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For Peer Review

# Modeling the Dynamics of EU Economic Sentiment Indicators: An Interaction-Based Approach\*<sup>†</sup>

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## Abstract

This paper estimates a simple univariate model of expectation or opinion formation in continuous time adapting a ‘canonical’ stochastic model of collective opinion dynamics (Weidlich and Haag, 1983; Lux, 1995, 2009a). This framework is applied to a selected data set on survey-based expectations from the rich EU business and consumer survey database for twelve European countries. The model parameters are estimated through maximum likelihood and numerical solution of the transient probability density functions for the resulting stochastic process. The model’s success is assessed with respect to its out-of-sample forecasting performance relative to univariate time series models of the ARMA( $p, q$ ) and ARFIMA( $p, d, q$ ) varieties. These tests speak for a slight superiority of the canonical opinion dynamics model over the alternatives in the majority of cases.

**JEL classification:** E32, C83, C53.

**Keywords.** *Expectation formation, survey-based expectations, opinion dynamics, Fokker-Planck equation, forecasting.*

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## I. Introduction

It is widely believed that expectations play a major role in determining macroeconomic outcomes. Unfortunately, there is no consensus about the appropriate modeling of *expectation formation* (Pesaran, 1984, 1987; Evans and Honkapohja, 2001). Many theories and approaches have been suggested in the literature to formalize this important ingredient of economic models. Over the last decades, the rational expectations hypothesis has become the dominant paradigm of macroeconomic theory and survey data have been used to test for rational expectations of respondents, mostly with not too much support for rationality.<sup>1</sup> However, little has been done to test *alternative* theories of expectation formation using the vast amount of survey data on empirical expectations that are regularly published by private and academic institutes or governments in most developed countries.

Branch (2004), Carroll (2003) and Roberts (1998) are some of the rare examples that consider alternative theories of expectation formation that do not impose homogeneous rational expectations. While there is a scarcity of theoretical models of boundedly rational expectation formation, extant empirical research on survey-based expectations is quite rich. As an obvious research question a large number of papers investigates the predictive capacity of survey data for consumer spending or output (e.g., Lemmens *et al.*, 2005; Taylor and McNabb, 2007; Gelper *et al.*, 2007; Malgarini and Margani, 2007). Others seek for determinants of sentiment in macroeconomic or political data (Vuchelen, 1995) or even in compact measures of the generally optimistic or pessimistic disposition of a society (e.g. Zullo, 1991, who uses indices of positive and negative moods in pop songs and news articles).<sup>2</sup> The later studies are close to our approach in so far as they presume some kind of propagation of a dominant mood via direct or indirect interaction. Popular culture and mass media might, then, both reflect and reinforce overall mass-psychological trends in a society. Our goal here is to contribute to such a behavioral theory of sentiment formation by moving from a purely statistical analysis to an explicit modeling of the interaction effects in consumer or business surveys. Such an attempt at modeling and testing alternative hypotheses of opinion and expectation formation is a relatively recent strand of literature. We follow closely the recent work by Lux (2009a) and Franke (2008) who both estimate (with different econometric techniques) the parameters of a ‘canonical’ opinion dynamic model introduced below for a particular German business survey.

<sup>1</sup>Cf., Acemoglu and Scott, 1994; Delorme *et al.*, 2001, and the survey by Nardo, 2003.

<sup>2</sup>Other instances of empirical research utilizing EU survey-based expectations data include, e.g. Madsen (1996), who explores how producers form their inflation expectations, Cotsomitis and Kwan (2006), who examine the ability of consumer confidence indices to forecast household spending within a multicountry framework, and Drakos (2008), who investigates the predictive content of survey-based expectations on investment.

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8 This study provides an empirical assessment of this opinion formation  
9 model on the base of social interaction using the rich EU business and  
10 consumer survey database for twelve European countries as collected and  
11 released by the European Commission Directorate-General For Economic  
12 and Financial Affairs [henceforth, the Commission]. In particular, a simple  
13 univariate model of opinion or expectation formation in continuous time is  
14 postulated in the spirit of Weidlich and Haag (1983). Following the method-  
15 ology of Lux (2009a), based on previous contributions by Poulsen (1999) and  
16 Hurn *et al.* (2006), the model parameters are then estimated via approxi-  
17 mate maximum likelihood. Since no closed-form solution of the transient  
18 density of this model is available, our ML algorithm will be based on the  
19 numerical solution of the relevant Fokker-Planck equation (the partial dif-  
20 ference equation governing the dynamics of the pdf) using a finite difference  
21 approximation. The model's goodness-of-fit is checked with respect to its  
22 out-of-sample forecasting performance relative to standard univariate time  
23 series models of the ARMA( $p, q$ ) and ARFIMA( $p, d, q$ ) varieties. The results  
24 of these tests speak for the moderate superiority of the canonical continuous-  
25 time model over the alternatives.

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29 The paper proceeds as follows. Section II introduces briefly the content  
30 of the survey data under investigation. Section III sketches the theoretical  
31 framework suggested to model such data. Section IV provides the empirical  
32 analysis and investigates goodness-of-fit of the model vis-à-vis pure time se-  
33 ries models. Section V concludes and indicates further directions of research  
34 under the framework of this paper.

## 35 36 37 **II. Overview of the EU Business and Consumer Survey Data**

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39 National institutes in the EU Member States and candidate countries reg-  
40 ularly<sup>3</sup> conduct business and consumer surveys on behalf of the Joint Har-  
41 monised EU Programme of Business and Consumer Surveys [henceforth,  
42 BCS programme].<sup>4</sup> The collected data are compiled and published in the  
43 media by the Commission and are freely available. The purpose of the BCS  
44 programme is twofold. On the one hand, the database provides essential  
45 information for economic surveillance, short-term forecasting, economic re-  
46 search, and, in general, monitoring of economic developments at the member  
47 state, EU and Euro-area level. On the other hand, with these data, the Com-  
48 mission builds composite indicators to track cyclical movements in a specific  
49 sector or in the economy as a whole with the aim of detecting turning points  
50 in the economic cycle.

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54 The surveys are usually conducted in the following areas: manufacturing  
55 industry, construction, consumers, retail trade and services. The sample size

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57 <sup>3</sup>On monthly and quarterly bases.

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59 <sup>4</sup>The programme was set up in 1961 and is currently managed by the Commission.  
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of each survey varies across countries according to the heterogeneity of their economies, and is generally positively related to their respective population size.<sup>5</sup>

The responses from the surveys are aggregated in the form of “balances” or diffusion indices. Balances are constructed as the difference between the percentages of respondents giving positive and negative replies. ‘Neutral’ answers are ignored. For example, if among the total number of  $N^*$  respondents<sup>6</sup> (for some specific question) ‘positive’ (‘negative’) answers are given by  $N^+$  ( $N^-$ ) individuals, then the *balance*,  $B$ , is computed as follows

$$B = (N^+ - N^-)/N^*.$$

The *balance series*<sup>7</sup> constitute the major part of the output data of the BCS programme. These series are further used to build *composite indicators* like (a) various *confidence indicators* that provide information on economic developments in different sectors; (b) the *Economic Sentiment Indicator* [ESI], whose purpose is to track GDP growth at Member State, EU and euro-area level; and (c) the factor model-based *Business Climate Indicator* [BCI], which uses the results of the industry survey and is designed to assess cyclical developments in the euro area.

While many studies use selected entries of the EU survey data for single countries, surprisingly little work exists on cross-sections of data. An exception is the paper by Lemmens *et al.* (2005), exploring the predictive content of production surveys and Clar *et al.* (2007) who compare the forecasts from a variety of simple time series models for out-of-sample survey data themselves. The later study is very close to our approach here: Like Clar *et al.*, we are interested in forecasting sentiment itself on the base of past observations. We also use time series models (as a benchmark), but compare them to forecasts from a behavioral model of opinion dynamics that could, in principle, capture the intrinsic build-up of an optimistic or pessimistic mood in society (or in business).

From the vast amount of the available EU survey data the particular questions<sup>8</sup> chosen for the analysis in this paper are the following since they relate to future expectations:

<sup>5</sup>About 125 000 firms and almost 40 000 consumers are currently surveyed every month across the EU. Source: [http://ec.europa.eu/economy\\_finance/indicators/business\\_consumer\\_surveys/userguide\\_en.pdf](http://ec.europa.eu/economy_finance/indicators/business_consumer_surveys/userguide_en.pdf).

<sup>6</sup>Note that  $N^*$  includes the number of ‘neutral’ agents,  $N^\sim$ , i.e.  $N^* = N^+ + N^- + N^\sim$ .

<sup>7</sup>Balance series are usually referred to as “opinion index”, “climate index”, or “diffusion index” in the literature.

<sup>8</sup>In the monthly surveys, the total number of questions referring to future expectations is 18. The sectoral breakdown of these is as follows: 3 in industry, 3 in services, 6 in consumer, 4 in retail trade, and 2 in construction surveys. See Section 4 on the selection criterion for the countries. Since the survey in the service sector had only been launched during the 90’s, we did not include it in our selection.

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- Industry Survey, Q5: *How do you expect your production to develop over the next 3 months? It will...*  
+ increase            = remain unchanged            – decrease
  - Construction Survey, Q4: *How do you expect your firm's total employment to change over the next 3 months? It will...*  
+ increase            = remain unchanged            – decrease
  - Retail Trade Survey, Q4: *How do you expect your business activity (sales) to change over the next 3 months? It (They) will...*  
+ improve (increase)            = remain unchanged            – deteriorate (decrease)
  - Consumer Survey, Q4: *How do you expect the general economic situation in this country to develop over the next 3 months? It will...*<sup>9</sup>  
++ get a lot better            = stay the same            – get a little worse  
+ get a little better            N don't know            -- get a lot worse

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These questions refer to respondents' expectations on either the overall economic conditions (consumers) or their own economic situation (in the case of the questions selected from the industry, construction, and retail trade surveys). In contrast, a number of other questions are either backward looking (asking for past economic conditions or performance) or pertain to specific issues such as inflation or unemployment. We would expect responses to the more general questions to be more likely affected by social interactions.

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Figure 1 below provides information on the evolution of the balance series for the relevant questions in the case of Germany. The dynamics of various composite indicators are also superimposed on these series for purely illustrative purposes. It should be noted that the individual balances and the composite series are not directly comparable.

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The questions that the next sections attempt to answer are: How could we model expectation formation of agents faced with the above questions? Do agents independently form expectations or can we identify some sort of social interaction between respondents? Can we predict future expectations? How good are our forecasts? What could be done in order to improve predictions?

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<sup>9</sup>Note that in the case of the last question the balance is calculated as

$$B = [(N^{++} + 1/2N^+) - (1/2N^- + N^{--})] / N^*$$

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with the intuitive notation of  $N^{++}$  ( $N^{--}$ ) being the number of 'very optimistic' ('very pessimistic') respondents.



