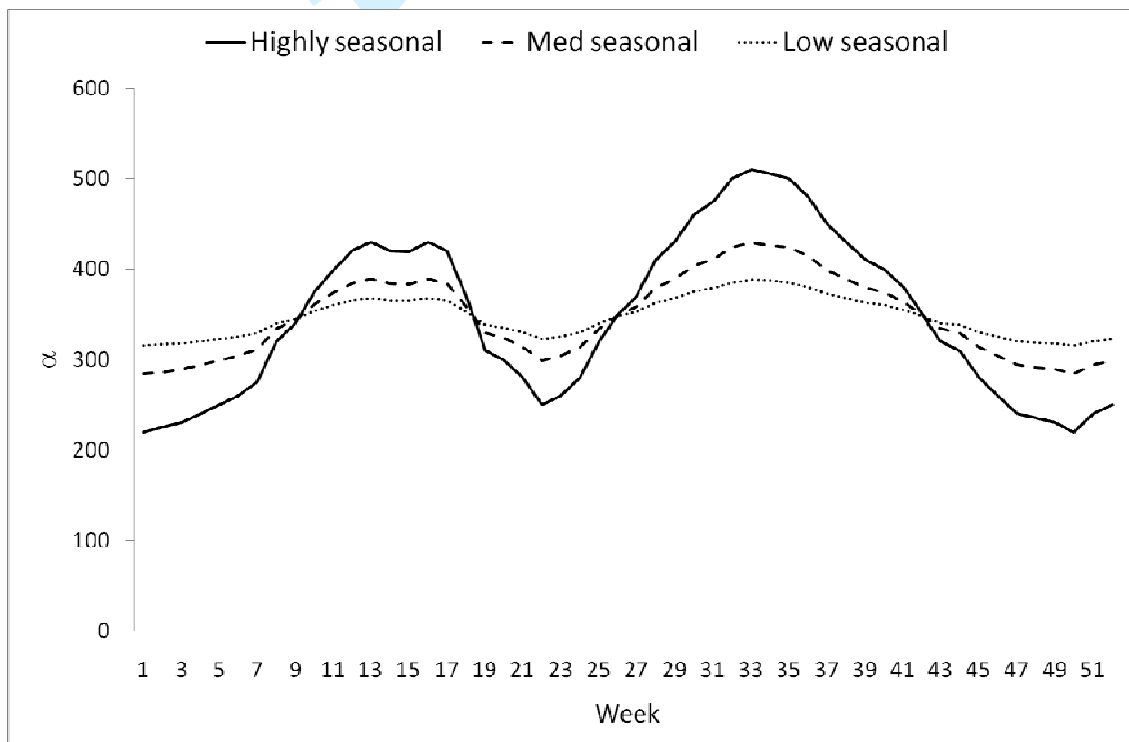


Figure 6. α profiles

The combination of the four models and the previous parameters gives rise to 48 possible situations. For each one of the 12 parameter combinations, different instances were obtained by randomly modifying the values of α (the original values were multiplied by a random number between 0.95 and 1.05).

For each one of the 12 scenarios the four models were run and the value of the objective function (the profit) was saved. Model M is considered to represent the starting situation, M-w and M-p intermediate situations and M-wp the final one, in which all decisions are integrated. The results of the model-scenario analysis are included in Table 2, where for each one of the 12 parameter combinations the following information is provided:

- ***Incr M-w/M***: shows the average increase in the value of the objective function (in %) when model M-w is used instead of model M (i.e., the number of workers is considered as a decision variable).
- ***Incr M-p/M***: shows the average increase in the value of the objective function (in %) when model M-p is used instead of model M (i.e., the selling price is considered as a decision variable).
- ***Incr M-wp/M***: shows the average increase in the value of the objective function (in %) when model M-wp is used instead of model M (i.e., the number of workers and the selling price are considered as decision variables).

[INSERT TABLE 2. SCENARIO ANALYSIS RESULTS (% INCR. OBJECTIVE FUNCTION)]

Values in Table 2 lead to some managerial insights. Even though most of them are reasonable, the experiments are a proof and a quantification of them. The most important remarks are the following ones:

- When the market is sensible to changes in prices (the γ parameter is not low), the possibility of integrating selling price decisions in AP lead to high increments in profit (objective function). This is reasonable, because if the sensibility of the market to changes in selling price is high, the impact that these changes have on the demand (and, hence, on the incomes and the profit) are also high.
- The increasing of profit due to considering selling price in AP is higher for seasonal demand profiles (the α parameter is seasonal). This can be explained as follows: model M use as selling price the value that would be optimal if the

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selling price had to be the same for the whole horizon. When the selling price can be changed at any period, if the demand is almost constant (the α parameter is not seasonal) the optimal value for the selling price is also almost constant during the planning horizon (i.e., similar to those used in models with a fixed selling price). Hence, the increment in the profit when considering that the selling price can be changed during the planning horizon is short for low seasonal demands.

- The increasing of profit due to considering selling price in AP is higher for high labour cost scenarios. When labour costs are high and demand cannot be changed (i.e., selling price is fixed), the unique alternatives to meet demand are subcontracting (which has an important cost), overtime or modifying the number of workers (which means, since labour costs are high, an important cost). But if the selling price can be changed then the so high labour costs (overtime and/or hiring/firing costs) can be reduced by modifying the demand (i.e., adapting, at least a bit, the demand to the capacity and not only the capacity to the demand) and a great increase in the profit can be obtained.
- Considering hiring and firing in AP is always interesting, and, as it was expected, the increasing of profit is higher for seasonal demand profiles (the α parameter is seasonal) in which the need of adapting capacity to demand is high.
- The results seem to suggest that when the demand is seasonal (highly or medium), the increasing of profit due to considering hiring and firing is higher for high labour cost scenarios. This could lead to false conclusions because what it really happens is the following: when the demand (the α parameter) increases, adapting capacity to demand by means of hiring has a large cost in high labour cost scenarios (thus the increasing in the profit is small) and a small cost in the low labour cost scenarios (thus the increasing in the profit is large). Instead, when the demand (the α parameter) decreases, adapting capacity to demand by means of firing may lead to a large profit increment in high labour cost scenarios (because even if the firing costs are high, during the low demand periods the cost of workers is significantly reduced) and a small cost saving in the low labour cost scenarios (thus the increasing in the profit is small).

5. Computational experiment

The model may have a large number of integer variables and constraints. To check whether it can be solved in reasonable computing times, we carried out a computational experiment.

The basic data set of the experiment (shown in Table 3) was the same as the one used in the numerical example described in the previous section. To generate instances of different sizes, different values for the number of products (Q) and for the number of admissible values for the price of the products in each period ($|P_{qt}|$) were used: 5, 10 and 50 products and 6, 11 and 51 number of admissible prices. For each of the nine combinations, 10 instances were generated by setting the parameter α_{qt} at random.

**[INSERT TABLE 3. PARAMETER VALUES USED IN THE
COMPUTATIONAL EXPERIMENT]**

The number of variables and constraints are given in Table 4. Table 5 summarizes the results of the experiment and gives the minimum, average and maximum computing times. Since all the instances were solved to optimality in very short times (especially if we consider the kind of problem being solved), it can be concluded that the model can be solved in a very efficient way and that could be used as a decision tool.

**[INSERT TABLE 4. NUMBER OF VARIABLES/CONSTRAINTS OF THE
INSTANCES IN THE COMPUTATIONAL EXPERIMENT]**

**[INSERT TABLE 5. SOLVING TIMES (IN SECONDS) FOR THE
COMPUTATIONAL EXPERIMENT (MINIMUM, AVERAGE, AND
MAXIMUM)]**

6. Conclusions and future research

In this paper, a planning model that integrates production, staff and marketing decisions is proposed. Demand is assumed to be a nonlinear function of the selling price, as are the incomes. The nonlinear functions are linearized by means of mixed integer linear programming tools. The model has a high number of binary variables, but is solved to optimality within a few seconds, using standard optimization software (CPLEX 11.0) on a PC. Hence, the model could be the basis of a decision tool that could be used by many companies. The results of a numerical example and the analysis of different scenarios show the solutions given by the model and the advantages of integrating production, human resources and marketing decisions and also the situations in which this integration is more desirable.

Future research should try to integrate other decisions such as supplies (in a multi-supplier environment, considering different prices, quality and lead time conditions for different suppliers), working time (under flexible working time schemes such as working time accounts), cash management and hiring and firing in a more complex way (considering learning periods, firing costs depending on the worker being fired, etc.). The final goal is to design planning models which integrate the most relevant areas of a company. In addition, future research includes testing other demand functions (some of them proposed by other authors).

On the other hand, the model proposed in this paper is suitable for companies that have no advertising or advertising with a little impact and also for companies in which advertisement is decided beforehand (and the effects are considered to be included in the α demand parameter). In the future, we could extend the model including advertisement investments as decision variables that have an influence on the demand (i.e., to consider a demand function depending both on the selling price and on the advertisement).

Acknowledgements

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References

- Bhatnagar, R., Saddikutti, V., Rajgopalan, A. (2007). Contingent manpower planning in a high clock speed industry. *International Journal of Production Research*, 45, 2051-2072.
- Damon, W., Schramm, R. (1972), A simultaneous decision model for production, marketing and finance. *Management Science*, 19, 161-172.
- Deng, S., Yano, C.A. (2006). Joint Production and Pricing decisions with Setup Cost and Capacity Constraints. *Management Science*, 52, 741-756.
- Freeland, J. R. (1980). Coordination strategies for production and marketing in a functionally decentralized firm. *AIIE Transactions*, 12, 126-132.
- González-Ramírez, R., Smith, N. R., Askin, R. G. (2010). A heuristic approach for a multi-product capacitated lot-sizing problem with pricing. *International Journal of Production Research*, Published on line, DOI: 10.1080/00207540903524482.
- Gilbert, S.M. (1999). Coordinating of pricing and multi-period production for constant priced goods. *European Journal of Operational Research*, 114, 330-337.
- Gilbert, S.M. (2000). Coordination of Pricing and Multiple-Period Production Across Multiple Constant Priced Goods. *Management Science*, 46, 1602-1616.
- Guillén, G., Badell, M., Puigjaner, L. (2007). A holistic framework for short-term supply chain management integrating production and corporate financial planning. *International Journal of Production Economics*, 106, 288-306.

