



HAL
open science

On the elastic fields produced by nonuniformly moving dislocations: a revisit

Markus Lazar

► **To cite this version:**

Markus Lazar. On the elastic fields produced by nonuniformly moving dislocations: a revisit. *Philosophical Magazine*, 2011, pp.1. 10.1080/14786435.2011.579584 . hal-00712752

HAL Id: hal-00712752

<https://hal.science/hal-00712752>

Submitted on 28 Jun 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



On the elastic fields produced by nonuniformly moving dislocations: a revisit

Journal:	<i>Philosophical Magazine & Philosophical Magazine Letters</i>
Manuscript ID:	TPHM-10-Nov-0485.R1
Journal Selection:	Philosophical Magazine
Date Submitted by the Author:	31-Mar-2011
Complete List of Authors:	Lazar, Markus; Institute of Condensed Matter Physics, Darmstadt University of Technology
Keywords:	dislocation dynamics, elastic waves, elasticity
Keywords (user supplied):	nonuniform motion, elastodynamics

SCHOLARONE™
Manuscripts

On the elastic fields produced by nonuniformly moving dislocations: a revisit

Markus Lazar^{a,b,*}

^a Heisenberg Research Group,
Department of Physics,
Darmstadt University of Technology,
Hochschulstr. 6,
D-64289 Darmstadt, Germany

^b Department of Physics,
Michigan Technological University,
Houghton, MI 49931, USA

March 31, 2011

Abstract

We investigate the nonuniform motion of straight dislocations in infinite media using the theory of incompatible elastodynamics. The equations of motion are derived for nonuniformly moving screw dislocation, gliding edge and climbing edge dislocations. The exact closed-form solutions of the elastic fields are calculated. The fields of the elastic velocity and elastic distortion surrounding the arbitrarily moving dislocations are given explicitly in the form of integral representations free of non-integrable singularities. The elastic fields describe the response in the form of nonuniformly moving elastic waves caused by the dislocation motion.

Keywords: dislocation dynamics; nonuniform motion; elastodynamics.

1 Introduction

The investigation of the nonuniform motion of dislocations has attracted the interest of researchers in different fields such as material science, continuum mechanics, seismology and earthquake engineering (see, e.g., [1, 2]). Dislocation movements can be considered

**E-mail address:* lazar@fkp.tu-darmstadt.de (M. Lazar).

1
2
3
4
5 as one of the dominant wavesources. It should be emphasized that the problem consid-
6 ered in this paper is important for the understanding of several physical phenomena such
7 as acoustic emission signals, dislocation avalanches and elastic waves in geophysics (see,
8 e.g., [3]). The motion of dislocations is usually investigated in the framework of incom-
9 compatible elastodynamics where the dislocation density and dislocation current tensors are
10 given as source terms of the elastic fields (see, e.g., [4, 5, 6, 7, 8, 9]). It is known that in
11 continuum elastodynamics the behaviour of a dislocation is somehow particular, because
12 at any time the fields are determined not only by the instantaneous values of the velocity
13 (or higher derivatives of the position with respect to time), but also by the values in the
14 past [10, 11]. As Eshelby [10] succinctly put it: ‘The dislocation is haunted by its past’.
15 This fact is based on the physical property that Huygens’ principle is not valid in two
16 dimensions (see, e.g., [12]).

17
18
19 Using an electromagnetic analogy, Eshelby [11] derived simple solutions of inhomogeneous
20 wave equations for the velocity and elastic distortion fields in terms of ‘stress
21 functions’ of a nonuniformly moving screw dislocation. The solution is given in the form
22 of time integrals. Eshelby [11] pointed out that the displacement field is of less physical
23 importance than the elastic fields of a moving screw dislocation. The direct solution of
24 the elastic fields given in [11] involves much simpler integrals than the integrals occurring
25 for the displacement of a nonuniformly moving screw dislocation given by Eshelby [10]
26 and Nabarro [13]. Later, Kiusalaas and Mura [14] found solutions in terms of ‘stress (or
27 potential) functions’ for the velocity and elastic distortion fields for a gliding edge disloca-
28 tion moving nonuniformly (see also [1, 9]). Weertman and Weertman [1] pointed out that
29 the problem of the climbing edge dislocation which moves at an arbitrary velocity has not
30 been considered in any published paper. Furthermore, to the knowledge of the author of
31 this paper a similar solution for a nonuniformly climbing edge dislocation has been still
32 lacking in the literature. Thus, the case of a climbing edge dislocation is an outstand-
33 ing problem. In addition, a systematic presentation of the elastic fields produced by the
34 nonuniform motion of screw and edge dislocations is missing in the literature so far. Even
35 in standard books of dislocation theory [6, 15, 16, 17, 18] this topic is not covered. This
36 is surprising because the nonuniform motion of dislocations is a fundamental problem in
37 dislocation theory like the Liénard-Wiechert potentials in electromagnetodynamics. The
38 reason lies in the complexity of the nonuniform motion of dislocations. In the present
39 paper, we give a clear derivation for the elastic fields of nonuniformly moving dislocations
40 using the ‘improved’ Mura equations. We investigate a screw dislocation, a gliding edge
41 dislocation and a climbing edge dislocation. For the first time, we give the elastic fields
42 of a nonuniformly climbing edge dislocation.

43
44
45 Markenscoff [19] (see also [21]) pointed out that some care is necessary in the cal-
46 culation of the elastic fields produced by nonuniformly moving dislocations in order to
47 avoid non-integrable singularities. Markenscoff [19] proved that the general expressions
48 for the velocity and elastic distortion fields of dislocations given by Mura [4, 5] are not
49 free of non-integrable singularities. The reason is that the integration and differentiation
50 cannot be changed in these cases. As a consequence the differentiation has to be outside
51 the integral in order to obtain correct results. In the application to screw and gliding
52 edge dislocations, Kiusalaas and Mura [14, 22] used the correct interpretation with a
53 differentiation outside the integrals.

The paper is organized as follows: in Section 2, we present the framework of incompatible elastodynamics and we give the equations of motion, Mura's integral expressions for the elastic velocity and elastic distortion fields as well as the two-dimensional Green tensors. The elastic fields of nonuniformly moving screw dislocation, gliding edge and climbing edge dislocations are presented in Sections 3, 4 and 5, respectively. The static limit of the elastic fields of the nonuniformly moving dislocations is given in Section 6.