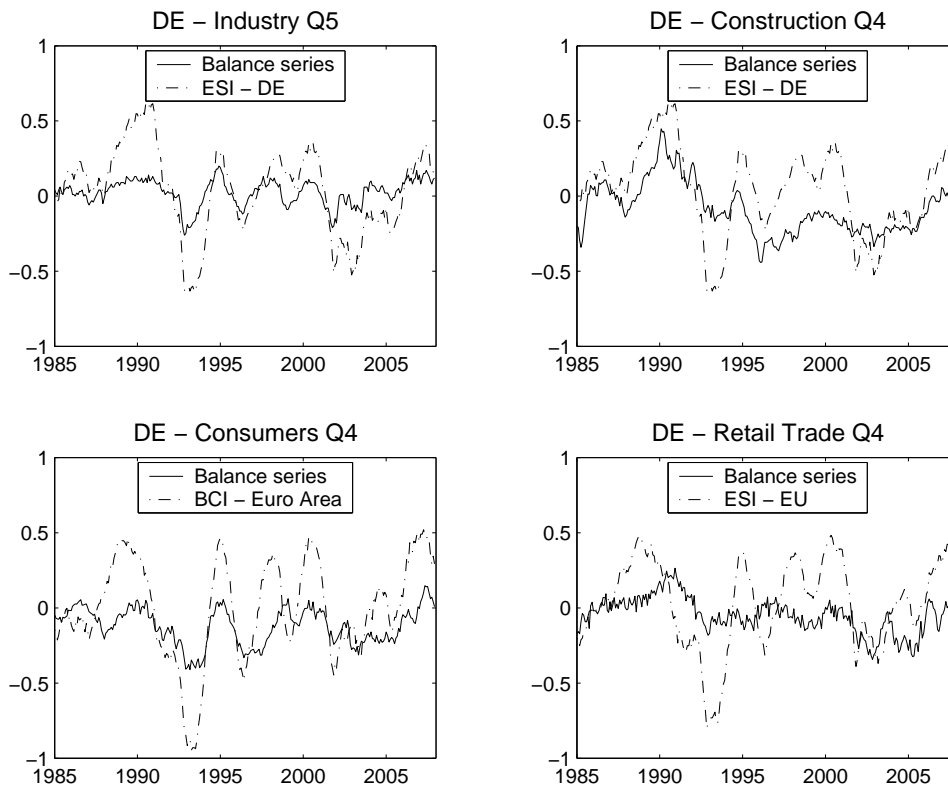


Figure 1: Balance series for Germany and composite indicators with appropriate scaling. These series are monthly observations over the 23 year period 01.1985 – 12.2007. Note that (a) ESI is calculated for the individual member countries as well as on the EU level, and (b) BCI is computed only for the Euro Area.

### III. A Framework for Collective Opinion Formation

As a model of expectation formation we adopt a stochastic framework along the lines of Weidlich and Haag (1983) and Lux (1995). The model is stylized and is based on a set of mass-statistical regularities governing respondents' switches between two possible opinions.

We begin our exposition by assuming that (A1) the total number of respondents is constant and, without loss of generality, is given by  $N^* = 2N$ , and that (A2) the respondents are allowed to have only two relevant opinions or expectations, denoted by + and -.

Let us denote by  $n_t^+$  and  $n_t^-$  the numbers of agents holding at time  $t$  the positive and negative expectations, respectively. We define next the *configuration*,  $n_t$ , as follows

$$n_t := (n_t^+ - n_t^-)/2, \quad (1)$$

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8  $-N \leq n_t \leq N$ , and introduce the notion of *aggregate* or *average expectations*  
9 as the ratio

$$10 \quad x_t := n_t/N, \quad (2)$$

11 with  $-1 \leq x_t \leq 1$ . Since all agents have equal weight in the population, we  
12 can interpret the state  $x_t = 0$  as representing the balance between overall  
13 optimism and pessimism, with the states  $x_t > 0$  and  $x_t < 0$  describing the  
14 cases of optimistic and pessimistic *majorities*, respectively. This opinion  
15 index is our proxy for the balance series (see remark below).  
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18 As time passes individual agents may change their opinions about the  
19 relevant questions. Thus they might switch from being optimistic to becoming  
20 pessimistic and *vice versa*. These switches are theoretically governed by  
21 the next two assumptions: (A3) The respondents have the same individual  
22 probabilities of reactions and interactions in the expectation formation  
23 process, and (A4) the probability that more than one agent will change the  
24 opinion per (infinitesimal) unit time period is zero.  
25

26 Let  $p(n; t)$  denote the probability that at time  $t$  the configuration is equal  
27 to  $n$ . Obviously, the condition  
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$$29 \quad \sum_{n=-N}^N p(n; t) = 1$$

30 holds for all  $t$ . Since changes of opinion of agents might happen at any  
31 point in time, we adopt a continuous-time framework for the dynamics of  
32 this opinion index. Let  $\omega(j \rightarrow i)$  denote the *transition rate* per unit time  
33 for a change of the configuration defined in Eq. (1) from state  $j$  to state  $i$ ,  
34 for all  $i, j \in [-N, N]$ . Then the equation of motion for  $p(n_t; t)$  is given by  
35 the following so called *Master equation*:<sup>10</sup>  
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$$40 \quad \frac{dp(i; t)}{dt} = \sum_j [\omega(j \rightarrow i)p(j; t) - \omega(i \rightarrow j)p(i; t)]. \quad (3)$$

41 The first term of the right hand side of (3) describes the probability flux  
42 from all states  $j$  into state  $i$  and the second term describes the probability  
43 flux from state  $i$  into all states  $j$ .  
44

45 Assumption (A4) ensures that it suffices to consider only the following  
46 transitions:  
47

$$48 \quad n \rightarrow (n + 1) \text{ and } n \rightarrow (n - 1).$$

49 We, therefore, define  
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$$51 \quad \omega_{\uparrow}(n) := \omega(n \rightarrow n + 1)$$

$$52 \quad \omega_{\downarrow}(n) := \omega(n \rightarrow n - 1)$$

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56 <sup>10</sup>The Master equation represents the general and exact system of equations tracking  
57 the flow of probabilities between states, see Weidlich and Haag (1983) or van Kampen  
58 (2007).  
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and set:

$$\omega(n \rightarrow n') = 0 \text{ for } n' \neq n \pm 1.$$

Under this new notation, Eq. (3) can be written as

$$\begin{aligned} \frac{dp(n; t)}{dt} = & \omega_{\uparrow}(n-1)p(n-1; t) - \omega_{\uparrow}(n)p(n; t) \\ & + \omega_{\downarrow}(n+1)p(n+1; t) - \omega_{\downarrow}(n)p(n; t). \end{aligned} \quad (4)$$

It can be shown that an equivalent description can be given for the opinion index:

$$\begin{aligned} \frac{dP(x; t)}{dt} = & w_{\uparrow} \left( x - \frac{1}{N} \right) P \left( x - \frac{1}{N}; t \right) - w_{\uparrow}(x) P(x; t) \\ & + w_{\downarrow} \left( x + \frac{1}{N} \right) P \left( x + \frac{1}{N}; t \right) - w_{\downarrow}(x) P(x; t), \end{aligned} \quad (5)$$

where  $P(x, t)$  denotes the probability that at time  $t$  the configuration is equal to  $x$ .<sup>11</sup>

The official statistics provides us only with the following data: the balance series at time  $t$ ,  $B_t$ , and the overall number of respondents,  $N^* = 2N$ . Note that the definitions of our opinion index and that of the diffusion indices of the BCS differ slightly due to the possibility of a ‘neutral’ opinion in the surveys. However, we can bridge this gap in a relatively straightforward way: Since in the theoretical model the following identity must hold,

$$n_t^+ + n_t^- = N^*,$$

we might use  $n_t^+$  as a proxy for  $N_t^+ + \frac{1}{2}N_t^{\sim}$  and, similarly,  $n_t^-$  as a proxy for  $N_t^- + \frac{1}{2}N_t^{\sim}$ . Under this assumption,

1.  $n_t^+ + n_t^- = N_t^+ + \frac{1}{2}N_t^{\sim} + N_t^- + \frac{1}{2}N_t^{\sim} = N^*$  and
2.  $x_t = \frac{n_t^+ - n_t^-}{N^*} = \frac{N_t^+ - N_t^-}{N^*} = B_t$ .

Thus, if we assign half of the ‘neutral’ agents to the ‘optimistic’ and ‘pessimistic’ groups, respectively, then the information given by the diffusion index  $B_t$  is exactly represented by the theoretical index  $x_t$ .<sup>12</sup>

<sup>11</sup>Note the notational change:  $p(n, t) \rightarrow P(x, t)$  and  $\omega(n) \rightarrow w(x)$ .

<sup>12</sup>It might, however, be mentioned that we could also design a slightly modified framework allowing for a neutral disposition along with the “+” and “-” choices. We leave this for future research.

## Behavioral Assumptions

By treating  $x$  as a continuous variable and using a Taylor series expansion we can approximate the Master equation of Eq. (5) by the so-called *Fokker-Planck equation*:<sup>13</sup>

$$\frac{\partial P(x;t)}{\partial t} = -\frac{\partial}{\partial x} \{A(x)P(x;t)\} + \frac{1}{2} \frac{\partial^2}{\partial x^2} \{D(x)P(x;t)\}. \quad (6)$$

where

$$A(x) = \frac{1}{N} [w_{\uparrow}(x) - w_{\downarrow}(x)],$$

$$D(x) = \frac{1}{N^2} [w_{\uparrow}(x) + w_{\downarrow}(x)].$$

$A(x)$  and  $D(x)$  are the drift and diffusion terms that govern the dynamics of the first and second moments.

As a next step we specify the transition rates. Utilizing Poisson probabilities in continuous time to jump from the “+” to the “−” group or *vice versa* within the next instant, a simple stochastic process of individual moves between groups is obtained. Here we follow the earlier literature and assume the following ‘canonical’ representation (see Weidlich and Haag (1983), Lux (1995, 1997, 2009a)):

$$w_{\uparrow}(x) = \frac{n_-}{2N} v \exp \{U(x)\} = (1-x)v \exp \{U(x)\},$$

$$w_{\downarrow}(x) = \frac{n_+}{2N} v \exp \{-U(x)\} = (1+x)v \exp \{-U(x)\} \quad (7)$$

with

$$U(x) = \alpha_0 + \alpha_1 x. \quad (8)$$

In Eq. (7),  $\frac{n_-}{2N}$  and  $\frac{n_+}{2N}$  are the fractions of currently pessimistic or optimistic respondents who constitute the pool of those who could potentially switch to the “+” or “−” opinion, respectively. The remainder of the expression,  $v \exp \{U(x)\}$  or  $v \exp \{-U(x)\}$  determines the switching rate per individual. The function  $U(\cdot)$  might be labeled the ‘forcing function’ for transitions.<sup>14</sup> We have the following model parameters:  $v$  determines the frequency (time scale) of moves between groups,  $\alpha_0$  generates a *bias* towards the choice of “+” (“−”) opinions if positive (negative), and  $\alpha_1$  formalizes

<sup>13</sup>See Weidlich and Haag (1983), Lux (1997, 2009a), Gardiner (2004), and van Kampen (2007) for more details.

<sup>14</sup>Our function  $U(\cdot)$  resembles the utility function within a discrete choice framework (cf. Brock and Durlauf, 2001). However, there is no clear utility component to survey responses so that we prefer the notion of a ‘forcing function’. The major difference of our framework to studies of discrete choice problems with social interaction (DSCI) is that we investigate a *dynamic* model of aggregate opinion formation while DSCI models are typically applied to cross-sections of micro data.

the *degree of group pressure* if it is positive (if negative, it would rather imply a tendency towards *non-conformity*).