1
2
3 Haugen, K. K., Olstad, A., Pettersen, B. I. (2007a). Solving large-scale profit
4 maximization capacitated lot-size problems by heuristic methods. *Journal of*
5 *Mathematical Modelling and Algorithms*, 6 (1), 135-149.

6
7
8
9
10 Haugen, K. K., Olstad, A., Pettersen, B. I. (2007b). The profit maximizing capacitated
11 lot-size (PCLSP) problem. *European Journal of Operational Research*, 176 (1), 165-176.

12
13
14 Kirca, Ö., Murat, M. (1996). An integrated production and financial planning model and
15 an application. *IIE transactions*, 28, 677-686.

16
17
18
19 Kim, D.D., Lee, W.J. (1998a). Optimal coordination strategies for production and
20 marketing decisions, *Operations Research Letters*, 22, 41-47.

21
22
23 Kim, D.D., Lee, W.J. (1998b). Optimal joint pricing and lot sizing with fixed and
24 variable capacity. *European Journal of operational Research*, 109, 212-227.

25
26
27
28 Lee, W.J., K.C. Lee. (1999a). A meta decision support system approach to coordinating
29 production/marketing decisions. *Decision Support Systems*, 25, 239-250.

30
31
32 Lee, W.J., Lee, K.C. (1999b). PROMISE: a distributed DSS approach to coordinating
33 production and marketing decisions. *Computers & Operations Research*, 26, 901-920.

34
35
36
37 Leitch, R. A. (1974). Marketing strategy and the optimal production schedule.
38 *Management Science*, 21, 302-312.

39
40
41
42 Lusa, A., Corominas, A., Pastor, R. (2008). An exact procedure to plan holidays and
43 working time under annualised hours considering cross-trained workers with different
44 efficiencies. *International Journal of Production Research*, 46, 2123-2142.

45
46
47
48 Lusa, A., Corominas, A., Olivella, J., Pastor, R. (2009). Production planning under a
49 working time accounts scheme. *International Journal of Production Research*, 47,
50 3435-3451.

51
52
53
54 O'Leary-Kelly, S. W., Flores, B. E. (2002). The integration of manufacturing and
55 marketing/sales decisions: impact on organizational performance. *Journal of Operations*
56 *Systems*, 2, 221-240.
57
58
59
60

1
2
3 Sadjadi, S.J., Oroujrr, M., Aryanezhad, M.B. (2005). Optimal Production and Marketing
4 Planning. *Computational Optimization and Applications*, 20, 195-203.
5
6

7
8 Salviotti, L., Smith, N. R. (2008). A profit-maximizing economic lot scheduling
9 problem with price optimization. *European Journal of Operational Research*, 184 (3),
10 900-914.
11
12

13
14 Saphiro, B. P. (1977). Can marketing and manufacturing coexist? *Harvard Business*
15 *Review*, 55, 104-114.
16
17

18
19 Singhal, J., Singhal, K. (2007). Holt, Modigliani, Muth, and Simon's work and its role
20 in the renaissance and evolution of operations management. *Journal of Operations*
21 *Management*, 25, 300-309.
22
23

24
25 Smith, N. R., Limón, J., Cardenas-Barrón, L. E. (2009). Optimal pricing and production
26 master planning in a multiperiod horizon considering capacity and inventory
27 constraints. *Mathematical Problems in Engineering*, published on line,
28 <http://www.hindawi.com/journals/mpe/2009/932676.html>.
29
30
31

32
33 Thomas, J. (1970). Price-production decisions with deterministic demand. *Management*
34 *Science*, 16, 747-750.
35
36

37
38 Upasani, A., Uzsoy, R. (2008). Incorporating manufacturing lead times in join
39 production-marketing models: A review and some future directions. *Annals of*
40 *Operations Research*, 161, 171-188.
41
42

43
44 Welam, U. P. (1977a). On a simultaneous decision model for marketing, production and
45 finance. *Management Science*, 23, 1005-1009.
46
47

48
49 Welam, U. P. (1977b). Synthesizing Short Run Production and Marketing Decisions.
50 *AIIE Transactions*, 93, 1005-1009.
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Table 1. Parameter values used in the example

T	52 weeks	cp_t	2 monetary units/product unit
Q	1 product	cs	0.5 monetary units/ product unit-week
ρ_t	0.5 product units/hour-worker	cw_t	100 monetary units/worker
US	300 product units	ch_t	100 monetary units/worker ($=cw_t$)
h_t	40 hours	cd_t	110 monetary units/worker ($=1.1cw_t$)
P	{5, 5.5, 6, 6.5,..., 29.5, 30}	ce	2 monetary units/hour ($=0.02cw_t$)
$W0$	8 workers	B	10,000 monetary units
$LW,$ UW	5, 15 workers	i_t^b, i_t^d, i_t^a	0.0015, 0.0004, 0.0006
E_t	10 hours	B_0	0 monetary units
α_t	A seasonal profile has been generated, with the values of α_t set between 220 and 510 units (the values are shown in Figure 1).	S_0	0 product units
β_t	20	BF_t	0 monetary units
γ_t	0.8		

Table 2. Scenario analysis results (% Incr. objective function)

		High labour costs	Low labour costs
Highly seasonal α	High γ	<i>Incr M-w/M: 70.65</i> <i>Incr M-p/M: 172.51</i> <i>Incr M-wp/M: 183.48</i>	<i>Incr M-w/M: 15.86</i> <i>Incr M-p/M: 40.56</i> <i>Incr M-wp/M: 43.45</i>
	Low γ	<i>Incr M-w/M: 9.30</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 9.31</i>	<i>Incr M-w/M: 4.91</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 4.91</i>
Medium seasonal α	High γ	<i>Incr M-w/M: 5.94</i> <i>Incr M-p/M: 24.40</i> <i>Incr M-wp/M: 24.40</i>	<i>Incr M-w/M: 4.08</i> <i>Incr M-p/M: 10.65</i> <i>Incr M-wp/M: 11.21</i>
	Low γ	<i>Incr M-w/M: 1.47</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 1.47</i>	<i>Incr M-w/M: 1.03</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 1.03</i>
Low seasonal α	High γ	<i>Incr M-w/M: 0.31</i> <i>Incr M-p/M: 6.64</i> <i>Incr M-wp/M: 6.64</i>	<i>Incr M-w/M: 1.54</i> <i>Incr M-p/M: 3.56</i> <i>Incr M-wp/M: 3.56</i>
	Low γ	<i>Incr M-w/M: 0.20</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 0.20</i>	<i>Incr M-w/M: 0.28</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 0.28</i>

Table 3. Parameter values used in the computational experiment

T	52 weeks	γ_{qt}	0.8
Q	5, 10 and 50 products	cp_{qt}	2 monetary units/product unit
ρ_t	0.5 product units/hour-worker	cs_q	0.5 monetary units/ product unit-week
US	300 volume units	cw_t	100 monetary units/worker
v_q	1 volume unit/product unit	ch_t	100 monetary units/worker
US_q	US/Q product units	cd_t	110 monetary units/worker
h_t	40 hours	ce	2 monetary units/hour
P_{qt}	From 15 to 20, in steps of 1, 0.5 and 0.1 monetary units (this gives 6, 11 and 51 possible values)	B	10,000 monetary units
$W0$	8 workers	i_t^b, i_t^d, i_t^a	0.0015, 0.0004, 0.0006
LW, UW	5, 15 workers	b_0	0 monetary units
E_t	10 hours	s_{0q}	0 product units
α_{qt}	$\alpha_t \in U[220, 510]; \alpha_{qt} = \alpha_t / Q$	BF_t	0 monetary units
β_{qt}	$20/Q$		

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Table 4. Number of variables/constraints of the instances in the computational experiment

Number of products	Number of admissible values for the price (for each product)		
	6	11	51
5	3,078 / 1,464	4,378 / 1,464	14,778 / 1,464
10	5,683 / 2,509	8,283 / 2,509	29,083 / 2,509
50	26,523 / 10,869	39,523 / 10,869	143,523 / 10,869

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Table 5. Solving times (in seconds) for the computational experiment (minimum, average, and maximum)

Number of products	Number of admissible values for the price (for each product)								
	6			11			51		
5	0.29	0.55	1.38	0.34	0.58	0.81	0.98	1.88	6.7
10	0.59	0.92	1.48	0.89	1.36	2	1.94	3.95	13.96
50	2.3	6.7	12.94	3.56	8.89	30.3	12	35.46	132.5

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FIGURE CAPTIONS

- **Figure 1.** Values of α_t in the example
- **Figure 2.** Optimal selling price
- **Figure 3.** Staff size for the optimal solution
- **Figure 4.** Demand, production capacity, production (normal hours and overtime), subcontracting and inventory for the optimal solution
- **Figure 5.** Cash inflows, cash outflows and credit account balance for the optimal solution
- **Figure 6.** α profiles

An integral planning model that includes production, selling price, cash flow management and flexible capacity

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Abstract

Many authors consider that production and marketing decisions should be integrated. In this paper, we discuss an aggregate planning problem that includes production, selling price, cash management and flexible capacity (by means of hiring and firing and with the possibility of unlimited production subcontracting). The demand is considered to be a nonlinear function of the product selling price. The problem, which is modelled as a mixed integer linear program, can be solved using standard optimization software. The results of a computational experiment and a numerical example are shown to illustrate the performance of the proposed model and obtain some managerial insights.