2 Equations of motion of dislocations

In this section, we give the equations of motion for dislocations in the framework of incompatible elastodynamics (see, e.g., [4, 5, 6, 23]). In elasticity without external forces, the equilibrium condition reads¹

$$\dot{p}_i = \sigma_{ij,j}, \quad (1)$$

where \mathbf{p} and $\boldsymbol{\sigma}$ are the linear momentum vector and the force stress tensor, respectively. In the incompatible linear elasticity, the momentum vector \mathbf{p} and the stress tensor $\boldsymbol{\sigma}$ can be expressed in terms of the incompatible elastic velocity vector \mathbf{v} and the incompatible elastic distortion tensor $\boldsymbol{\beta}$ by means of the following constitutive relations

$$p_i = \rho v_i, \quad (2)$$

$$\sigma_{ij} = C_{ijkl} \beta_{kl}, \quad (3)$$

where ρ is the mass density and C_{ijkl} is the tensor of elastic moduli. The tensor C_{ijkl} possesses the following symmetry properties

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}. \quad (4)$$

If we substitute the constitutive relations (2) and (3) in Eq. (1), we obtain the equilibrium condition expressed in terms of the elastic fields \mathbf{v} and $\boldsymbol{\beta}$

$$\rho \dot{v}_i = C_{ijkl} \beta_{kl,j}. \quad (5)$$

The presence of dislocations makes the elastic fields incompatible which means that they are not anymore simple gradients or time derivatives of a displacement vector \mathbf{u} . In the incompatible elasticity, the elastic fields can be given in terms of the displacement field \mathbf{u} , the plastic distortion $\boldsymbol{\beta}^P$ and the plastic velocity \mathbf{v}^P [24, 25]

$$\beta_{ij} = u_{i,j} - \beta_{ij}^P, \quad (6)$$

$$v_i = \dot{u}_i - v_i^P. \quad (7)$$

The plastic distortion $\boldsymbol{\beta}^P$ is a well-known quantity in the dislocation theory. The plastic velocity \mathbf{v}^P was introduced by Kossecka [24] and its existence has not been widely recognized. However, for elasto-plastic deformations caused by dislocations a part of the

¹We use the usual notation $\beta_{ij,k} := \partial_k \beta_{ij}$ and $\dot{\beta}_{ij} := \partial_t \beta_{ij}$.

1
2
3
4
5 motion may be of plastic nature [26]. In general, the plastic fields may be the discontinuous parts of the total fields. Nowadays, the plastic fields can be better understood
6 as gauge fields in the framework of dislocation gauge theory [27, 28]. In contrast to the
7 displacement field and the plastic fields, the elastic fields are physical state quantities of
8 dislocations. For that reason, we deal only with the calculation of the elastic fields in this
9 paper.
10

11 The dislocation density and dislocation current tensors are defined by
12

$$13 \quad T_{ijk} = \beta_{ik,j} - \beta_{ij,k}, \quad (8)$$

$$14 \quad I_{ij} = \dot{\beta}_{ij} - v_{i,j}, \quad (9)$$

15 or they read in terms of the plastic fields
16

$$17 \quad T_{ijk} = -\beta_{ik,j}^P + \beta_{ij,k}^P, \quad (10)$$

$$18 \quad I_{ij} = -\dot{\beta}_{ij}^P + v_{i,j}^P, \quad (11)$$

19 and $T_{ijk} = -T_{ikj}$. Moreover, they fulfill the Bianchi identities
20

$$21 \quad \epsilon_{jkl} T_{ijk,l} = 0, \quad (12)$$

$$22 \quad \dot{T}_{ijk} + I_{ij,k} - I_{ik,j} = 0, \quad (13)$$

23 which are ‘conservation’ laws. Here ϵ_{jkl} denotes the Levi-Civita tensor. Eq. (12) states
24 that dislocations cannot end inside the medium and Eq. (13) means that the time evolution
25 of the dislocation density tensor \mathbf{T} is determined by the ‘curl’ of the dislocation current
26 tensor \mathbf{I} . For straight dislocations we have also the relation
27

$$28 \quad I_{ij} = V_k T_{ijk}, \quad (14)$$

29 where V_k denotes the velocity of the moving dislocation.
30

31 By differentiating Eq. (5) with respect to x_m and substituting Eqs. (8) and (9) into it,
32 we find the dynamic elasticity-theory equation for the determination of the incompatible
33 elastic distortion tensor β
34

$$35 \quad \rho \ddot{\beta}_{im} - C_{ijkl} \beta_{km,jl} = C_{ijkl} T_{kml,j} + \rho \dot{I}_{im}. \quad (15)$$

36 This is the equation of motion for β where the dislocation density and the dislocation
37 current tensors are the sources. Eq. (15) is an anisotropic tensorial Navier equation for
38 β .
39

40 Similarly, performing the differentiation of Eq. (5) with respect to time and using
41 Eq. (9), we obtain the dynamic elasticity-theory equation for the determination of the
42 incompatible elastic velocity vector \mathbf{v}
43

$$44 \quad \rho \ddot{v}_i - C_{ijkl} v_{k,jl} = C_{ijkl} I_{kl,j}. \quad (16)$$

45 Eq. (16) is the equation of motion for \mathbf{v} with the dislocation current tensor as source
46 term. Eq. (16) is an anisotropic vectorial Navier equation for \mathbf{v} .
47
48
49
50
51
52
53
54
55
56
57
58
59
60

The solutions of Eqs. (15) and (16) can be represented as convolution integrals [4, 5, 24, 29]. In an unbounded medium and under the assumption of zero initial conditions, which means that $\beta(\mathbf{r}, t_0)$ and $\mathbf{v}(\mathbf{r}, t_0)$ and their first time derivative are zero for $t_0 \rightarrow -\infty$, the solutions of β and \mathbf{v} are given in terms of the Green tensor. Therefore, the convolution integrals for the elastic fields, which are free of non-integrable singularities, are given by (see also [19])

$$\begin{aligned} \beta_{im}(\mathbf{r}, t) = & \partial_k \int_{-\infty}^t \int_{-\infty}^{\infty} C_{jklm} G_{ij}(\mathbf{r} - \mathbf{r}', t - t') T_{lmn}(\mathbf{r}', t') d\mathbf{r}' dt' \\ & + \partial_t \int_{-\infty}^t \int_{-\infty}^{\infty} \rho G_{ij}(\mathbf{r} - \mathbf{r}', t - t') I_{jm}(\mathbf{r}', t') d\mathbf{r}' dt' \end{aligned} \quad (17)$$

and

$$v_i(\mathbf{r}, t) = \partial_k \int_{-\infty}^t \int_{-\infty}^{\infty} C_{ijklm} G_{ij}(\mathbf{r} - \mathbf{r}', t - t') I_{lm}(\mathbf{r}', t') d\mathbf{r}' dt'. \quad (18)$$

Here, G_{ij} is the dynamic Green tensor of the anisotropic Navier equation defined by

$$[\delta_{ik} \rho \partial_{tt} - C_{ijkl} \partial_j \partial_l] G_{km} = \delta_{im} \delta(t) \delta(\mathbf{r}), \quad (19)$$

where $\delta(\cdot)$ denotes the Dirac delta function and $G_{ij} = G_{ji}$. The expressions for the elastic fields given in Eqs. (17) and (18) differ from the expressions given by Mura [4, 5, 6], Kossecka [24] and Teodosiu [29] in that the differentiation is outside the integral, as required to obtain correct results for dislocations. In the expressions given by Mura [4, 5, 6] the differentiation of the Green tensor leads to non-integrable singularities as pointed out by Markenscoff [19]. We would like to mention that Mura [4, 5], Bross [7], Teodosiu [29] and Markenscoff [19] assumed $\mathbf{v}^P = 0$, so that they made no distinction between \mathbf{v} and $\dot{\mathbf{u}}$.

