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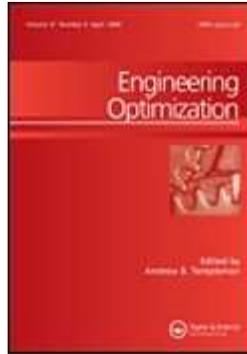
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## **Novel local rules of Cellular Automata applied to topology and size optimization**

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Cellular Automata are mathematical idealization of physical systems in which the design domains are divided into lattices of cells, states of which are updated synchronously in discrete time steps according to some local rules. The principle of the Cellular Automata is that global behaviour of the system is governed by cells that only interact with their neighbours. Because of its simplicity and versatility the method has been found as useful tool for structural design, especially that Cellular Automata methodology can be adopted for both optimal sizing and topology optimization. This article presents the application of the Cellular Automata concept to topology optimization of plane elastic structures. As to the optimal sizing, the design of columns exposed to loss of stability is also discussed. A new local update rule is proposed, selected optimal design problems are formulated, and finally the article is illustrated by results of numerical optimization.

Keywords: Cellular Automata, topology optimization, optimal sizing

### **1. Introduction**

One can easily observe that there are many optimization processes in nature. One of the biologically inspired optimization techniques that has recently aroused the interest of designers are Cellular Automata. Cellular Automata (CA) are mathematical idealizations of physical systems in which the design domains are divided into lattices of cells, states of which are updated synchronously in discrete time steps according to some local rules. The principle of the Cellular Automata is that global behavior of the system is governed by cells that only interact with their neighbors. This is analogous to the behaviour of biological tissues. The concept of Cellular Automata was introduced in late 1940s by Von Neumann (1966) and Ulam (1952), and developed afterwards for example by Wolfram (1994). The characteristic feature of Cellular Automata, which is modeling of complex systems by simple local rules, has attracted

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researchers from various disciplines like physics, biology, social sciences, transport or logistics. Cellular Automata have been found suitable for description of different phenomena like diffusion of gaseous systems, crystal growth in solids, solidification, hydrodynamic flow or turbulence. There is also a broad field of application of Cellular Automata in computer science where computer graphics and image analysis may serve only as examples. An extensive literature review on the subject can be found for example in Ganguly *et al.* (2001) or Tovar *et al.* (2006a).

Cellular Automata have been found to be a useful tool also for structural optimization. The first application of Cellular Automata to optimization of structures was proposed in the mid-1990s. The basic idea was described by Inou *et al.* (1994). In their approach, the values of elastic modulus of cells are used as the design variables which are updated by applying a local rule implementing the difference between a current stress value and its target value. The cells with a low elastic modulus are removed resulting in modification of topology of the structure during the iteration process. In the article by Kita and Toyoda (2000) the values of cell thicknesses stand for the design variables. As the objective to be minimized a combination of weight and deviation of current stress value from an admissible one have been chosen. The authors implemented a so-called “CA-constraint” responsible for exchange of local information between cells. In Tatting and Gurdal (2000) authors propose a new scheme, in which analysis and design are performed simultaneously. This technique called simultaneous analysis and design (SAND) has been then modified for afterwards example in Canyurt and Hajela (2007) where the concept of a Cellular Genetic Algorithm has been developed. In order to improve computational efficiency of that approach the accelerated simultaneous analysis and design technique (ASAND), which uses additional gradient information, was presented by Cortes *et al.*

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(2005). In Missoum *et al.* (2005) another update scheme, a “repeat approach”, was proposed, which requires locally performed updating of state and design variables repeated a given number of times. Some original updating schemes are also proposed in Hajela and Kim (2001). They were built using a genetic algorithm whereas overall optimization was performed using the CA technique. The iteration process required long computational times, so parallel computations were implemented. It is worth noting here that the natural suitability for large-scale parallel implementation is an important advantage of using Cellular Automata. This is of course due to local data flow and information storage, since the update rules are the same for all cells and they are processed simultaneously. The parallel implementation and SAND approach are widened in Setoodeh *et al.* (2006), where the pipeline concept was adopted. The topology optimization using Cellular Automata has been developed by Tovar and co-workers in a series of articles. A new Cellular Automata technique is inspired by phenomenological approaches implemented to simulate bone functional adaptation. The method presented for example in Tovar *et al.* (2004) is referred to as a hybrid Cellular Automata. The local update rules are taken from control theory and the design task is to minimize the error between target local strain energy and current average strain energy, sensed in a neighborhood around each cell. This approach has been expanded in Tovar *et al.* (2006a, 2006b) and Narvaez *et al.* (2007) where implementation of the Fuzzy Logic technique can also be found. The articles discussed above show application of the Cellular Automaton to optimization of 2D isotropic continua and trusses. The article by Abdalla and Gurdal (2004) deserves special attention, since the authors consider optimization of a 1D structure, namely design of a column under buckling constraints using the SAND approach. A similar problem but formulated in a different way is considered in detail in this article.

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The application of Cellular Automata in structural optimization in most cases is concerned with generation of optimal topologies. Since the early article by Bendsoe and Kikuchi (1988) one can find numerous approaches for generating optimal topologies in the literature based both on optimality criteria and evolutionary methods. The broad discussion on topology optimization concepts can be found in many survey articles *e.g.* Rozvany (2001), Eschenauer and Olhoff (2001), Bendsoe and Sigmund (2003), Arora and Wang (2005) or Rozvany (2008), as well as in hundreds of articles dealing with specific methods ranging from gradient based approaches to biologically inspired algorithms and level set method, *e.g.* Wang *et al.* (2003), presenting numerous solutions including classic Michell examples as well as complicated spatial engineering structures. In particular heuristic, evolutionary methods have gained widespread popularity among researchers, and have been recently intensively developed. This is because they are friendly for numerical implementation, do not require gradient information, and one can easily combine this type of algorithm with any finite element structural analysis code. Generally the proposed approaches are similar to the fully stressed design criterion, where inefficient material is gradually removed from the structure to approach the optimal topology. Among others there are hard killing and soft killing methods, *e.g.* Mattheck (1997), evolutionary structural optimization ESO, *e.g.* Xie and Steven (1997) or bi-directional evolutionary structural optimization BESO, *e.g.* Querin and Young (2000). Although computationally very effective these approaches demonstrate also some drawbacks. In some cases they fail to achieve convergent optimal solutions, and also usually require many iterations to perform. This is the reason for further research in this area, and there is still some space for presentation of an efficient alternative to existing algorithms.

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The aim of this article is to present a new optimization algorithm, the performance of which is based on local rules of Cellular Automata. Comparing with existing evolutionary optimization approaches, the contributions of the present article are twofold. Firstly, a fast converging, easy to implement algorithm is proposed and secondly its application to both topology optimization and optimal sizing is presented.

**2. The idea of Cellular Automata**

The idea of Cellular Automata is based on modeling of a complex problem by a sequence of relatively simple decision making rules. The engineering implementation requires decomposition of the considered domain into a set of cells which form a uniform lattice. The particular cell together with cells to which it is connected is called a neighbourhood. It is assumed that the cells interact only within their neighbourhood. The examples of 1D and 2D neighbourhoods are presented in figures 1-2, respectively.

The evolution of each state is governed by a local homogeneous rule. The rules are identical for all neighbourhoods and are applied simultaneously to each of them. The rules operate over a large number of cells that carry only local information. By applying the rules repetitively to locally updated physical quantities the process converges to a description of the global behaviour of the system. A new value for each cell can be calculated based on already updated values found for cell neighbours (Gauss-Seidel iteration mode) or the cell updates its state based on the states of the surrounding cells determined in the previous iteration (Jacobi iteration mode). Each cell has the same neighbourhood, therefore those at the boundary have neighboring cells that lay outside the design domain. It is important to specify how to establish for them values of the design variables. The simplest and the most often used approach

Figure 1  
Figure 2

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sets all these values to zero, but as the alternatives, periodic, reflecting or adiabatic boundary conditions can also be adopted, see *e.g.* Tovar *et al.* (2006a).

### 3. The novel approach

#### 3.1. Concept

Topology optimization via CA rules is discussed in this section. The power-law approach known as SIMP (Solid Isotropic Material with Penalization, Bendsoe 1989 and Zhou and Rozvany 1991), commonly used while generating optimal topologies, is adopted within the Cellular Automata formalism. The design variables are relative densities of a material, and the elastic modulus of each cell element is modelled as a function of relative density  $d_i$  see equation (1), where elastic modulus  $E_0$  stands for a solid material. The power  $p$  penalizes intermediate densities and drives the design to a black-and-white structure.

$$E_i = d_i^p E_0, \quad d_{\min} \leq d_i \leq 1 \quad (1)$$

A typical topology optimization problem is minimization of a structure compliance subject to a volume constraint. Such a problem can be solved for example using optimality criteria approach as presented by many authors *e.g.* Bendsoe (1995), Sigmund (2001) or Groenwold and Etman (2007). This approach can be also implemented into problems formulated within the framework of the Cellular Automata formalism (see *e.g.* Tovar *et al.* 2004, Bochenek and Tajs-Zielińska 2009).