Keywords: integral planning, aggregate planning, marketing, mixed integer linear programming

1. Introduction

Planning is a required process when there is a significant time since decisions are taken until they are applied. Usually this process is taken under a hierarchic framework. This way, plans and schedules are included in successive levels and each level obtains, as an output, the input of the following level. In the Aggregate Planning (AP) level, which corresponds to a medium term (for example, one year) divided in periods (for example, months), products and resources are considered in an aggregate way (for example, in families). It is usually assumed that AP is focused in the operations (or production) area: AP receives from marketing a demand forecast and tries to adjust production at a minimum cost, considering as variables, normally, the production levels, the overtime and, sometimes, the staff size.

However, as it is stated by Singhal and Singhal (2007), AP could (and should) play a more relevant role, since it can be seen as a meeting point of main company areas activities planning. In fact, the integration of decisions regarding different areas into a single model is a current trend in planning. This integration is possible due to the improvements that have been made in hardware and software capacity over the last decades.

There are several examples of this line of work in the literature. For example, the model by Bhatnagar et al. (2007) that includes sub-processes, skills of workers, overtime and the number of contingent workers hired, the models by Lusa et al. (2008 and 2009) in which production and working time (under a flexible working time scheme) decisions in AP are integrated in a single model or production planning models that include cash management (Kirca and Murat, 1996; Guillén et al., 2007).

This paper is mainly focused on the integration of production and marketing (selling price) decisions in AP, including also flexible capacity (by means of hiring and firing workers and overtime) and cash management. Companies and the academic community began to consider the need to coordinate production and marketing in the late 1970s (Saphiro, 1977; Welam, 1977a-b; Freeland, 1980). Although few empirical studies quantify the improvement that can be obtained when production and marketing areas are integrated (O'Leary-Kelly et al., 2002), many authors consider that such integration is advantageous (Leitch, 1974; Sadjadi, 2005; Upasani, 2008). In fact, it seems clear

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3 that the inclusion of marketing strategy in production planning may reduce costs and
4 increase benefits.
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8 Some authors have proposed a planning model that includes production and price
9 decisions. Kim and Lee (1998a-b), Lee and Lee (1998a-b) and Sadjadi et al. (2005)
10 propose a price-production model for a short-term, single-period planning horizon.
11 Thomas (1970), Damon and Schramm (1972), Freeland (1980), Welam (1997b), Deng
12 and Yano (2006), Haugen et al. (2007a-b), Smith et al. (2009) and Gonzalez-Ramirez et
13 al. (2010) proposed a model for determining product prices and a production plan for a
14 multi-period planning problem. Damon and Schramm (1972), Freeland (1980), Welam
15 (1997b), Lee and Lee (1998a-b), Sadjadi et al. (2005) and Deng and Yano (2006) also
16 suggest that expenditures should be included as decision variables. Gilbert (1999, 2000)
17 studied the case of companies that work with a catalogue of products for which the price
18 should be constant throughout the whole planning period.
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27
28 With regard to capacity variables, only few authors, among those dealing with planning
29 problems in which production and marketing (selling price) are integrated, consider the
30 quantity of the workforce (e.g., Welam, 1977b; Kim and Lee, 1998b) or the number of
31 working hours (basically, overtime; e.g., Damon and Schramm, 1972) as decision
32 variables. Furthermore, few authors consider limitations of capacity as a constraint in
33 their models (e.g. Kim and Lee, 1998b; Gilbert, 2000; Deng and Yano, 2006; Haugen et
34 al. 2007a-b; Smith et al, 2009; Gonzalez-Ramirez et al., 2010).
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42 The aim of this paper is to help AP to become a suitable tool for medium-term
43 forecasting of activities in the main company areas and to ensure their coordination. We
44 propose an AP model that includes production, staff size, cash management and
45 marketing decisions (selling price). Aggregate planning that fails to consider cash
46 management can lead to cost unfavourable situations, and even to liquidity needs that
47 cannot be attained by the company. The financial implications of decisions about
48 production, inventory and hiring and firing are included in the proposed model. None of
49 the aforementioned papers include all of the decisions considered herein.
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57 After this introduction, the paper is organized as follows: Section 2 describes the
58 planning problem. Section 3 includes the optimization planning model, which is a
59 mixed integer linear program. Section 4 describes the characteristics and the solution of
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3 an example that illustrates the performance of the model. Section 5 contains the main
4 results of a computational experiment. Finally, Section 6 contains the conclusions and
5 the prospects of future research.
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10 11 12 **2. Problem description** 13

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15 The problem consists of determining for each period the production of the different
16 products (or family of products if they are aggregated), the overtime, the number of
17 workers to be hired/fired and the selling price of the products, which must be kept
18 within a lower and an upper bound.
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23 The demand for each product, which is assumed to be a function of its selling price, has
24 to be met in each period. To achieve this, production can be subcontracted without
25 limitation and new workers can be hired, provided that the number of workers does not
26 exceed an upper bound. Production capacity is assumed to be proportional to the
27 number of workers and the number of working hours, but there is a lower bound for the
28 number of workers (below which the system does not run) and an upper bound (due to
29 space reasons or because with the existing equipments hiring more workers would not
30 lead to capacity increments). There is also an upper bound for overtime.
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38 Several demand functions have been proposed in the literature, depending on the
39 characteristics of the manufacturer and the reaction of the market to changes in the price
40 of products. Some authors have worked with a linear demand function (Thomas, 1970;
41 Kim and Lee, 1998a; Lee and Lee, 1998a; Gonzalez-Ramirez et al., 2010), while others
42 have used a power demand function (for example: Damon and Schramm, 1972; Welam,
43 1977a-b; Kim and Lee, 1998a-b; Sadjadi et al., 2005) or exponential demand function
44 (Salvietti and Smith, 2008; Smith et al, 2009). In addition, the case of seasonal demand
45 is addressed by Gilbert (1999, 2000) and Deng and Yano (2006). The studies by Gilbert
46 (1999, 2000) and Deng and Yano (2006) involve a price function that is the inverse of
47 the demand function; i.e. the price depends linearly on the demand.
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57 Here, we assume the case of a manufacturer who operates in a market in which there
58 may be other competitors. Of course, the demand might depend on the price of products
59 offered by the competitors. However, as justified in Deng and Yano (2006), a
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3 downward sloping demand curve can be considered representative of the market's broad
4 reaction to increasing price set by the manufacturer.
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8 It is also assumed that the manufacturer's customers do not hold inventory of any of the
9 products (this may occur with durable or semi-durable products like tools, equipment
10 and appliances). Otherwise, demand curves should be considered dependent across
11 periods, since an increase in demand in a given period, due to a price reduction, may
12 lead to a decrease in demand in the following periods, as customers may have held
13 inventory during the low price period.
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19 In reality, as stated by Deng and Yano (2006), there is some intertemporal substitution.
20 However, the degree depends upon the nature of the product, the willingness of
21 customers to search for longer and delay their purchases, and the extent of the price
22 fluctuations that may influence the perceived value of searching.
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28 The products can be stored and there is an upper bound for the inventory of each
29 product. In addition, the warehouse is assumed to have a limited capacity, so the volume
30 of stored products must be considered. The objective is to maximize the difference
31 between the income and the costs. The costs to be considered include production costs,
32 subcontracting costs, inventory holding costs, staff costs (wages), hiring costs, firing
33 costs, overtime costs and bank interests.
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39 Cash flows are managed by means of a credit account. Three different interest rates are
40 applied: to the borrowed amount, to the deposited amount and to the available credit
41 that is not used.
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48 **3. Model**

49 **3.1. Optimization model**

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51 A mixed integer linear program is proposed to solve the problem. The parameters,
52 variables and model equations are detailed below.
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57 **Parameters**

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59 *B* Maximum amount that can be borrowed from the credit account.
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- b_0 Initial amount of the balance of the credit account (which, of course, may be positive, null or negative, provided that it is $\geq -B$).
- BF_t Balance of cashing and payments, in period t ($t=1, \dots, T$), that are fixed (i.e. they do not depend on the decisions involved in the model). They may have positive (when cashing is greater than payments), null or negative values. BF_t may include past sales, fixed payments, etc.
- ce Cost of one hour of overtime.
- cd_t Firing cost in period t ($t=1, \dots, T$).
- ch_t Hiring cost in period t ($t=1, \dots, T$).
- cp_{qt} Production cost (exclusive labour cost) per unit for product q in period t ($q=1, \dots, Q; t=1, \dots, T$).
- cr_{qt} Subcontracting cost per unit for product q in period t ($q=1, \dots, Q; t=1, \dots, T$).
- cs_q Inventory holding cost per period for product q ($q=1, \dots, Q$).
- cw_t Cost per worker in period t ($t=1, \dots, T$).
- E_t Maximum overtime in period t ($t=1, \dots, T$), per worker.
- h_t Number of ordinary working hours available in period t ($t=1, \dots, T$).
- i_t^b, i_t^d, i_t^a ($< i_t^b$) Rates of interest, per period, that apply, respectively, to the borrowed amount (the interest is credited), to the deposited amount (balance of the account, when it is positive; in this case the company obtains an interest) and to the credit available and not used (B minus the borrowed amount). We assume that the interests are credited or debited in the account in the same periods that they are earned or due ($t=1, \dots, T$).
- LP_{qt}, UP_{qt} Lower and upper bound for the selling price of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).
- LW, UW Lower and upper bound for the number of workers.