For an isotropic material the tensor of elastic moduli reduces to

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \quad (20)$$

where λ and μ being the Lamé constants. Substituting Eq. (20) in Eq. (19), we obtain the isotropic Navier equation for the dynamic Green tensor

$$[\delta_{ik} \rho \partial_{tt} - \delta_{ik} \mu \Delta - (\lambda + \mu) \partial_i \partial_k] G_{km} = \delta_{im} \delta(t) \delta(\mathbf{r}), \quad (21)$$

where Δ denotes the Laplacian. For the calculation of the elastic fields of straight dislocations, the problem is two-dimensional (plane strain and anti-plane strain), consequently we need the two-dimensional Green tensors. If the material is infinitely extended, the dynamic Green tensor of plane strain reads [30, 31]

$$\begin{aligned} G_{ik} = & \frac{1}{2\pi\rho} \left\{ \frac{x_i x_k}{r^4} \left(\frac{[2t^2 - r^2/c_L^2]}{\sqrt{t^2 - r^2/c_L^2}} H(t - r/c_L) - \frac{[2t^2 - r^2/c_T^2]}{\sqrt{t^2 - r^2/c_T^2}} H(t - r/c_T) \right) \right. \\ & \left. - \frac{\delta_{ik}}{r^2} \left(\sqrt{t^2 - r^2/c_L^2} H(t - r/c_L) - \frac{t^2}{\sqrt{t^2 - r^2/c_T^2}} H(t - r/c_T) \right) \right\} \end{aligned} \quad (22)$$

and the dynamic Green tensor of anti-plane strain is given by

$$G_{zz} = \frac{1}{2\pi\rho c_T^2} \frac{H(t - r/c_T)}{\sqrt{t^2 - r^2/c_T^2}}, \quad (23)$$

where $H(\cdot)$ denotes the Heaviside step function and $r = \sqrt{x^2 + y^2}$. The constants c_L and c_T denote the velocities of the longitudinal and transversal elastic waves (sometimes called P and S-waves). The sound velocities can be given in terms of the Lamé constants ($c_T < c_L$)

$$c_L = \sqrt{\frac{2\mu + \lambda}{\rho}}, \quad c_T = \sqrt{\frac{\mu}{\rho}}. \quad (24)$$

Let us now discuss some important properties of the Green tensors of plane strain and anti-plane strain. The Green tensor of anti-plane strain G_{zz} diverges to $+\infty$ at $r \rightarrow c_T t^-$ and converges to $1/(2\pi\rho c_T^2 |t|)$ at $r \rightarrow 0$ (see Fig. 1a). On the other hand, the Green tensor of plane strain (22) consists of two characteristic pieces. Note that the expression

$$\frac{1}{r^2} \left(\frac{[2t^2 - r^2/c_L^2]}{\sqrt{t^2 - r^2/c_L^2}} H(t - r/c_L) - \frac{[2t^2 - r^2/c_T^2]}{\sqrt{t^2 - r^2/c_T^2}} H(t - r/c_T) \right)$$

diverges to $+\infty$ at $r \rightarrow c_L t^-$ and it converges to 0 at $r \rightarrow 0$. At $r = c_T t$ it possesses a jump: it diverges to $-\infty$ at $r \rightarrow c_T t^-$ and converges to a finite-limit value of $(2c_L^2 - c_T^2)/(c_L^2 c_T^2 \sqrt{(c_L^2 - c_T^2)/c_L^2} |t|)$ as $r \rightarrow c_T t^+$ (see Fig. 1b). Thus, discontinuities exist for both wavefronts. The piece

$$\frac{1}{r^2} \left(\frac{t^2}{\sqrt{t^2 - r^2/c_T^2}} H(t - r/c_T) - \sqrt{t^2 - r^2/c_L^2} H(t - r/c_L) \right)$$

converges to 0 at $r \rightarrow c_L t^-$ and converges to a finite-limit value of $(c_L^2 + c_T^2)/(2c_L^2 c_T^2 |t|)$ at $r \rightarrow 0$. At $r = c_T t$ it possesses a jump: it diverges to $+\infty$ at $r \rightarrow c_T t^-$ and converges to a finite-limit value of $-1/(c_T^2 |t|) \sqrt{(c_L^2 - c_T^2)/c_L^2}$ as $r \rightarrow c_T t^+$ (see Fig. 1c). Therefore, a discontinuity exists for the wavefront at $r = c_T t$. The combination of the two pieces gives a finite value at $r = c_T t$ and at $r = c_L t$ it diverges to $+\infty$ (see Fig. 1d). A discontinuity exists only for the wavefront at $r = c_L t$. Thus, in general, at the wavefronts $r = c_T t$ and $r = c_L t$ the Green tensors possess singularities and discontinuities and for small r they possess a long tail of the typical two-dimensional pulse. Physically, there is afterglow implying that Huygens' principle is not valid in two dimensions. Although two waves with the velocities c_L and c_T exist, there is never complete separation between the two. These properties of the Green tensors will influence also the properties of the 'potential functions' (retarded potentials) and the elastic fields of dislocations in that way that they are influenced or haunted by their past. Thus, the expression (22) shows that the disturbances are propagated with the velocities c_T and c_L . At the wavefronts $r = c_T t$ and $r = c_L t$ the Green tensor (22) shows algebraic singularities. In Fig. 1 the Green tensors G_{zz} , G_{xy} for $\varphi = \pi/4$, G_{xx} for $\varphi = \pi/2$ and G_{xx} for $\varphi = 0$ are plotted over r for fixed time t .

It should be noted that for the calculation of the elastic fields produced by dislocation loops and curved dislocations the three-dimensional Green tensor G_{ij} is needed (see also [20, 21]).

