Calculation of DEP force on spherical particle in non-uniform electric fields

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***Abstract* — The effective moment method is used to calculate the dielectrophoretic forces exerted on a homogeneous spherical particle by external non-uniform electric fields. We examine the problem of a spherical particle placed in the center of two types of microsystems, to asses the accuracy of the dipole moment method for situations of slightly and strongly non-uniform electric fields. The results are compared against net force calculations using the Maxwell stress tensor method.**

I. INTRODUCTION

When a dielectric particle is placed in a medium of different electrical properties and is subjected to a spatially non-uniform electric field, a force is exerted on the particle. This force called the dielectrophoretic (DEP) force, may be used to manipulate and characterize many types of particles, such as biological cells under special conditions [3].

Here we calculate the DEP force on a homogeneous spherical particle in non-uniform fields, which are generated by two practical microsystems beyond the most commonly used to perform DEP experiments. The point-plate electrode structure (Fig. 2) enables to generate highly non-uniform field suitable for particle manipulations such as feedback-controlled DEP levitation [2]. The four electrode chamber (Fig. 1) is another microsystem configuration used in dielectrophoretic experiments such as cell sorting and trapping [3]. First we introduce the force calculation approaches and describe the electrode systems, where the particle of interest is placed. Then the results for the force obtained from the dipole approximation are presented and compared with the corresponding results obtained from the rigorous Maxwell stress tensor method both for verification and for determination of higher order force terms contribution to the total dielectrophoretic force.

II. DIELECTROPHORETIC FORCE CALCULATION

In the particle electromechanics theory, the simplest and most direct approach to calculate the force and the torque is a two-step process known as the effective dipole moment method, whereby first one calculates the dipole moment induced in the particle by the electric field and then the force on the particle follows from standard formulas of classical electrodynamics [1]. Consider a homogeneous spherical particle with relative permittivity ε_2 and conductivity σ_2 immersed in a dielectric fluid with relative permittivity ε_1 and conductivity σ_1. According to the effective dipole moment method, the DEP force expression is given by,

$$\mathbf{F} = (\mathbf{p}^{(1)} \cdot \nabla)\mathbf{E}, \quad \text{where} \quad \mathbf{p}^{(1)} = 4\pi\varepsilon_1 K^{(1)}\mathbf{E}, \qquad (1)$$

where $\mathbf{p}^{(1)}$ is the effective dipole moment, $\mathbf{E}$ the imposed electric field and $K^{(1)} = (\underline{\varepsilon}_2 - \underline{\varepsilon}_1)/(\underline{\varepsilon}_2 + 2\underline{\varepsilon}_1)$ the complex frequency-dependent Clausius-Mossotti factor. The complex permittivity of different materials in the system is given by $\underline{\varepsilon}_i = \varepsilon_0\varepsilon_i - j\frac{\sigma_i}{2\pi f}$ where $j^2 = -1$, ε_0 the vacuum permittivity and f the electric field frequency.

Equation (1) for the force on a dipole is not exact for any situation of particle in a non-uniform electric field. This approximation breaks down in some situations described by Jones [1]. The particle shape effect was partly treated in [5]. Here electric field non-uniformity effect is examined. The multipole moment expansion enables to extend the dipole moment approach, introducing higher-order terms [1]:

$$\mathbf{F} = \sum_{n}^{N} F^{(n)} = \mathbf{p}^{(1)} \cdot \nabla\mathbf{E} + \mathbf{p}^{(2)} \cdot \nabla\nabla\mathbf{E} + \ldots \qquad (2)$$

where $\mathbf{p}^{(1)}, \mathbf{p}^{(2)}$ are the first multipole moments [1]. A practical method for the multipole moments factors estimation for any particle shape has been introduced in [5].

From (2) a force expression may be derived for spherical particles using linear multipoles, given by [1]

$$F_z = \sum_{n}^{N} \frac{2\pi\varepsilon_1 R^{2n+1}}{n!(n-1)!} Re(K^{(n)}) \frac{\partial}{\partial z} \left(\frac{\partial^{n-1} E_z}{\partial z^{n-1}} \right)^2 \qquad (3)$$

where $K^{(n)} = (\underline{\varepsilon}_2 - \underline{\varepsilon}_1)/(n\underline{\varepsilon}_1 + (n+1)\underline{\varepsilon}_2)$ is the generalized Clausius-Mossotti factor. Another method for calculating the DEP force used by Sauer and Schlögl and studied in [4] is based on integration the Maxwell stress tensor over the surface of the particle.

A. *Particle in a slightly non-uniform electric field*

A variety of four-electrode configurations and correction factors for the field strength are available in the literature, and it is well established that the most homogeneous fields are generated by the electrodes of polynomial geometry where the polynomials defining the electric potential are derived from Laplace's equation analysis [3].

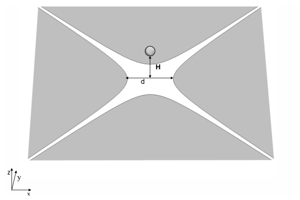

Fig. 1. A spherical particle placed in microsystem of polynomial electrodes.

