



HAL
open science

Subproblem h-Conform Magnetodynamic Finite Element Formulation for Accurate Model of Multiply Connected Thin Regions

Vuong Quoc Dang, Patrick Dular, Ruth V. Sabariego, Laurent Krähenbühl,
Christophe Geuzaine

► **To cite this version:**

Vuong Quoc Dang, Patrick Dular, Ruth V. Sabariego, Laurent Krähenbühl, Christophe Geuzaine. Subproblem h-Conform Magnetodynamic Finite Element Formulation for Accurate Model of Multiply Connected Thin Regions. Numélec 2012, Jul 2012, Marseille, France. pp.72. hal-00714494

HAL Id: hal-00714494

<https://hal.science/hal-00714494>

Submitted on 5 Jul 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Subproblem h -Conform Magnetodynamic Finite Element Formulation for Accurate Model of Multiply Connected Thin Regions

Vuong Q. Dang¹, Patrick Dular^{1,2}, Ruth V. Sabariego¹, Laurent Krähenbühl³ and Christophe Geuzaine¹

¹University of Liège, Dept. of Electrical Engineering and Computer Science, ACE, B-4000 Liège, Belgium

²Fonds de la Recherche Scientifique - F.R.S.-FNRS, B-1000 Brussels, Belgium

³University of Lyon, Ampère (CNRS UMR5005), École Centrale de Lyon, F-69134 Écully Cedex, France
E-mail: quocvuong.dang@ulg.ac.be

Abstract — A subproblem h -conform eddy current finite element method is proposed for correcting the inaccuracies inherent to thin shell models. Such models replace thin volume regions by surfaces but neglect border effects in the vicinity of their edges and corners. The developed surface-to-volume correction problem is defined as a step of the multiple subproblems applied to a complete problem, consisting of inductors and magnetic or conducting regions, some of these being thin regions. The general case of multiply connected thin regions is considered.

I. INTRODUCTION

Thin shell (TS) finite element (FE) models [1], [2] [3], assume that the fields in the thin regions are approximated by a *a priori* 1-D analytical distributions along the shell thickness. In the frame of the FE method, their interior is thus not meshed and is rather extracted from the studied domain, being reduced to a zero-thickness double layer with interface conditions (ICs) linked to the inner analytical distributions. This means that corner and edge effects are neglected.

To overcome these drawbacks, the subproblem method (SPM) for the h -conform FE formulation has been already developed by authors [5] for simply connected TS regions, proposing a surface-to-volume local correction. The method is herein extended to multiply connected TS regions, i.e. regions with holes, for both the associated surface model (alternative to the method in [3]) and its volume correction. The global currents flowing around the holes and their associated voltages are naturally coupled to the local quantities, via some cuts for magnetic scalar potential discontinuities at both TS and correction steps.

A reduced model (SP q) with the inductors alone is first considered before adding the TS (SP p), followed by the volumic correction SP (SP k). From SP q to SP p , the solution q contributes to the surface sources (SSs) for the added TS, with TS ICs. From SP p to SP k , SSs and volume sources (VSs) allow to suppress the TS and cut discontinuities and simultaneously add the actual volume of the thin region. Each SP requires a proper adapted mesh of its regions. The method is illustrated and validated on a practical problem.

II. FROM THIN SHELL TO VOLUME MODEL

A. h -formulation with source and reaction magnetic fields

The magnetic field \mathbf{h}_i at step i of the SPM, is defined in a domain Ω_i , with boundary $\partial\Omega_i = \Gamma_i = \Gamma_{h,i} \cup \Gamma_{b,i}$, as

$$\mathbf{h}_i = \mathbf{h}_{s,i} + \mathbf{h}_{r,i} - \text{grad } \phi_i, \quad \text{curl } \mathbf{h}_{s,i} = \mathbf{j}_{s,i}, \quad (1a-b)$$

where $\mathbf{h}_{s,i}$ is a source magnetic field due to the fixed current density $\mathbf{j}_{s,i}$, $\mathbf{h}_{r,i}$ is the reaction field in conducting regions $\Omega_{c,i}$ and ϕ_i is the reaction magnetic scalar potential in non-conducting regions $\Omega_{c,i}^C$. Potential ϕ_i in $\Omega_{c,i}^C$ is multivalued (Fig. 1) and made singlevalued via the definition of cuts through each hole of $\Omega_{c,i}$ [6]. The \mathbf{h}_i - ϕ_i magnetodynamic formulation of the SPs $i = q, p$ and k is obtained from the weak form of Faraday's equation [5], i.e.,

This work was supported by the Belgian Science Policy (IAP P6/21).