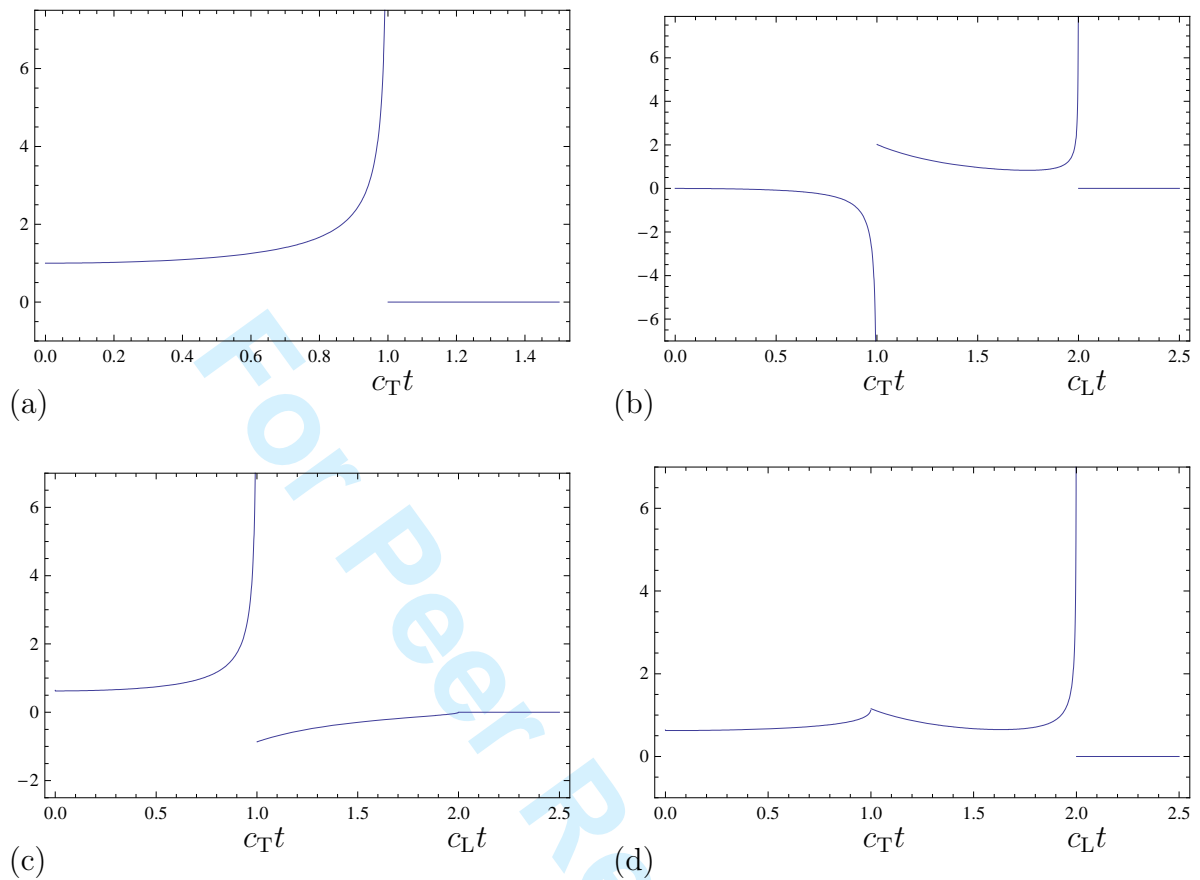


Figure 1: Two-dimensional Green tensors as functions of r for fixed time t : (a) G_{zz} , (b) G_{xy} for $\varphi = \pi/4$, (c) G_{xx} for $\varphi = \pi/2$ and (d) G_{xx} for $\varphi = 0$. In the plots we used $c_T = 1$ and $c_L = 2$ for numeric convenience.

3 Screw dislocation

Let us consider a nonuniformly moving screw dislocation at the position $(\xi(t), \eta(t))$ at time t . The dislocation line and the Burgers vector b_z are parallel to the z -axis. The components of the dislocation velocity are: $V_x = \dot{\xi}(t)$, $V_y = \dot{\eta}(t)$. The non-vanishing components of the dislocation density and dislocation current tensors are

$$T_{zxy} = b_z \delta(x - \xi(t))\delta(y - \eta(t)), \quad (25)$$

$$I_{zx} = V_y T_{zxy}, \quad (26)$$

$$I_{zy} = -V_x T_{zxy}. \quad (27)$$

For a screw dislocation, Eqs. (17) and (18) reduce to the non-vanishing components of the elastic fields

$$v_z(\mathbf{r}, t) = \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{zxzx} G_{zz}(\mathbf{r} - \mathbf{r}', t - t') I_{zx}(\mathbf{r}', t') d\mathbf{r}' dt' + \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{zyzy} G_{zz}(\mathbf{r} - \mathbf{r}', t - t') I_{zy}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (28)$$

$$\beta_{zx}(\mathbf{r}, t) = \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{zyzy} G_{zz}(\mathbf{r} - \mathbf{r}', t - t') T_{zxy}(\mathbf{r}', t') d\mathbf{r}' dt' + \partial_t \int_{-\infty}^t \int_{-\infty}^{\infty} \rho G_{zz}(\mathbf{r} - \mathbf{r}', t - t') I_{zx}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (29)$$

$$\beta_{zy}(\mathbf{r}, t) = \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{zxzx} G_{zz}(\mathbf{r} - \mathbf{r}', t - t') T_{zyx}(\mathbf{r}', t') d\mathbf{r}' dt' + \partial_t \int_{-\infty}^t \int_{-\infty}^{\infty} \rho G_{zz}(\mathbf{r} - \mathbf{r}', t - t') I_{zy}(\mathbf{r}', t') d\mathbf{r}' dt'. \quad (30)$$

Substituting Eqs. (20), (23) and (25)–(27) into Eqs. (28)–(30) and carrying out the integration in \mathbf{r}' , we find the following time integral representations

$$v_z = A_{y,x} - A_{x,y}, \quad (31)$$

$$\beta_{zx} = \phi_{,y} + \frac{1}{c_T^2} \dot{A}_y, \quad (32)$$

$$\beta_{zy} = -\left(\phi_{,x} + \frac{1}{c_T^2} \dot{A}_x\right), \quad (33)$$

where the ‘potential’ functions are defined by

$$A_i = \mu b_z \int_{-\infty}^t V_i(t') G_{zz}(\mathbf{r} - \mathbf{r}''(t'), t - t') dt', \quad (34)$$

$$\phi = \mu b_z \int_{-\infty}^t G_{zz}(\mathbf{r} - \mathbf{r}''(t'), t - t') dt' \quad (35)$$

with $\mathbf{r}'' = (\xi, \eta)$. Substituting the Green function (23) in Eqs. (34) and (35), the ‘potential’ functions read

$$A_x = \frac{b_z}{2\pi} \int_{-\infty}^{t_T} \frac{V_x(t')}{S_T} dt', \quad (36)$$

$$A_y = \frac{b_z}{2\pi} \int_{-\infty}^{t_T} \frac{V_y(t')}{S_T} dt', \quad (37)$$

$$\phi = \frac{b_z}{2\pi} \int_{-\infty}^{t_T} \frac{1}{S_T} dt' \quad (38)$$

with the notations

$$\begin{aligned} \bar{x} &= x - \xi(t'), & \bar{y} &= y - \eta(t') & \bar{t} &= t - t', & \bar{R}^2 &= \bar{x}^2 + \bar{y}^2, \\ S_T^2 &= \bar{t}^2 - \frac{\bar{R}^2}{c_T^2}, & t_T &= t - \frac{\bar{R}}{c_T}. \end{aligned} \quad (39)$$

The ‘potential’ functions (36)–(38) play the physical role of the retarded potentials of a screw dislocation moving nonuniformly. Note that t_T is called retarded time, which is the root of the equation $S_T^2 = 0$ and is less than t , and (\bar{x}, \bar{y}) is the distance between the field point (x, y) and the position (ξ, η) of the dislocation. Here, $\xi(t')$ and $\eta(t')$ are the positions in the x and y directions of the dislocation at time t' , $V_x(t')$ and $V_y(t')$ are the velocity components of the dislocation at the same time t' . The retarded time t_T is the time before t , when the dislocation caused an excitation of the elastic field, which moves from (ξ, η) to (x, y) in the time \bar{R}/c_T . Thus, $t - t_T = \bar{R}/c_T$ is the time if the elastic dislocation fields move from (ξ, η) to (x, y) with velocity c_T . Eqs. (31)–(38) agree with the expressions given by Eshelby [11] (see also [9, 17]). Eshelby [11] derived the elastic fields using an electromagnetic analogy. Here we derived the elastic fields in a more straightforward manner as solutions of the equations of motion (15) and (16). The solutions (31)–(38) on the position (x, y) at time t depend on all contributions emitted at all times t' from $-\infty$ up to t_T . That is why Eshelby [10] said: ‘The dislocation is haunted by its past’. The physical interpretation is that disturbances from remote positions on the dislocation line continue to arrive at other positions on the line as the dislocation moves forward. Thus, the motion of a screw dislocation produces only transversal elastic waves.