Let  $\theta$  denote the parameter vector,  $\theta = (v, \alpha_0, \alpha_1)'$ . Then, highlighting the dependence on  $\theta$ , the stochastic dynamics of our opinion model is finally specified by Eq. (6) with<sup>15</sup>

$$\begin{aligned} A(x; \theta) &= v(1-x)e^{\alpha_0 + \alpha_1 x} - v(1+x)e^{-\alpha_0 - \alpha_1 x}, \\ D(x; \theta) &= [v(1-x)e^{\alpha_0 + \alpha_1 x} + v(1+x)e^{-\alpha_0 - \alpha_1 x}]/N. \end{aligned}$$

The Fokker-Planck equation (6) corresponds to the representation of the opinion dynamics,  $x_t$ , as a solution to the stochastic differential equation (SDE)

$$dx_t = A(x_t; \theta)dt + \sqrt{D(x_t; \theta)}dW_t, \quad (9)$$

where  $W_t$  denotes the standard Wiener process.<sup>16</sup>

### Model Properties

The limiting behavior of our stochastic model can be characterized by its *unconditional density* which can be obtained by setting the left hand side of Eq. (6) equal to zero,

$$\frac{\partial P(x; t)}{\partial t} = 0. \quad (10)$$

We do not reproduce the closed-form solution of the stationary density here, but summarize its important properties:<sup>17</sup>

1. For  $\alpha_1 \leq 1$ , the stationary distribution of the process  $x_t$  is characterized by a unique maximum (mode). If  $\alpha_0 = 0$ , this maximum is located at  $x^* = 0$ . It shifts to the right (left) for  $\alpha_0 > 0$  ( $< 0$ ).
2. For  $\alpha_1 > 1$  and  $\alpha_0$  not too large, the stationary distribution has two maxima (two modes)  $x_+ > 0$  and  $x_- < 0$ . If  $\alpha_0 = 0$ , the bimodal distribution is symmetric around 0. It becomes asymmetric if  $\alpha_0 \neq 0$  with right-hand (left-hand) skewness and more concentration of probability mass in the right (left) maximum if  $\alpha_0 > 0$  ( $< 0$ ) holds.

<sup>15</sup>Using the hyperbolic trigonometric functions, the drift and diffusion function can also be written as:

$$\begin{aligned} A(x; \theta) &= 2v \cosh(\alpha_0 + \alpha_1 x) \{ \tanh(\alpha_0 + \alpha_1 x) - x \}, \\ D(x; \theta) &= 2v \cosh(\alpha_0 + \alpha_1 x) \{ 1 - x \tanh(\alpha_0 + \alpha_1 x) \} / N. \end{aligned}$$

<sup>16</sup>Note that this is only an approximation to our population dynamics in that the microscopic sources of randomness have been proxied by a macroscopic noise factor  $W_t$ . See Gardiner (2004, Ch. 3) and the appendix of Lux (2009a) for technical details on the diffusion approximation to Markov jump processes.

<sup>17</sup>Cf. Weidlich and Haag (1983), Lux (2009a).

3. If  $|\alpha_0|$  becomes very large, the smaller mode vanishes and the stationary distribution becomes uni-modal again. This happens if  $|\alpha_0|$  increases beyond the bifurcation value  $\bar{\alpha}_0$  given by

$$\cosh^2(\bar{\alpha}_0 - \sqrt{\alpha_1(\alpha_1 - 1)}) = \alpha_1 \quad (11)$$

where  $\cosh(\cdot)$  denotes the hyperbolic cosine,

$$\cosh(x) = (\exp(x) + \exp(-x))/2.$$

These properties are illustrated in Figure 2 below. For more details we refer to Lux (2009a).

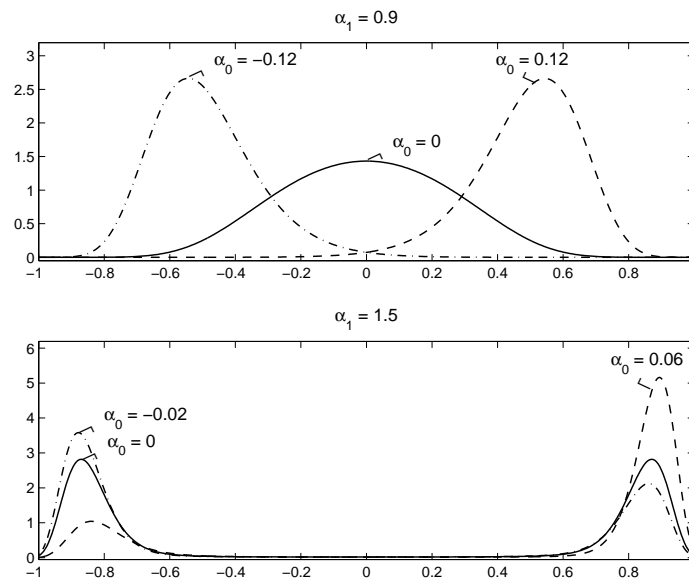


Figure 2: Equilibrium densities for various parameters.

### Estimation

The temporal development of the aggregate dynamics of our model of opinion formation is characterized by the diffusion approximation, Eq. (9), and its associated Fokker-Planck equation, Eq. (6). The latter tracks the development of the transitory density  $P(x; t)$  conditional on some initial value and can, therefore, be used for a likelihood-based estimation approach.

We have available discrete observations of the balance series in order to estimate the parameters of our continuous-time process of opinion formation. For a sample of observations  $x_0, \dots, x_T$  we can estimate these parameters

most efficiently via maximum likelihood. The log likelihood of our sample of observations, then, amounts to

$$\log P_0(x_0|\theta) + \sum_{s=0}^{T-1} \log P(x_{s+1}|x_s, \theta)$$

Note that no closed-form solutions are available for the conditional probabilities  $P(x_{s+1}|x_s, \theta)$ . However, they can be obtained by numerical solution of the Fokker-Planck equation over a unit time interval taking  $x_s$  as the initial condition. Since we do not have a previous observation for  $x_0$ , we have to use the unconditional probability  $P_0(x_0|\theta)$  to evaluate this component (however, since its influence is negligible, we will simply skip this observation in our empirical applications).

For the numerical approximation of the Fokker-Planck equation we follow the methodology developed by Lux (2009a) and suggested earlier by Poulsen (1999) and Hurn *et al.* (2006) in a different context. First, the Fokker-Planck equation (6) is solved numerically via a Crank-Nicolson finite difference scheme. Then, the log-likelihood function is evaluated for the in-sample observations and is numerically maximized with respect to the unknown parameters. More details on the numerical aspects can be found in Lux (2009a). Computations have been performed in GAUSS. The results are summarized in Tables 3 – 14.

#### IV. Empirical Results

The main objective of this paper is to estimate the parameters of our behavioral model for the selected balance series and assess the performance of this model as a hypothesized data-generating process for the BCS sentiment data. Since the EU Business and Consumer Survey Data is huge we have chosen only the four questions described in Section II for only those twelve countries for which the series were available from 1985. Thus, for each single question-country pair we have a sample of 276 monthly observations. In order to test the forecasting power of the model we have chosen the first 192 observations as our in-sample, covering the 16 year period 01.1985 – 12.2000. The number of out-of-sample observations is 84, covering the next 7 year period 01.2001 – 12.2007. These data series are seasonally adjusted by the provider.

Previous experience indicates the need to consider different versions of the basic opinion dynamics model (9) (see Lux (2009a)). Therefore the following set of four models have been estimated:

**M1:** The parameter vector to be estimated is  $\theta = (v, \alpha_0, \alpha_1)'$ , with the number of respondents fixed at the ‘official’ number given in the doc-

umentation of the BCS programs.<sup>18</sup> Model 1 is exactly the full model specified above as Eq. (9) and will henceforth be referred to as the *canonical model*.

**M2:**  $N$  is fixed and is given as in Model 1. The parameter vector to be estimated is  $\theta = (v, \alpha_1)'$ . Here we neglect the bias parameter  $\alpha_0$ . The reason is that for relatively weak interaction ( $\alpha_1$  small) with small fluctuations of  $x_t$ , approximate collinearity between  $\alpha_0$  and  $\alpha_1$  (and  $v$  and  $N$ ) could impede our estimation.

**M3:** Under Model 3  $N$  is no longer fixed. The parameter vector to be estimated is thus  $\theta = (v, \alpha_0, \alpha_1, N)'$ . Here we let the in-sample data provide the information about the implied ‘effective’ number of respondents.<sup>19</sup>

**M4:** As in Model 3,  $N$  is also not fixed here. The parameter vector to be estimated is  $\theta = (v, \alpha_1, N)'$ . We neglect the effect of the bias parameter  $\alpha_0$ .

The total number of models that we estimate thus amounts to 144.

### Goodness-of-fit

The goodness-of-fit of all four models is checked with respect to their out-of-sample forecasting performance relative to a benchmark. In particular, one month out-of-sample forecasts are constructed for all models and two types of forecasting errors are computed: *root mean-squared errors* (RMSE) and *mean absolute errors* (MAE). The same quantities are calculated for the forecasts based on the univariate ARMA( $p, q$ ) and ARFIMA ( $p, d, q$ ) time series models, which serve as our benchmarks.

### Forecasting

The notion of a prediction derived from a model like ours needs special attention. Taking the mathematical expectation of  $x_{t+i}$  conditional on time  $t$  information would certainly be an obvious choice for a uni-modal distribution, but in the multi-modal case (e.g. in the case representing the lower panel of Fig. 2) there also exist plausible alternative predictors. Note that

<sup>18</sup>This information can be obtained from the EU Business and Consumer Surveys database. It actually varies widely across countries and sections.

<sup>19</sup>The idea is that despite the inclusion of a social interaction term our model might not capture all correlation between respondents. For example, there might be groups that always switch simultaneously which would, indeed, reduce the number of effectively independent agents. Of course, the officially reported number should be an upper boundary to the ‘effective’ number. However, in quite a few cases we obtained higher estimates for  $N$  than this boundary. In the tables we report these estimates without applying the corresponding upper boundaries.



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8 for a bi-modal symmetric density like the ones in Fig. 2 the point estimate  
9 corresponding to the mean would coincide with the least likely value (the  
10 minimum of the density). The most likely values are the two modes that  
11 are quite different from the mean prediction. Taking these considerations  
12 into account, we also use as an alternative predictor besides the mean the  
13 value of the mode nearest to the last observation at time  $t$ . This choice is  
14 determined by the time dependency of our stochastic process: Because of  
15 the inertia of the opinion dynamics, the process will with a high probability  
16 remain within its current mode for some time before stochastic fluctuations  
17 will trigger a switch to the alternative mode. This paper, therefore, consid-  
18 ers two different one-month-ahead forecasts for the models M1–M4: *expected*  
19 *value* and *nearest maximum* of the predictive density function. The term  
20 ‘expected’ in the figures and tables below stands for the expected value of the  
21 opinion index  $x_{t+1}$  at one-month horizon conditional on its value one month  
22 earlier,  $x_t$ . The needed predictive density is again obtained via numerical  
23 solution of the Fokker-Planck equation with the previously estimated pa-  
24 rameter vector  $\hat{\theta}$ . A similar procedure applies to the computation of the  
25 ‘nearest’ forecast, which is the nearest maximum of the predictive density,  
26 and therefore represents the most likely mode of the opinion index at some  
27 future date. These ‘expected’ and ‘nearest’ forecasts are calculated for the  
28 out-of-the-sample data of respective balance series.  
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33 In order to set benchmarks, we have also estimated the best ARMA( $p, q$ )  
34 and ARFIMA( $p, d, q$ ) in-sample. For ARMA we have set  $p, q \leq 5$ , for  
35 ARFIMA,  $p, q \leq 1$  (as the longer lags should be captured by the parameter  
36 of fractional differentiation).<sup>20</sup> From the range of the ARMA and ARFIMA  
37 models within this set, the one that minimizes the Akaike information cri-  
38 terium is chosen for forecasting. Then, based on the fitted models, out-of-  
39 sample one-month-ahead forecasts have been computed. The results of our  
40 extensive estimation study of the out-of-sample forecasting performance of  
41 the competing models are summarized in Tables 1 and 2. Complete results  
42 are available in Tables 3 through 14. Tables 1 and 2 report the numbers of  
43 winners across all questions and countries and also report the  $p$ -values of the  
44 sign-tests for the null hypothesis that the predictive accuracy of the opinion  
45 models is not different from those of the time series models in cross-section.  
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47

48 Empirical results show the following regularities:  
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- 50 1. ARMA forecasting accuracy is usually outperformed by both, the  
51 predictive power of ARFIMA and that of models M1–M4 (both for  
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53 <sup>20</sup>ARMA models have been estimated via standard maximum likelihood, while for  
54 ARFIMA models first the parameter of fractional differentiation,  $d$ , has been estimated  
55 from the frequency spectrum and the remaining parameters have been estimated via max-  
56 imum likelihood. This approach proved to be more robust than full maximum likelihood  
57 in previous studies with small samples (cf. Lux and Kaizoji, 2007).  
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Table 1: Summary of the forecasting performances, RMSE criterion.