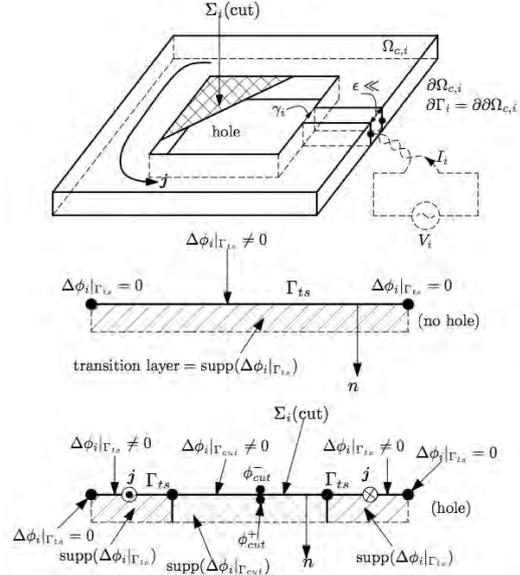


Fig. 1. 3D plate and its section without and with a hole, with the associated cut and transition layer for $\Delta\phi_i$.

$$\partial_t(\mu_i \mathbf{h}_i, \mathbf{h}'_i)_{\Omega_i} + (\sigma_i^{-1} \text{curl } \mathbf{h}_i, \text{curl } \mathbf{h}'_i)_{\Omega_{c,i}} + \langle [\mathbf{n} \times \mathbf{e}_i]_{\Gamma_i}, \mathbf{h}'_i \rangle_{\Gamma_i} + (\sigma_i^{-1} \mathbf{j}_{s,i}, \text{curl } \mathbf{h}'_i)_{\Omega_{s,i}} = 0, \quad \forall \mathbf{h}'_i \in F_i^1(\Omega_i), \quad (2)$$

where μ_i is the magnetic permeability, σ_i is the conductivity and $F_i^1(\Omega_i)$ is a curl-conform function space (built with edge FEs at the discrete level) defined on Ω_i and containing the basis functions for \mathbf{h}_i (coupled to ϕ_i) as well as for the test function \mathbf{h}'_i . The notation $[\cdot]_{\Gamma_i} = \cdot|_{\gamma_i^+} - \cdot|_{\gamma_i^-}$ refers to the discontinuity of a quantity through an interface γ_i (with sides γ_i^+ and γ_i^-) in Ω_i , defining ICs. The term $\langle [\mathbf{n} \times \mathbf{e}_i]_{\Gamma_i}, \mathbf{h}'_i \rangle_{\Gamma_i}$ is used to weakly express the electric field TS IC proper to the TS model [2]. It can be extended to a global condition defining a voltage V_i [4], with $\mathbf{h}'_i = \mathbf{c}_i = -\text{grad } q_i$ in $\Omega_{c,i}^C$ made simply connected by cut Σ_i (Fig. 1). Potential q_i is fixed to 1 on one side of the associated cut and to 0 on the other side. One gets [4]

$$\langle \mathbf{n} \times \mathbf{e}_i, \mathbf{c}_i \rangle_{\partial\Omega_{c,i}} = \oint_{\partial\Gamma_i} q_i \mathbf{e}_i \cdot d\mathbf{l} = \int_{\gamma_i} \mathbf{e}_i \cdot d\mathbf{l} = V_i, \quad (3)$$

