



Determining the Number of Zones in a Pick-and-sort Order Picking System

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Determining the Number of Zones in a Pick-and-sort Order Picking System

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Abstract

In this study we consider a pick-and-sort order picking system, in which batches of orders are picked simultaneously from different (work) zones by a group of order pickers. After picking, the orders are transported by a conveyor to a next station for consolidation and packing. Packing can only occur when an order has been picked completely. For a given number of workers, each assigned to a single zone, a larger number of zones reduces pick time (since travel time reduces), but increases waiting time for completion at the packing stations, because more partial batches needing assembly arrive at the packing stations. Our aim is to determine the optimal number of zones such that the total (picking and packing) time to complete a batch is minimized. We solve this problem by optimally assigning items to pick routes in each zone. We illustrate the method with data taken from a distribution center of one of the largest online retailers in the Netherlands.

Keywords: Order picking; Zone picking; Warehousing; Logistics; Order fulfillment and distribution

1. Introduction and literature review

Warehouses play an important role in companies' supply chains. Among many activities carried out in a warehouse, order picking - *the process of retrieving products from storage in response to a specific customer request* - is the most critical. It has long been identified as a very labor intensive operation in manual systems, and a very capital intensive operation in automated systems (Goetschalckx and Ashayeri, 1989). It may consume as much as 60% of all labor activities in the warehouse, and, for a typical warehouse, the cost of order picking is estimated to be as much as 55% of the total warehouse operating expense (Tompkins et al., 2003). For these

1
2 reasons, warehousing professionals consider order picking as the highest-priority activity for
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4 productivity improvements.
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8 We can make a distinction between two types of order picking systems: conventional (manual
9 picking) and automated picking. In this research we focus on conventional picking systems and
10 exclude unit load retrieval. Four operational decision problems influencing the performance of
11 order picking systems have received attention from researchers.
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- 15 • *Storage assignment.* Storage assignment methods assign stock keeping units (SKUs) to
16 storage locations. This assignment affects the order-picking throughput time. The main
17 storage policies mentioned in the literature are randomized, class-based and dedicated
18 storage. The easiest storage method is to randomly allocate incoming products to available
19 storage locations. However, we can reduce the expected travel time of a picking tour by
20 locating high-demand products near the input/ output (I/O) point (depot) of the warehouse.
21 There are two ways of doing that: either on group or on item basis. In practice, pick-
22 frequency class-based storage strategies (see for example, Hausman et al., 1976) are the
23 most popular. Such a strategy divides products and locations into classes, ranks product
24 classes in decreasing order of pick frequency, and then assigns them in that order to the
25 location classes nearest to the I/O point. Within a class, storage is random. A dedicated
26 storage strategy (see for example Caron et al., 1998, 2000) ranks the items individually to
27 some criterion (for example pick frequency) and then assigns them in that order to the
28 locations nearest to the I/O point. The cube-per-order index (COI) rule, which is attributed
29 to Heskett (1964), is an example of such a dedicated storage strategy. The COI is the ratio
30 of the space requirement (cube) of a SKU to its turnover rate. Two common types of
31 assignment of product classes to a low level picker-to-parts warehouse, within-aisle storage
32 and across-aisle storage, are illustrated by De Koster et al.(2007).
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- *Layout problem.* This is the problem of finding a good aisle configuration (i.e. the optimal number and length of aisles) minimizing order picking time. Little research has been done in this area. Roodbergen (2001) proposes a non-linear objective function (i.e. average travel time in terms of number of picks per route and pick aisles) for determining the aisle configuration for random storage warehouses (including single and multiple blocks) that minimizes the average tour length. Also considering minimization of the average tour length as the major objective, Caron et al. (2000) consider 2-block warehouses (i.e., one middle cross aisle) under the COI-based storage assignment. Parikh and Meller (2008) develop analytical models to estimate blocking in an order picking system with wide picking aisles. For small (up to 2-blocks) class-based storage warehouses, Le-Duc and De Koster (2005) propose a travel time model and a local search procedure for determining optimal storage zone boundaries as well as the number of storage aisles. Gue and Meller (2006) propose two innovations in warehouse design, diagonal cross aisles and picking aisles having different orientations, to increase order picking throughput. For a one-block warehouse, Roodbergen and Vis (2006) present analytical formulas to show the relationship between the order picking area and the average length of a picking route. The effect of pick density and congestion on order picking system performance has been studied by Gue et al.(2006).
 - *Routing order pickers.* This is the problem of determining the optimal sequence of visits to pick up a number of requested items as quickly as possible. Optimal methods to route (order) pickers depend on the warehouse layout and the location of the depot. Optimal methods for simple warehouse layouts (single or two blocks) are mentioned by Ratliff and Rosenthal (1983), Goetschalckx and Ratliff (1988), De Koster and Van der Poort (1998), and Roodbergen and De Koster (2001b). The disadvantage of the exact algorithm is that it depends on the layout and depot location and the resulting routes may be too complicated for pickers to follow (see De Koster et al., 1998 and Dekker et al. 2004). For large and