Q	Number of products.
T	Planning horizon.
US	Capacity of the warehouse (m^3).
US_q	Upper bound for the number of units of product q stored at any time ($q=1, \dots, Q$). This upper bound is not related to the warehouse capacity but, for example, to the desired stock turnover. Of course, $US_q \cdot v_q \leq US$.
v_q	Physical volume of one unit of product q ($q=1, \dots, Q$), in m^3 .
$W0$	Number of workers at the beginning of the planning horizon.
ρ_{qt}	Number of units of product q produced per worker per hour in period t ($q=1, \dots, Q, t=1, \dots, T$).

Decision variables

$he_t \in \square^+$	Overtime done by all workers in period t ($t=1, \dots, T$).
$p_{qt} \in \square^+$	Selling price of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).
$wd_t \in \square^+$	Number of workers fired at the beginning of period t ($t=1, \dots, T$).
$wh_t \in \square^+$	Number of workers hired at the beginning of period t ($t=1, \dots, T$).
$x_{qt} \in \square^+$	Production of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).

Other variables

$b_t \in \square$	Balance of the credit account at the end of period t ($t=1, \dots, T$). Since these variables can be negative, null or positive, b_t is replaced with $b_t^+ - b_t^-$, being $b_t^- \in \square^+$ the credited amount and $b_t^+ \in \square^+$ the deposited amount (i.e., absolute values of the nonpositive and nonnegative balances, respectively), at the end of period t ($t=1, \dots, T$).
$D_{qt} \in \square^+$	Demand of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).

$I_{qt} \in \square^+$ Incomes due to sales of product q in period t ($q=1, \dots, Q; t=1, \dots, T$).

$r_{qt} \in \square^+$ Units of product q to be subcontracted at period t ($q=1, \dots, Q; t=1, \dots, T$).

$s_{qt} \in \square^+$ Inventory of product q at the end of period t ($q=1, \dots, Q; t=1, \dots, T$).

$w_t \in \square^+$ Number of workers available in period t ($t=1, \dots, T$).

Model

$$[MAX] z = \sum_{t=1}^T \sum_{q=1}^Q (I_{qt} - cp_{qt} \cdot x_{qt} - cs_q \cdot s_{qt} - cr_{qt} \cdot r_{qt}) + \sum_{t=1}^T (i_t^d \cdot b_t^+ - (i_t^b - i_t^a) \cdot b_t^- - cw_t \cdot w_t - ch_t \cdot wh_t - cd_t \cdot wd_t - ce \cdot he_t) \quad (1)$$

$$(b_t^+ - b_t^-) = (b_{t-1}^+ - b_{t-1}^-) + (BF_t + b_{t-1}^+ \cdot i_t^d + I_{qt}) - \left(\begin{array}{l} i_t^b \cdot b_{t-1}^- + i_t^a \cdot (B - b_{t-1}^-) + \\ \sum_{q=1}^Q (cp_{qt} \cdot x_{qt} + cs_q \cdot s_{qt} + cr_{qt} \cdot r_{qt}) + \\ cw_t \cdot w_t + ch_t \cdot wh_t + cd_t \cdot wd_t + ce \cdot he_t \end{array} \right) \quad t = 1, \dots, T \quad (2)$$

$$I_{qt} = p_{qt} \cdot D_{qt}(p_{qt}) \quad q = 1, \dots, Q; t = 1, \dots, T \quad (3)$$

$$s_{q,t-1} + x_{qt} + r_{qt} = D_{qt}(p_{qt}) + s_{qt} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (4)$$

$$\sum_{q=1}^Q \frac{x_{qt}}{\rho_{qt}} \leq W \cdot h_t + he_t \quad t = 1, \dots, T \quad (5)$$

$$he_t \leq w_t \cdot E_t \quad t = 1, \dots, T \quad (6)$$

$$w_t = w_{t-1} + wh_t - wd_t \quad t = 1, \dots, T \quad (7)$$

$$\sum_{q=1}^Q v_q \cdot s_{qt} \leq US \quad t = 1, \dots, T \quad (8)$$

$$s_{qt} \leq US_q \quad q = 1, \dots, Q; t = 1, \dots, T \quad (9)$$

$$LP_{qt} \leq p_{qt} \leq UP_{qt} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (10)$$

$$b_t^- \leq B \quad (11)$$

$$\begin{aligned} x_{qt}, s_{qt}, wh_t, wd_t, he_t, p_{qt}, D_{qt}, I_{qt}, b_t^-, b_t^+ &\geq 0 \\ w_t &\in \mathbb{Z}^+ \end{aligned} \quad (12)$$

Equation (1) is the objective function to be maximized and corresponds to the income (sales plus bank interests) minus the costs due to production, inventories, bank interests, wages, hiring, firing and overtime; (2) corresponds to the credit account balance: the balance of the credit account in period t (b_t , which has been replaced with $b_t^- - b_t^+$) is equal to the balance in period $t-1$ ($b_{t-1}^- - b_{t-1}^+$) plus the balance of cashing and payments that are fixed before the beginning of the planning horizon (BF_t , which can be positive, null or negative) plus inputs (interest of the deposited amount of the credit account and incomes due to sales) minus the outputs (interest of the borrowed amount from the credit account, interest of the unused available credit and payments due to production, inventory, subcontracting, personnel, hiring and firing and overtime); (3) is the income function. Equation (4) is the production, inventory and demand balance; (5) imposes the upper bound for production, depending on the number of working hours; (6) imposes the upper bound for the total overtime in a period. Equation (7) corresponds to the staff balance. For the sake of simplicity, hiring is supposed to be instantaneous; a different situation could be modelled, for example, by including a lag between the period in which a worker is hired and the period in which the worker is available to work. Equations (8) and (9) impose an upper bound to the inventory. Equation (10) imposes the lower and the upper bound for the selling price; (11) imposes the upper bound for the negative balance of the credit account. Finally, (12) expresses the nonnegative, integer and binary character of some variables.

In the solutions of the computational experiments, either b_t^+ or b_t^- was always equal to zero. This happens because the interests that are paid for credited amount are greater than those gained for the deposited amount (if the balance of the credit account is equal to, for example, 600, it is preferable to assign 600 to b_t^+ and 0 to b_t^- than any other incorrect possibility in which both variables are positive as, for example, assigning 700 to b_t^+ and 100 to b_t^-). In case that the values of the interests do not guarantee that this happens, new variables and constraints should be added to the model, as follows:

$y_t \in \{0,1\}$ Auxiliary binary variable to guarantee that either b_t^+ or b_t^- are equal to zero.

If $b_t^+ > 0$ then y_t takes value 1, and if $b_t^- > 0$, y_t takes value 0 ($t=1, \dots, T$).

$$\left. \begin{array}{l} b_t^+ \leq M_t \cdot y_t \\ b_t^- \leq B \cdot (1 - y_t) \end{array} \right\} \quad t = 1, \dots, T \quad (13)$$

where M_t is a large enough number (an upper bound for b_t^+).

3.2. Demand function, $D_{qt}(p_{qt})$

On the basis of the assumptions stated at section 2, the following demand function is considered:

$$D_{qt}(p_{qt}) = \alpha_{qt} - \beta_{qt} \cdot p_{qt}^{\gamma_{qt}} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (14)$$

Where $\alpha_{qt}, \beta_{qt}, \gamma_{qt}$ ($q=1, \dots, Q; t=1, \dots, T$) are the parameters of the demand function, with $\alpha_{qt} > 0$, $\beta_{qt} > 0$, $\gamma_{qt} \geq 0$. It is assumed that, as normally occurs, the demand decreases according to the price (the higher the price, the lower the demand).

Note that the potential variability in demand due to exogenous factors can be considered by means of the function parameters, whose value may vary over time.

Of course, demand cannot take negative values; this means that $p_{qt} \leq \left(\frac{\alpha_{qt}}{\beta_{qt}} \right)^{1/\gamma_{qt}}$ and,

hence, that $UP_{qt} \leq \left(\frac{\alpha_{qt}}{\beta_{qt}} \right)^{1/\gamma_{qt}}$.

The parameter α corresponds to the maximum demand, which would be obtained with a selling price equal to 0 (i.e., if the company gave the product for free). The parameter β corresponds to the demand scaling factor and represents the fact that the company does not operate in a monopolistic market. Finally, the parameter γ is the price elasticity of demand and represents the sensitivity of the market to changes in the selling price.

Using the demand function displayed in equation (14) and considering that there is no lost or delayed demand, the income function, which is also nonlinear, is detailed in Equation (15).

$$I_{qt} = p_{qt} \cdot D_{qt}(p_{qt}) = p_{qt} \cdot \alpha_{qt} - \beta_{qt} \cdot p_{qt}^{\gamma_{qt}+1} \quad (15)$$

In next section the linearisation of demand and income functions is detailed.

3.3. Linearising demand and income functions

The proposed demand function (equation (14)) is convex for $\gamma_{qt} \leq 1$ (for $\gamma_{qt} = 1$ the function would be linear) and concave otherwise, while the income function is concave for all parameter values. When the demand function is convex, it can be linearised by approximating it to a stepwise function. However, since generally speaking its convexity cannot be assumed, the linearisation of demand and income has been done by considering them as discrete functions.

For doing so, a finite set of values has been considered for the selling price values. Actually, this can be interesting for many companies, since allow considering other features that may be quite difficult to model (psychological criteria), such as considering prices close to a value (the usual X.99 price), prices that may look appealing for customers or prices that are multiple of a certain value (even though this particular condition can be modelled in different ways). Of course, it is possible to consider a wide range of selling prices by designing a wide enough set of admissible values.

For linearising the demand and income functions, the following additional data and variables are defined:

Data:

P_{qt} Set of admissible values for the price of product q in period t ($q=1, \dots, Q$; $t=1, \dots, T$).