Cases	Number of Winnings		Opinion Models (OM)	H <sub>0</sub> : OM ≈ TS	
	Time Series Models (TS)			<i>p</i> -values for sign-tests	
	ARMA	ARFIMA		one-sided	two-sided
			Nearest Mode (NM)		
M1	2	12	22	0.1215	0.2430
M2	3	14	19	0.4340	0.8679
M3	2	12	22	0.1215	0.2430
M4	3	14	19	0.4340	0.8679
			Expected Value (EV)		
M1	3	10	23	0.0662	0.1325
M2	3	14	19	0.4340	0.8679
M3	2	10	24	0.0326	0.0652
M4	3	14	19	0.4340	0.8679
			Best of NM & EV		
M1	2	10	24	0.0326	0.0652
M2	3	13	20	0.3089	0.6177
M3	2	9	25	0.0144	0.0288
M4	3	13	20	0.3089	0.6177
			Best of NM & EV over all M1–M4		
	2	7	27	0.0020	0.0039

Notes: “H<sub>0</sub>: OM ≈ TS” stands for the null hypothesis of equal predictive accuracy of opinion and time series models. The alternative hypotheses are (i) H<sub>1</sub>: OM is better than TS (or worse, depending on the context) and (ii) H<sub>1</sub>: OM and TS have different forecasting powers, for one-sided and two-sided sign-tests, respectively.

the nearest mode and expected value forecasts) with respect to both, RMSE and MAE criteria (see Tables 1 and 2).

- Another general observation is that the expected value (EV) forecasts of the opinion models typically record more winning cases against the time series alternatives than the nearest mode (NM) forecasts, irrespective of the error criteria (RMSE/MAE). Nevertheless, the EVs do not always outperform NMs in all instances as reflected by the higher numbers of winners of the best of EVs and NMs compared to the ARMA/ARFIMA forecasts (see lower panels of Tables 1 and 2).
- When comparing the canonical M1 with the time series models under the RMSE criterion (see Table 1), ARMA wins 3, ARFIMA 10, and M1 23 of the 36 cases with respect to the expected value forecasts in

Table 2: Summary of the forecasting performances, MAE criterion.

Cases	Number of Winnings		Opinion Models (OM)	H <sub>0</sub> : OM $\approx$ TS	
	Time Series Models (TS)	ARFIMA		$p$ -values for sign-tests	
	ARMA			one-sided	two-sided
			<u>Nearest Mode (NM)</u>		
M1	3	15	18	0.5660	1.0000
M2	3	17	16	0.3089	0.6177
M3	2	15	19	0.4340	0.8679
M4	3	17	16	0.3089	0.6177
			<u>Expected Value (EV)</u>		
M1	3	13	20	0.3089	0.6177
M2	3	16	17	0.4340	0.8679
M3	2	15	19	0.4340	0.8679
M4	3	16	17	0.4340	0.8679
			<u>Best of NM &amp; EV</u>		
M1	3	13	20	0.3089	0.6177
M2	3	15	18	0.5660	1.0000
M3	2	15	19	0.4340	0.8679
M4	3	16	17	0.4340	0.8679
			<u>Best of NM &amp; EV</u>		
			<u>over all M1–M4</u>		
	2	12	22	0.1215	0.2430

Notes: “H<sub>0</sub>: OM  $\approx$  TS” stands for the null hypothesis of equal predictive accuracies of the opinion and the time series models. The alternative hypotheses are (i) H<sub>1</sub>: OM is better than TS (or worse, depending on the context) and (ii) H<sub>1</sub>: OM and TS have different forecasting powers, for one-sided and two-sided sign-tests, respectively.

cross-section. M3 performs slightly better in the same setting. Its 24 winnings out of 36 has a  $p$ -value of 6.52% in the two-sided sign-test with the alternative hypothesis that the predictive accuracies of the opinion and the time series models are different, but it has a  $p$ -value of 3.26% in the one-sided test using the alternative hypothesis that the opinion model forecasts are better than those of the competitors. These results speak for the statistically significant cross-sectional superiority of the opinion model’s forecasting power over its alternatives.

- The MAE criterion does not provide sufficient evidence for such a statistically significant superior performance of the opinion models’ forecasts in cross-section. Sign-tests, both one-sided and two-sided, cannot reject the null hypothesis of equal predictive accuracy of the

opinion and the time series models at the conventional significance level. Still, our study clearly attributes higher winnings to the opinion models in the forecasting race under the MAE criterion too.

5. Considering the full set of our interaction-based models we find better fits of at least one specification with respect to RMSE (resp., MAE) than the best ARMA/ARFIMA performance in 27 (resp., 22) cases out of 36.
6. Considering the range of our model specifications M1 to M4, we find that M1 and M3 perform better than their counterparts M2/M4. It, thus, seems that the bias term  $\alpha_0$  is a necessary ingredient to cover a positive or negative prevailing tendency in our series. In contrast, the gain in adding the overall number of respondents,  $N^*$ , as an endogenous parameter is more negligible. This lack of considerable improvements of the in-sample fits is surprising since allowing for  $N^*$ , the ‘effective’ number of participants, as a free parameter provided for a significant improvement of goodness-of-fit in the case of a German sentiment index (Lux, 2009a).<sup>21</sup>
7. We have also conducted tests for equal predictive accuracy for individual series. As it turned out, the Diebold-Mariano test<sup>22</sup> could not reject the null hypothesis of equal predictive accuracy at the 5% level between (a) the expected value and ARFIMA forecasts in 90.3% of cases, (b) the nearest value and ARFIMA forecasts in 88.9% cases, and (c) the expected value and the nearest value forecasts in 88.2% of cases, when the total number of 144 forecasting error measurements is taken into account.<sup>23</sup> To put it in a nutshell, the forecasting performance, as measured by the Diebold-Mariano test, is typically not significantly better for the opinion model than for the time series models. This outcome can be contrasted with the observations from point 3 above.
8. For the parameters of the opinion model, we typically find the entry for the intensity of interaction,  $\alpha_1$ , to be in the close vicinity of its bifurcation value 1 (at which the behavior of the system switches from uni-modal to bi-modal) for Model 1. However, allowing for endogeneity of  $N$ , this value mostly turns out to be lower. Similar findings

<sup>21</sup>Note also that often large standard errors are associated with the estimates of  $N$ . This happens mostly if the estimated values of  $\alpha_1$  in Models 3 and 4 are much lower than 1. As has been demonstrated in Lux (2009a), in this case the relationship between parameters  $v$  and  $N$  is close to collinear so that one of both could be fixed without much deterioration of the likelihood.

<sup>22</sup>See Diebold and Mariano (1995).

<sup>23</sup>We considered only one-month-ahead forecasts.

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8 have been reported in Lux (2009a). It appears that there is a trade-off  
9 between the number of independent agents and their interaction  
10 intensity: If we insist on  $N$  in accordance with the official numbers,  
11 the model can only reproduce the fluctuations of the index with  $\alpha_1$   
12 close to its critical value. If we allow for a lower number of “effectively  
13 independent” agents, lower interaction intensity will be sufficient in  
14 the best fit to our model.<sup>24</sup>  
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- 17 9. In contrast to our pilot application of the present estimation method-  
18 ology in Lux (2009a), different specifications of the model (M1–M4)  
19 make little difference to its in-sample and out-of-sample fit. In partic-  
20 ular, we found little improvement through adding a ‘momentum’ effect  
21 to the opinion dynamics. In order to account for the momentum effects  
22 in our setting we have simply expanded the forcing function according  
23 to the following two specifications, **M5**:  $U(x_t) = \alpha_0 + \alpha_1 x_t + \alpha_2 \Delta x_t$ ,  
24 and **M6**:  $U(x_t) = \alpha_1 x_t + \alpha_2 \Delta x_t$ . Here  $\Delta x_t$  denotes the difference be-  
25 tween the time  $t$  and the time  $(t - 1)$  observations of the index which  
26 stays fixed over the time interval  $[t, t + 1)$ . The idea is that respondents  
27 might react not only to the net influence of their environment but also  
28 be particularly sensitive to changes of the index itself. Note that in  
29 the specification  $U(x_t) = \alpha_1 x_t + \alpha_2 \Delta y_t$ , where  $y_t$  tracks some changes  
30 in the fundamentals, the term  $\alpha_2 \Delta y_t$  can be interpreted as the *time*  
31 *varying trend* or *bias* fixed over the period  $[t, t + 1)$ .  
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36 Combining the results from the sign-tests and the Diabold-Mariano tests  
37 for the empirical forecasting performance we conclude: While the opinion  
38 model does typically not generate forecasts that are clearly superior to those  
39 of its rivals, on the whole it produces more winning cases than the alternative  
40 time series models. Therefore, although we would be careful not to draw too  
41 strong conclusions from Tables 1 and 2 because multiple comparisons have  
42 been conducted, we would like to characterize these results as encouraging.<sup>25</sup>  
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46 <sup>24</sup>Note that using the large official numbers of respondents would lead to very low  
47 predicted volatility due to the law of large numbers. This can to a certain degree be  
48 overcome by high sensitivity of the system to changes. This is what characterizes the  
49 neighborhood of  $\alpha_1$  while moving away from this benchmark in both directions leads to  
50 more persistent macroscopic dynamics.

51 <sup>25</sup>As an additional test for predictive accuracy we have used the Kullback–Leibler in-  
52 formation criterion (following the methodology of Mitchell and Hall, 2005) to compare  
53 out-of-sample predictive densities of all models considered in this paper. The overall con-  
54 clusion is that, while ARMA density forecasts are quite poor, in more than fifty percent  
55 of the cases the hypothesis of equal predictive accuracy between opinion and ARFIMA  
56 models can be rejected at five percent level. The tests again point towards the superiority  
57 of the opinion model density forecasts.  
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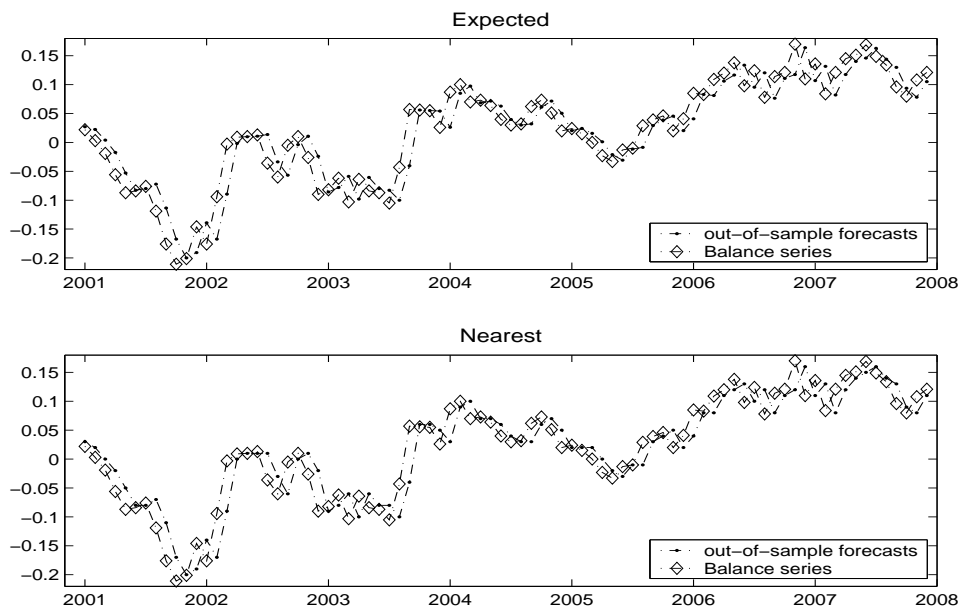


Figure 3: One-month-ahead forecasting performance of the canonical model for the German industry survey, question 5. ‘Expected’ represents the prediction based on conditional expected value of the predictive density. ‘Nearest’ corresponds to the nearest maximum of the predictive density.