The alternative to gradient based optimality criteria approach is selection of some heuristic local rules, and application of so-called evolutionary design. In the articles by Tovar and co-workers (Tovar *et al.* 2006a, Tovar *et al.* 2006b) various proposals of local rules for compliance minimization problems are presented. A design variable represented by relative material density is updated based on

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information that is derived from comparison of the local average compliance and a specified value of compliance chosen for the design process. This idea comes from the concept of uniform stress design for which for highly stressed regions material is added whereas from low stressed it is removed. In the case of compliance-based topology optimization, if local compliance is lower than a specified threshold value material density is lowered, tending to zero (forming a void region), otherwise its value increases towards unity (creating a solid region). In practice a lower bound is implemented in order to deal with voids numerically.

This article proposes a novel CA formulation which can be treated as a generalization of the above idea. Instead of dealing with neighbourhood average compliance, the local update rules applied to design variables  $d_i$  associated with central cells are constructed based on individual information gathered from adjacent cells forming the neighbourhood. They take the form of a linear combination of design value corrections with coefficients, values of which are influenced by states of the neighbouring cells surrounding each central cell, as presented in equation (2)

$$d_i^{(t+1)} = d_i^{(t)} + \delta d_i^{(t)}, \quad \delta d_i^{(t)} = (\alpha_0 + \sum_{k=1}^N \alpha_k) m = \tilde{\alpha} m \quad (2)$$

In what follows, the objective values calculated for iteration ( $t$ ) for a central  $U_i$  and neighbouring cells  $U_k$  are compared with a selected threshold value  $U^*$  and, depending whether they are larger or smaller than the one selected, a positive or negative coefficient  $C_{\alpha_0}$  and  $C_\alpha$  is transferred to the design variable update rule, according to equation (3).

$$\alpha_0 = \begin{cases} -C_{\alpha_0} & \text{if } U_i^{(t)} < U^* \\ C_{\alpha_0} & \text{if } U_i^{(t)} \geq U^* \end{cases} \quad \alpha_k = \begin{cases} -C_\alpha & \text{if } U_k^{(t)} < U^* \\ C_\alpha & \text{if } U_k^{(t)} \geq U^* \end{cases} \quad k = 1 \dots N \quad (3)$$

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It is worth noting that since there are many combinations of positive and negative coefficients, an increase or decrease of the design variable value can be achieved at larger or smaller rate of change, within specified move limit  $m$ .

### **3.2. Algorithm**

The above presented local rules have been implemented into a numerical algorithm. The two main approaches can be considered, the sequential approach which is based on a design update rule applied for an exact displacement field, typically obtained by a global finite element analysis, and simultaneous analysis and design approach for which structural analysis and design are performed locally at the cell level. For the latter case the use of local equilibrium equations eliminates the need for finite element analysis since the optimizer simultaneously drives the local field variables to target values. This however generates significant computational cost; even hundreds of thousands of iterations might be required to achieve convergence since the field variable information propagates usually very slowly. Therefore the sequential approach, which for practical problems is much more effective, has been adopted in the present article, meaning that, for each iteration, the structural analysis performed for the optimized element as a whole is followed by the local updating process. Simultaneously a global volume constraint is applied for specified volume fraction  $\kappa$ .

Practical implementation of proposed local rules requires specification of introduced parameters. The value of  $C_{\alpha 0}$  is selected first and then  $C_{\alpha}$  is calculated as  $1 - C_{\alpha 0}$  divided by a number of neighbouring cells. Based on numerous numerical tests  $C_{\alpha 0} = 0.2$  seems to be a good choice. In figure 3 possible combinations of cells with positive and negative coefficients for an extended Moore (1D) neighbourhood are presented, whereas figure 4 shows allowable variants for a 2D case and von Neumann neighbourhood.

*Engineering Optimization*Figure 3  
Figure 4

An acceleration strategy can also be adopted, namely an additional multiplier in equation (2) may be implemented. In what follows, if for 3 of 4 (von Neumann) or at least 6 of 8 (Moore) neighbouring cells coefficients  $\alpha_k$  have the same sign, then  $\tilde{\alpha}$  is multiplied by a specified, greater than 1, value of the additional coefficient  $\delta$  (e.g.  $\delta=2.0$ ).

The value for move limit has to be also specified. As the default  $m=0.2$  is used.

**3.3. Performance**

The topology optimization of plane elastic structures has been performed. The important issues reported in the literature are the checkerboard effect and mesh dependency. Since the updating scheme according to proposed local rules is influenced by all cells within the neighbourhood the checkerboard effect is practically absent. As confirmed by numerical tests changing meshes from crude to fine does not influence significantly the obtained topologies either.

Figure 5  
Figure 6

Two illustrative examples are presented. In the figure 5 and figure 6 the process of generation of optimal topologies is presented. In both cases the structure consists of 3200 cells (80×40 quadrangular elements in mesh discretization). The initial structures, selected intermediate, and final topologies are shown. Once the solutions have been found the calculations are repeated for the same structures but for different number of cells. In what follows, the crude mesh of 800 cells (40×20) and the fine one of 12800 cells (160×80) have been chosen. The final topologies obtained for the three above cases are presented in the figure 7, showing in all cases the same type of structure.

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Figure 7

### 3.4. Efficiency

In order to present the efficiency of the proposed Cellular Automata local rules with other approaches two recently published examples are chosen for a comparison.

In the article by Zhu *et al.* (2007) the structure that consists of  $40 \times 40$  quadrangular elements, clamped at the left end and loaded by a vertical force applied at the bottom right corner has been optimized using the element replaceable method, a new evolutionary topology optimization approach. The final structure has been found after 55 iterations, for which the compliance of  $1.05 \cdot 10^{-6}$  Nm has been obtained. The same example has been solved with the approach of the present article resulting in the compliance of  $1.04 \cdot 10^{-6}$  Nm found already in 18 iteration. The overview of iteration history is presented in the figure 8.

Figure 8

The next example is taken from the article by Xu *et al.* (2010), where the implementation of a volume-preserving nonlinear density filter based on the Heaviside function is discussed. The authors show that their approach allows, at least for the presented example being a cantilever beam which consists of 4800 cells ( $120 \times 40$ ), to obtain the topology with the smallest minimal compliance value as compared with results of other authors. It appears that the proposed novel CA approach of this article works also well for this problem. The compliance value is 179.3 Nmm which is almost the same as 179.1 Nmm reported by Xu *et al.* (2010). It is worth noting that the number of 70 iterations required by the CA approach is significantly smaller than the 359 needed by the optimality criteria algorithm with density filtering. The overview of iteration history is presented in figure 9.

Figure 9

## 4. Numerical examples

### 4.1. Generating optimal topologies of plane structures

Some examples of compliance-based topologies generated using the approach presented in this article have been discussed in the previous section. In order to complement them, three additional ones are described in this section.

The well known Michell structure, for the case of two immovable supports, presented in figure 10 is discussed first. Another optimized structure is a bridge structure with non-design domain shown in figure 11, and finally optimal topology for the L-shaped structure has been found. The selected intermediate topologies as well as the final one for the latter case are presented in the figure 12.

Figure 10  
Figure 11  
Figure 12

The obtained results correspond to optimal topologies presented in the literature. As to the first example the recent articles by Kaveh *et al.* (2008) and Victoria *et al.* (2009) can be mentioned, where the application of ant colony methodology and isolines topology design have been presented, respectively. The optimal topology for all cases presents the same Michell truss as obtained with the approach of the present article. The topology of the second example converges towards the arch bridge known from bridge engineering and closely resembles, for example, the result found for the same initial structure shown in Victoria *et al.* (2009).

The numerical results found for various volume fractions can be also compared to analytical solutions obtained for numerous benchmark problems and presented among others by Rozvany (1998), or Lewiński and Rozvany (2008). The numerical solution shown in figure 12 found within the framework of the approach presented in this article can be compared with analytical result presented by Lewiński and Rozvany (2008).

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#### 4.2. Optimal sizing of columns exposed to loss of stability

For optimal sizing, the design of columns exposed to loss of stability is considered.

The buckling load of a structure is a global quantity therefore it is not that obvious to apply here the CA algorithm, which by nature requires local formulation of the design problem. Fortunately one can observe that for the optimal column, for which critical load has been maximized, the maximal bending stress is uniformly distributed along the column axis. Thus it is possible to replace conventional maximization of buckling load by a problem formulated within the framework of the proposed approach similarly to the fully stressed design.