1
2 more complicated layouts (more than two blocks) several heuristics are documented. The
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4 best routing heuristic known so far is probably the combined heuristic (Roodbergen and De
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6 Koster, 2001a). This method combines two basis methods: either traversing a visited aisle
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8 from one end to the other or entering and leaving the aisle from the same aisle's end. The
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10 choices are made with dynamic programming. Le-Duc and De Koster (2007) develop a
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12 model to estimate the first and the second moment of travel time for a 2-block warehouse
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14 with S-shape routing policy and random storage method. The estimation enables them to
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16 determine the optimal batch size.
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21 • *Batching* and *zoning*. Batching determines which orders are released together (in batch
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23 picking, multiple orders are picked together in one pick tour and need to be sorted by order
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25 later). Order batching reduces the average travel time per order by sharing a pick tour with
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27 other orders. Basically two criteria for batching exist: proximity of pick locations batching
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29 and time-window batching. Proximity batching, the clustering of orders based on retrieval
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31 locations, is studied in, for example, Hwang et al. (1988), Gibson and Sharp (1992),
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33 Elsayed et al. (1993), Rosenwein (1994), Elsayed and Lee (1996), De Koster et al. (1999),
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35 Gademann et al. (2001), and Gademann and Van de Velde (2005). With time-window
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37 batching the number of orders per batch can be fixed or variable. Variable time-window
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39 batching groups all orders that arrive during the same time interval or window. With fixed-
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41 number-of-orders time-window batching, a time window is the variable length until a batch
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43 has a predetermined number of orders (Le-Duc and De Koster, 2003, 2006). Zoning is
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45 closely related to batching; it divides the pick area into sub-divisions (or zones), each with
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47 one or few pickers dedicated to it. The major advantages of zoning are: reduction of the
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49 travel time (because of the smaller traversed area and also the familiarity of the picker with
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51 the zone) and less traffic congestion. Depending on the pick process sequence, zoning can
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53 be further classified as progressive zoning or synchronized zoning. With progressive
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55 zoning, orders are sequentially picked zone by zone (this system is also called pick-and-
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2 pass); a batch is finished when all (order) lines of the orders in the batch are picked. In
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4 contrast, in synchronized (or parallel) zoning, pickers in all zones can work on the same
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6 batch at the same time. In this case the picking process has to be followed by an order
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8 assembly process, which is often combined with packing the orders for the customers.
9
10 Choe and Sharp (1991) notice that zoning has received little attention in the literature
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12 despite its important impact on the performance of order picking systems. Mellema and
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14 Smith (1988) examine the effects of the aisle configuration, stocking policy and batching
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16 and zoning rules by using a simulation model. They suggest that a combination of batching
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18 and zoning can significantly increase the productivity (pieces per man-hour). Russell and
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20 Meller (2003) provide a design aid for potential automation of the sorting process. Their
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22 model is based on demand levels, labor rates, order sizes, and other factors. Choe et al.
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24 (1993) and Van Nieuwenhuysse and De Koster (2009) study the effects of various order
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26 picking strategies in an aisle-based system (like single-order-pick, sort-while-pick, and
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28 pick-and-sort, pick batch size and the capacity of the picking and sorting operations). They
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30 propose analytical tools for the planner to quickly evaluate various alternatives without
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32 using simulation. Parikh and Meller (2008) focus on the problem of selecting between a
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34 batch picking and a zone picking strategy. For this problem, they propose a cost model to
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36 estimate the cost of each type of picking strategy. Chen et al. (2009) study the effect of
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38 similar combinations and additionally take layout and order due times into account in
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40 design choices.
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50 A largely unsolved problem associated with zoning is to define the zone storage capacity (or
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52 zone borders). More specifically, for a given layout, operational policies (routing, batching
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54 method) and a storage assignment policy, the problem is to divide the picking area into zones
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56 such that a certain objective is maximized or minimized. Example objectives include the system
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58 throughput time (Petersen, 2002) and the load balance between zones (Jane and Laih, 2005, Yu
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60 and De Koster, 2008, 2009). If we assume that all aisles are identical and all zones are of the

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2 same size (an equal number of identical aisles), then the zone partitioning problem becomes the
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4 problem of determining the optimal number of aisles constituting a zone. It should be mentioned
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6 here that this problem has not been studied in the literature. The most related publication is
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8 Petersen (2002), where the effects on the travel distance within a zone of the number of aisles in
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10 the zone, of storage assignment methods, and of the number of items in the pick list are
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12 investigated (by using simulation). However, the zone storage capacity is fixed (i.e. aisle length
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14 varies with the number of aisles). Therefore, the problem essentially differs from the problem of
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16 determining the number of zones (or zone storage capacity). In this study, we consider the
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18 problem of finding the optimal number of zones to minimize the system throughput time for
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20 batch-order picking and subsequent order sortation and packing (synchronized zone picking).
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22 This system can be found in many companies; one of them serves as an example in the result
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24 section. The rest of the paper is organized as follows. In the next section, we describe the order
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26 picking operation. Then, we present a mathematical model for the problem of optimally
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28 assigning items-to-routes in each zone in Section 3. We apply the model to find the optimal
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30 number of zones for a mail-order company in Section 4. In Section 5 we discuss our model
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32 limitations and extensions. Finally, in Section 6, we conclude and propose further research.
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42 **2. Order picking system**

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44 The schematic layout of the order picking system that we consider is shown in Figure 1.
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46 Basically, we have two functional areas: one area for picking and one for packing. Items are
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48 stored in rectangular bin-shelving storage racks. Order pickers pick the batched orders
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50 simultaneously in different zones in the picking area. After an order picker has completed a pick
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52 tour, the picked items are deposited on a conveyor and transported to the buffer area. When all
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54 items of an order have been picked, they are sorted and packed.
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- *Batch generation*: orders (requests from a customer consist of one or several *items*²) arriving within a predetermined time interval are grouped together in one *batch* for joint release to the order pickers. Within a batch, orders are spread over the zones based on the storage locations of their items. They are consolidated later at the packing area.
 - *Picking operation*: all batched items from the same zone are picked by one order picker or a group of order pickers designated to the zone. An order picker can only be assigned to at most one zone (zone picking). Because each order picker can only pick a limited number of items in one pick route (e.g. due to the capacity limitation of the picking cart), the batched items from a zone may require t *pick shifts* to be completed, where $1 \leq t \leq \tau$, with $\tau = \max_{zones} \{t\}$. (In the case of a single order picker per zone, the number of pick shifts required is the number of pick routes.) The order picker starts a batch by obtaining a picking cart and pick lists (each is a list of items to be picked in one pick route) from a central location. The order picker then goes to the left-most aisle in the zone (see Figure 1) to start a pick route. After picking all requested items, the order pickers place them on the transportation conveyor, and go back to the left-most aisle to start a new pick route. The transportation conveyor runs continuously to move all picked items to the buffer area.
 - *Packing operation*: incomplete orders (an order is called incomplete if not all of its items are picked) are buffered on the conveyor. Orders only enter the sorter when they are complete. A group of packers manually pack the orders. After packing, orders are transported to the shipping docks for delivery to the customers.

[Insert Figure 1 here]

With a given work force level (the number of workers at both picking and packing stages), the objective of our study is to minimize the total time to complete a batch of orders (*throughput*

² 'Item' here means stock keeping unit (SKU), in the literature it is also called 'order-line.'

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2 *time*). There are two decision problems that may impact the overall time to complete an order
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4 batch.