**An Example.** Next we visualize the out-of-sample forecasting performances of competing models. The data come from the German industry survey Q5 for the (out-of-sample) period 01.2001 – 12.2007. On the one hand, we consider the canonical model 1 with two potential one-month-ahead forecasts: *expected value* and *nearest maximum* of the predictive density function (see Figure 3). On the other hand, the best ARMA and ARFIMA forecasts are presented in Figure 4. For this case the preferred time series models were ARMA(2,1) and ARFIMA(1, $\hat{d}$ ,1) models with  $\hat{d} = 0.7202$ .<sup>26</sup>

This example highlights the overall impression that ARMA models perform poorly for the balance series, whereas both the canonical model and the ARFIMA model track the dynamics of the future values in this case quite well. Both, expected and nearest forecasts show better performance than ARFIMA with respect to RMSE and MAE (see details in Table 5).

In order to illustrate what the potential added explanatory power of our opinion model could be, we exhibit some more details in the case of German consumers. This is particularly interesting because the fitted canonical model displays the possibilities for *phase transitions*. In particular, both

<sup>26</sup>Note that the value of  $\hat{d}$  outside of the interval  $(-0.5, 0.5)$  indicates that the series might be non-stationary (see Brockwell and Davis, 1991).

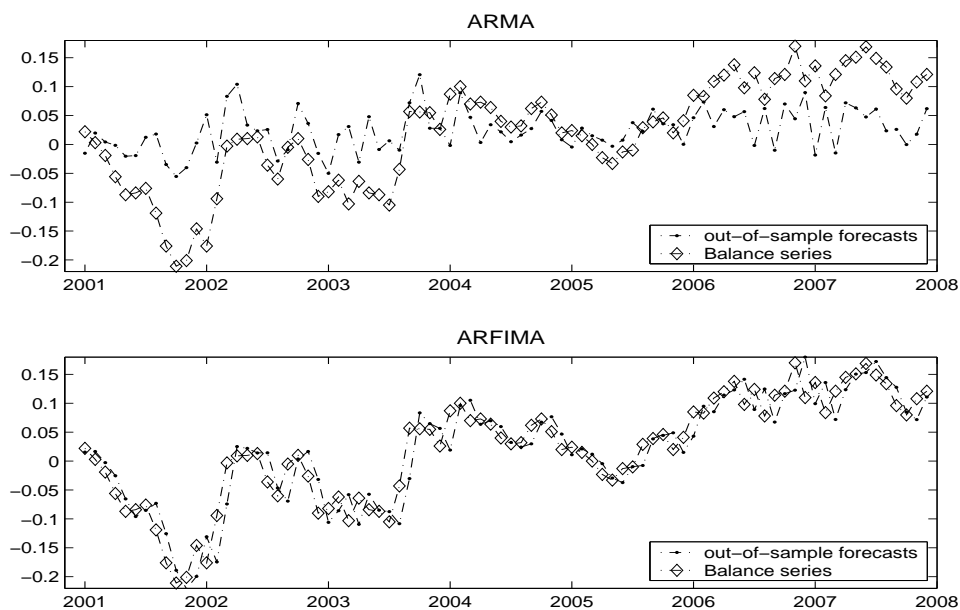


Figure 4: One-month-ahead forecasting performance of the  $ARMA(2,1)$  and  $ARFIMA(1,\hat{d},1)$  models,  $\hat{d} = 0.7202$  for the German industry survey, Q5.

M1 and M2 have  $\alpha_1$  parameters higher than unity. This setting corresponds to the bi-modal equilibrium distribution of consumer opinions, switching from optimistic to pessimistic long-run equilibria and *vice versa*. In Figure 5 below we have displayed in-sample and out-of-sample observations for the series. We have also superimposed two standard deviation bounds on the evolution of the predictive density conditional on the very last in-sample observation,  $x_0 = -0.03$ . As parameters we have chosen the simple averages of the two models, M1 and M2:  $v = .5475$ ,  $\alpha_0 = -0.0006$ ,  $\alpha_1 = 1.0109$ ,  $N = 1000$ . As can be seen from the graph, we are able to track the global maximum of the predictive density. This is represented by the dashed black curve in the out-of-sample interval starting in 2001. The evolution of the mean is given by the green line and it stays closely in the neighborhood of the initial condition,  $x_0 = -0.03$ . The evolution of the global maximum, on the contrary, diverges from the mean downwards a strongly negative configuration. Why is the global maximum not capable of pulling the mean with it? What is the force that keeps the dynamics of the mean almost unchanged? The answers to these questions are provided by Figure 6. It displays the complete evolution of the predictive density. We see how a second local maximum develops in the positive quadrant. This is exactly the reason for the observed dynamics of the mean that roughly corresponds to the average between both modes.

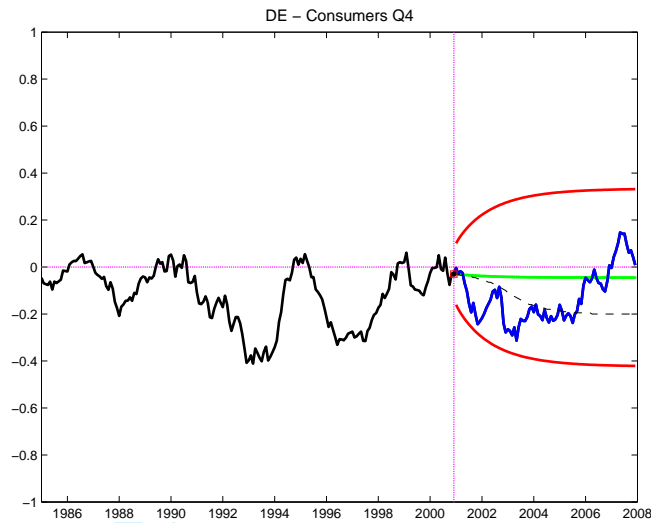


Figure 5: The evolution of the moments of the predictive density conditional on the last in-sample observation,  $x_0 = -0.03$ , see Table 3, Consumers. The parameters are the simple averages of the two models,  $M1$  and  $M2$ :  $v = .5475$ ,  $\alpha_0 = -0.0006$ ,  $\alpha_1 = 1.0109$ ,  $N = 1000$ . The red bands represent the two standard deviation bounds. The green line is the (conditional) mean process. The dashed black line denotes the evolution of the global maximum.

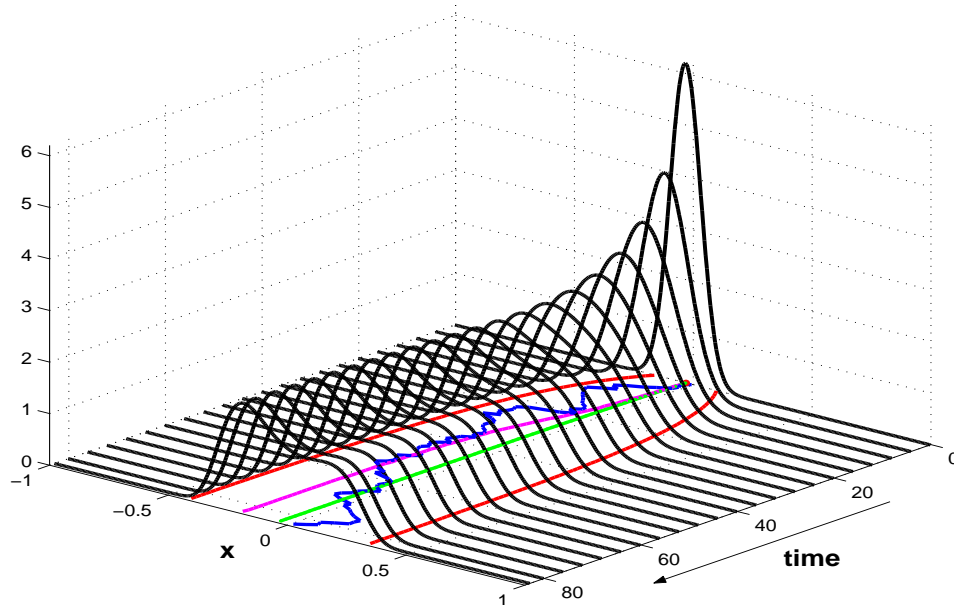


Figure 6: The evolution of the predictive density conditional on the last in-sample observation,  $x_0 = -0.03$ , see Table 3, Consumers. The parameters are the simple averages of the two models,  $M1$  and  $M2$ :  $v = .5475$ ,  $\alpha_0 = -0.0006$ ,  $\alpha_1 = 1.0109$ ,  $N = 1000$ .



## V. Conclusions

This paper has explored the explanatory and predictive power of a non-rational model of opinion formation among interacting agents for European business and consumer sentiment data. Applying the canonical model of opinion formation by Weidlich and Haag (1983) to four selected indices across 12 core countries of the European Union, we found the following:

With respect to its forecasting performance out-of-sample, the endogenous opinion model typically did better than an ARMA model. Compared to the more persistent ARFIMA class, predictive power was mostly not significantly different for single series (as judged by the Diebold-Mariano test). However, for the cross-section of data as a whole, we find a dominance of the opinion model in about two thirds of all cases (although its advantage over ARFIMA might be small).

It is worthwhile to note that it is not really clear whether we could expect more predictive power if this model were the ‘true’ data generating process. On the one hand, the model’s output is characterized by stochastic switches between two maxima of its probability density in the case of strong interaction. Although our model could help in understanding such transitions between prevailing optimism and pessimism, the stochasticity of these swings would prevent successful prediction of changes of the public’s mood. On the other hand, if interaction is relatively weak ( $\alpha_1 < 1$ ), the persistency of the stochastic ARFIMA model might be a good approximation to the behavioral persistency of the opinion model.<sup>27</sup> Both aspects need to be explored in order to get an idea of the potential forecasting performance of such models.

A certain deficit of our present approach is the uni-variate nature of our models. Of course, the opinion dynamics will not be decoupled from other economic data and might be influenced by exogenous news about economic and possibly political conditions. In order to get a handle on such factors, we could let them enter the formalization of transition rates<sup>28</sup> or we could combine our opinion dynamics with additional dynamic components formalizing the time development of, for example, GDP, interest rates etc. One would, then, hope to disentangle the influence of objective factors from intrinsic propagation of moods among the population of respondents. This daunting task is left for our future research.

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<sup>27</sup>Alfarano and Lux (2007) demonstrate that a closely related model mimics the long-term dependency that is the defining feature of ARFIMA models. Lux (2009b) shows that both a behavioral opinion model and a parsimonious diffusion process provide nearly equivalent fits to a financial sentiment index.

<sup>28</sup>Lux (2009a) considered various macroeconomic factors in the analysis of a German business climate index but found surprisingly little value added compared to the ‘canonical’ model.