The set of equations describing the critical state of the column can be presented in the following form:

$$\frac{dw}{dx} = \varphi, \quad \frac{d\varphi}{dx} = -\frac{m}{d^4}, \quad \frac{dm}{dx} = t + p\varphi, \quad \frac{dt}{dx} = 0 \quad (4)$$

The dimensionless quantities in equation (4), namely independent variable  $x$ , deflection  $w$ , angle of rotation  $\varphi$ , bending moment  $m$ , transverse force  $t$ , and critical load  $p$  are defined by dimensional ones according to the relations:

$$x = \frac{X}{L} \quad w = \frac{W}{L} \quad m = \frac{ML}{EJ_0} \quad t = \frac{TL^2}{EJ_0} \quad p = \frac{PL^2}{EJ_0} \quad (5)$$

Dimensional quantities are represented by capital letters, respectively. In addition  $L$  stands for the column length and  $EJ_0$  represents the bending rigidity of a reference prismatic column. It has also been assumed that the column cross section is a square with the side width  $d$ .

Instability analysis conducted by integration of the above state equations requires boundary conditions to be specified. In what follows, for a cantilever, these are:

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$$w(0) = \varphi(0) = m(1) = t(1) = 0 \quad (6)$$

For a clamped-clamped column the two first buckling modes (1, 2) are taken into account, and these are distinguished by setting boundary conditions for a middle section of a column:

$$w_1(0) = \varphi_1(0) = \varphi_1(0.5) = t_1(0.5) = 0, \quad w_2(0) = \varphi_2(0) = w_2(0.5) = m_2(0.5) = 0 \quad (7)$$

The column is divided into  $n$  prismatic segments and the optimal values of their cross-sections are sought during optimization process. The local update rule takes the form of equation (8) with

$$\alpha_0 = \begin{cases} -C_{\alpha_0} & \text{if } S_i^{(t)} < S^* \\ C_{\alpha_0} & \text{if } S_i^{(t)} \geq S^* \end{cases} \quad \alpha_k = \begin{cases} -C_{\alpha} & \text{if } S_k^{(t)} < S^* \\ C_{\alpha} & \text{if } S_k^{(t)} \geq S^* \end{cases} \quad k = 1 \dots N \quad (8)$$

where

$$S = \lambda \left( \frac{m}{d^3} \right)^2 \quad (9)$$

for unimodal optimization, where only one buckling mode is considered, and

$$S = \lambda \left[ \left( \frac{m_1}{d^3} \right)^2 + \mu \left( \frac{m_2}{d^3} \right)^2 \right] \quad (10)$$

for the bimodal case, that is two buckling modes are taken into account, respectively.

The value of quantity  $\lambda$  is determined from the global constraint of constant total volume of the optimized column. For the bimodal case an additional global constraint ensures equal values of critical loads for both considered buckling modes, which results in selection of an appropriate value of  $\mu$ .

Optimizing the cantilever column, the critical load value of 2.467 for a prismatic column has been increased to 3.288 for the optimal column presented below in figure 10 for which a uniform distribution of maximal bending stress along column axis has been obtained.

Figure 13

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For the clamped-clamped column the actual optimum is bimodal as reported by Olhoff and Rasmussen (1977), meaning that the first two buckling modes have the same critical load value. The bimodal critical value of 52.33 has been obtained with the approach proposed in this article for the optimal column presented in figure 14. The uniform distribution of  $S$  defined by equation (10) is associated with this solution.

Figure 14

It is worth noting that the CA approach to buckling load maximization presented by Abdalla and Gurdal (2004) does not allow bimodal solutions to be found.

## 5. Closing remarks

This article discusses application of the Cellular Automata concept to structural optimization. A novel proposal for local update rules is presented, which can be adapted to both topology and size optimization. Selected optimal design problems are considered, among which there are compliance-based topology optimization of plane elastic structures as well as problems of optimal sizing of columns under stability constraints.

The performance of the algorithm based on the novel local rules seems to be good. The numerical results obtained for various structural elements are in good agreement with the ones presented in the literature obtained with use of various optimization techniques. Among others there is the evolutionary technique called the element replaceable method, with numerical processing based on optimality criteria with a special nonlinear density filter based on Heaviside function and isolines topology design ITD algorithm. The main advantage of the Cellular Automata algorithm is that it is a fast convergent technique and usually requires far fewer iterations as compared to other approaches to achieve the solution. What is also

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3 important it does not require any additional density filtering. There are not many  
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5 parameters to adjust, and it is very easy to implement parallel computations in  
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7 Cellular Automata algorithms. Finally, for topology optimization problems, changing  
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9 mesh density does not influence the resulting topologies and solutions are free from  
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11 the checkerboard effect.  
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15 The algorithm presented in the article is quite general, which allows its easy  
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17 application to 3D problems. The local update rules are simple so they can be easily  
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19 implemented into professional FEM analysis codes, allowing for solving practical  
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21 engineering optimization problems.  
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25 Minimization of structure compliance is the typical topology optimization  
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27 problem. Apart from that the topology optimization problems can be also formulated  
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29 with stress constraints imposed. The approach presented above allows also for that.  
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31 The compliance in the local rule should therefore be replaced by the appropriate stress  
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37 The development of CA based size optimization is possible. Some results of  
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39 optimal design of truss structures are discussed in Bochenek and Zielińska (2010).  
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41 Although having many advantages, Cellular Automata algorithms have also  
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43 some limitations. Their performance is based on local exchange of information, which  
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45 suits well for locally formulated problems such as these dealing with local compliance  
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47 or stresses, but restricts their direct application to problems with global objectives or  
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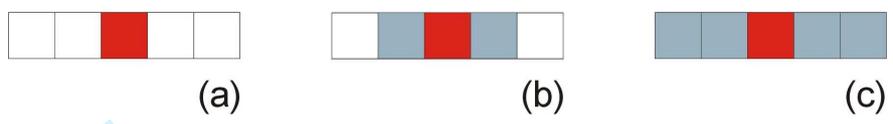
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Figure 1. Neighbourhoods for one-dimensional problems: (a)-empty, (b)-Moore, (c)-extended Moore

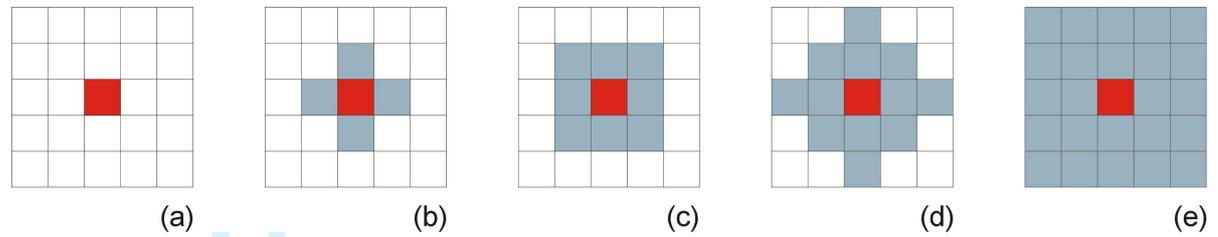


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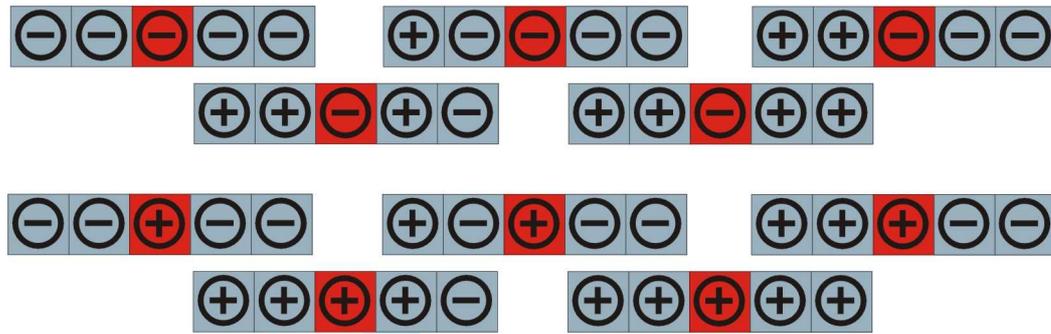
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Figure 2. Neighbourhoods for one-dimensional problems: (a)-empty, (b)-von Neumann, (c)-Moore, (d)-MvonN, (e)-extended Moore