Variables:

$y_{nqt} \in \{0,1\}$ Takes value 1 if the price of product q in period t (p_{qt}) is equal to n ($q=1,\dots,Q; t=1,\dots,T; n \in P_{qt}$).

The demand and the income nonlinear functions are modelled as follows:

$$\left. \begin{aligned} p_{qt} &= \sum_{n \in P_{qt}} y_{nqt} \cdot n \\ \sum_{n \in P_{qt}} y_{nqt} &= 1 \end{aligned} \right\} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (16), (17)$$

$$D_{qt} = \alpha_{qt} - \beta_{qt} \cdot \sum_{n \in P_{qt}} y_{nqt} \cdot n^{\gamma_{qt}} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (18)$$

$$I_{qt} = p_{qt} \cdot \alpha_{qt} - \beta_{qt} \cdot \sum_{n \in P_{qt}} y_{nqt} \cdot n^{\gamma_{qt}+1} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (19)$$

Equations (16) and (17) force the binary variable y_{nqt} be equal to 1 when the price of the product q in period t is equal to n , and 0 otherwise. Equations (18) and (19) assign the demand and income values, respectively, according to the binary variables (the price is replaced by n when the binary variable that indicates whether the price is equal to n or not takes value 1).

Now, the optimization model can be formulated as a mixed integer linear model. For doing so, the following steps should be done:

- Add P_{qt} as a parameter and y_{nqt} as variables
- Add equations (16) and (17)
- Replace equation (3) by equation (19)
- Replace $D_{qt}(p_{qt})$ in equation (4) by the expression on the right of equation (18)
- Delete equation (10) (obviously, the first value in P_{qt} will be equal to LP_{qt} and the last one to UP_{qt}).
- In equation (12), add $y_{nqt} \in \{0,1\}$

4. Numerical examples and managerial insights

In this section, we provide the results of a numerical example. The most relevant parameters used for the experiment are given in Table 1 and Figure 1 (as it can be observed in this figure, a seasonal demand has been considered). Since real data was not available, the different parameters were designed with the aim of obtaining realistic scenarios. The model was solved by means of CPLEX 11.0 in less than three seconds using a standard PC.

[INSERT TABLE 1. PARAMETER VALUES USED IN THE EXAMPLE]

[INSERT FIGURE 1. Values of α_t in the example]

Figures 2, 3 and 4 present the optimal solution that was obtained. For each week, the selling price is shown in Figure 2, the number of workers is shown in Figure 3 and the demand, production capacity, production (both in normal hours and in overtime), subcontracting and inventory are shown in Figure 4.

Figure 2 also shows that there are some periods in which it could be considered that the change in the selling price is large. Smaller changes can be ensured in order to avoid unforeseen market behaviour by including the following constraint:

$$-k_{qt} \leq p_{qt} - p_{q,t-1} \leq k_{qt} \quad q = 1, \dots, Q; t = 1, \dots, T \quad (20)$$

where k_{qt} is the maximum change allowed in the price of product q in period t (note that in this case p_{q0} should also be defined).

[INSERT FIGURE 2. Optimal selling price]

[INSERT FIGURE 3. Staff size for the optimal solution]

[INSERT FIGURE 4. Demand, production capacity, production (normal hours and overtime), subcontracting and inventory for the optimal solution]

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6 Figure 1 show a seasonal profile of the demand (α data). Analysing Figures 1 and 2 it
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8 can be clearly seen that when α goes up, also the price goes up so the demand does not
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10 increase too much; similarly, when α goes down, the price also goes down to prevent a
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12 large decrease in the demand. Changing the selling price helps having demand under (a
13
14 relative) control and, hence, avoiding large subcontracting costs (which would occur if
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16 demand increased too much) or large reductions in the income (if demand decreases too
17
18 much).

19
20 In Figures 3 and 4 it can be observed that, as expected, when the demand goes up it is
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22 desirable to hire workers, while when the demand goes down the number of workers
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24 decreases. It can be seen, also, that when there is a small change in the demand (as the
25
26 one in week 14) the number of workers does not change; this is reasonable if we take
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28 into account that hiring and firing workers has a significant cost. The fact that the
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30 capacity (number of workers and overtime) is adapted to the demand can be clearly
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32 observed comparing demand and capacity lines in Figure 4.

33
34 Finally, Figure 5 shows the evolution of the cash inflows, cash outflows and credit
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36 account balance (this has been divided by 10 so that changes in inflows and outflows
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38 are clearer). Both inflows and outflows follow the activity level of the company, and
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40 analysing the outflows it can be seen that in some periods there is a peak, which
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42 corresponds to hiring or firing workers.

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44
45 **[INSERT FIGURE 5. Cash inflows, cash outflows and credit account balance for**
46
47 **the optimal solution]**
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51 The convenience (or benefit) of using a model such as the one proposed here, in which
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53 production, hiring and firing and product price decisions are integrated in Aggregate
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55 Planning (AP), may depend on the company parameters. To analyse this issue, different
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57 models and scenarios were considered in a new experiment:

58
59 Models:
60

- M: Number of workers and selling price is considered to be fixed (i.e., production planning)
- M-w: The number of workers can be modified; the selling price is fixed (staff decisions are included)
- M-p: The number of workers is fixed; the selling price can be modified (marketing decisions are included)
- M-wp: Both number of workers and selling price can be modified (i.e., coincides with the model proposed in this paper).

In models M, M-w and M-p the decisions that are supposed to be fixed are considered to be constant during the planning horizon and, among all the possible values, those that optimize the objective function have been set (i.e., in the M model, the number of workers and the selling price for the whole horizon are included as two decision variables; for models M-w and M-p an analogue thing is done with the selling price and the number of workers, respectively).

Scenarios:

- α : highly seasonal, medium seasonal, low seasonal (see Figure 6)
- γ : high (0.8; the market is highly sensible to changes in selling price), low (0.5; the sensitivity to changes in price is low)
- Labour costs: high ($c_{w_i}=200$), low ($c_{w_i}=100$)

[INSERT FIGURE 6. α profiles]

The combination of the four models and the previous parameters gives rise to 48 possible situations. For each one of the 12 parameter combinations, different instances were obtained by randomly modifying the values of α (the original values were multiplied by a random number between 0.95 and 1.05).

For each one of the 12 scenarios the four models were run and the value of the objective function (the profit) was saved. Model M is considered to represent the starting situation, M-w and M-p intermediate situations and M-wp the final one, in which all decisions are integrated. The results of the model-scenario analysis are included in

Table 2, where for each one of the 12 parameter combinations the following information is provided:

- ***Incr M-w/M***: shows the average increase in the value of the objective function (in %) when model M-w is used instead of model M (i.e., the number of workers is considered as a decision variable).
- ***Incr M-p/M***: shows the average increase in the value of the objective function (in %) when model M-p is used instead of model M (i.e., the selling price is considered as a decision variable).
- ***Incr M-wp/M***: shows the average increase in the value of the objective function (in %) when model M-wp is used instead of model M (i.e., the number of workers and the selling price are considered as decision variables).

[INSERT TABLE 2. SCENARIO ANALYSIS RESULTS (% INCR. OBJECTIVE FUNCTION)]

Values in Table 2 lead to some managerial insights. Even though most of them are reasonable, the experiments are a proof and a quantification of them. The most important remarks are the following ones:

- When the market is sensible to changes in prices (the γ parameter is not low), the possibility of integrating selling price decisions in AP lead to high increments in profit (objective function). This is reasonable, because if the sensibility of the market to changes in selling price is high, the impact that these changes have on the demand (and, hence, on the incomes and the profit) are also high.
- The increasing of profit due to considering selling price in AP is higher for seasonal demand profiles (the α parameter is seasonal). This can be explained as follows: model M use as selling price the value that would be optimal if the selling price had to be the same for the whole horizon. When the selling price can be changed at any period, if the demand is almost constant (the α parameter is not seasonal) the optimal value for the selling price is also almost constant during the planning horizon (i.e., similar to those used in models with a fixed

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2
3 selling price). Hence, the increment in the profit when considering that the
4 selling price can be changed during the planning horizon is short for low
5 seasonal demands.
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7

- 8
9 • The increasing of profit due to considering selling price in AP is higher for high
10 labour cost scenarios. When labour costs are high and demand cannot be
11 changed (i.e., selling price is fixed), the unique alternatives to meet demand are
12 subcontracting (which has an important cost), overtime or modifying the number
13 of workers (which means, since labour costs are high, an important cost). But if
14 the selling price can be changed then the so high labour costs (overtime and/or
15 hiring/firing costs) can be reduced by modifying the demand (i.e., adapting, at
16 least a bit, the demand to the capacity and not only the capacity to the demand)
17 and a great increase in the profit can be obtained.
18
- 19 • Considering hiring and firing in AP is always interesting, and, as it was
20 expected, the increasing of profit is higher for seasonal demand profiles (the α
21 parameter is seasonal) in which the need of adapting capacity to demand is high.
22
- 23 • The results seem to suggest that when the demand is seasonal (highly or
24 medium), the increasing of profit due to considering hiring and firing is higher
25 for high labour cost scenarios. This could lead to false conclusions because what
26 it really happens is the following: when the demand (the α parameter) increases,
27 adapting capacity to demand by means of hiring has a large cost in high labour
28 cost scenarios (thus the increasing in the profit is small) and a small cost in the
29 low labour cost scenarios (thus the increasing in the profit is large). Instead,
30 when the demand (the α parameter) decreases, adapting capacity to demand by
31 means of firing may lead to a large profit increment in high labour cost scenarios
32 (because even if the firing costs are high, during the low demand periods the
33 cost of workers is significantly reduced) and a small cost saving in the low
34 labour cost scenarios (thus the increasing in the profit is small).
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56 **5. Computational experiment**

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58 The model may have a large number of integer variables and constraints. To check
59 whether it can be solved in reasonable computing times, we carried out a computational
60 experiment.