B. Field Discontinuities for Multiply Connected TS Regions

A volume shell initially in $\Omega_{c,i}$ is extracted from Ω_i and then considered with the double layer TS surface $\Gamma_{ts,i}$ [2]. In addition to the electric field IC weakly defined in (2), the TS model requires a magnetic field discontinuity $[\mathbf{h}_i]_{\Gamma_{ts,i}} = \mathbf{h}_{d,i}$ strongly defined in $F_i^1(\Omega_i)$ via an IC on both sides of

the TS via $\mathbf{h}_i|_{\Gamma_{ts,i}^+} = \mathbf{h}_{c,i}|_{\Gamma_{ts,i}} + \mathbf{h}_{d,i}|_{\Gamma_{ts,i}}$ and $\mathbf{h}_i|_{\Gamma_{ts,i}^-} = \mathbf{h}_{c,i}|_{\Gamma_{ts,i}}$ [2]. This can be also formulated via a discontinuity $[\phi_i]_{\Gamma_{ts,i}} = \Delta\phi_i|_{\Gamma_{ts,i}} = \phi_{d,i}|_{\Gamma_{ts,i}}$ (Fig. 1), with $\phi_i|_{\Gamma_{ts,i}^+} = \phi_{c,i}|_{\Gamma_{ts,i}} + \phi_{d,i}|_{\Gamma_{ts,i}}$ and $\phi_i|_{\Gamma_{ts,i}^-} = \phi_{c,i}|_{\Gamma_{ts,i}}$. The discontinuities $\phi_{d,i}$ of ϕ_i are constant on each cut and can be written as

$$[\phi_i]_{cut,i} = \phi_i|_{\Gamma_{cut,i}^+} - \phi_i|_{\Gamma_{cut,i}^-} = \phi_{d,i}|_{\Gamma_{cut,i}} = I_i, \quad (4)$$

where I_i is the global current flowing around the cut [4]. Discontinuities $\phi_{d,i}|_{\Gamma_{ts,i}}$ and $\phi_{d,i}|_{\Gamma_{cut,i}}$ have to be matched at the TS-cuts intersections.

C. Volume Correction

The obtained TS solution in SP p is then corrected by SP k solution that overcomes the TS assumptions [2]. The SPM provides the tools to implement such a model refinement, thanks to simultaneous SSs and VSs. A fine volume mesh of the shell is now required and is extended to its neighborhood without including the other regions of previous SPs. This allows to focus on the fineness of the mesh only in the shell. SSs related to ICs [2], [5] compensate the TS and cut discontinuities, i.e. $\phi_{d,p}|_{\Gamma_{ts,p}}$ and $\phi_{d,p}|_{\Gamma_{cut,p}}$, and $[\mathbf{n} \times \mathbf{e}_p]_{\Gamma_{ts,p}}$ to suppress the TS representation via SSs opposed to ICs, i.e. $\mathbf{h}_{d,k} = -\mathbf{h}_{d,p}$ and $\phi_{d,k} = -\phi_{d,p}$, and $[\mathbf{n} \times \mathbf{e}_k]_{\Gamma_{ts,k}} = -[\mathbf{n} \times \mathbf{e}_p]_{\Gamma_{ts,p}}$ in parallel to VSs [5] in the added volume shell that account for volume change of μ_p and σ_p in SP p to μ_k and σ_k in SP k (with $\mu_p = \mu_0$, $\mu_k = \mu_{r,plate}$, $\sigma_p = 0$ and $\sigma_k = \sigma_{plate}$).

III. APPLICATION

The 3D test problem is based on TEAM problem 7: an inductor placed above a thin plate with a hole (Fig. 2) ($\mu_{r,plate} = 1$, $\sigma_{plate} = 35.26$ MS/m). A SP scheme considers the TS model followed by its volume correction.

Distributions of eddy current densities \mathbf{j}_i on the TS SP p and volume correction SP k are shown in Fig. 2, with plate thickness $d = 19$ mm and frequency $f = 200$ Hz (skin depth $\delta = 6$ mm). The TS error on \mathbf{j}_p locally reaches 43%. The inaccuracy on the Joule power loss densities of TS SP p is pointed out by the importance of the correction SP k (Fig. 3). It reaches several tens of percents along the borders for some critical parameters: e.g., 53% (Fig. 3, *top*) or 61% (Fig. 3, *bottom*), for $\delta = 6$ mm in both cases. Significant errors on the Joule losses for TS SP p are shown in Table I. They increase with both the thickness of the plate and the frequency.