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Figure 3. Cells with positive and negative coefficients for 1D extended Moore neighbourhood ( $C_{\alpha 0}=C_{\alpha}=0.2$ ), first two rows from left to right  $\tilde{\alpha} = -1.0, \tilde{\alpha} = -0.6,$   
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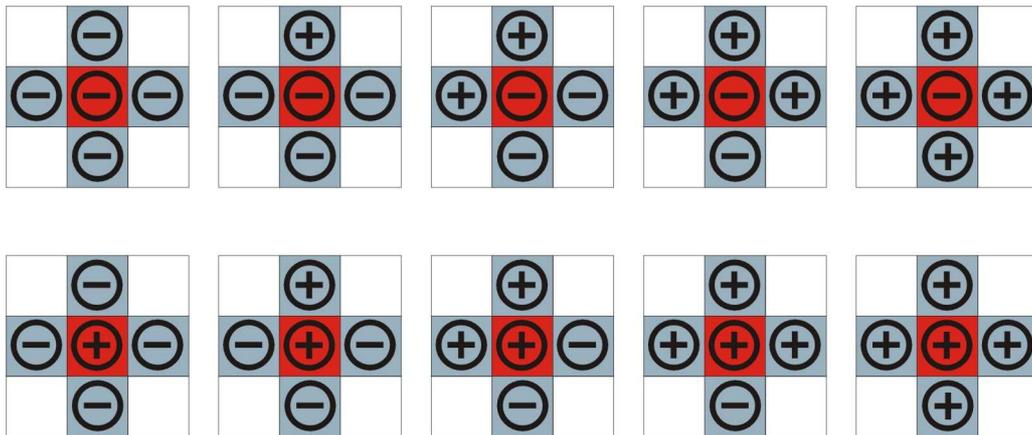


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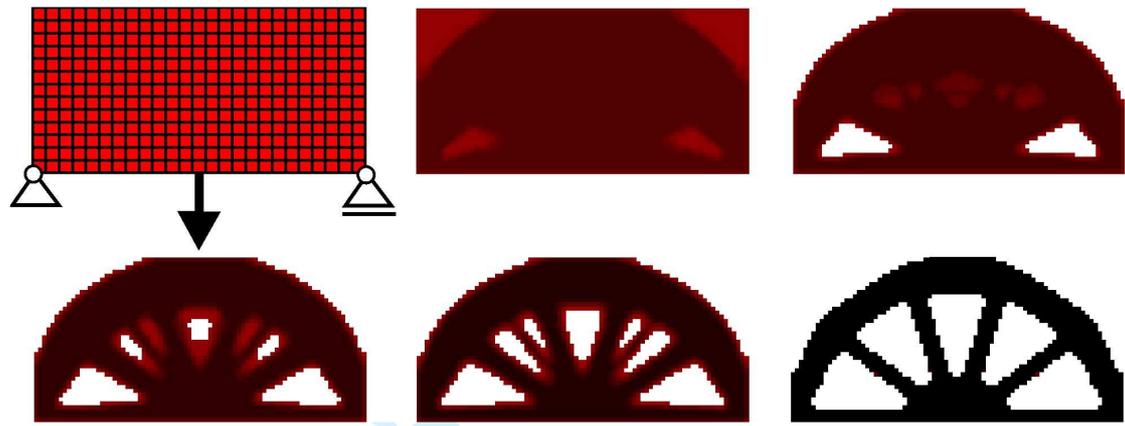
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Figure 4. Cells with positive and negative coefficients for von Neumann 2D neighbourhood ( $C_{\alpha 0}=C_{\alpha}=0.2$ ). First row from left to right  $\tilde{\alpha} = -1.0$ ,  $\tilde{\alpha} = -0.6$ ,  $\tilde{\alpha} = -0.2$ ,  $\tilde{\alpha} = 0.2$ ,  $\tilde{\alpha} = 0.6$ , second row from left to right  $\tilde{\alpha} = -0.6$ ,  $\tilde{\alpha} = -0.2$ ,  $\tilde{\alpha} = 0.2$ ,  $\tilde{\alpha} = 0.6$ ,  $\tilde{\alpha} = 1.0$



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Figure 5. The initial, selected intermediate (iterations: 2, 8, 10, 12) and final (iteration 35) topology of a Michell-type structure  $80 \times 40$  cells ( $1 \text{ mm} \times 1 \text{ mm}$ ), force  $100 \text{ N}$ , material data  $E=10 \text{ GPa}$ ,  $\nu = 0.3$ , volume fraction  $\kappa = 0.5$ , compliance =  $14.04 \text{ Nmm}$ .

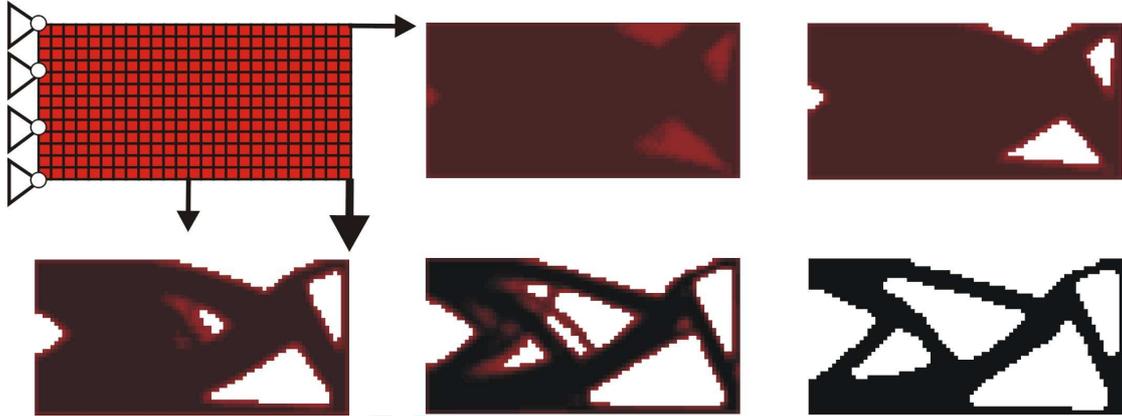


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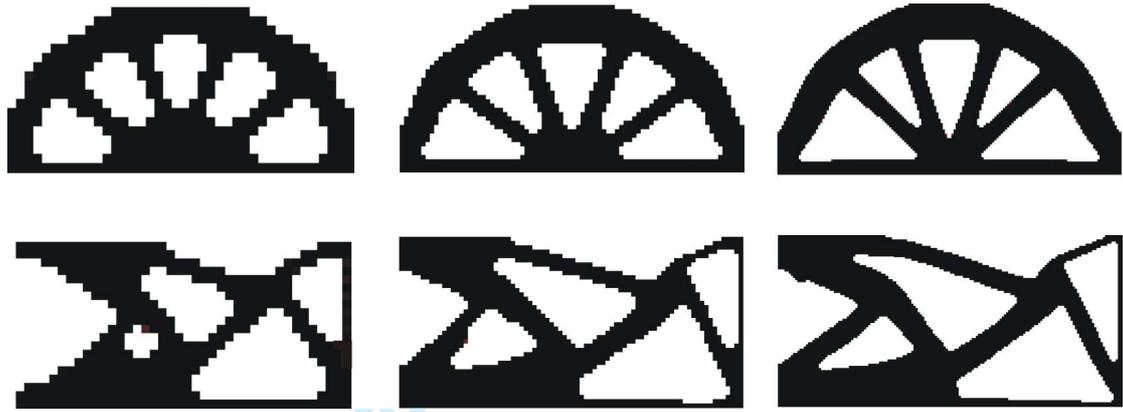
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Figure 7. Generation of topologies for crude and fine meshes. From left to right: 800 cells, 3200 cells, 12800 cells.