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7 At the execution level, the problem is to assign items to different routes in each zone (recall that
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9 completion of a batch in one zone may require more than one pick route to be completed). The
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11 item-to-route assignment and resulting sequence in which items are picked in each zone affects
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13 later order consolidation. Assume for example, that we have two picking zones A and B, each
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15 with one order picker, with pick capacity of six items per pick route. We have to complete three
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17 orders: order 1 including five items in zone A and six items in zone B, order 2 including four
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19 items in each zone, and order 3 including two items in zone A and one item in zone B. A pair
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21 $(X_1 \rightarrow X_2, Y_1 \rightarrow Y_2)$ denotes that order X_1 is picked before order X_2 in zone A and order Y_1 is
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23 picked before order Y_2 in zone B. For this situation we have four possible pick sequences:
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28 (Order1 \rightarrow Order2&3, Order1 \rightarrow Order2&3), (Order1 \rightarrow Order2&3, Order 2&3 \rightarrow Order1),
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30 (Order2&3 \rightarrow Order1, Order1 \rightarrow Order2&3) and (Order 2&3 \rightarrow Order1, Order2&3 \rightarrow Order 1). The
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32 second and third sequence will result in longer throughput times, because there is no order to
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34 pack after the first pick shift. In the general case, when we have a set of orders, a given layout
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36 (number of zones, the size of zones), and a work force level at both the picking and packing area,
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38 we can formulate this problem as a mixed integer-linear program. We will discuss this in the
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40 next section.
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45 At the design level, we have to decide the number of zones into which the overall picking area
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47 should be divided (in other words, how large the zone size should be). A larger number of zones
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49 will result in a shorter picking time and consequently there will be a larger number of unpacked
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51 orders waiting on the sorter to be packed after the last period of picking. Hence, a larger number
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53 of zones reduces the picking time but on the other hand increases the packing time after the last
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55 period of picking. When the zone size increases, the route time (to pick a given number of items)
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57 also increases. Consequently, the throughput time may also increase. However, large zones
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59 reduce the consolidation problem, because orders are spread over fewer zones. This makes it
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2 simpler to arrange the pick sequence (item-to-route assignment in each zone) in such a way that
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4 the number of complete orders arriving at the packing area (per time unit) increases. And thus,
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6 the throughput time may be shorter. As an example, consider a warehouse with two aisles: one
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8 with two items of order a and two items of order b , and the other with one item of both orders.
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10 Pick capacity is three items per route. If the warehouse consists of two zones, one order picked in
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12 the second aisle (zone) will always have to wait for completion until the second pick route in the
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14 first aisle has finished. If the warehouse would consist of one zone, two routes can be picked
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16 with complete orders a and b respectively, avoiding waiting for completion. The best zoning
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18 scheme is the one that brings the best compromise between these two opposite effects. Changing
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20 zone sizes and reallocating workers usually does not require investments and is supported in
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22 many warehouse management systems. Therefore, also this problem can be considered short-
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24 term.
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31 Finding an optimal zone size linked with optimal allocation of workers over zones is an
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33 interesting problem and can be encountered in many environments. Many food retail warehouses
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35 employ such a zoned system, with equal zone sizes within a storage system in order to balance
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37 workers. They work on a fixed departure schedule per group of stores and therefore release
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39 orders based on this schedule.
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44 In practice, the number of aisles in a warehouse is limited. Therefore, when we assume zones are
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46 identical, we can choose from only a limited number of possible zone sizes (number of aisles per
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48 zone). For example, if we have 20 aisles then we have the following zone-size possibilities: 1, 2,
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50 4, 5, 10, and 20 aisles (with 20, 10, 5, 4, 2, and 1 zones respectively). Because of that, our
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52 solution strategy is as follows. For each zoning scheme, we first solve the item-to-route
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54 assignment problem. In a next step, we vary the zone sizes and choose the zone size that
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56 provides the shortest overall throughput time. In the next section we formulate a mathematical
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58 model for the item-to-route assignment problem and discuss a solution approach.
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3. Mathematical model for the item-to-route assignment problem

We make the following assumptions:

- A zone is a set of adjacent entire (identical) aisles (i.e. one aisle can not belong to more than one zone). All zones have the same number of aisles; this assumption is consistent with common practice as it makes balancing the workload between zones easier.
- Picking is in batches of a given size. Each order picker picks his or her part of the batch (that part that falls in the picker's zone) in a number of routes. After picking, the items need to be sorted and packed per order. Only complete orders can be packed, incomplete orders are buffered on the sorter. Its capacity is sufficiently large to buffer all orders needed.
- We assume the number of items an order picker picks in a pick route is identical for all order pickers (except possibly for the last route in each zone). In practice, this number would be determined by the capacity of the picking cart. We do not consider an unequal allocation of workers to zones as this makes the problem more complex and for many companies it is not really relevant. Within a given storage system, most companies try to balance work over the zones. The simplest way to do this is by sizing the zones equally, having the same number of workers per zone, and giving the zones an identical (on average) pick profile. If one zone would contain more fast movers than another, an attempt to balance zones by having more workers in this zone may lead to congestion.
- Order pickers always start from the left-most aisle (of the assigned zones). Within a zone, the average route length depends only on the number of items per route, the zone size, the storage assignment, and the routing method. We assume random storage.
- The travel time from one side of the aisle to the other is negligible, meaning that an order picker can pick items from both sides of the aisles in a single pass. No additional time is needed to reach the higher-level storage locations in an aisle.

- Multiple order pickers can work in one zone at the same time; traffic congestion is negligible.
- Routes between order pickers in different zones are synchronized; they start simultaneously. This is caused by the constant, identical number of picks per route, in combination with deterministic picking and travel speeds. In many companies, such behavior is enforced by releasing orders in “waves” to the pickers to be able to better control throughput times.
- The item transportation time on the conveyor (t_c) between the picking and packing area is a constant. The packing time per order is also constant.
- Incomplete orders are buffered. The buffer capacity is sufficiently large to buffer all orders needed. Suppose after the first shift all items belonging to one specific order have not been picked so far; the picked items of this order cannot yet be packed and have to wait on the sorter until all the remaining items in that order have been picked.

Data

q	the maximum number of items that an order picker can pick in a pick route.
a	number of aisles per zone
L	length (in travel time units) of a storage aisle
w_b	centre-to-centre distance (in travel time units) between two consecutive storage aisles
t_s	set-up time of a pick route
t_c	transportation (conveyor) time
r_{pi}	picking rate (number of items per time unit that an order picker can pick). It is identical and constant for all order pickers.
r_{pa}	overall -constant- packing rate (number of orders per time unit). In the experiments, it varies with the average order size.