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8 **Remark on Tables 3–14:** The symbol  $\mathfrak{L}$  stands for the logarithmic maximum likelihood. AIC and BIC represent the Akaike and Bayesian information criteria, respectively. The minimal forecast errors within the columns of single questions are given in bold. The global minimal values within the questions are emphasized by stars. In a few cases, standard errors could not be obtained which is indicated by the sign ‘–’.

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Table 3: Belgium [BE]

Parameter Estimates and Standard Errors								
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5								
M1	0.6880 (0.0733)	-0.0016 (0.0023)	0.9683 (0.0195)	775.0	340.1	-674.2	-664.4	
M2	0.6858 (0.0728)		0.9720 (0.0187)	775.0	339.8	-675.6	-669.1	
M3	0.0209 (0.0629)	-0.0312 (0.0702)	-0.5213 (4.3190)	23.8 (69.5)	340.5	-672.9	-659.9	
M4	0.0143 (0.0165)		-1.0567 (2.0992)	16.6 (18.2)	340.4	-674.7	-665.0	
Cst.Q4								
M1	0.3266 (0.0347)	0.0007 (0.0045)	0.9256 (0.0357)	440.0	357.5	-709.0	-699.2	
M2	0.3270 (0.0347)		0.9241 (0.0342)	440.0	357.5	-711.0	-704.5	
M3	0.0656 –	0.0043 –	0.5685 –	88.6 –	357.5	-706.9	-693.9	
M4	0.0533 –		0.4492 –	71.9 –	357.5	-709.0	-699.2	
Cns.Q4								
M1	0.9323 (0.0987)	-0.0005 (0.0025)	1.0073 (0.0141)	800.0	306.0	-606.0	-596.2	
M2	0.9303 (0.0979)		1.0091 (0.0107)	800.0	305.9	-607.9	-601.4	
M3	0.0230 –	-0.0564 –	-0.1585 –	19.5 –	308.7	-609.5	-596.5	
M4	0.0836 –		0.7582 –	70.2 –	308.4	-610.7	-600.9	
RTr.Q4								
M1	1.9951 (0.2618)	0.0070 (0.0018)	0.8702 (0.0197)	575.0	251.4	-496.8	-487.0	
M2	1.8437 (0.2242)		0.9093 (0.0170)	575.0	243.9	-483.8	-477.3	
M3	0.2140 –	0.0660 –	-0.2613 –	62.4 –	251.0	-494.1	-481.1	
M4	0.3746 (0.1301)		0.6287 (0.1212)	117.5 (37.6)	242.8	-479.5	-469.7	
<b>Comparison of the out-of-sample one month forecasting errors:</b>								
	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	0.03308	0.02676	0.03265	0.02660	0.08242	0.06719	0.03343	0.02764
M2	0.03296	0.02702	0.03268	0.02676				
M3	0.03249	<b>0.02638*</b>	<b>0.03254</b>	<b>0.02646</b>				
M4	<b>0.03247*</b>	0.02655	0.03257	0.02657				
Cst.Q4								
M1	0.03025	0.02363	0.03013	0.02322	0.17509	0.15137	<b>0.02925*</b>	<b>0.02262*</b>
M2	0.03019	<b>0.02339</b>	0.03014	0.02322				
M3	0.03062	0.02387	<b>0.03011</b>	0.02319				
M4	<b>0.03010</b>	0.02339	0.03012	<b>0.02318</b>				
Cns.Q4								
M1	0.05661	0.04392	0.05589	0.04331	0.14855	0.12578	0.05648	0.04366
M2	0.05652	0.04387	0.05604	0.04347				
M3	<b>0.05513</b>	<b>0.04294</b>	<b>0.05498*</b>	<b>0.04234*</b>				
M4	0.05555	0.04325	0.05543	0.04279				
RTr.Q4								
M1	0.06277	0.04783	0.06292	<b>0.04784</b>	0.07086	0.05266	0.06184	<b>0.04645*</b>
M2	0.06048	0.04981	<b>0.06048</b>	0.04963				
M3	0.06298	<b>0.04783</b>	0.06297	0.04784				
M4	<b>0.06046*</b>	0.04986	0.06056	0.05009				

Table 4: Denmark [DK]

Parameter Estimates and Standard Errors								
	$\hat{\nu}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\bar{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5								
M1	0.2675 (0.0320)	0.0309 (0.0098)	0.6984 (0.0783)	250.0	329.7	-653.4	-643.7	
M2	0.2520 (0.0288)		0.8878 (0.0514)	250.0	324.9	-645.8	-639.3	
M3	0.0670 –	0.1326 –	-0.3066 –	63.3 –	330.2	-652.3	-639.3	
M4	0.0808 –		0.6132 –	80.4 –	324.8	-643.6	-633.8	
Cst.Q4								
M1	1.1220 (0.1327)	-0.0029 (0.0026)	0.9333 (0.0196)	375.0	232.0	-458.0	-448.3	
M2	1.1138 (0.1309)		0.9390 (0.0190)	375.0	231.3	-458.6	-452.1	
M3	0.1994 –	-0.0169 –	0.5479 –	67.0 –	231.9	-455.7	-442.7	
M4	0.1804 –		0.5365 –	61.1 –	231.1	-456.1	-446.4	
Cns.Q4								
M1	0.8919 (0.1059)	-0.0046 (0.0023)	0.9190 (0.0230)	750.0	311.3	-616.5	-606.7	
M2	0.8690 (0.1008)		0.9423 (0.0200)	750.0	309.0	-613.9	-607.4	
M3	0.0055 (0.0044)	-0.7620 (0.3699)	-7.1684 (2.3278)	6.1 (4.0)	315.1	-622.2	-609.2	
M4	0.0509 (0.0572)		-0.1166 (1.2107)	44.2 (48.9)	308.8	-611.6	-601.8	

## Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	0.04854	0.03737	0.04814	0.03717	0.06842	0.05192	0.05169	0.04039
M2	0.05035	0.03851	0.05043	0.03875				
M3	<b>0.04811</b>	<b>0.03689*</b>	<b>0.04801*</b>	<b>0.03709</b>				
M4	0.05038	0.03839	0.05045	0.03873				
Cst.Q4								
M1	0.03790	0.03110	0.03788	0.03129	0.47826	0.40125	0.04067	0.03129
M2	<b>0.03712*</b>	<b>0.03055*</b>	<b>0.03741</b>	<b>0.03086</b>				
M3	0.03861	0.03195	0.03825	0.03164				
M4	0.03763	0.03102	0.03777	0.03122				
Cns.Q4								
M1	<b>0.05798</b>	0.03645	0.05790	0.03625	0.15902	0.13965	<b>0.05350*</b>	<b>0.03321*</b>
M2	0.05851	0.03469	0.05798	<b>0.03435</b>				
M3	0.05810	0.04074	<b>0.05726</b>	0.03682				
M4	0.05857	<b>0.03464</b>	0.05811	0.03437				

Table 5: Germany [DE]

Parameter Estimates and Standard Errors								
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5								
M1	0.5587 (0.0611)	0.0010 (0.0017)	0.9703 (0.0187)	1800.0	437.2	-868.4	-858.6	
M2	0.5579 (0.0609)		0.9733 (0.0180)	1800.0	437.1	-870.2	-863.7	
M3	0.0275 (0.1929)	0.0269 (0.2322)	0.1979 (5.5973)	88.9 (619.0)	437.4	-866.8	-853.8	
M4	0.0202	-	0.0135	65.4	437.2	-868.5	-858.7	
Cst.Q4								
M1	0.4772 (0.0487)	0.0000 (0.0041)	1.0309 (0.0168)	700.0	355.3	-704.6	-694.8	
M2	0.4772 (0.0487)		1.0310 (0.0117)	700.0	355.3	-706.6	-700.1	
M3	0.0619	-0.0326	0.7503	86.9	359.3	-710.7	-697.7	
M4	0.0531	-	0.8164	75.0	358.6	-711.2	-701.4	
Cns.Q4								
M1	0.5482 (0.0570)	-0.0012 (0.0030)	1.0085 (0.0176)	1000.0	383.5	-761.1	-751.3	
M2	0.5469 (0.0567)		1.0132 (0.0129)	1000.0	383.4	-762.7	-756.2	
M3	0.0512	-0.0278	0.7101	92.0	385.0	-762.0	-748.9	
M4	0.0229	-	0.8610	40.6	384.0	-762.0	-752.2	
RTr.Q4								
M1	0.5346 (0.0592)	-0.0001 (0.0035)	0.8558 (0.0377)	405.0	311.7	-617.3	-607.6	
M2	0.5346 (0.0592)		0.8559 (0.0376)	405.0	311.7	-619.3	-612.8	
M3	0.6038 (2.8489)	-0.0001 (0.0033)	0.8733 (0.6385)	457.4 (2157.5)	311.7	-615.3	-602.3	
M4	0.0745 (0.0898)		-0.0676 (1.2590)	56.6 (67.4)	311.6	-617.1	-607.3	

## Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	0.03299	0.02580	<b>0.03323</b>	<b>0.02621</b>	0.07729	0.06118	0.03399	0.02696
M2	<b>0.03241*</b>	<b>0.02544*</b>	0.03328	0.02627				
M3	0.03303	0.02585	0.03325	0.02625				
M4	0.03283	0.02582	0.03333	0.02636				
Cst.Q4								
M1	0.03245	0.02361	0.03218	0.02351	0.50231	0.47217	0.03338	0.02497
M2	0.03245	0.02361	<b>0.03218*</b>	<b>0.02351</b>				
M3	<b>0.03239</b>	<b>0.02351*</b>	0.03278	0.02416				
M4	0.03276	0.02377	0.03280	0.02419				
Cns.Q4								
M1	0.03394	0.02820	0.03355	0.02810	0.08820	0.07452	0.03400	<b>0.02787*</b>
M2	0.03386	0.02820	0.03360	0.02819				
M3	0.03409	<b>0.02811</b>	<b>0.03354*</b>	<b>0.02795</b>				
M4	<b>0.03376</b>	0.02820	0.03375	0.02833				
RTr.Q4								
M1	0.05461	0.04405	0.05511	0.04447	0.13057	0.10435	0.05830	0.04828
M2	0.05463	0.04417	0.05516	0.04450				
M3	0.05480	0.04417	0.05520	0.04456				
M4	<b>0.05437*</b>	<b>0.04381*</b>	<b>0.05458</b>	<b>0.04392</b>				

Table 6: Ireland [IE]

Parameter Estimates and Standard Errors								
	$\hat{\nu}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\bar{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5								
M1	2.5303 (0.3167)	0.0093 (0.0021)	0.9309 (0.0142)	550.0	217.5	-429.1	-419.3	
M2	2.2776 (0.2574)		0.9775 (0.0100)	550.0	207.8	-411.6	-405.1	
M3	1.3496 (2.2901)	0.0181 (0.0320)	0.8515 (0.2888)	292.3 (495.5)	217.6	-427.3	-414.2	
M4	4.2625 (2.2347)		0.9961 (0.0122)	1029.9 (534.1)	208.3	-410.5	-400.7	
Cst.Q4								
M1	2.2200 (0.2635)	0.0040 (0.0023)	1.0272 (0.0080)	250.0	147.5	-289.0	-279.2	
M2	2.1975 (0.2582)		1.0304 (0.0079)	250.0	146.0	-288.1	-281.6	
M3	3.2175 (0.7754)	0.0032 (0.0016)	1.0341 (0.0064)	362.3 (78.9)	148.6	-289.1	-276.1	
M4	2.9496 (0.7402)		1.0355 (0.0068)	335.4 (76.6)	146.7	-287.4	-277.6	
Cns.Q4								
M1	0.7146 (0.0769)	-0.0005 0.0024	0.9828 (0.0150)	650.0	319.9	-633.8	-624.1	
M2	0.7137 (0.0766)		0.9834 (0.0148)	650.0	319.9	-635.8	-629.2	
M3	0.0660 (0.3682)	0.0016 (0.0311)	0.6496 (2.0290)	60.9 (337.8)	319.0	-630.1	-617.1	
M4	0.0691 (0.1753)		0.6641 (0.8980)	63.6 (160.7)	319.1	-632.1	-622.3	