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3 The basic data set of the experiment (shown in Table 3) was the same as the one used in
4 the numerical example described in the previous section. To generate instances of
5 different sizes, different values for the number of products (Q) and for the number of
6 admissible values for the price of the products in each period ($|P_{qt}|$) were used: 5, 10 and
7 50 products and 6, 11 and 51 number of admissible prices. For each of the nine
8 combinations, 10 instances were generated by setting the parameter α_{qt} at random.
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18 **[INSERT TABLE 3. PARAMETER VALUES USED IN THE**
19 **COMPUTATIONAL EXPERIMENT]**
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25 The number of variables and constraints are given in Table 4. Table 5 summarizes the
26 results of the experiment and gives the minimum, average and maximum computing
27 times. Since all the instances were solved to optimality in very short times (especially if
28 we consider the kind of problem being solved), it can be concluded that the model can
29 be solved in a very efficient way and that could be used as a decision tool.
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38 **[INSERT TABLE 4. NUMBER OF VARIABLES/CONSTRAINTS OF THE**
39 **INSTANCES IN THE COMPUTATIONAL EXPERIMENT]**
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45 **[INSERT TABLE 5. SOLVING TIMES (IN SECONDS) FOR THE**
46 **COMPUTATIONAL EXPERIMENT (MINIMUM, AVERAGE, AND**
47 **MAXIMUM)]**
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55 **6. Conclusions and future research**

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57 In this paper, a planning model that integrates production, staff and marketing decisions
58 is proposed. Demand is assumed to be a nonlinear function of the selling price, as are
59 the incomes. The nonlinear functions are linearized by means of mixed integer linear
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3 programming tools. The model has a high number of binary variables, but is solved to
4 optimality within a few seconds, using standard optimization software (CPLEX 11.0) on
5 a PC. Hence, the model could be the basis of a decision tool that could be used by many
6 companies. The results of a numerical example and the analysis of different scenarios
7 show the solutions given by the model and the advantages of integrating production,
8 human resources and marketing decisions and also the situations in which this
9 integration is more desirable.

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17 Future research should try to integrate other decisions such as supplies (in a multi-
18 supplier environment, considering different prices, quality and lead time conditions for
19 different suppliers), working time (under flexible working time schemes such as
20 working time accounts), cash management and hiring and firing in a more complex way
21 (considering learning periods, firing costs depending on the worker being fired, etc.).
22 The final goal is to design planning models which integrate the most relevant areas of a
23 company. In addition, future research includes testing other demand functions (some of
24 them proposed by other authors).

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32 On the other hand, the model proposed in this paper is suitable for companies that have
33 no advertising or advertising with a little impact and also for companies in which
34 advertisement is decided beforehand (and the effects are considered to be included in
35 the α demand parameter). In the future, we could extend the model including
36 advertisement investments as decision variables that have an influence on the demand
37 (i.e., to consider a demand function depending both on the selling price and on the
38 advertisement).

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References

Bhatnagar, R., Saddikutti, V., Rajgopalan, A. (2007). Contingent manpower planning in a high clock speed industry. *International Journal of Production Research*, 45, 2051-2072.

Damon, W., Schramm, R. (1972), A simultaneous decision model for production, marketing and finance. *Management Science*, 19, 161-172.

Deng, S., Yano, C.A. (2006). Joint Production and Pricing decisions with Setup Cost and Capacity Constraints. *Management Science*, 52, 741-756.

Freeland, J. R. (1980). Coordination strategies for production and marketing in a functionally decentralized firm. *AIIE Transactions*, 12, 126-132.

González-Ramírez, R., Smith, N. R., Askin, R. G. (2010). A heuristic approach for a multi-product capacitated lot-sizing problem with pricing. *International Journal of Production Research*, Published on line, DOI: 10.1080/00207540903524482.

Gilbert, S.M. (1999). Coordinating of pricing and multi-period production for constant priced goods. *European Journal of Operational Research*, 114, 330-337.

Gilbert, S.M. (2000). Coordination of Pricing and Multiple-Period Production Across Multiple Constant Priced Goods. *Management Science*, 46, 1602-1616.

Guillén, G., Badell, M., Puigjaner, L. (2007). A holistic framework for short-term supply chain management integrating production and corporate financial planning. *International Journal of Production Economics*, 106, 288-306.

Haugen, K. K., Olstad, A., Pettersen, B. I. (2007a). Solving large-scale profit maximization capacitated lot-size problems by heuristic methods. *Journal of Mathematical Modelling and Algorithms*, 6 (1), 135-149.

Haugen, K. K., Olstad, A., Pettersen, B. I. (2007b). The profit maximizing capacitated lot-size (PCLSP) problem. *European Journal of Operational Research*, 176 (1), 165-176.

1
2
3 Kirca, Ö., Murat, M. (1996). An integrated production and financial planning model and
4 an application. *IIE transactions*, 28, 677-686.

7 Kim, D.D., Lee, W.J. (1998a). Optimal coordination strategies for production and
8 marketing decisions, *Operations Research Letters*, 22, 41-47.

11 Kim, D.D., Lee, W.J. (1998b). Optimal joint pricing and lot sizing with fixed and
12 variable capacity. *European Journal of operational Research*, 109, 212-227.

15 Lee, W.J., K.C. Lee. (1999a). A meta decision support system approach to coordinating
16 production/marketing decisions. *Decision Support Systems*, 25, 239-250.

19 Lee, W.J., Lee, K.C. (1999b). PROMISE: a distributed DSS approach to coordinating
20 production and marketing decisions. *Computers & Operations Research*, 26, 901-920.

23 Leitch, R. A. (1974). Marketing strategy and the optimal production schedule.
24 *Management Science*, 21, 302-312.

27 Lusa, A., Corominas, A., Pastor, R. (2008). An exact procedure to plan holidays and
28 working time under annualised hours considering cross-trained workers with different
29 efficiencies. *International Journal of Production Research*, 46, 2123-2142.

32 Lusa, A., Corominas, A., Olivella, J., Pastor, R. (2009). Production planning under a
33 working time accounts scheme. *International Journal of Production Research*, 47,
34 3435-3451.

37 O'Leary-Kelly, S. W., Flores, B. E. (2002). The integration of manufacturing and
38 marketing/sales decisions: impact on organizational performance. *Journal of Operations
39 Systems*, 2, 221-240.

42 Sadjadi, S.J., Oroujrr, M., Aryanezhad, M.B. (2005). Optimal Production and Marketing
43 Planning. *Computational Optimization and Applications*, 20, 195-203.

46 Salviotti, L., Smith, N. R. (2008). A profit-maximizing economic lot scheduling
47 problem with price optimization. *European Journal of Operational Research*, 184 (3),
48 900-914.

1
2
3 Saphiro, B. P. (1977). Can marketing and manufacturing coexist? *Harvard Business*
4 *Review*, 55, 104-114.
5
6

7
8 Singhal, J., Singhal, K. (2007). Holt, Modigliani, Muth, and Simon's work and its role
9 in the renaissance and evolution of operations management. *Journal of Operations*
10 *Management*, 25, 300-309.
11
12

13
14 Smith, N. R., Limón, J., Cardenas-Barrón, L. E. (2009). Optimal pricing and production
15 master planning in a multiperiod horizon considering capacity and inventory
16 constraints. *Mathematical Problems in Engineering*, published on line,
17 <http://www.hindawi.com/journals/mpe/2009/932676.html>.
18
19
20
21

22 Thomas, J. (1970). Price-production decisions with deterministic demand. *Management*
23 *Science*, 16, 747-750.
24
25

26
27 Upasani, A., Uzsoy, R. (2008). Incorporating manufacturing lead times in join
28 production-marketing models: A review and some future directions. *Annals of*
29 *Operations Research*, 161, 171-188.
30
31
32

33 Welam, U. P. (1977a). On a simultaneous decision model for marketing, production and
34 finance. *Management Science*, 23, 1005-1009.
35
36

37
38 Welam, U. P. (1977b). Synthesizing Short Run Production and Marketing Decisions.
39 *AIIE Transactions*, 93, 1005-1009.
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TABLES

Table 1. Parameter values used in the example

T	52 weeks	cp_t	2 monetary units/product unit
Q	1 product	cs	0.5 monetary units/ product unit-week
ρ_t	0.5 product units/hour-worker	cw_t	100 monetary units/worker
US	300 product units	ch_t	100 monetary units/worker ($=cw_t$)
h_t	40 hours	cd_t	110 monetary units/worker ($=1.1cw_t$)
P	{5, 5.5, 6, 6.5,..., 29.5, 30}	ce	2 monetary units/hour ($=0.02cw_t$)
$W0$	8 workers	B	10,000 monetary units
$LW,$ UW	5, 15 workers	i_t^b, i_t^d, i_t^a	0.0015, 0.0004, 0.0006
E_t	10 hours	B_0	0 monetary units
α_t	A seasonal profile has been generated, with the values of α_t set between 220 and 510 units (the values are shown in Figure 1).	S_0	0 product units
β_t	20	BF_t	0 monetary units
γ_t	0.8		

Table 2. Scenario analysis results (% Incr. objective function)