N_k	number of order pickers in zone k
t, i, o, k	indices of period, item, order and zone
K	set of zones
O	set of all orders
I_o	set of all items in order o
I_k	set of all items in zone k
I	set of all items, $I = \bigcup_{o \in O} I_o = \bigcup_{k \in K} I_k$

Intermediate variables

τ	the maximum number of required pick shifts in the zones, $\tau = \max_{k \in K} \left\{ \left\lceil \frac{ I_k }{qN_k} \right\rceil \right\}$.
$T(q, a)$	expected time needed to finish a pick route of q items (or picks) in a zone containing a aisles and return to the left-most aisle of the assigned zone. It consists of four components: travel time, setup time, picking time and correction time. (Note that the number of items in the last pick route (in each zone) can be less than the route's capacity.) If the random storage assignment and the S-shape routing method are used, then it can be calculated by the following deterministic expression (see Appendix A for details):

$$T(q, a) = La \left[1 - \left(1 - \frac{1}{a} \right)^q \right] + 2w_b \sum_{i=1}^a (i-1) \left[\left(\frac{i}{a} \right)^q - \left(\frac{i-1}{a} \right)^q \right] + CR(q, a) + t_s + \frac{q}{r_{pi}} \quad (1)$$

Decision variables

$$x_{ti} = \begin{cases} 1 & \text{if item } i \text{ is picked in period } t (t = 1 \dots \tau) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{to} = \begin{cases} 1 & \text{if order } o \text{ has been completely picked in or before period } t (t = 1 \dots \tau) \\ 0 & \text{otherwise} \end{cases}$$

TL_{to} total number of items of order o completely picked at the end of period $t (t = 1 \dots \tau)$

NCO_t number of newly completed orders in period $t(t=1\dots\tau)$

UCO_t number of complete (but unpacked) orders transferred from period $t(t=1\dots\tau)$ to period $t+1$. In a period of length $T(q,a)+t_c$, we can only pack a limited number of complete orders: $P := \left\lfloor \frac{[T(q,a)+t_c] \cdot r_{pa}}{1} \right\rfloor$.

PAC_t Number of orders packed in period $t(t=1\dots\tau)$. $PAC_t \leq P$

The whole batch is completed only when all orders have been packed. Therefore, the throughput time, the overall time (ψ) to complete a batch, is the summation of time required to pick all items (the total picking time), the transportation (for all pick shifts) and the time needed to pack all remaining unpacked orders after the last pick shift. The throughput time can be calculated by:

$$\psi = (\tau - 1)\{T(q,a)+t_c\} + \{T(q_M,a)+t_c\} + UCO_\tau / r_{pa} \quad (2)$$

where $T(q_M,a)$ is the longest pick-route time in period τ and q_M is the maximum number of items which need to be picked from some zone in period τ . $T(q,a)$, for all periods $t=1,\dots,\tau-1$, is the time it takes to pick q items in aisles. In the last period fewer items are picked and consequently the maximum resulting picking time equals $T(q_M,a)$.

Since the first two components in the objective function and r_{pa} are constant, minimizing ψ also means minimizing UCO_τ . We now can formulate the item-to-route assignment problem as follows.

MODEL

Objective $Min UCO_\tau$

Such that

$$\sum_{i=1}^{\tau} x_{ii} = 1 \quad \forall (k \in K, i \in I_k) \quad (3)$$

$$\sum_{i \in I_k} x_{ii} \leq qN_k \quad \forall (k \in K, t=1\dots\tau) \quad (4)$$

$$TL_{to} = \sum_{j=1}^t \sum_{i \in I_o} x_{ji} \quad \forall (o \in O, t = 1 \dots \tau) \quad (5)$$

$$|I_o| - TL_{to} \leq M_1 (1 - y_{to}) \quad \forall (o \in O, t = 1 \dots \tau) \quad (6)$$

$$-|I_o| + TL_{to} \leq y_{to} - 1 \quad \forall (o \in O, t = 1 \dots \tau) \quad (7)$$

$$M_1 = \max_{o \in O} \{|I_o|\} \quad (8)$$

$$NCO_t = \sum_{o \in O} \sum_{j=1}^t y_{jo} - \sum_{o \in O} \sum_{j=1}^{t-1} y_{jo} \quad \forall (t = 1 \dots \tau) \quad (9)$$

$$PAC_1 = 0 \quad (10)$$

$$PAC_t \leq P \quad \forall (t = 2 \dots \tau) \quad (11)$$

$$PAC_t \leq UCO_{t-1} \quad \forall (t = 2 \dots \tau) \quad (12)$$

$$UCO_t = NCO_t + UCO_{t-1} - PAC_t \quad \forall (t = 1 \dots \tau) \quad (13)$$

$$UCO_0 = 0 \quad (14)$$

$$UCO_t \geq 0 \quad \forall (t = 1 \dots \tau) \quad (15)$$

$$x_{ii}, y_{to} \in \{0, 1\} \quad \forall (o \in O, t = 1 \dots \tau, i \in I_k, k \in K) \quad (16)$$

Constraint (3) ensures that each item is assigned to exactly one pick route. Constraint (4) is the capacity constraint. It indicates that the maximum number of items that can be picked from zone k by N_k order pickers in one period cannot exceed the total capacity of the N_k order pickers.

Constraints (5)-(8) indicate that $y_{to} = 1$ if order o is completed by the end of period t (meaning that all items belonging to order o have been picked in or before pick shift t), and $y_{to} = 0$ otherwise. M_1 is a large constant; we have taken the smallest possible value. Constraints (9)-(15) indicate that the number of complete orders left over at the end of period t equals the number of newly completed orders during period t plus the number of complete orders left over from period $t-1$ minus the number of orders that have been packed in period t . The last constraint defines the nonnegative and binary property of variables x_{ii} and y_{to} . The model is a mixed

1 integer-linear program. Constraints (5)-(16) keep track of the ‘inventory’ level after each period.
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4 In Appendix B we prove the problem is strongly NP-complete.
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6 Our computational experience with this model suggests that the running time of the model
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8 mainly depends on four factors: the total number of items, the order size (average number of
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10 items per order), and the number of periods and zones. For a problem size of 6 zones, 1000 items
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12 to be picked in 4 periods, 10 items per order on average, the time required to run the model to
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14 optimality was about 15 seconds (using LINGO release 8.0, 2.4 GHz CPU). However, for larger
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16 instances the running time went up very rapidly; it increased to more than 41 hours when the
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18 number of periods increased to 7. For real-life warehouses, the number of periods (per batch) in
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20 each zone can be rather few. However, the number of items per order can be large. A heuristic
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22 approach may be needed for solving large instances.
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30 **4. Case study and numerical experiments**

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32 In this section, we first introduce the case we have investigated. Then we discuss the results
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34 obtained by using the model described in Section 3.
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37