4 Gliding edge dislocation

In this section, we investigate the problem of an edge dislocation at the position $(\xi(t), 0)$ at time t gliding in x -direction with arbitrary dislocation velocity $V_x(t) = \dot{\xi}(t)$ and Burgers vector b_x . The non-vanishing components of the dislocation density and dislocation current tensors read

$$T_{xxy} = b_x \delta(x - \xi(t))\delta(y), \quad (40)$$

$$I_{xy} = -V_x T_{xxy}. \quad (41)$$

For a gliding edge dislocation, Eqs. (17) and (18) reduce to the non-vanishing components

$$v_x(\mathbf{r}, t) = \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xyxy} G_{xx}(\mathbf{r} - \mathbf{r}', t - t') I_{xy}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yxxy} G_{xy}(\mathbf{r} - \mathbf{r}', t - t') I_{xy}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (42)$$

$$v_y(\mathbf{r}, t) = \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yxxy} G_{yy}(\mathbf{r} - \mathbf{r}', t - t') I_{xy}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xyxy} G_{yx}(\mathbf{r} - \mathbf{r}', t - t') I_{xy}(\mathbf{r}', t') d\mathbf{r}' dt' \quad (43)$$

and

$$\begin{aligned} \beta_{xx}(\mathbf{r}, t) = & \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xyxy} G_{xx}(\mathbf{r} - \mathbf{r}', t - t') T_{xy}(\mathbf{r}', t') d\mathbf{r}' dt' \\ & + \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yxxy} G_{xy}(\mathbf{r} - \mathbf{r}', t - t') T_{xy}(\mathbf{r}', t') d\mathbf{r}' dt', \end{aligned} \quad (44)$$

$$\begin{aligned} \beta_{yx}(\mathbf{r}, t) = & \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xyxy} G_{yx}(\mathbf{r} - \mathbf{r}', t - t') T_{xy}(\mathbf{r}', t') d\mathbf{r}' dt' \\ & + \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yxxy} G_{yy}(\mathbf{r} - \mathbf{r}', t - t') T_{xy}(\mathbf{r}', t') d\mathbf{r}' dt', \end{aligned} \quad (45)$$

$$\begin{aligned} \beta_{xy}(\mathbf{r}, t) = & \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xxxx} G_{xx}(\mathbf{r} - \mathbf{r}', t - t') T_{yx}(\mathbf{r}', t') d\mathbf{r}' dt' \\ & + \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yyxx} G_{xy}(\mathbf{r} - \mathbf{r}', t - t') T_{yx}(\mathbf{r}', t') d\mathbf{r}' dt' \\ & + \partial_t \int_{-\infty}^t \int_{-\infty}^{\infty} \rho G_{xx}(\mathbf{r} - \mathbf{r}', t - t') I_{xy}(\mathbf{r}', t') d\mathbf{r}' dt', \end{aligned} \quad (46)$$

$$\begin{aligned} \beta_{yy}(\mathbf{r}, t) = & \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xxxx} G_{yx}(\mathbf{r} - \mathbf{r}', t - t') T_{yx}(\mathbf{r}', t') d\mathbf{r}' dt' \\ & + \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yyxx} G_{yy}(\mathbf{r} - \mathbf{r}', t - t') T_{yx}(\mathbf{r}', t') d\mathbf{r}' dt' \\ & + \partial_t \int_{-\infty}^t \int_{-\infty}^{\infty} \rho G_{yx}(\mathbf{r} - \mathbf{r}', t - t') I_{xy}(\mathbf{r}', t') d\mathbf{r}' dt'. \end{aligned} \quad (47)$$

Substituting Eqs. (20), (22), (40) and (41) in Eqs. (42)–(47) and carrying out the integration in \mathbf{r}' , we find the following time integral representations for the elastic fields of a gliding edge dislocation

$$v_x = -\frac{b_x c_T^2}{2\pi} (A_{xy,x} + A_{xx,y}), \quad (48)$$

$$v_y = -\frac{b_x c_T^2}{2\pi} (A_{yy,x} + A_{yx,y}), \quad (49)$$

and

$$\beta_{xx} = \frac{b_x c_T^2}{2\pi} (\phi_{xy,x} + \phi_{xx,y}), \quad (50)$$

$$\beta_{yx} = \frac{b_x c_T^2}{2\pi} (\phi_{yy,x} + \phi_{yx,y}), \quad (51)$$

$$\beta_{xy} = -\frac{b_x c_L^2}{2\pi} (\phi_{xx,x} + \phi_{xy,y}) + \frac{b_x c_T^2}{\pi} \phi_{xy,y} - \frac{b_x}{2\pi} \dot{A}_{xx}, \quad (52)$$

$$\beta_{yy} = -\frac{b_x c_L^2}{2\pi} (\phi_{yx,x} + \phi_{yy,y}) + \frac{b_x c_T^2}{\pi} \phi_{yy,y} - \frac{b_x}{2\pi} \dot{A}_{yx}, \quad (53)$$

where the following ‘potential’ functions are defined by

$$A_{ij} = 2\pi\rho \int_{-\infty}^t V_x(t') G_{ij}(\mathbf{r} - \mathbf{r}''(t'), t - t') dt', \quad (54)$$

$$\phi_{ij} = 2\pi\rho \int_{-\infty}^t G_{ij}(\mathbf{r} - \mathbf{r}''(t'), t - t') dt' \quad (55)$$

with $\mathbf{r}'' = (\xi, 0)$, $A_{ij} = A_{ji}$ and $\phi_{ij} = \phi_{ji}$. Substituting the Green tensor (22), they finally read

$$A_{xx} = \int_{-\infty}^{t_L} \frac{V_x(t')}{\bar{R}^4} \left(\frac{\bar{x}^2 \bar{t}^2}{S_L} - \bar{y}^2 S_L \right) dt' + \int_{-\infty}^{t_T} \frac{V_x(t')}{\bar{R}^4} \left(\frac{\bar{y}^2 \bar{t}^2}{S_T} - \bar{x}^2 S_T \right) dt', \quad (56)$$

$$A_{yy} = \int_{-\infty}^{t_L} \frac{V_x(t')}{\bar{R}^4} \left(\frac{\bar{y}^2 \bar{t}^2}{S_L} - \bar{x}^2 S_L \right) dt' + \int_{-\infty}^{t_T} \frac{V_x(t')}{\bar{R}^4} \left(\frac{\bar{x}^2 \bar{t}^2}{S_T} - \bar{y}^2 S_T \right) dt', \quad (57)$$

$$A_{xy} = \int_{-\infty}^{t_L} \frac{V_x(t')}{\bar{R}^4} \bar{x} \bar{y} \left(\frac{\bar{t}^2}{S_L} + S_L \right) dt' - \int_{-\infty}^{t_T} \frac{V_x(t')}{\bar{R}^4} \bar{x} \bar{y} \left(\frac{\bar{t}^2}{S_T} + S_T \right) dt', \quad (58)$$

with

$$\begin{aligned} \bar{x} &= x - \xi(t'), & \bar{y} &= y, & \bar{t} &= t - t', & \bar{R}^2 &= \bar{x}^2 + \bar{y}^2, \\ S_T^2 &= \bar{t}^2 - \frac{\bar{R}^2}{c_T^2}, & t_T &= t - \frac{\bar{R}}{c_T}, & S_L^2 &= \bar{t}^2 - \frac{\bar{R}^2}{c_L^2}, & t_L &= t - \frac{\bar{R}}{c_L}. \end{aligned} \quad (59)$$