The SPM allows to accurately correct any TS solution. In particular, accurate correction of eddy current and power loss density are obtained at the edges and corners of multiply connected thin regions. Details on the efficiency of the proposed method and the simplification of SP meshes will be given in the extended paper.

TABLE I. JOULE LOSSES IN THE PLATE

Thickness d (mm)	Frequency f (Hz)	Thin Shell P_{thin} (W)	Volume P_{vol} (W)	Error (%)
2	50	14.45	13.82	4.36
19	50	5.86	7.95	26.18
2	200	50.44	47.33	6.44
19	200	8.88	15.19	41.51

REFERENCES

[1] L. Krähenbühl and D. Müller, "Thin layers in electrical engineering. Examples of shell models in analyzing eddy-currents by boundary and finite element methods," IEEE Trans. Magn., vol. 29, no. 2, pp. 1450–1455, 1993.

[2] C. Geuzaine, P. Dular, and W. Legros, "Dual formulations for the modeling of thin electromagnetic shells using edge elements," IEEE Trans. Magn., vol. 36, no. 4, pp. 799–802, 2000.

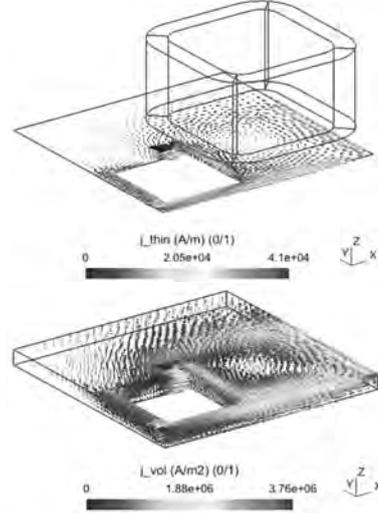


Fig. 2. TEAM problem 7; eddy current density on TS model (*top*) and volume correction (*bottom*) ($d = 19$ mm, $f = 200$ Hz).

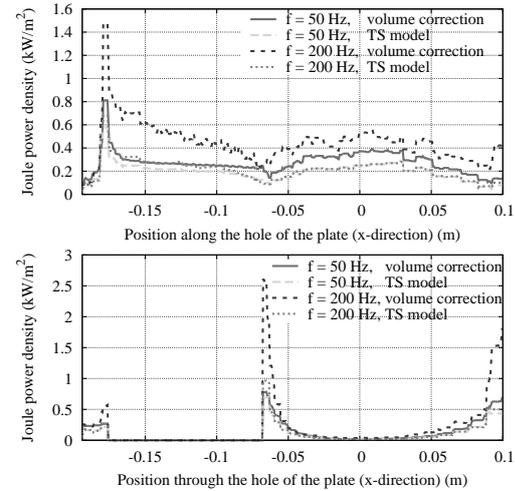


Fig. 3. Joule power loss density for TS model and volume correction along hole and plate border (*top*) and through the hole ($d = 19$ mm).

[3] C. Guérin and G. Meunier, "3-D Magnetic Scalar Potential Finite Element Formulation for Conducting Shells Coupled With an External Circuit," IEEE Trans. Magn., vol. 48, no. 2, pp. 323–326, 2012.

[4] P. Dular and W. Legros, "Coupling of Local and Global Quantities in Various Finite Element Formulations and its Application to Electrostatics, Magnetostatics and Magnetodynamics," IEEE Trans. Magn., vol. 34, no. 5, pp. 3078–3081, 1998.

[5] Vuong Q. Dang, P. Dular, R. V. Sabariego, L. Krähenbühl and C. Geuzaine, "Subproblem Approach for Thin Shell Dual Finite Element Formulations," IEEE Trans. Magn., vol. 48, no. 2, pp. 407–410, 2012.

[6] A. Bossavit, "Magnetostatic problems in multiply connected regions: some properties of the curl operator," IEE Proc. vol. 135, no. 3, pp. 179–187, 1988.