38 ***Introduction***

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40 The case we consider is based on one of the distribution centers of Wehkamp, a large online
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42 retailer in the Netherlands. Its mission is “being an innovative home-shopping organization with
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44 a wide assortment of consumer products at competitive prices and recognizably better service”.
45
46 The company uses a pick and pack system (which was simplified and sketched in Figure 1).
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48 About 25000 orders have to be picked per day, each containing 1.6 items (in total 2.3 units per
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50 order) on average. Since the picking and packing department have a limited capacity
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52 (particularly the sorting process before packing), orders received from customers are processed
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54 several times (in batches) a day; each batch contains about 1000 items in total. The picking
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56 process is described in Section 2. The order picker starts a batch by picking up a picking cart and
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58 obtaining pick lists from the central location. Order pickers always start at the left-most aisle in
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1
2 the zone, and follow the S-shape routing method. The picked items are dropped on the
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4 transportation conveyor, which conveys them to the packaging area. At the packaging area,
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6 complete orders are sorted automatically by packing destination station and then manually per
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8 order, while items that belong to incomplete orders are buffered until they are complete (see
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10 Figure 1). In this case, all the buffering takes place at the packing station. When an order at the
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12 packing station is complete, a light indicator turns on to signal the packers that packaging can
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14 start.
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18 As previously discussed, the zone size may strongly influence the system throughput time.
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20 Therefore, it is a crucial decision for the manager to decide how large zones should be, or,
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22 equivalently, how many zones the pick area should be divided into, so that the throughput time
23
24 of the system is minimized. In the next section, we will use the model of Section 3 to answer this
25
26 question for the case.
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28

29 30 31 *Numerical experiments and results*

32
33 Table 1 shows the current operational data as well as the size of the picking area. The company
34
35 has 36 storage aisles and uses 18 order pickers. Therefore, there are 6 possible zoning schemes
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37 (see Table 2). The company uses large, specially designed picking carts that can contain about 40
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39 line items. Arriving orders vary in size, however, the packing rate only depends on the average
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41 order size (average number of items per order) and the average packing time per unit; we
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43 estimated them at 8, 3, 1 and 0.5 order(s) per minute for order sizes of 1.6, 5, 10, and 20 items
44
45 respectively.
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49
50 [Insert Table 1 here]
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53 In order to determine the optimal number of zones, we carried out a number of experiments. We
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55 considered four pick-list sizes (10, 20, 30 and 40 items per pick route), and four order sizes (1.6,
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57 5, 10, and 20 items per order on average). Combining this with 6 zoning schemes, we have 96
58
59 scenarios in total, including the current situation (18 zones, 1.6 items per order, maximum 40
60
items per pick route).

[Insert Table 2 here]

An order batch was generated as follows. We fixed the number of items per batch. Next, for each item, a storage location (in one of the 36 aisles) and an order (to which the item belongs, from 1 to κ) were randomly drawn from respective uniform distributions. The average order size was controlled by adjusting κ : $\#orders \approx \kappa \left[1 - (1 - 1/\kappa)^{\#items} \right]$. After solving the item-to-route assignment problem mentioned in Section 3 for each batch-run, we calculated the average throughput time value for the scenario. The average travel time per pick route can be calculated, based on the zone size, the number of items per route, and the routing method used. In our case, the S-shape method is used and the route time is calculated by using formulation (1). The route times for the different pick-list and zone sizes are tabulated in Appendix C.

We used LINGO (version 8.0) to solve the item-to-route assignment problem (discussed in Section 3). For each scenario, we generated 10 order batches. Thus, each throughput time in Table 3 represents the average of 10 runs. We could get optimal solutions in 72 scenarios out of 96, for all 10 runs (2.4 MHz Pentium CPU). For the remaining 24 scenarios, we found the optimal solutions in 50% of the 10 runs depending on the order batch. It means that for these 24 scenarios each number representing the throughput time in table 3 is the average of both optimal and truncated solutions.

The gap between the feasible solution and the best lower bound provided by LINGO after about 5 minutes of running is small (less than on average 5% for small, medium, and large order sizes). 10% is the largest gap we found between a feasible solution and the lower bound, meaning the gap between a feasible solution and the optimal one is smaller. The results of the experiments are presented in Table 3, where the truncated solutions are printed bold and italic.

[Insert Table 3 here]

Figure 2(a) shows the impact of pick list size on average order throughput time with different zone settings. It appears from Figure 2 that the average throughput time is a convex function of

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2 the pick list size for a fixed number of zones and order size. This result is consistent with
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4 findings of Chew and Tang (1999) and of Le-Duc and De Koster (2007) who considered a
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6 single-block and two-block warehouse respectively. We can explain this behavior as follows.
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8 There are mainly two elements that affect the average throughput time of the system. They are
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10 overall picking time of a batch and the packing time (including waiting). When the pick list size
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12 is small, the overall picking time is small, but the arrival rate of picked but unpacked orders to
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14 the packing area becomes high the cumulative number of complete orders, which have to be
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16 packed in the last period when the picking is completed, will grow (potentially) - leading to high
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18 utilization of servers in packing area and long packing time. In contrast, when the pick list size is
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20 large, the overall traveling time and the resulting picking time of orders are large, but the
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22 packing time in the last period can be small. Thus, there exists a trade-off between picking time
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24 and packing time when increasing the pick-list size. This trade-off indicates that the optimal pick
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26 list size exists.
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33 [Insert Figure 2 here]
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37 Figure 2(b) shows that for a given order size, the average throughput time decreases and then
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39 increases with an increasing pick list size. The reason for this phenomenon can be explained as
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41 follows: with an increasing pick list size the consolidation time decreases sharply at first. Beyond
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43 the minimum point, the increase of traveling and therefore picking time dominates the reduction
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45 of consolidation time, and leads to an increase of average throughput time.
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49 Table 3 shows that for the current demand situation (1.6 items per order on average and pick list
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51 size of 40) the 18-zone configuration gives the shortest throughput time for the system³. In
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53 conclusion, the company already uses the optimal zone configuration, which has been found by
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55 experimentation.
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³ the 18-zone option would even become more favorable if we take aisle congestion into account; in an 18-zone configuration, each zone has only one order picker, thus it is free from the travel congestion.