		High labour costs	Low labour costs
Highly seasonal α	High γ	<i>Incr M-w/M: 70.65</i> <i>Incr M-p/M: 172.51</i> <i>Incr M-wp/M: 183.48</i>	<i>Incr M-w/M: 15.86</i> <i>Incr M-p/M: 40.56</i> <i>Incr M-wp/M: 43.45</i>
	Low γ	<i>Incr M-w/M: 9.30</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 9.31</i>	<i>Incr M-w/M: 4.91</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 4.91</i>
Medium seasonal α	High γ	<i>Incr M-w/M: 5.94</i> <i>Incr M-p/M: 24.40</i> <i>Incr M-wp/M: 24.40</i>	<i>Incr M-w/M: 4.08</i> <i>Incr M-p/M: 10.65</i> <i>Incr M-wp/M: 11.21</i>
	Low γ	<i>Incr M-w/M: 1.47</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 1.47</i>	<i>Incr M-w/M: 1.03</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 1.03</i>
Low seasonal α	High γ	<i>Incr M-w/M: 0.31</i> <i>Incr M-p/M: 6.64</i> <i>Incr M-wp/M: 6.64</i>	<i>Incr M-w/M: 1.54</i> <i>Incr M-p/M: 3.56</i> <i>Incr M-wp/M: 3.56</i>
	Low γ	<i>Incr M-w/M: 0.20</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 0.20</i>	<i>Incr M-w/M: 0.28</i> <i>Incr M-p/M: 0</i> <i>Incr M-wp/M: 0.28</i>

Table 3. Parameter values used in the computational experiment

T	52 weeks	γ_{qt}	0.8
Q	5, 10 and 50 products	cp_{qt}	2 monetary units/product unit
ρ_t	0.5 product units/hour-worker	cs_q	0.5 monetary units/ product unit-week
US	300 volume units	cw_t	100 monetary units/worker
v_q	1 volume unit/product unit	ch_t	100 monetary units/worker
US_q	US/Q product units	cd_t	110 monetary units/worker
h_t	40 hours	ce	2 monetary units/hour
P_{qt}	From 15 to 20, in steps of 1, 0.5 and 0.1 monetary units (this gives 6, 11 and 51 possible values)	B	10,000 monetary units
$W0$	8 workers	i_t^b, i_t^d, i_t^a	0.0015, 0.0004, 0.0006
$LW,$ UW	5, 15 workers	b_0	0 monetary units
E_t	10 hours	s_{0q}	0 product units
α_{qt}	$\alpha_t \in U[220, 510]; \alpha_{qt} = \alpha_t / Q$	BF_t	0 monetary units
β_{qt}	$20/Q$		

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Table 4. Number of variables/constraints of the instances in the computational experiment

Number of products	Number of admissible values for the price (for each product)		
	6	11	51
5	3,078 / 1,464	4,378 / 1,464	14,778 / 1,464
10	5,683 / 2,509	8,283 / 2,509	29,083 / 2,509
50	26,523 / 10,869	39,523 / 10,869	143,523 / 10,869

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Table 5. Solving times (in seconds) for the computational experiment (minimum, average, and maximum)

Number of products	Number of admissible values for the price (for each product)								
	6			11			51		
5	0.29	0.55	1.38	0.34	0.58	0.81	0.98	1.88	6.7
10	0.59	0.92	1.48	0.89	1.36	2	1.94	3.95	13.96
50	2.3	6.7	12.94	3.56	8.89	30.3	12	35.46	132.5