The explicit expressions for ϕ_{ij} can be obtained from the corresponding expressions for A_{ij} if $V_x(t')$ is replaced by 1. If we do this replacement, we observe the relations: $v_x(V_x(t') = 1) = -\beta_{xx}$ and $v_y(V_x(t') = 1) = -\beta_{yx}$. The potential functions A_{ij} and ϕ_{ij} are the retarded potentials for a gliding edge dislocation. The elastic fields of a gliding edge dislocation are given as time integrals and they are more complicated than the expressions for a screw dislocation. The elastic fields contain terms proportional to S_T , S_L , $1/S_T$ and $1/S_L$. The appearance of two velocities of elastic waves c_T and c_L makes the results more complicated and is also the reason of two retarded times t_T and t_L which are the roots of $S_T^2 = 0$ and $S_L^2 = 0$, respectively. Eqs. (48)–(58) agree with the expressions given by Kiusalaas and Mura [14]. For the calculation of the elastic fields, Kiusalaas and Mura [14] used the three-dimensional Green tensor in contrast to the two-dimensional Green tensor (22) that is used in the present paper thereby making the calculation easier and more straightforward as well as appropriate for the calculation of the elastic fields of a climbing edge dislocation. The structure of the potential functions (56)–(58) is immediately a consequence of the form of the two-dimensional Green tensor (22). It is clear that the gliding of the edge dislocation causes transversal as well as longitudinal elastic waves with the velocities c_T and c_L , respectively.

5 Climbing edge dislocation

In this section, for the first time we investigate the problem of a climbing edge dislocation at the position $(0, \eta(t))$ at time t climbing in y -direction with arbitrary dislocation velocity

$V_y(t) = \dot{\eta}(t)$ and Burgers vector b_x . The non-vanishing components of the dislocation density and dislocation current tensors are given by

$$T_{xxy} = b_x \delta(x) \delta(y - \eta(t)), \quad (60)$$

$$I_{xx} = V_y T_{xxy}. \quad (61)$$

Thus, for a climbing edge dislocation, Eqs. (17) and (18) reduce to the non-vanishing components of the elastic fields

$$v_x(\mathbf{r}, t) = \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xxxx} G_{xx}(\mathbf{r} - \mathbf{r}', t - t') I_{xx}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yyxx} G_{xy}(\mathbf{r} - \mathbf{r}', t - t') I_{xx}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (62)$$

$$v_y(\mathbf{r}, t) = \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xxxx} G_{yx}(\mathbf{r} - \mathbf{r}', t - t') I_{xx}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yyxx} G_{yy}(\mathbf{r} - \mathbf{r}', t - t') I_{xx}(\mathbf{r}', t') d\mathbf{r}' dt' \quad (63)$$

and

$$\beta_{xx}(\mathbf{r}, t) = \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xyxy} G_{xx}(\mathbf{r} - \mathbf{r}', t - t') T_{xxy}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yxxy} G_{xy}(\mathbf{r} - \mathbf{r}', t - t') T_{xxy}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_t \int_{-\infty}^t \int_{-\infty}^{\infty} \rho G_{xx}(\mathbf{r} - \mathbf{r}', t - t') I_{xx}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (64)$$

$$\beta_{yx}(\mathbf{r}, t) = \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xyxy} G_{yx}(\mathbf{r} - \mathbf{r}', t - t') T_{xxy}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yxxy} G_{yy}(\mathbf{r} - \mathbf{r}', t - t') T_{xxy}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_t \int_{-\infty}^t \int_{-\infty}^{\infty} \rho G_{yx}(\mathbf{r} - \mathbf{r}', t - t') I_{xx}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (65)$$

$$\beta_{xy}(\mathbf{r}, t) = \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xxxx} G_{xx}(\mathbf{r} - \mathbf{r}', t - t') T_{xyx}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yyxx} G_{xy}(\mathbf{r} - \mathbf{r}', t - t') T_{xyx}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (66)$$

$$\beta_{yy}(\mathbf{r}, t) = \partial_x \int_{-\infty}^t \int_{-\infty}^{\infty} C_{xxxx} G_{yx}(\mathbf{r} - \mathbf{r}', t - t') T_{xyx}(\mathbf{r}', t') d\mathbf{r}' dt' \\ + \partial_y \int_{-\infty}^t \int_{-\infty}^{\infty} C_{yyxx} G_{yy}(\mathbf{r} - \mathbf{r}', t - t') T_{xyx}(\mathbf{r}', t') d\mathbf{r}' dt'. \quad (67)$$

If we substitute Eqs. (20), (22), (60) and (61) in Eqs. (62)–(67) and we carry out the integration in \mathbf{r}' , we obtain the following time integral representations for the elastic fields

of a climbing edge dislocation

$$v_x = \frac{b_x c_L^2}{2\pi} (A_{xx,x} + A_{xy,y}) - \frac{b_x c_T^2}{\pi} A_{xy,y}, \quad (68)$$

$$v_y = \frac{b_x c_L^2}{2\pi} (A_{yx,x} + A_{yy,y}) - \frac{b_x c_T^2}{\pi} A_{yy,y}, \quad (69)$$

and

$$\beta_{xx} = \frac{b_x c_T^2}{2\pi} (\phi_{xy,x} + \phi_{xx,y}) + \frac{b_x}{2\pi} \dot{A}_{xx}, \quad (70)$$

$$\beta_{yx} = \frac{b_x c_T^2}{2\pi} (\phi_{yy,x} + \phi_{yx,y}) + \frac{b_x}{2\pi} \dot{A}_{yx}, \quad (71)$$

$$\beta_{xy} = -\frac{b_x c_L^2}{2\pi} (\phi_{xx,x} + \phi_{xy,y}) + \frac{b_x c_T^2}{\pi} \phi_{xy,y}, \quad (72)$$

$$\beta_{yy} = -\frac{b_x c_L^2}{2\pi} (\phi_{yx,x} + \phi_{yy,y}) + \frac{b_x c_T^2}{\pi} \phi_{yy,y}, \quad (73)$$

where the ‘potential’ functions are defined by

$$A_{ij} = 2\pi\rho \int_{-\infty}^t V_y(t') G_{ij}(\mathbf{r} - \mathbf{r}''(t'), t - t') dt', \quad (74)$$

$$\phi_{ij} = 2\pi\rho \int_{-\infty}^t G_{ij}(\mathbf{r} - \mathbf{r}''(t'), t - t') dt', \quad (75)$$