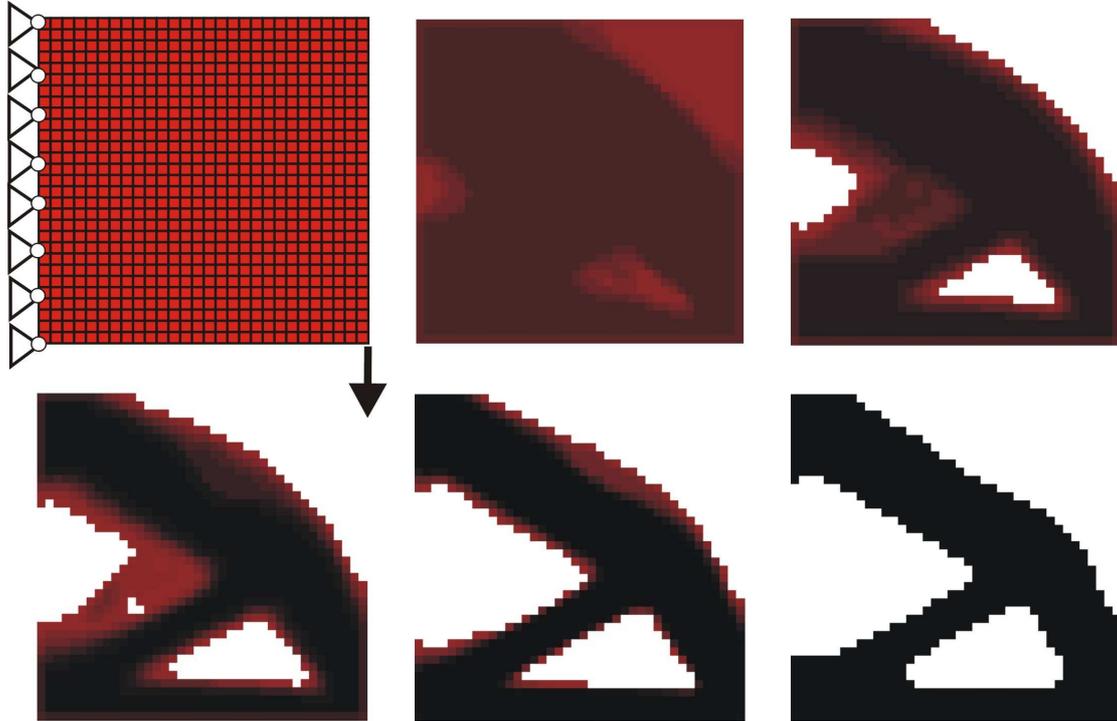


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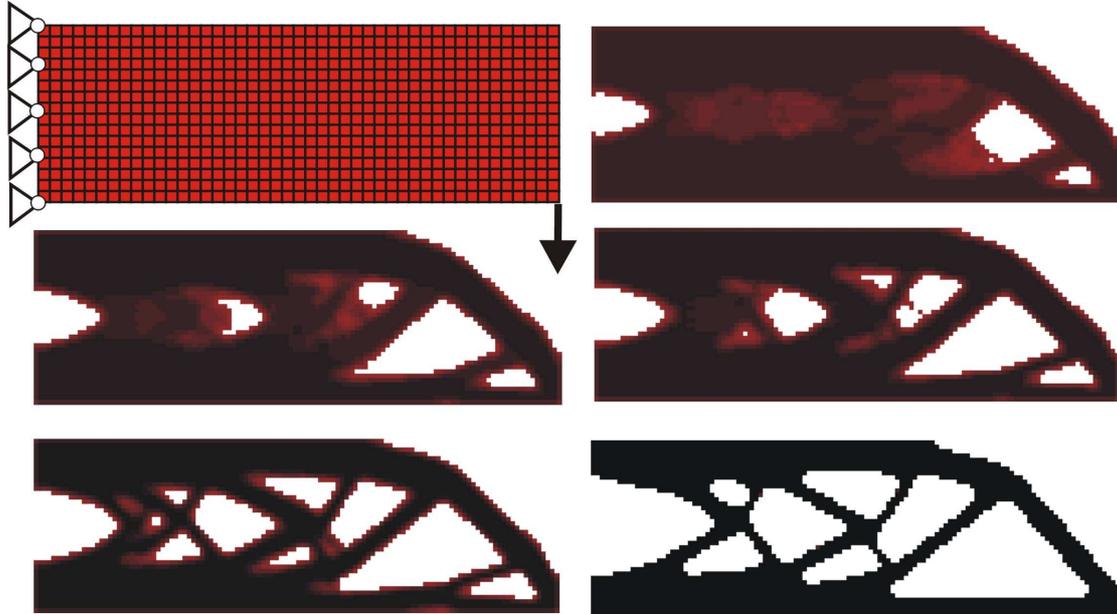
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Figure 8. The initial, selected intermediate (iterations: 2, 5, 7, 9) and final (iteration 18) topology of a Michell-type structure  $40 \times 40$  cells ( $10 \text{ cm} \times 10 \text{ cm}$ ), force  $100 \text{ N}$ , material data  $E=20 \text{ GPa}$ ,  $\nu = 0.3$ , volume fraction  $\kappa = 0.5$ , compliance  $=1.04 \cdot 10^{-6} \text{ Nm}$



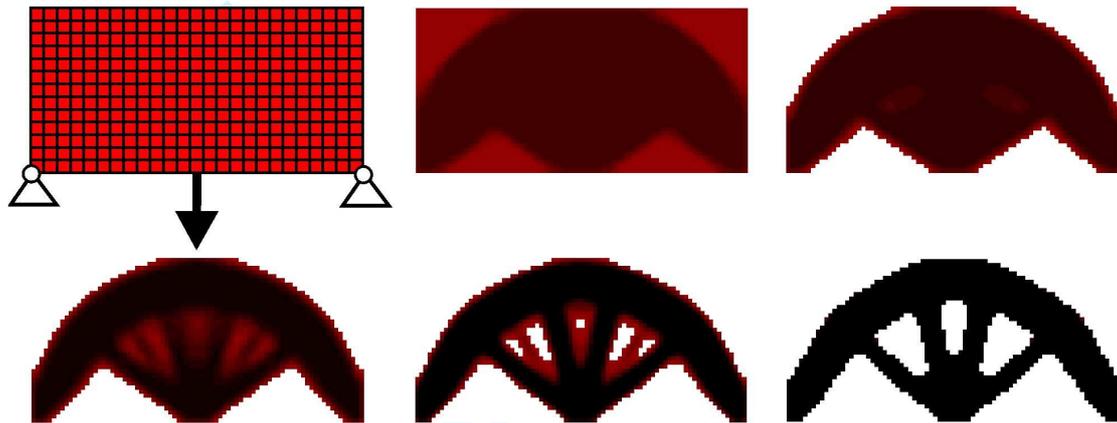
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Figure 9. The initial, selected intermediate (iterations: 5, 8, 10, 14) and final (iteration 70) topology of a cantilever beam  $120 \times 40$  cells ( $1 \text{ mm} \times 1 \text{ mm}$ ), force  $100 \text{ N}$ , material data  $E=10 \text{ GPa}$ ,  $\nu = 0.3$ , volume fraction  $\kappa = 0.5$ , compliance  $=179.3 \text{ Nmm}$



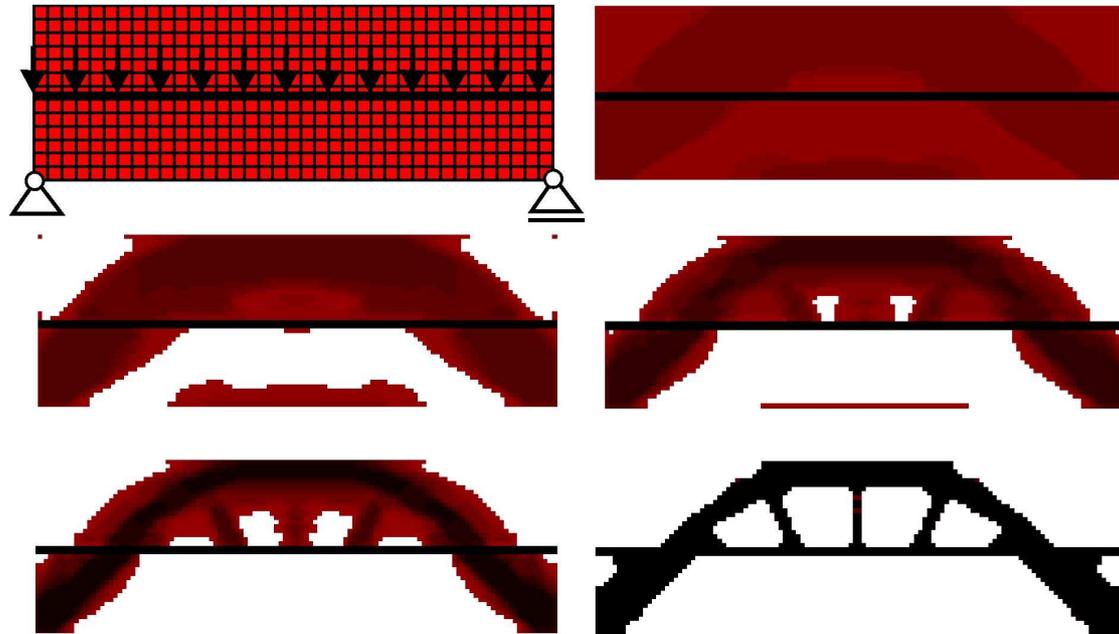
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Figure 10. The initial, selected intermediate (iterations: 4, 10, 16, 20) and final (iteration 40) topology of a Michell-type structure  $80 \times 40$  cells ( $1 \text{ mm} \times 1 \text{ mm}$ ), force  $100 \text{ N}$ , material data  $E=10 \text{ GPa}$ ,  $\nu = 0.3$ , volume fraction  $\kappa = 0.5$ , compliance  $=9.83 \text{ Nmm}$