**Comparison of the out-of-sample one month forecasting errors:**

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	0.10111	0.08199	0.09976	0.08094	<b>0.09770*</b>	<b>0.07931*</b>	0.09786	0.08026
M2	0.10698	0.08673	0.10489	0.08527				
M3	<b>0.10008</b>	<b>0.08151</b>	<b>0.09936</b>	<b>0.08053</b>				
M4	0.10773	0.08758	0.10570	0.08590				
Cst.Q4								
M1	<b>0.20797</b>	<b>0.16905</b>	<b>0.20356</b>	0.16777	0.20172	0.17083	<b>0.19222*</b>	<b>0.15244*</b>
M2	0.20852	0.17014	0.20424	0.16918				
M3	0.20867	0.16905	0.20374	<b>0.16769</b>				
M4	0.20904	0.16955	0.20438	0.16903				
Cns.Q4								
M1	<b>0.05228*</b>	<b>0.04273*</b>	<b>0.05266</b>	<b>0.04297</b>	0.06989	0.05719	0.05407	0.04462
M2	0.05230	0.04273	0.05278	0.04305				
M3	0.05268	0.04296	0.05294	0.04317				
M4	0.05268	0.04296	0.05290	0.04314				



Table 7: Greece [EL]

Parameter Estimates and Standard Errors									
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathcal{L}$	AIC	BIC		
Ind.Q5									
M1	1.1746 (0.1360)	0.0168 (0.0057)	0.9571 (0.0227)	850.0	316.5	-627.0	-617.3		
M2	1.1182 (0.1241)		1.0215 (0.0068)	850.0	312.3	-620.5	-614.0		
M3	0.1330 –	0.2217 –	0.1122 –	96.2 –	316.7	-625.3	-612.3		
M4	0.1220 –		0.9644 –	98.0 –	307.4	-608.8	-599.0		
Cst.Q4									
M1	1.5145 (0.1680)	0.0039 (0.0030)	1.0068 (0.0126)	220.0	149.7	-293.4	-283.6		
M2	1.5011 (0.1653)		1.0116 (0.0121)	220.0	148.9	-293.8	-287.3		
M3	0.7291 (0.5179)	0.0075 (0.0079)	0.9589 (0.0698)	104.0 (73.2)	150.9	-293.8	-280.8		
M4	0.6752 (0.5166)		0.9620 (0.0731)	97.0 (73.6)	150.3	-294.5	-284.8		
Cns.Q4									
M1	0.7327 (0.0812)	-0.0076 (0.0034)	0.9558 (0.0204)	750.0	335.0	-663.9	-654.2		
M2	0.7161 (0.0778)		0.9909 (0.0135)	750.0	332.2	-660.3	-653.8		
M3	0.0779 –	-0.0750 –	0.4387 –	80.9 –	333.9	-659.8	-646.8		
M4	0.0776 (0.3034)		0.7892 (0.8767)	82.5 (321.3)	330.8	-655.6	-645.8		
Comparison of the out-of-sample one month forecasting errors:									
	Nearest Mode		Expected Value		ARMA		ARFIMA		
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	
Ind.Q5									
M1	0.03549	<b>0.02854*</b>	<b>0.03533*</b>	<b>0.02856</b>	0.10063	0.08716	0.03666	0.02928	
M2	0.03647	0.02920	0.03625	0.02906					
M3	<b>0.03543</b>	0.02877	0.03546	0.02861					
M4	0.03866	0.03108	0.03749	0.02992					
Cst.Q4									
M1	0.10630	<b>0.08255</b>	0.10639	<b>0.08246*</b>	0.11216	<b>0.09168</b>	0.11301	0.08833	
M2	<b>0.10517*</b>	0.08279	<b>0.10618</b>	0.08364					
M3	0.10693	0.08288	0.10754	0.08325					
M4	0.10765	0.08474	0.10775	0.08478					
Cns.Q4									
M1	0.04704	0.03517	<b>0.04662*</b>	<b>0.03486</b>	0.04722	0.03523	0.04685	<b>0.03453*</b>	
M2	0.04691	0.03517	0.04739	0.03564					
M3	<b>0.04678</b>	<b>0.03469</b>	0.04693	0.03505					
M4	0.04802	0.03593	0.04794	0.03596					

Table 8: France [FR]

	Parameter Estimates and Standard Errors				$\mathcal{L}$	AIC	BIC
	$\hat{\nu}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$			
Ind.Q5							
M1	1.7768 (0.1942)	0.0013 (0.0010)	0.9960 (0.0076)	2000.0	340.3	-674.6	-664.8
M2	1.7705 (0.1927)		1.0004 (0.0068)	2000.0	339.4	-674.8	-668.3
M3	0.0690 –	0.0385 –	0.5681 –	78.2 –	339.8	-671.7	-658.6
M4	0.0632 (0.1927)		0.6761 (1.0165)	71.3 (214.6)	338.6	-671.3	-661.5
Cst.Q4							
M1	0.7898 (0.0821)	0.0023 (0.0019)	1.0458 (0.0078)	1500.0	360.8	-715.6	-705.8
M2	0.7989 (0.0828)		1.0402 (0.0063)	1500.0	360.2	-716.5	-710.0
M3	0.0016 (0.0014)	0.2988 (0.4187)	-1.9889 (1.3183)	4.0 (3.2)	375.5	-743.1	-730.0
M4	0.0005 (0.0013)		-2.9117 (0.8679)	1.3 (3.2)	375.0	-744.0	-734.2
Cns.Q4							
M1	1.3952 (0.1557)	-0.0009 (0.0022)	1.0106 (0.0108)	1650.0	342.3	-678.5	-668.8
M2	1.3912 (0.1548)		1.0144 (0.0054)	1650.0	342.1	-680.3	-673.8
M3	0.0631 (0.1275)	-0.0932 (0.1945)	0.3581 (1.3685)	73.6 (147.9)	342.8	-677.6	-664.6
M4	0.0628 –		0.7619 –	75.7 –	340.6	-675.3	-665.5
RTr.Q4							
M1	1.0343 (0.0056)	-0.0026 (0.0015)	0.9964 (0.0028)	1875.0	-319.6	645.2	655.0
M2	1.0031 (0.0042)		1.0002 (0.0002)	1875.0	-327.4	658.9	665.4
M3	0.1108 (0.1589)	-0.0405 (0.0785)	0.8644 (0.3726)	38.9 (55.6)	233.0	-458.0	-445.0
M4	0.1133 (0.1789)		0.9643 (0.2183)	40.2 (63.0)	232.3	-458.6	-448.8

## Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	0.04003	0.03068	0.03990	0.03050	<b>0.03833*</b>	<b>0.02939*</b>	0.04042	0.03113
M2	0.03996	0.03049	0.04011	0.03049				
M3	<b>0.03949</b>	<b>0.03042</b>	<b>0.03964</b>	<b>0.03044</b>				
M4	0.04021	0.03104	0.03999	0.03065				
Cst.Q4								
M1	0.02903	0.02386	0.02897	0.02372	0.29539	0.27448	0.02672	<b>0.02192*</b>
M2	0.02789	0.02271	0.02781	0.02259				
M3	0.02681	0.02219	<b>0.02671</b>	<b>0.02209</b>				
M4	<b>0.02641*</b>	<b>0.02210</b>	0.02677	0.02208				
Cns.Q4								
M1	0.05028	0.03668	0.04973	0.03613	0.09868	0.07829	0.05068	0.03860
M2	0.05035	0.03704	0.05002	0.03649				
M3	<b>0.04948</b>	<b>0.03596</b>	<b>0.04905*</b>	<b>0.03534*</b>				
M4	0.05070	0.03749	0.05049	0.03731				
RTr.Q4								
M1	0.05095	0.03974	0.05077	0.03956	0.22687	0.19629	0.05223	<b>0.03830*</b>
M2	<b>0.05080</b>	0.03888	<b>0.05066*</b>	0.03887				
M3	0.05191	0.04050	0.05152	0.04020				
M4	0.05107	<b>0.03871</b>	0.05098	<b>0.03867</b>				

Table 9: Italy [IT]

Parameter Estimates and Standard Errors								
	$\hat{\nu}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\bar{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5								
M1	1.4947 (0.1607)	0.0027 (0.0016)	0.9979 (0.0096)	2050.0	362.8	-719.6	-709.8	
M2	1.4773 (0.1572)		1.0110 (0.0057)	2050.0	361.6	-719.1	-712.6	
M3	1.0080 (1.0467)	0.0045 (0.0062)	0.9853 (0.0422)	1379.4 (1427.3)	362.8	-717.6	-704.6	
M4	0.0456 (0.1167)		0.7950 (0.5654)	64.3 (164.1)	359.8	-713.5	-703.7	
Cst.Q4								
M1	1.8635 (0.2075)	-0.0057 (0.0028)	0.9671 (0.0153)	250.0	145.9	-285.8	-276.0	
M2	1.8086 (0.1953)		0.9831 (0.0132)	250.0	143.8	-283.6	-277.1	
M3	0.0235 (0.0138)	-0.3857 (0.1915)	-3.5683 (1.4569)	3.8 (1.8)	155.3	-302.7	-289.6	
M4	0.0253 (0.0135)		-2.3553 (1.0157)	4.1 (1.8)	152.0	-298.1	-288.3	
Cns.Q4								
M1	1.2686 (0.1366)	-0.0026 (0.0017)	0.9778 (0.0137)	1000.0	304.0	-602.1	-592.3	
M2	1.2522 (0.1333)		0.9890 (0.0116)	1000.0	302.7	-601.4	-594.9	
M3	0.1287 –	-0.0299 –	0.5444 –	98.6 –	306.8	-605.6	-592.6	
M4	0.0911 –		0.5386 –	71.1 –	305.2	-604.4	-594.6	

**Comparison of the out-of-sample one month forecasting errors:**

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	<b>0.03771*</b>	<b>0.02865</b>	0.03789	0.02856	0.04521	0.03583	0.03830	0.02947
M2	0.03851	0.02968	0.03845	0.02918				
M3	0.03803	0.02920	<b>0.03787</b>	<b>0.02855*</b>				
M4	0.03917	0.02980	0.03889	0.02945				
Cst.Q4								
M1	0.06481	0.05183	0.06689	0.05406	0.13318	0.11536	<b>0.05618*</b>	<b>0.04518*</b>
M2	<b>0.05776</b>	<b>0.04614</b>	<b>0.05793</b>	<b>0.04683</b>				
M3	0.08885	0.07236	0.07331	0.05919				
M4	0.06283	0.05186	0.05952	0.04867				
Cns.Q4								
M1	<b>0.03122</b>	<b>0.02427*</b>	<b>0.03111*</b>	<b>0.02436</b>	0.03790	0.03030	0.03148	0.02494
M2	0.03272	0.02620	0.03187	0.02552				
M3	0.03148	0.02520	0.03163	0.02525				
M4	0.03243	0.02663	0.03274	0.02665				

Table 10: Luxembourg [LU]