1
2 Nevertheless, if the company were not using the optimal configuration, they could move to the
3 optimal configuration at no cost. Assuming the number of pickers is fixed regardless of the
4 optimal configuration and also the packing rate is constant, changing the zoning scheme is only a
5 zoning configuration and also the packing rate is constant, changing the zoning scheme is only a
6 matter of dividing the pickers to the zones differently and defining a new pick list for each
7 picker.
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12 The results show that throughput time is not a convex function of the number of zones. However,
13 as the number of potential zone configurations is small, it is still possible to find the optimal
14 configuration rapidly, for a given order size and pick list size.
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24 **5. Model limitations and extensions**

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26 In this section, we discuss some limitations of our model and possible extensions. A first main
27 limitation of the model is we assume zones are identical. This is usually (approximately) true
28 within a single storage system (rack system), but sometimes picks come from different storage
29 systems which operate in different ways. Zones in different systems are not necessarily balanced.
30 In that respect our model gives an optimistic estimate, as unbalanced systems will have longer
31 lead times.
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40 In the current situation, random storage and S-shape routing are used. However it is possible to
41 use other storage and routing strategies as long as routes are synchronized. This will only impact
42 the travel time $T(q, a)$ to pick a route of q items in a aisles. Closed-form expressions for class-
43 based storage exist (Le-Duc and De Koster, 2005), however, they are somewhat less accurate
44 and more complicated. In addition, turnover-based storage or other routing policies do not bring
45 many advantages if the batch size is relatively large and when a is small. We can compare
46 $T(q, a)$ with the throughput time for class-based storage and S-shape routing using the website
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58 <http://www.fbk.eur.nl/OZ/LOGISTICA/ShelfArea.html>⁴.
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⁴ This site only works with Firefox.

The results are shown in Table 4. To finish a pick route of q items (with q relatively large; in our case at least 10 items) the picking time under across-aisle class-based storage is the same or even longer compared to random storage. For within-aisle class-based storage the total picking time is shorter in some cases and in some cases it is longer (e.g for $q=40$, number of aisles/picker=3 the traveling time of within-aisle class-based storage is longer while for the same pick-cart capacity and number of aisles/picker=6 it is shorter). In the case company, the zones consist of 2 aisles and random storage is used in combination with S-shape travel. Table 4 shows other storage policies do not bring any advantage.

[Insert Table 4 here]

Though we found the optimal solutions for most scenarios investigated, it does not guarantee that we can find a 'good' solution for the item-to-route assignment problem when the problem size increases (i.e. more aisles, periods, items). We may obtain better quality solutions using Lagrangean relaxation (LR). The constraints that complicate the solution of the model are constraints (4), the capacity constraints. By relaxing constraints (4) by introducing a set of non-negative Lagrange multipliers $\lambda = (\lambda_{tk})_{\tau \times |K|}$ we obtain a Lagrangean relaxed model as follows:

$$\min \quad \psi + \sum_{t=1}^{\tau} \sum_{k \in K} \lambda_{tk} \left(\sum_{i \in I_k} x_{ti} - qN_k \right) \quad (17)$$

subject to Constraints (3) and (5-16).

The question left is how to find $(\lambda_{tk})_{\tau \times |K|} \geq 0$ that can provide a good (i.e. close to optimal) solution. The sub-gradient method may be used here to update $(\lambda_{tk})_{\tau \times |K|} \geq 0$.

The proposed model can also be used to minimize the number of pickers (a good proxy for operational cost) for a given allowed throughput time and packing rate (i.e. number of packers).

The following algorithm enumerates all possible zone configurations and chooses the configuration with the smallest number of pickers subject to a given allowed throughput time (ψ_{\max}). S is a set of all possible zoning schemes (assuming all zones have equal size). N_s is the

number of pickers in each of the s zones. M is a big number in which a feasible solution is assured subject to the allowed throughput time for $s = 1$ (i.e. one zone).

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1   $s^* = \max_{s \in S} \{s\}$ 
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9   $N_{s^*}^* = M$ 
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11 for  $s \in S (1 \dots \max_{s \in S} \{s\})$ 
12     for  $N_s = 1 \dots M$ 
13         if  $sN_s < s^* N_{s^*}^*$ 
14             obtain  $\psi(s, N_s)$  by solving the model
15             if  $\psi(s, N_s) < \psi_{\max}$ 
16                  $s^* \leftarrow s$ 
17                  $N_{s^*}^* \leftarrow N_s$ 
18             end
19         end
20     end
21 end
22
23 end
24
25 output  $s^*$  and  $N_{s^*}^*$ .
26

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For instance, for $\psi_{\max} = 120$ minutes and the situation with 10 items per order and maximum 20 items per pick route, the result of the algorithm which minimizes the number of pickers (i.e. cost) is $s^* = 6$ and $N_6^* = 3$ (see Table 3).

6. Concluding remarks

In this paper, we have elaborated the problem of choosing the right number of zones at a manual pick-and-sort OP system. At the first phase, we formulated the problem of assigning items to pick routes in each zone such that the throughput time is minimized, as a mixed-integer-linear program. At the second phase, we used this problem as a tool for evaluating different zone-size options to find the optimal one. We illustrated the method with the pick-and-sort order picking of a distribution center of the one of an online retailer in the Netherlands.

Our results show that for a given order size, the average throughput time decreases and then increases with an increasing pick list size. The throughput time is not convex in the number of zones, for a given order size and pick list size. For our case company 18 zones, each consisting of 2 aisles in combination with S-shape routing appears to yield the best results. In general the method can easily be applied to companies with identical zones and synchronized batches. In

future work it may be interesting to take congestions in the aisles into account, for zones with more than one order picker. Another interesting problem is to consider the effect of imbalanced zones and non-synchronized release policies.

Appendix A Picking time estimation

Travel time consists of three components: travel time in the cross-aisles aisles, travel time within the storage aisles, and travel time back to the left-most aisle of the zone (e.g. to start a new pick route). As we assume that the order picker always starts a pick route from the left-most aisle of the zone, the last component equals the cross-aisle travel time. With the S-shape routing method and random storage, the average travel time within storage aisles can be estimated by $La \left[1 - \left(1 - \frac{1}{a} \right)^q \right] + CR(q, a)$, where the term in square brackets is 1 minus the probability that a certain aisle does not contain pick items. The first term therefore estimates the travel time within aisles, assuming all aisles are traversed entirely. $CR(q, a)$ is a correction time; it takes into account the fact that from the last pick position (in the last visited aisle) the order picker has to return to the head of the aisle (the transportation conveyor), instead of traversing it entirely. Such a turn has to be made if and only if the number of visited aisles is odd. $CR(q, a)$ can be estimated by (see Roodbergen, 2001):

$$CR(q, a) = \sum_{g \in G: \text{odd}} \left[\binom{a}{g} \left(\frac{g}{a} \right)^q X(g) \left(2L \frac{\frac{q}{g} + 1}{g} - L \right) \right],$$

where $X(g)$ is 1 minus the probability that all q items are in less than g aisles ($g \in \{G \mid 1 \leq g \leq a, g \text{ is odd}\}$), conditional on the fact that all q items in at most g specific aisles:

$$X(g) = 1 - \sum_{j=1}^{g-1} (-1)^{j+1} \binom{g}{g-j} \left(\frac{g-j}{g} \right)^q.$$