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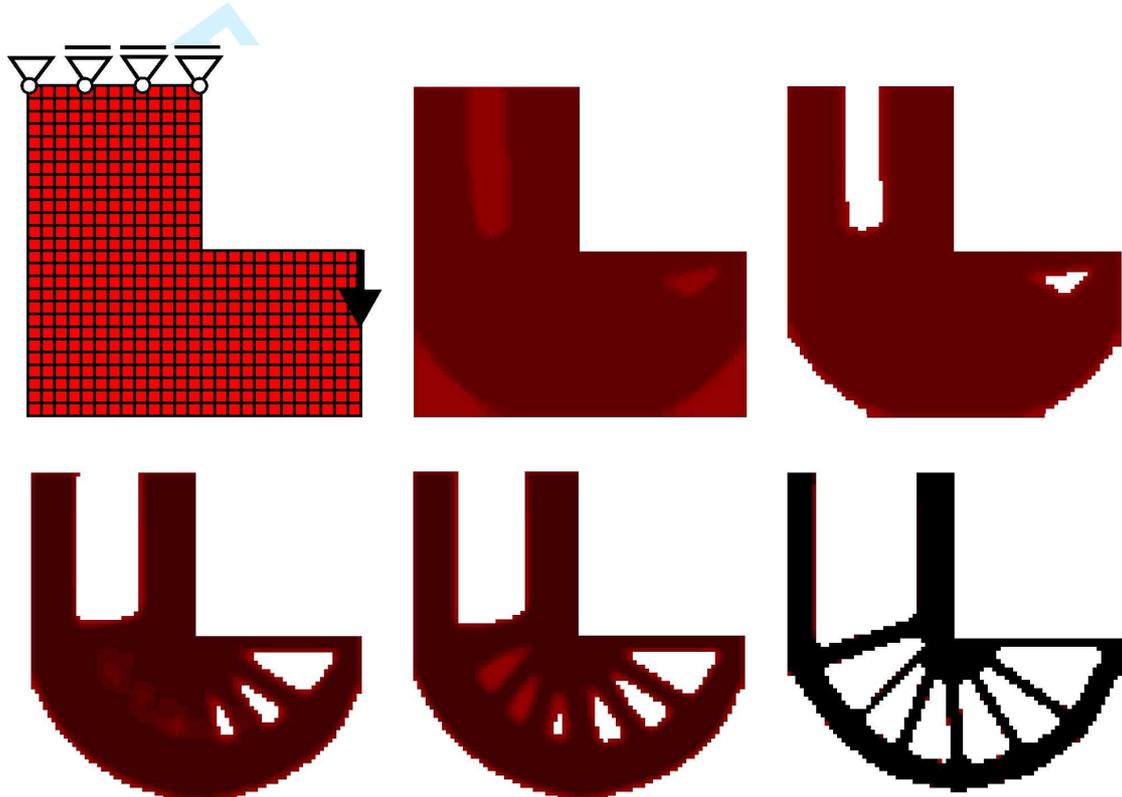
Figure 11. The initial, selected intermediate (iterations: 2, 5, 6, 8) and final (iteration 38) topology of a bridge structure  $120 \times 40$  cells ( $1 \text{ mm} \times 1 \text{ mm}$ ), force  $10 \text{ N/mm}$ , material data  $E=10 \text{ GPa}$ ,  $\nu = 0.3$ , volume fraction  $\kappa=0.3$ , compliance =  $11.25 \text{ Nmm}$ .



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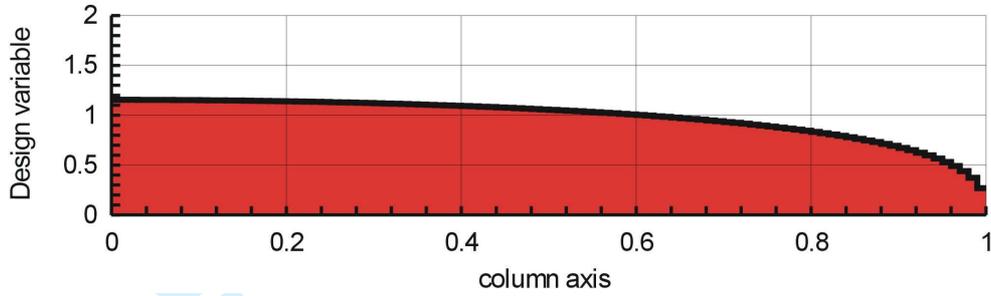
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Figure 12. The initial, selected intermediate (iterations: 4, 8, 25, 30) and final (iteration 80) topology of a L-shaped structure  $80 \times 40 + 40 \times 40$  cells ( $1 \text{ mm} \times 1 \text{ mm}$ ), force  $100 \text{ N}$ , material data  $E=10 \text{ GPa}$ ,  $\nu = 0.3$ , volume fraction  $\kappa = 0.3$ , compliance =  $96.86 \text{ Nmm}$ .



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Figure 13. Optimal cantilever column, unimodal solution.

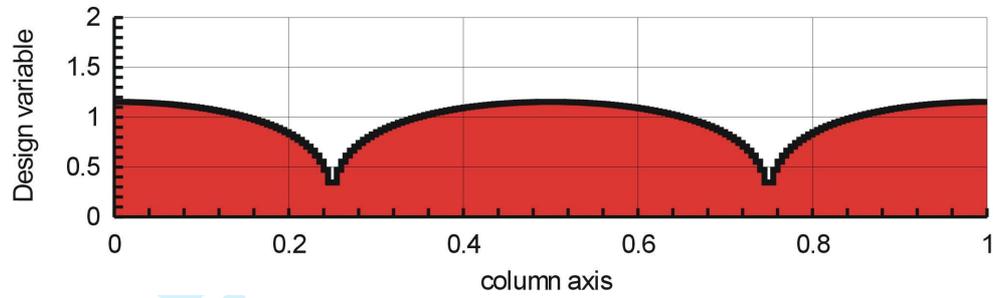


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Figure 14. Optimal clamped-clamped column, bimodal solution.



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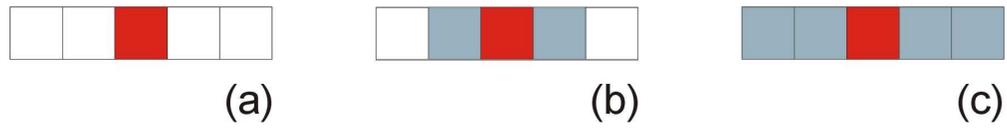
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Figure 11. The initial, selected intermediate (iterations: 2, 5, 6, 8) and final (iteration 38) topology of a bridge structure  $120 \times 40$  cells ( $1 \text{ mm} \times 1 \text{ mm}$ ), force 10 N/mm, material data  $E=10 \text{ GPa}$ ,  $\nu = 0.3$ , volume fraction  $\kappa=0.3$ , compliance =11.25 Nmm.

Figure 12. The initial, selected intermediate (iterations: 4, 8, 25, 30) and final (iteration 80) topology of a L-shaped structure  $80 \times 40 + 40 \times 40$  cells ( $1 \text{ mm} \times 1 \text{ mm}$ ), force 100 N, material data  $E=10 \text{ GPa}$ ,  $\nu = 0.3$ , volume fraction  $\kappa = 0.3$ , compliance =96.86 Nmm.

Figure 13. Optimal cantilever column, unimodal solution.

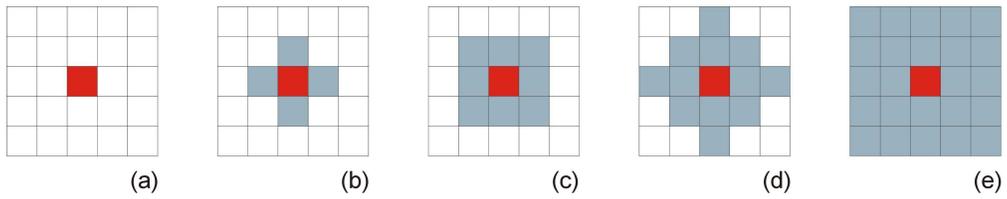
Figure 14. Optimal clamped-clamped column, bimodal solution.