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FIGURES

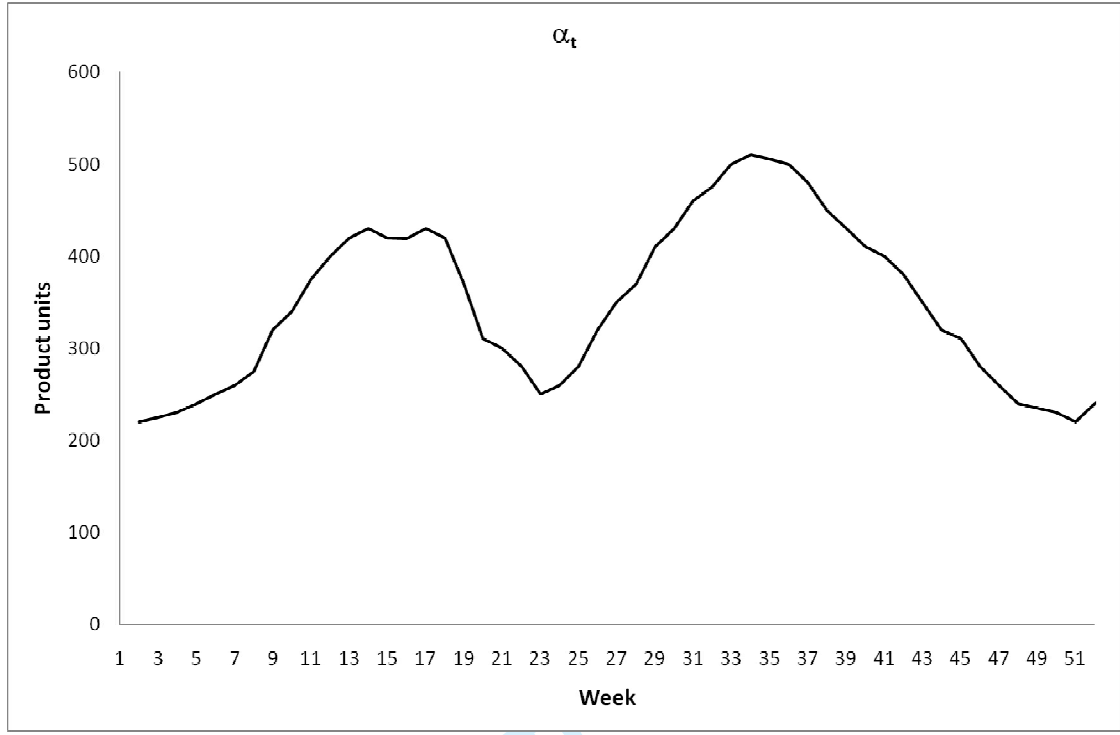


Figure 1. Values of α_t in the example

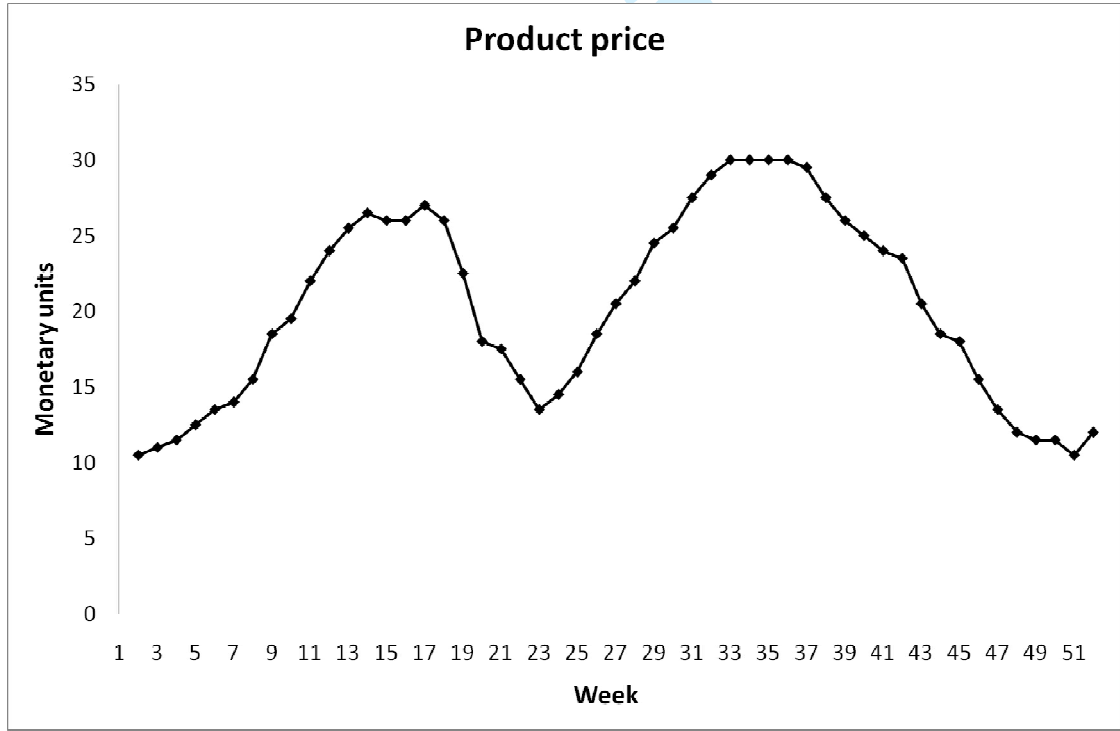


Figure 2. Optimal selling price

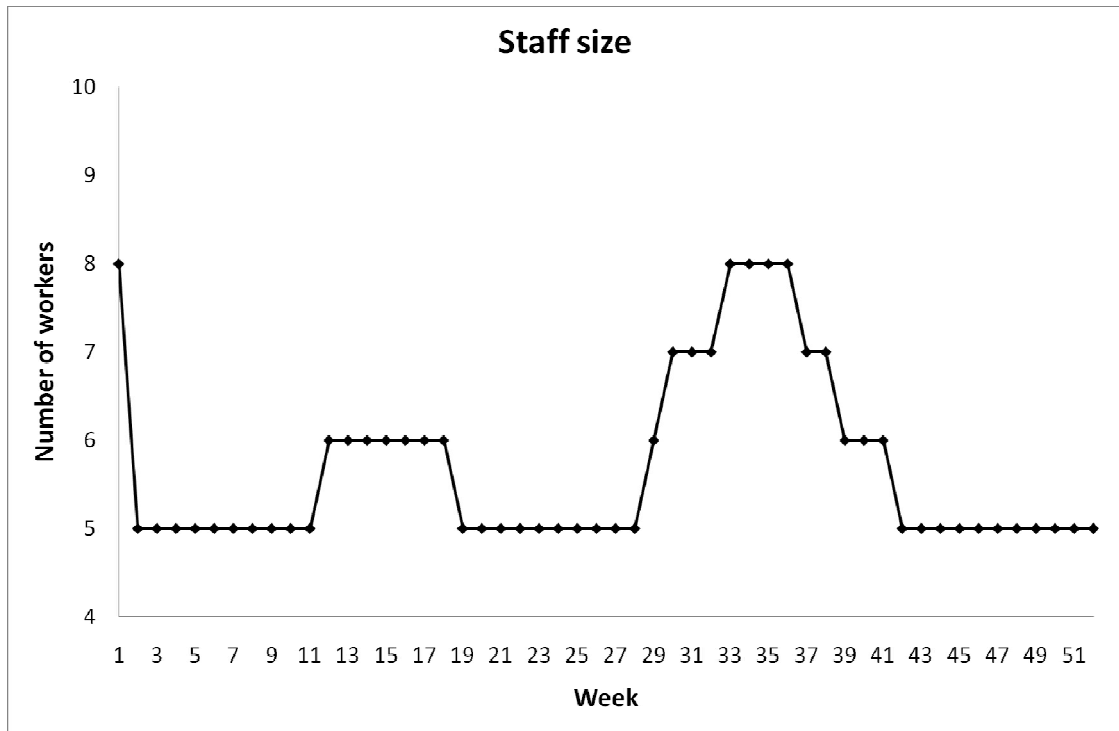


Figure 3. Staff size for the optimal solution

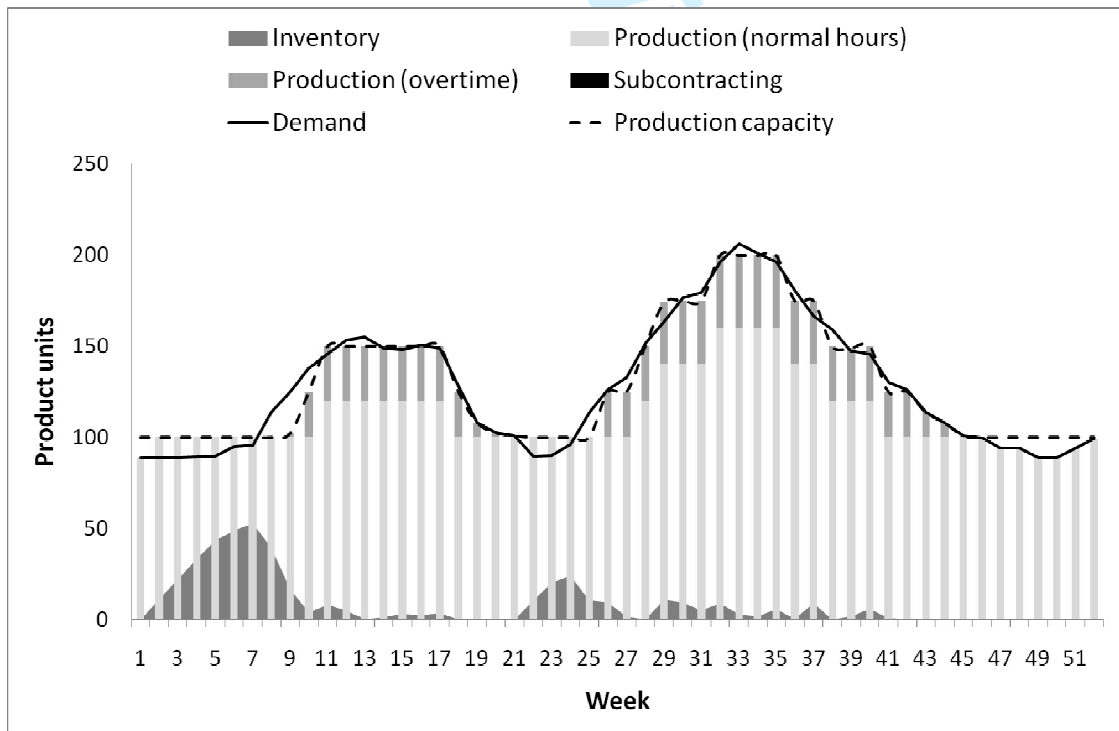


Figure 4. Demand, production capacity, production (normal hours and overtime), subcontracting and inventory for the optimal solution

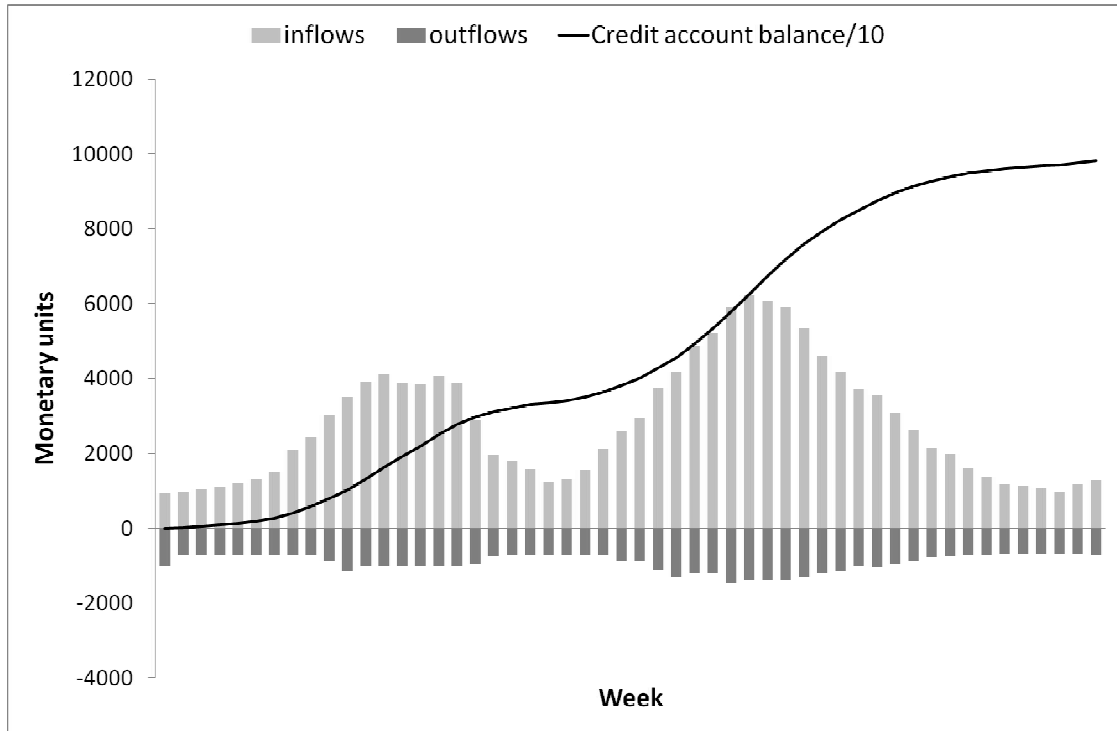


Figure 5. Cash inflows, cash outflows and credit account balance for the optimal solution

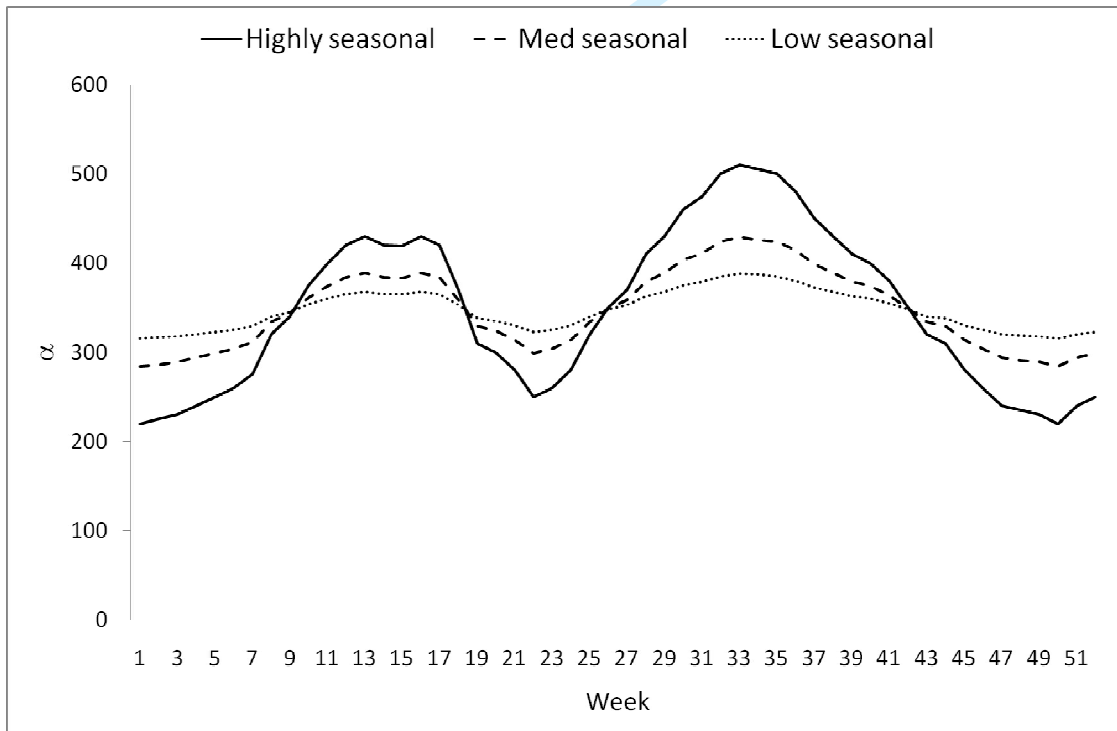


Figure 6. α profiles

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