If we number the aisles of a zone from 1 to a (from the left to the right), the cross-aisle travel time can be estimated by $w_b \sum_{i=1}^a (i-1) \left[\left(\frac{i}{a} \right)^q - \left(\frac{i-1}{a} \right)^q \right]$. Where $\left(\frac{i}{a} \right)^q - \left(\frac{i-1}{a} \right)^q$ is the probability that q picks fall in aisles $1, \dots, i$ minus the probability that q picks fall in aisles $1, \dots, i-1$, and w_b is the travel time between two consecutive storage aisles (see Figure 1). Since the picker has to return to his or her home base, we have to multiply it with a factor 2.

Finally, $T(q, a)$ is estimated by summing the above parts and adding setup time and pick time as:

$$T(q, a) = La \left[1 - \left(1 - \frac{1}{a} \right)^q \right] + 2w_b \sum_{i=1}^a (i-1) \left[\left(\frac{i}{a} \right)^q - \left(\frac{i-1}{a} \right)^q \right] + CR(q, a) + t_s + \frac{q}{r_{pi}} \blacklozenge$$

It is possible to derive approximate closed-form expressions for class-based storage and S-shape routing (Le-Duc and De Koster, 2005), or for other routing policies (returns or largest gap).

However, these expressions are more complicated for class-based storage and less accurate in most cases for non S-shape routing and in view of the rather large number of picks per route in the case of batch picking, S-shape usually gives shorter routes than these other heuristics. This can readily be checked by using the website www.fbk.eur.nl/OZ/LOGISTICA

Appendix B

In order to show our problem is strongly NP-complete we only consider the objective function and constraints (3) and (4). If we prove that this adapted problem is strongly NP-complete then our more complex problem would also be strongly NP-complete. We construct a special case of the problem with only 1 zone, $3m$ orders, and 1 order picker. Let the set of orders be $A = \{a_1, a_2, \dots, a_{3m}\}$, the capacity of the picking cart is $q \geq 3$, and the number of lines per order $s(a_i)$ is between $q/4$ and $q/2$, and $\sum_{a_i \in A} s(a_i) = mq$.

Constraints (3) and (4) now translate into the following question:

Question: Can A be partitioned into m disjoint sets A_1, A_2, \dots, A_m such that, for $1 \leq j \leq m$,

$$\sum_{a_i \in A_j} s(a_i) = q \text{ (note that each } A_j \text{ must therefore contain exactly three orders from } A\text{)?}$$

We need to make two steps:

- (1) Show that the problem belongs to NP.
- (2) Show that a strongly NP-complete problem reduces to our problem.

To demonstrate (1) it suffices to remark that a nondeterministic algorithm can check in polynomial time whether a given solution is feasible (i.e. whether $\sum_{a_i \in A_j} s(a_i) = q$ for all j).

To demonstrate (2) we remark the above formulation exactly matches the 3-Partition problem in the paper by Garey and Johnson (1975), which is known to be strongly NP-complete.

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Appendix C Average route time (in minutes) with different zone and pick-list sizes

Pick-list size	1 zone	2 zones	3 zones	6 zones	9 zones	18 zones
1	7.29	5.79	5.29	4.79	4.63	4.46
2	9.65	7.63	6.94	6.22	5.94	5.54
3	11.46	9.13	8.30	7.33	6.88	6.15
4	13.07	10.53	9.59	8.40	7.76	6.61
5	14.52	11.78	10.72	9.30	8.53	7.02
6	15.93	13.00	11.81	10.10	9.18	7.41
7	17.25	14.14	12.82	10.84	9.74	7.79
8	18.54	15.23	13.78	11.53	10.24	8.16
9	19.78	16.28	14.68	12.17	10.70	8.54
10	20.99	17.29	15.54	12.76	11.13	8.92
11	22.16	18.25	16.36	13.32	11.54	9.29
12	23.32	19.19	17.13	13.83	11.94	9.67
13	24.44	20.09	17.88	14.32	12.33	10.04
14	25.54	20.96	18.59	14.78	12.72	10.42
15	26.62	21.80	19.28	15.22	13.10	10.79
16	27.67	22.61	19.94	15.65	13.49	11.17
17	28.71	23.40	20.58	16.06	13.86	11.54
18	29.72	24.17	21.19	16.47	14.24	11.92
19	30.72	24.92	21.79	16.87	14.62	12.29
20	31.69	25.64	22.36	17.26	15.00	12.67
21	32.65	26.35	22.92	17.65	15.37	13.04
22	33.60	27.03	23.47	18.04	15.75	13.42
23	34.52	27.70	24.00	18.42	16.12	13.79
24	35.43	28.35	24.52	18.80	16.50	14.17
25	36.33	28.99	25.03	19.18	16.87	14.54
26	37.21	29.62	25.52	19.56	17.25	14.92
27	38.08	30.22	26.01	19.94	17.62	15.29
28	38.93	30.82	26.48	20.32	18.00	15.67
29	39.77	31.41	26.95	20.70	18.37	16.04
30	40.59	31.98	27.41	21.08	18.75	16.42
31	41.41	32.54	27.85	21.45	19.12	16.79
32	42.21	33.09	28.29	21.83	19.50	17.17
33	43.00	33.63	28.73	22.20	19.87	17.54
34	43.77	34.17	29.16	22.58	20.25	17.92
35	44.54	34.69	29.58	22.96	20.62	18.29
36	45.29	35.21	30.00	23.33	21.00	18.67
37	46.04	35.71	30.41	23.71	21.37	19.04
38	46.77	36.22	30.82	24.08	21.75	19.42
39	47.50	36.71	31.22	24.46	22.12	19.79
40	48.21	37.20	31.62	24.83	22.50	20.17