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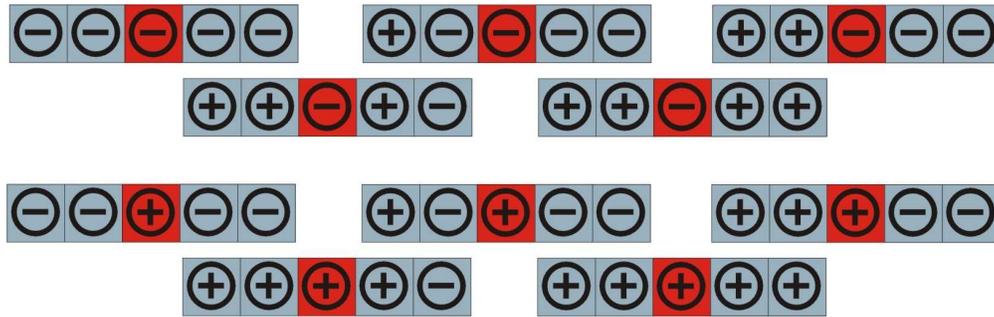
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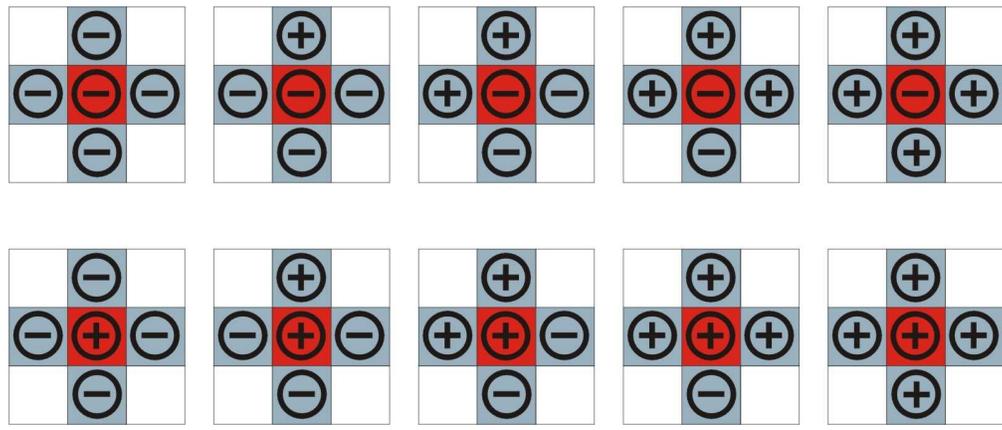
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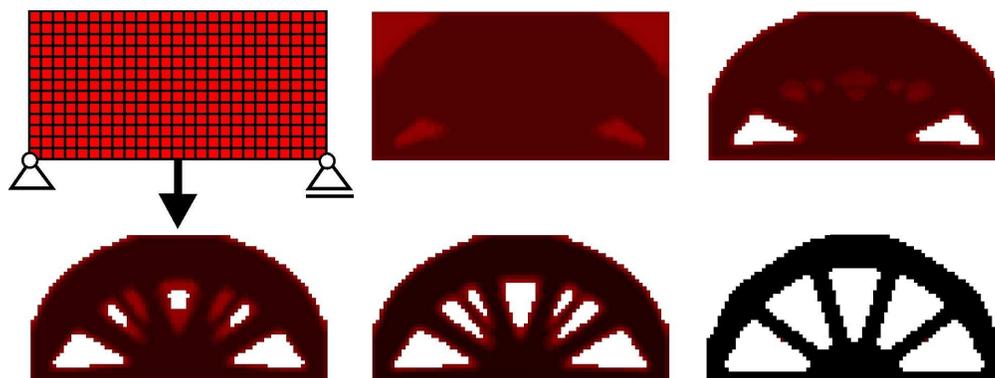
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135x57mm (300 x 300 DPI)

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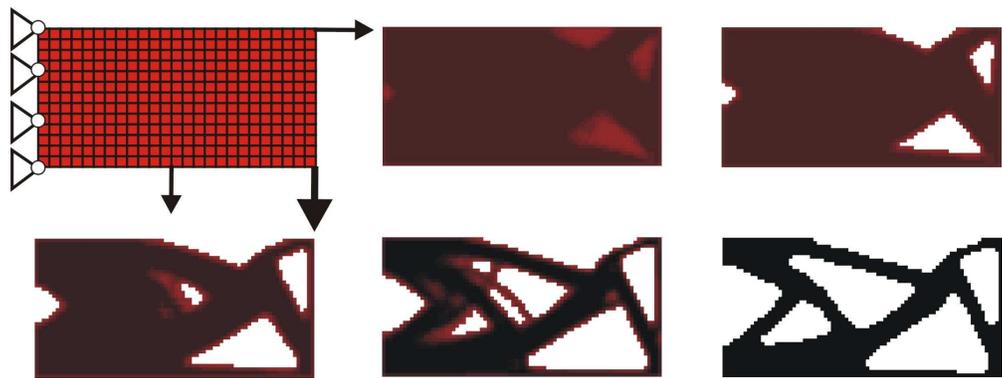


167x62mm (300 x 300 DPI)

Peer Review Only

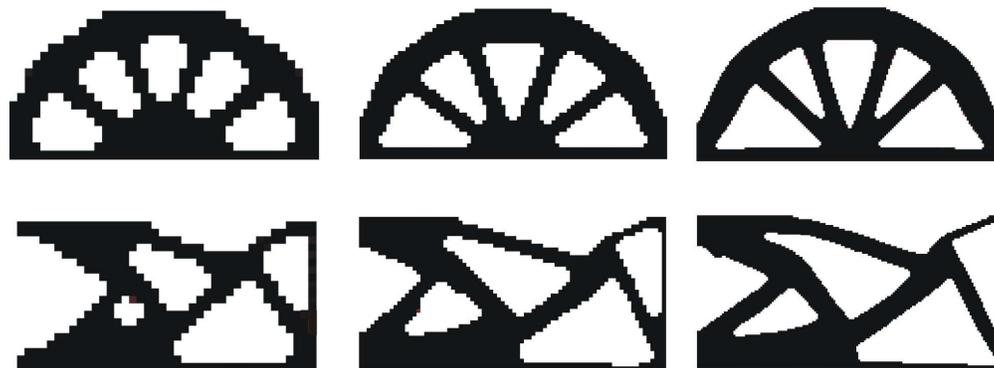
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179x67mm (300 x 300 DPI)

Peer Review Only

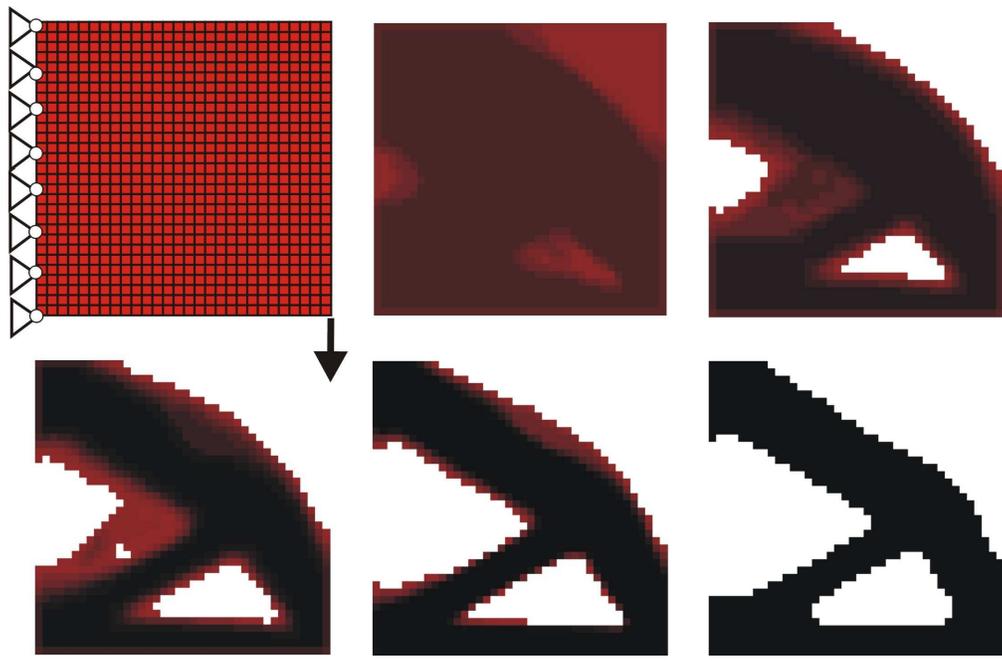


166x63mm (300 x 300 DPI)