Parameter Estimates and Standard Errors								
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5								
M1	0.2511 (0.0284)	-0.0119 (0.0140)	0.6984 (0.0845)	55.0	192.7	-379.4	-369.6	
M2	0.2501 (0.0281)		0.7119 (0.0831)	55.0	192.3	-380.6	-374.1	
M3	0.1168 (0.1723)	-0.0225 (0.0411)	0.3174 (1.0478)	25.7 (37.4)	192.8	-377.5	-364.5	
M4	0.3322 (0.8854)		0.7924 (0.6561)	73.1 (195.5)	192.2	-378.3	-368.6	
Cst.Q4								
M1	0.1232 (0.0132)	-0.0506 (0.0408)	0.7473 (0.1309)	20.0	162.8	-319.5	-309.8	
M2	0.1217 (0.0128)		0.8386 (0.1090)	20.0	161.9	-319.9	-313.3	
M3	0.0997 (0.1283)	-0.0619 (0.0883)	0.6643 (0.5723)	16.2 (20.5)	162.8	-317.6	-304.5	
M4	0.0840 (0.1898)		0.7185 (0.8798)	13.9 (30.7)	162.0	-317.9	-308.2	
<b>Comparison of the out-of-sample one month forecasting errors:</b>								
	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	0.08809	0.07317	0.08796	0.07296	0.09219	0.07562	0.08859	0.07497
M2	0.08784	<b>0.07281</b>	0.08772	0.07270				
M3	0.08791	0.07305	0.08794	0.07295				
M4	<b>0.08775</b>	0.07281	<b>0.08771*</b>	<b>0.07268*</b>				
Cstr.Q4								
M1	<b>0.07550</b>	<b>0.06063</b>	<b>0.07528*</b>	0.06042	0.13168	0.10302	0.07770	0.06180
M2	0.07731	0.06137	0.07699	0.06130				
M3	0.07567	0.06063	0.07533	<b>0.06041*</b>				
M4	0.07708	0.06163	0.07713	0.06139				

Table 11: Netherlands [NL]

Parameter Estimates and Standard Errors								$\mathcal{L}$	AIC	BIC
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$						
Ind.Q5										
M1	0.5087 (0.0630)	0.0258 (0.0048)	0.6224 (0.0628)	850.0	409.2	-812.4	-802.6			
M2	0.4392 (0.0479)		0.9147 (0.0348)	850.0	396.6	-789.2	-782.7			
M3	11.7242 –	0.0010 –	0.9887 –	20115.4 –	410.0	-812.0	-798.9			
M4	0.0371 (0.0243)		-0.0736 (0.5117)	72.0 (46.1)	396.4	-786.8	-777.1			
Cst.Q4										
M1	0.2535 (0.0279)	0.0121 (0.0064)	0.8630 (0.0624)	300.0	344.1	-682.3	-672.5			
M2	0.2523 (0.0276)		0.9092 (0.0578)	300.0	342.7	-681.3	-674.8			
M3	0.0761 –	0.0400 –	0.5119 –	90.1 –	344.4	-680.7	-667.7			
M4	0.0653 –		0.6068 –	77.7 –	342.9	-679.9	-670.1			
Cns.Q4										
M1	0.9296 (0.1000)	-0.0011 (0.0020)	0.9981 (0.0123)	750.0	304.2	-602.4	-592.7			
M2	0.9270 (0.0995)		0.9991 (0.0122)	750.0	304.0	-603.9	-597.4			
M3	0.0603 –	-0.0024 –	0.6071 –	48.6 –	304.7	-601.3	-588.3			
M4	0.0032 (0.0025)		-3.9698 (1.1338)	3.5 (2.5)	309.4	-612.7	-602.9			

**Comparison of the out-of-sample one month forecasting errors:**

	Nearest Mode		Expected Value		ARMA		ARFIMA		
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	
Ind.Q5									
M1	0.03186	0.02556	<b>0.03085</b>	0.02486	0.03560	0.02854	<b>0.02949*</b>	<b>0.02396</b>	
M2	0.03154	<b>0.02496</b>	0.03155	<b>0.02480</b>					
M3	<b>0.03088</b>	0.02527	0.03122	0.02545					
M4	0.03157	0.02496	0.03156	0.02480					
Cst.Q4									
M1	0.04184	0.03455	0.04197	0.03463	0.10610	0.08656	0.04527	0.03697	
M2	<b>0.04161*</b>	<b>0.03383*</b>	<b>0.04167</b>	<b>0.03388</b>					
M3	0.04184	0.03455	0.04211	0.03467					
M4	0.04161	0.03383	0.04184	0.03394					
Cns.Q4									
M1	<b>0.05691</b>	<b>0.04508</b>	<b>0.05678*</b>	<b>0.04489*</b>	0.16798	0.13644	0.05834	0.04628	
M2	0.05713	0.04527	0.05696	0.04497					
M3	0.05739	0.04532	0.05719	0.04524					
M4	0.05702	0.04518	0.05718	0.04516					

Table 12: Austria [AT]

Parameter Estimates and Standard Errors								
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\tilde{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5								
M1	0.1591 (0.0185)	0.0078 (0.0069)	0.8518 (0.0733)	405.0	409.2	-812.4	-802.6	
M2	0.1584 (0.0183)		0.8841 (0.0678)	405.0	408.6	-813.3	-806.8	
M3	0.0161 (0.0482)	0.0871 (0.3178)	-0.4479 (3.9818)	41.4 (121.4)	409.0	-810.1	-797.0	
M4	0.0284 (0.0410)		0.3505 (0.9232)	72.7 (104.4)	408.5	-811.0	-801.2	
<b>Comparison of the out-of-sample one month forecasting errors:</b>								
	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	0.03377	0.02696	<b>0.03347</b>	0.02674	0.07542	0.06324	0.03765	0.02998
M2	<b>0.03335*</b>	<b>0.02642*</b>	0.03372	0.02682				
M3	0.03373	0.02696	0.03347	<b>0.02672</b>				
M4	0.03343	0.02654	0.03374	0.02681				

Table 13: Finland [FI]

Parameter Estimates and Standard Errors								
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\tilde{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5								
M1	1.6466 (0.1936)	0.0077 (0.0026)	0.9538 (0.0153)	425.0	216.9	-427.9	-418.1	
M2	1.5751 (0.1777)		0.9837 (0.0118)	425.0	212.7	-421.4	-414.8	
M3	2.6658 (1.5102)	0.0046 (0.0033)	0.9803 (0.0256)	687.3 (382.9)	217.2	-426.5	-413.4	
M4	0.2960	-	0.8251	81.2	211.1	-416.1	-406.3	
Cst.Q4								
M1	0.4292 (0.0423)	-0.0182 (0.0108)	1.1490 (0.0326)	60.0	94.0	-181.9	-172.2	
M2	0.4248 (0.0417)		1.1551 (0.0327)	60.0	92.3	-180.6	-174.1	
M3	0.0208 (0.0251)	-0.1356 (0.2676)	0.1930 (0.7405)	2.5 (2.8)	129.1	-250.2	-237.2	
M4	0.0258 (0.0225)		0.3916 (0.4781)	3.1 (2.5)	128.9	-251.8	-242.0	
<b>Comparison of the out-of-sample one month forecasting errors:</b>								
	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
Ind.Q5								
M1	<b>0.10058*</b>	<b>0.07720*</b>	<b>0.10088</b>	<b>0.07815</b>	0.10803	0.08418	0.10227	0.07911
M2	0.10355	0.07965	0.10377	0.08033				
M3	0.10092	0.07777	0.10152	0.07896				
M4	0.10399	0.08011	0.10397	0.08040				
Cst.Q4								
M1	0.13602	0.10326	0.13352	0.10197	0.39744	0.35258	0.12240	0.09561
M2	0.13674	0.10438	0.13390	0.10256				
M3	<b>0.12511</b>	0.09755	0.12494	<b>0.09723</b>				
M4	0.12511	<b>0.09729</b>	<b>0.12490</b>	0.09727				

Table 14: United Kingdom [UK]

		Parameter Estimates and Standard Errors							
		$\hat{\nu}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathcal{L}$	AIC	BIC	
Ind.Q5									
M1	1.6677 (0.1807)	0.0029 (0.0018)	0.9947 (0.0101)	750.0	258.4	-510.8	-501.0		
M2	1.6480 (0.1766)		1.0041 (0.0082)	750.0	257.4	-510.7	-504.2		
M3	0.1676 (0.4734)	0.0339 (0.0997)	0.7176 (0.8708)	75.9 (213.1)	257.7	-507.5	-494.5		
M4	0.1774 (0.6034)		0.8407 (0.6247)	81.7 (276.6)	256.2	-506.3	-496.6		
Cst.Q4									
M1	0.4952 (0.0505)	-0.0006 (0.0037)	1.0602 (0.0154)	400.0	277.6	-549.2	-539.4		
M2	0.4949 (0.0504)		1.0604 (0.0154)	400.0	277.6	-551.1	-544.6		
M3	0.0022 (0.0021)	0.2287 (0.1974)	-2.5212 (0.7171)	2.4 (2.1)	297.6	-587.3	-574.3		
M4	0.0011 (0.0020)		-2.8015 (0.7066)	1.2 (2.2)	296.8	-587.6	-577.8		
Cns.Q4									
M1	1.5160 (0.1714)	-0.0038 (0.0017)	0.9594 (0.0139)	1000.0	296.5	-587.1	-577.3		
M2	1.4689 (0.1614)		0.9788 (0.0108)	1000.0	293.8	-583.7	-577.2		
M3	0.3173 (1.5598)	-0.0190 (0.0953)	0.7478 (1.3170)	208.7 (1025.1)	296.8	-585.6	-572.6		
M4	0.1127 -		0.5551 -	77.0 -	293.8	-581.6	-571.8		
RTr.Q4									
M1	0.3899 (0.0434)	0.0122 (0.0076)	0.9580 (0.0323)	250.0	287.0	-568.0	-558.2		
M2	0.3847 (0.0423)		0.9954 (0.0226)	250.0	285.8	-567.6	-561.1		
M3	0.1437 (0.3951)	0.0347 (0.1007)	0.8169 (0.6175)	92.8 (253.3)	286.3	-564.7	-551.7		
M4	0.8553 (0.4518)		1.0190 (0.0143)	553.3 (287.2)	286.4	-566.8	-557.0		

## Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA		
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	
Ind.Q5									
M1	0.06214	0.05171	0.06229	0.05148	0.23264	0.17157	<b>0.06057*</b>	<b>0.04841*</b>	
M2	0.06325	0.05193	0.06274	0.05105					
M3	<b>0.06173</b>	0.05048	<b>0.06167</b>	0.05058					
M4	0.06242	<b>0.05033</b>	0.06223	<b>0.05025</b>					
Cst.Q4									
M1	0.03732	0.03112	0.03738	0.03093	0.12580	0.11568	0.03707	0.03078	
M2	0.03698	0.03057	0.03744	0.03098					
M3	<b>0.03519*</b>	<b>0.02919*</b>	<b>0.03598</b>	<b>0.02980</b>					
M4	0.03643	0.03033	0.03611	0.02991					
Cns.Q4									
M1	<b>0.03789*</b>	<b>0.02921</b>	<b>0.03800</b>	<b>0.02886*</b>	0.03908	0.02955	0.03866	0.02892	
M2	0.03931	0.02948	0.03942	0.02961					
M3	0.03873	0.02948	0.03839	0.02906					
M4	0.04051	0.03064	0.04016	0.03007					
RTr.Q4									
M1	0.07965	0.06712	0.07982	0.06723	0.07971	0.06709	0.08056	0.06767	
M2	0.08032	0.06736	0.08038	0.06747					
M3	<b>0.07945*</b>	<b>0.06700*</b>	<b>0.07965</b>	<b>0.06708</b>					
M4	0.08098	0.06760	0.08077	0.06773					