where $\mathbf{r}'' = (0, \eta)$, $A_{ij} = A_{ji}$ and $\phi_{ij} = \phi_{ji}$. After substituting the Green tensor (22), they read

$$A_{xx} = \int_{-\infty}^{t_L} \frac{V_y(t')}{\bar{R}^4} \left(\frac{\bar{x}^2 \bar{t}^2}{S_L} - \bar{y}^2 S_L \right) dt' + \int_{-\infty}^{t_T} \frac{V_y(t')}{\bar{R}^4} \left(\frac{\bar{y}^2 \bar{t}^2}{S_T} - \bar{x}^2 S_T \right) dt', \quad (76)$$

$$A_{yy} = \int_{-\infty}^{t_L} \frac{V_y(t')}{\bar{R}^4} \left(\frac{\bar{y}^2 \bar{t}^2}{S_L} - \bar{x}^2 S_L \right) dt' + \int_{-\infty}^{t_T} \frac{V_y(t')}{\bar{R}^4} \left(\frac{\bar{x}^2 \bar{t}^2}{S_T} - \bar{y}^2 S_T \right) dt', \quad (77)$$

$$A_{xy} = \int_{-\infty}^{t_L} \frac{V_y(t')}{\bar{R}^4} \bar{x} \bar{y} \left(\frac{\bar{t}^2}{S_L} + S_L \right) dt' - \int_{-\infty}^{t_T} \frac{V_y(t')}{\bar{R}^4} \bar{x} \bar{y} \left(\frac{\bar{t}^2}{S_T} + S_T \right) dt', \quad (78)$$

with

$$\begin{aligned} \bar{x} &= x, & \bar{y} &= y - \eta(t') & \bar{t} &= t - t', & \bar{R}^2 &= \bar{x}^2 + \bar{y}^2, \\ S_T^2 &= \bar{t}^2 - \frac{\bar{R}^2}{c_T^2}, & t_T &= t - \frac{\bar{R}}{c_T}, & S_L^2 &= \bar{t}^2 - \frac{\bar{R}^2}{c_L^2}, & t_L &= t - \frac{\bar{R}}{c_L}. \end{aligned} \quad (79)$$

The explicit expressions for ϕ_{ij} can be obtained from the corresponding expressions for A_{ij} replacing $V_y(t')$ by 1. Using this replacement, we observe the relations: $v_x(V_y(t') = 1) = -\beta_{xy}$ and $v_y(V_y(t') = 1) = -\beta_{yy}$. Let us now discuss qualitatively the results. The expressions for a climbing edge dislocation (68)–(78) are similar to, but also different in some sense than, the expressions of a gliding edge dislocation (48)–(58). In particular, the elastic fields contain terms again proportional to S_T , S_L , $1/S_T$ and $1/S_L$. In the potential

functions (76)–(78) the dislocation velocity is $V_y(t')$ similar to $V_x(t')$ in Eqs. (56)–(58). The complexity and the type of integrals for a climbing edge dislocation in Eqs. (76)–(78) are the same as for a gliding edge dislocation in Eqs. (56)–(58). The structure of entering of the potential functions ϕ_{ij} in Eqs. (70)–(73) is the same as in Eqs. (50)–(53). However, there are some differences concerning the structure of entering of the potential functions A_{ij} in Eqs. (68)–(73) compared with Eqs. (48)–(53). Of course, the differences in the elastic fields (50)–(53) and (70)–(73) are expected due to the kind (gliding - climbing) of the edge dislocation. The difference between the elastic fields of a gliding edge dislocation and a climbing edge dislocation is mainly based on the specific component of the dislocation current tensor for gliding and climbing. For the gliding edge dislocation the component I_{xy} (see Eq. (41)) enters the Eqs. (42), (43), (46) and (47). However, for the climbing edge dislocation the component I_{xx} (see Eq. (61)) enters the Eqs. (62)–(65). What is also interesting and should be noted is the difference in the expressions for the velocity fields. In Eqs. (48) and (49) for the gliding edge dislocation only the transversal velocity c_T appears in the prefactors while in the prefactors of Eqs. (68) and (69) for the climbing edge dislocation both velocities c_T and c_L appear. Finally, the elastic fields of a climbing edge dislocation represent transversal and longitudinal elastic waves with the velocities c_T and c_L , respectively.

6 Static limit

In this section, we give the static limit of the elastic distortions of nonuniformly moving dislocations as a check of the above results and to show the connection between the dynamic solutions in this paper and the static solutions given by deWit [32]. Because all the potential functions (34), (35), (54), (55), (74) and (75) are defined as integrals of the Green tensors of plane strain and anti-plane strain, we will carry out the static limit for the Green tensors. The static limit of the Green tensors is defined by (see, e.g., [6])

$$G_{ij}(\mathbf{r} - \mathbf{r}') = \lim_{t \rightarrow \infty} \int_{-\infty}^t G_{ij}(\mathbf{r} - \mathbf{r}', t - t') dt'. \quad (80)$$

Although the obtained results provide the solutions of the elastic fields to the general motion of straight dislocations, their application to explicit problems demands some care to avoid the occurrence of divergent integrals. For the static limit it is possible to do this by supposing a limit $-T$ instead of $-\infty$. Then although G_{zz} in Eq. (23) and G_{ij} in Eq. (22) diverge if we let $T \rightarrow \infty$, their derivatives do not, so that the elastic distortions are well-defined in the static limit. The static limit of the derivative of the Green tensor (22) is calculated with

$$G_{ij,k}(\mathbf{r} - \mathbf{r}') = \lim_{T \rightarrow \infty} \partial_k \int_{-T}^t G_{ij}(\mathbf{r} - \mathbf{r}', t - t') dt' \quad (81)$$

as

$$G_{ij,k}(\mathbf{r} - \mathbf{r}') = -\frac{1}{8\pi\mu(1-\nu)} \left[(3-4\nu) \delta_{ij} \frac{R_k}{R^2} - \delta_{ik} \frac{R_j}{R^2} - \delta_{jk} \frac{R_i}{R^2} + 2 \frac{R_i R_j R_k}{R^4} \right], \quad (82)$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, ν is Poisson's ratio and $\lambda = 2\mu\nu/(1 - 2\nu)$. For the derivative of the Green function G_{zz}

$$G_{zz,k}(\mathbf{r} - \mathbf{r}') = \lim_{T \rightarrow \infty} \partial_k \int_{-T}^t G_{zz}(\mathbf{r} - \mathbf{r}', t - t') dt', \quad (83)$$

we obtain

$$G_{zz,k}(\mathbf{r} - \mathbf{r}') = -\frac{1}{2\pi\mu} \frac{R_k}{R^2}. \quad (84)$$

From Eq. (17) we recover the static Mura formula [4]

$$\beta_{im}(\mathbf{r}) = \int_{-\infty}^{\infty} C_{jklm} G_{ij,k}(\mathbf{r} - \mathbf{r}') T_{lmn}(\mathbf{r}') d\mathbf{r}'. \quad (85)$$