A spherical particle with relative permittivity $\varepsilon_2 = 2.55\mathrm{F/m}$ and conductivity $\sigma_2 = 10^{-2}\mathrm{S/m}$, suspended in a fluid medium with relative permittivity $\varepsilon_1 = 77.8\mathrm{F/m}$ and conductivity $\sigma_1 = 10^{-3}\mathrm{S/m}$, is placed on the vertical symmetry axis of the four electrode chamber with the particle center at heights $H = 80\mu\mathrm{m}$ and $H = 160\mu\mathrm{m}$ above the electrode plane. The distance distance between opposing electrode tips is $d = 400\mu\mathrm{m}$. The 3d electric field distribution is first calculated solving the Laplace equation in the system (without the cell) using the finite element analysis software COMSOL Multiphysics. The force is then estimated according to the dipole moment and Maxwell stress methods and the results are shown in Fig. 3.

B. Particle in a highly non-uniform electric field

The same particle and surrounding fluid medium properties as in subsection II.A. are considered. The particle is subjected to a highly non-uniform field generated by a point-plate electrode, with a distance d between the point and the plane electrode of $100\mu\mathrm{m}$, as shown in Fig. 2. The particle is positioned near the point (region of high field gradients), with its center at $h = 80\mu\mathrm{m}$ above the plane electrode.

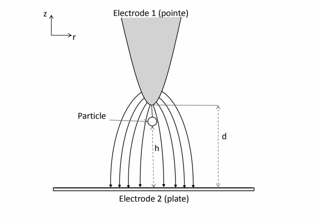

Fig. 2. Point-plate electrode system and a particle.

To calculate the electric field distribution to which the cell will be exposed in this chamber, we first solved the Laplace equation (with the suspending medium but without the cell) using the finite element method. Introducing a polar coordinates (r, ϕ, z) and taking advantage of the electrode structure symmetry, only a 2d calculation in the (r, z) plane is needed. The spherical particle of radius $R = 15\mu\mathrm{m}$ is positioned on the central axis.

III. NUMERICAL RESULTS

The force experienced by the particle placed in the slightly non-uniform electric field of the four-electrode chamber is calculated using the dipole approximation and the Maxwell stress tensor integrated over the surface of the particle and the results are shown in Fig. 3.

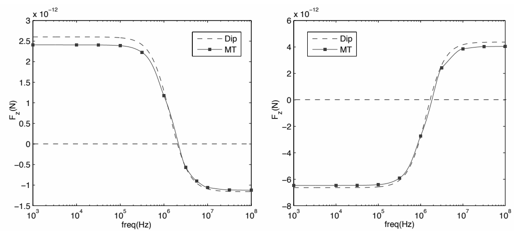

Fig. 3. Force calculation results using the effective dipole and the Maxwell stress tensor methods, for $H = 80\mu m$ (left) and $H = 160\mu m$ (right).

As suggested by the plots, the effective dipole method produces a result globally consistent with the Maxwell stress tensor approach. When the particle is closer to the electrodes ($H = 80\mu m$), the dipole moment method slightly overestimates the force at low frequencies with respect to the corresponding values for the Maxwell stress tensor analysis, while the approximation appear to be more accurate when the particle is far from the electrodes ($H = 160\mu m$).

The force experienced by the particle placed in the highly non-uniform electric field generated by the point-plate electrode system is calculated using the dipole approximation, the higher-order (quadrupole, octopole,...) moments and the Maxwell stress tensor. The results are shown in Fig. 4. It

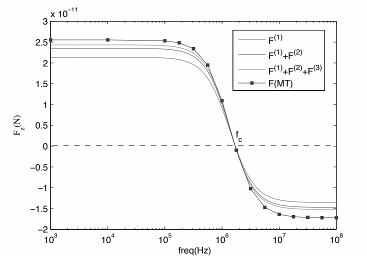

Fig. 4. Force calculation results using the multipole moment and Maxwell stress tensor methods.

may be seen from the data that, the force value in the more accurate Maxwell stress tensor spectra are shifted with respect to the corresponding values for the dipole analysis. It may be seen from the data that the higher-order term contribution corrects the dipole approximation witch underestimates the total force in particular at low and high frequencies. However, the crossover frequency f_c value matches well in different calculations. According to the dipole moment model $f_c = \frac{1}{2}\left(\frac{(\sigma_1-\sigma_2)(\sigma_2+2\sigma_1)}{(\varepsilon_2-\varepsilon_1)(\varepsilon_2+2\varepsilon_1)}\right)^{1/2}$. This frequency point at which the DEP force changes sign is of particular significance for the dielectric parameters identification from DEP spectra [6].

IV. CONCLUSION

We have calculated the dielectrophoretic force acting on a spherical particle in slightly and highly non-uniform electric fields generated by four polynomial electrodes and point-plate electrodes, using the dipole moment method, higher-order moments (quadrupole, octopole,..), comparing the results to the accurate Maxwell stress tensor analysis, showing that higher-order terms enables to correct the force estimation related to the electric field non-uniformity effect.

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