Peer Review Only

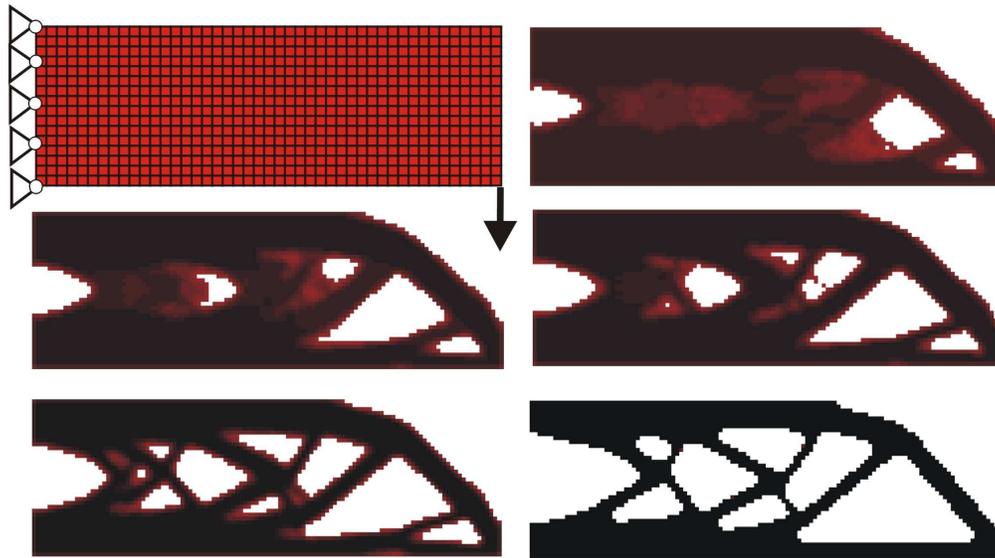
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190x124mm (300 x 300 DPI)

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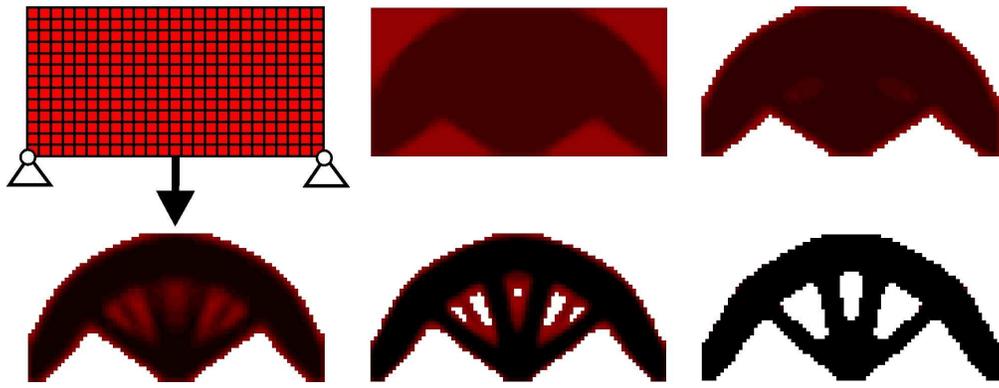


193x107mm (300 x 300 DPI)

Review Only

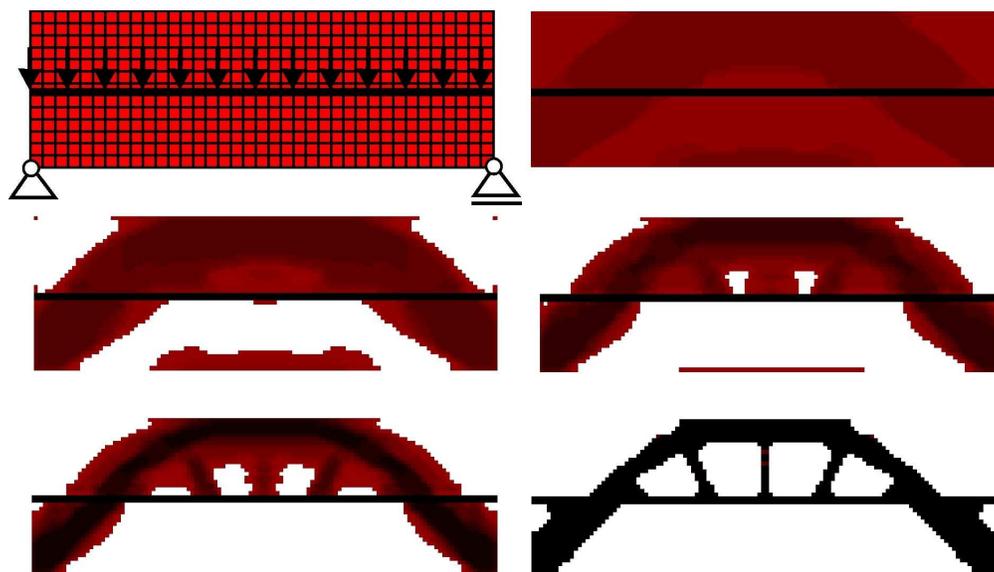
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167x63mm (300 x 300 DPI)

Peer Review Only

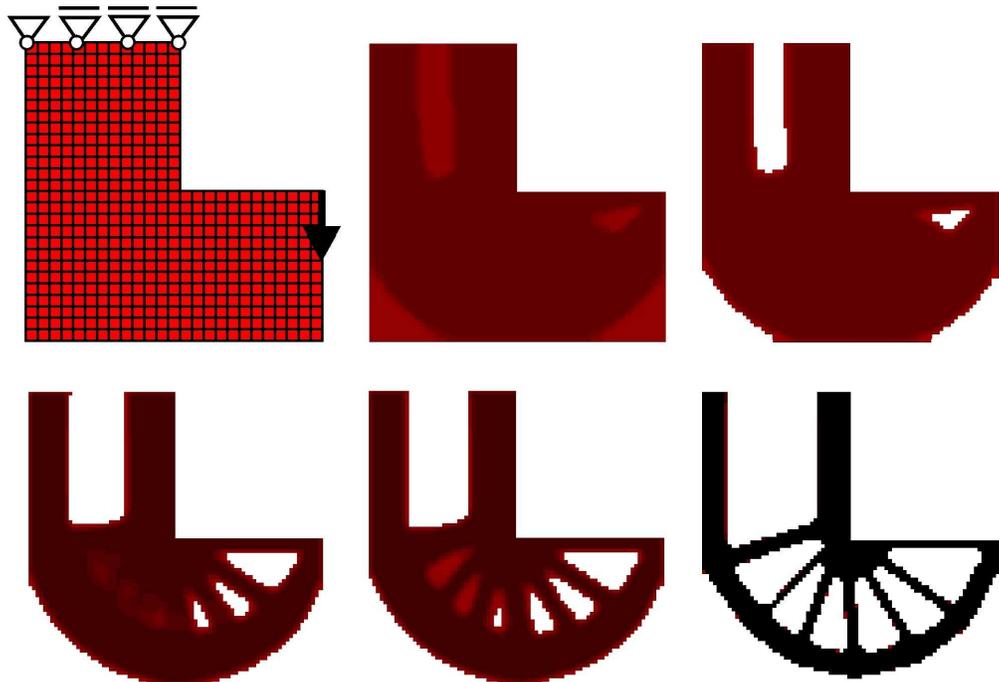


159x90mm (300 x 300 DPI)

Review Only

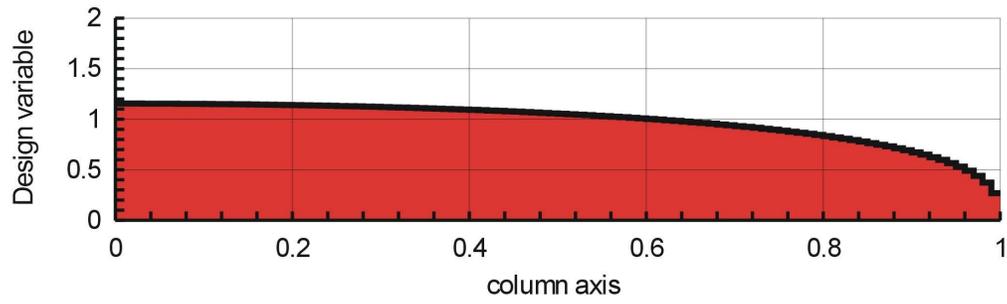
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167x115mm (300 x 300 DPI)

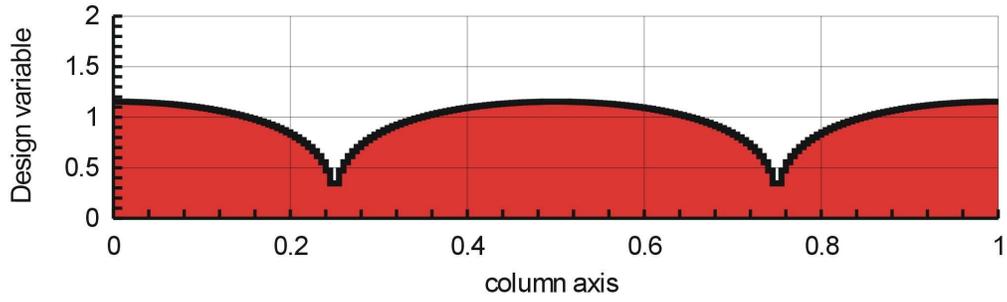
Review Only



171x50mm (300 x 300 DPI)

Peer Review Only

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171x50mm (300 x 300 DPI)

Peer Review Only