Substituting Eqs. (82) and (84) and the dislocation densities $T_{zxy} = b_z \delta(x)\delta(y)$ and $T_{xxy} = b_x \delta(x)\delta(y)$ into Eq. (85), we recover the expressions of the elastic distortion given by deWit [32]. The non-vanishing components of the elastic distortion of a screw dislocation are given by

$$\beta_{zx} = -\frac{b_z}{2\pi} \frac{y}{r^2}, \quad (86)$$

$$\beta_{zy} = \frac{b_z}{2\pi} \frac{x}{r^2}, \quad (87)$$

which are the static limits of Eqs. (32) and (33). The non-vanishing components of the elastic distortion tensor of an edge dislocation are obtained as

$$\beta_{xx} = -\frac{b_x}{4\pi(1-\nu)} \frac{y}{r^2} \left[(1-2\nu) + 2 \frac{x^2}{r^2} \right], \quad (88)$$

$$\beta_{yy} = -\frac{b_x}{4\pi(1-\nu)} \frac{x}{r^2} \left[(1-2\nu) + 2 \frac{y^2}{r^2} \right], \quad (89)$$

$$\beta_{xy} = \frac{b_x}{4\pi(1-\nu)} \frac{x}{r^2} \left[(3-2\nu) - 2 \frac{y^2}{r^2} \right], \quad (90)$$

$$\beta_{yy} = -\frac{b_x}{4\pi(1-\nu)} \frac{y}{r^2} \left[(1-2\nu) - 2 \frac{x^2}{r^2} \right], \quad (91)$$

which are the static limits of the gliding edge dislocation (50)–(53) and the climbing edge dislocation (70)–(73).

Acknowledgement

The author gratefully acknowledges the grants of the Deutsche Forschungsgemeinschaft (Grant Nos. La1974/1-2, La1974/1-3, La1974/2-1). The author wishes to express his gratitude to Dr. Eleni Agiasofitou for her constructive and very helpful remarks during the preparation of the paper. The author also wants to thank two anonymous reviewers for their helpful remarks on an earlier version of the paper.

References

- [1] J. Weertman and J.R. Weertman, Moving dislocations. In: *Dislocations in Solids, Vol. 3*. F.R.N. Nabarro, ed., North-Holland, Amsterdam (1980) pp. 1–59.
- [2] J.C. Savage, Dislocations in seismology. In: *Dislocations in Solids, Vol. 3*. F.R.N. Nabarro, ed., North-Holland, Amsterdam (1980) pp. 251–339.
- [3] K. Aki and P.G. Richards, *Quantitative Seismology*. 2nd Edition, University Science Books (2002).
- [4] T. Mura, Continuous distribution of moving dislocations. *Phil. Mag.* **8** (1963) 843–857.
- [5] T. Mura, On dynamic problems of continuous distribution of dislocations. *Int. J. Engng. Sci.* **1** (1963) 371–381.
- [6] T. Mura, *Micromechanics of Defects in Solids*, 2nd edition, Martinus Nijhoff, Dordrecht (1987).
- [7] H. Bross, Zur Theorie bewegter Versetzungen. *phys. stat. sol.* **5** (1964) 329–342.
- [8] H. Günther, *Zur Dynamik schneller Versetzungen*. Akademie-Verlag, Berlin (1973).
- [9] R.W. Lardner, *Mathematical Theory of Dislocations and Fracture*, University of Toronto Press, Toronto (1974).
- [10] J.D. Eshelby, The force on an elastic singularity. *Phil. Trans. Roy. Soc. London A* **244** (1951) 87–112.
- [11] J.D. Eshelby, The equation of motion of a dislocation. *Phys. Rev.* **90** (1953) 248–255.
- [12] W.S. Wladimirow, *Equations of Mathematical Physics*, Berlin: Deutscher Verlag der Wissenschaften, 1971 (in German).
- [13] F.R.N. Nabarro, The synthesis of elastic dislocation fields. *Phil. Mag.* **7** (1951) 1224–1231.
- [14] J. Kiusalaas and T. Mura, On the elastic field around an edge dislocation with application to dislocation vibration. *Phil. Mag.* **9** (1964) 1–7.
- [15] J.P. Hirth and J. Lothe, *Theory of Dislocations*, 2nd edition, John Wiley, New York (1982).
- [16] C. Teodosiu, *Elastic Models of Crystal Defects*, Springer-Verlag, Berlin (1982).
- [17] F.R.N. Nabarro, *Theory of Crystal Dislocations*, Dover, New York (1987).
- [18] S. Li and G. Wang, *Introduction to Micromechanics and Nanomechanics*, World Scientific, Singapore (2008).

- 1
2
3
4
5 [19] X. Markenscoff, On the dislocation fields in terms of the dynamic Green's function.
6 J. Elasticity **13** (1983) 237–241.
7
- 8 [20] X. Markenscoff and R.J. Clifton, Radiation from expanding circular dislocation loops
9 and elastic precursor decay. ASME Journal of Applied Mechanics **49** (1982) 792–796.
10
- 11 [21] X. Markenscoff and L. Ni, On the effect of dislocation loop curvature on elastic
12 precursor decay. ASME Journal of Applied Mechanics **51** (1984) 753–758.
13
- 14 [22] J. Kiusalaas and T. Mura, On the motion of a screw dislocation, in: Recent Advances
15 in Engineering Science (A. C., Eringen, ed.) Gordon & Breach, New York (1965) pp.
16 543–564.
17
- 18 [23] A.M. Kosevich, Crystal dislocations and the theory of elasticity. in: *Dislocations in*
19 *Solids Vol. 1*. F.R.N. Nabarro, ed., North-Holland, Amsterdam (1979) pp. 33–165.
20
- 21 [24] E. Kossecka, Theory of dislocation lines in a continuous medium. Arch. Mech.
22 Stos. **21** (1969) 167–190.
23
- 24 [25] E. Kossecka, Mathematical theory of defects. part II. Dynamics. Arch. Mech. **27**
25 (1975) 79–92.
26
- 27 [26] E. Kossecka and R. deWit, Disclination kinematics. Arch. Mech. **29** (1977) 633–651.
28
- 29 [27] M. Lazar and C. Anastassiadis, The gauge theory of dislocations: conservation and
30 balance laws. Phil. Mag. **88** (2008) 1673–1699.
31
- 32 [28] M. Lazar, On the fundamentals of the three-dimensional translation gauge theory
33 of dislocations, Mathematics and Mechanics of Solids (2011), in press, DOI:
34 10.1177/1081286510370889.
35
- 36 [29] C. Teodosiu, A dynamic theory of dislocations and its applications to the theory of the
37 elastic-plastic continuum. in *Fundamental Aspects of Dislocation Theory*, Vol. 2, Eds.
38 J.A. Simmons, R. deWit and R. Bullough, Nat. Bur. Stand. (U.S.), Spec. Publ. **317**
39 (1970) pp. 837–876.
40
- 41 [30] A.C. Eringen and S.S. Suhubi, *Elastodynamics, Volume II, Linear Theory*. Academic
42 Press, New York (1975).
43
- 44 [31] E. Kausel, *Fundamental Solutions in Elastodynamics*, Cambridge University Press
45 (2006).
46
- 47 [32] R. deWit, Theory of disclinations IV, J. Res. Nat. Bur. Stand. (U.S.) **77A** (1973)
48 607–658.
49
50
51
52
53
54
55
56
57
58
59
60