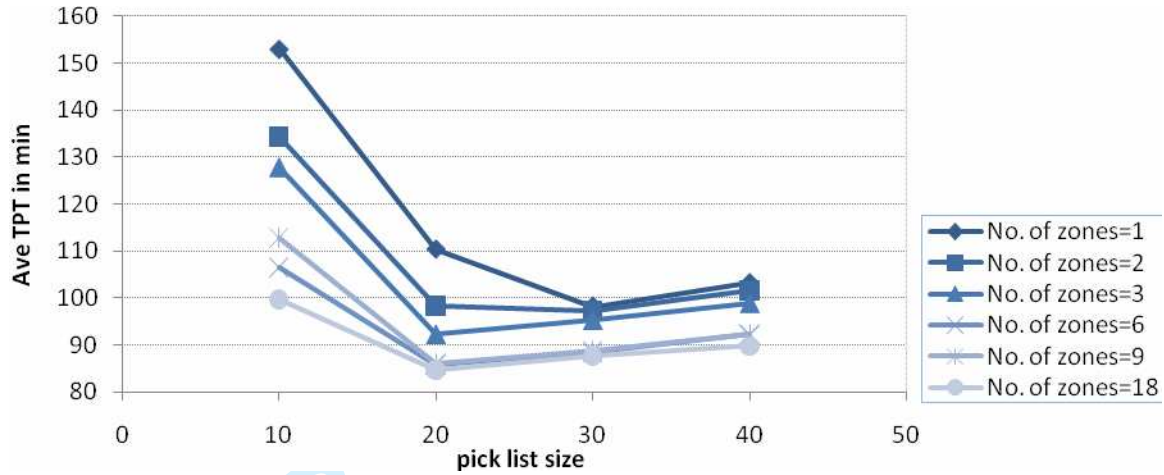
References

- Caron, F., Marchet, G. and Perego, A. (1998) Routing policies and COI-based storage policies in picker-to-part systems. *International Journal of Production Research*, 36, 713-732.
- Caron, F., Marchet, G. and Perego, A. (2000) Optimal layout in low-level picker-to-part systems. *International Journal of Production Research*, 38, 101-117.
- Chen, C.M., Gong, Y., De Koster, M.B.M. and Van Nunen, J.A.E.E. (2010) A Flexible Evaluative Framework for Order Picking Systems, *Production and Operations Management*, 19(1), 70-82.
- Chew, E.P. and Tang, L.C. (1999) Travel time analysis for general item location assignment in a rectangular warehouse. *European Journal of Operational Research*, 112, 582-597.
- Choe, K. and Sharp, G. P. (1991) Small parts order picking: design and operation. <http://www.isye.gatech.edu/logisticstutorial/order/article.htm> (Accessed: May 2005).
- Choe, K., Sharp, G.P. and Serfozo, R.S. (1993) Aisle-based order pick systems with batching, zoning and sorting, in *Progress in Material Handling Research: 1992*, The Material Handling Institute of America, Charlotte, NC, pp. 245-276.
- De Koster, R., Van der Poort, E.S. and Roodbergen, K.J., (1998) When to apply optimal or heuristic routing of order pickers, in: Fleischmann, B. et al., *Advances in Distribution Logistics*, Springer, Berlin, 375-401
- De Koster, R. and Van der Poort, E.S. (1998) Routing order pickers in a warehouse: a comparison between optimal and heuristic solutions. *IIE Transactions*, 30, 469-480.
- De Koster, R., Van der Poort, E.S. and Wolters, M. (1999) Efficient order batching methods in warehouse. *International Journal of Production Research*, 37, 1479-1504.
- De Koster, R., Le-Duc, T. and Roodbergen, K. J. (2007). Design and control of warehouse order picking: a literature review. *European Journal of Operational Research*, 182(2), 481-501.
- Dekker, R., De Koster, R., Roodbergen, K.J. and Van Kalleveen, H. (2004) Improving Order-Picking Response Time at Ankor's Warehouse. *Interfaces*, 34, 303-313.

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- Elsayed, E.A. and Lee, M.K. (1996) Order processing in automated storage/retrieval systems with due dates. *International Journal of Production Research*, 28, 567-577.
- Elsayed, E.A., Lee, M.K., Kim, S. and Scherer, E. (1993) Sequencing and batching procedures for minimizing earliness and tardiness penalty or order retrievals. *International Journal of Production Research*, 31, 727-738.
- Gademann, A.J.R.N., van den Berg, J.P. and van der Hoff, H.H. (2001) An order batching algorithm for wave picking in a parallel-aisle warehouse. *IIE Transactions*, 33, 385-398.
- Gademann, N. and van de Velde, S. (2005) Batching to minimize total travel time in a parallel-aisle warehouse. *IIE Transactions*, 37, 63-75.
- Garey, M.R. and Johnson, D.S. (1975) Complexity results for multiprocessor scheduling under resource constraints. *SIAM Journal on Computing*, 4(4), 397-411.
- Gibson, D.R. and Sharp, G.P. (1992) Order batching procedures. *European Journal of Operational Research*, 58, 57-67.
- Goetschalckx, M. and Ashayeri, J. (1989) Classification and design of order picking systems. *Logistics World*, June, 99-106.
- Goetschalckx, M. and Ratliff, D.H. (1988) Order picking in an aisle. *IIE Transactions*, 20, 531-562.
- Gue, K. R., Meller, R. and Skufca, J. D. (2006). The effects of pick density on order picking areas with narrow aisles. *IIE Solutions* 38: 859-868.
- Hausman, W.H., Schwarz, L.B. and Graves, S.C. (1976) Optimal storage assignment in automatic warehousing systems. *Management Science*, 22, 629-638.
- Heskett, J.L. (1964) Putting the cube-per-order index to work in warehouse layout. *Transport and Distribution Management*, 4, 23-30.
- Hwang, H., Baek, W. and Lee, M. (1988) Cluster algorithms for order picking in an automated storage and retrieval system. *International Journal of Production Research*, 26, 189-204.

- 1
2
3 Jane, C.C. and Laih, Y.W. (2005) A clustering algorithm for item assignment in a synchronized
4 zone order picking system. *European Journal of Operational Research*, 166(2), 489-496.
5
6
7
8 Le-Duc, T. and De Koster, R. (2003) An approximation for determining the optimal picking
9 batch size for order picker in single aisle warehouses, in *Progress in Material Handling*
10 *Research: 2002*, The Material Handling Institute of America, Charlotte, North Carolina,
11 pp. 267-286.
12
13
14
15
16
17 Le-Duc, T. and De Koster, R. (2005) Travel distance estimation and storage zone optimization in
18 a two-block class-based storage strategy warehouse. *International Journal of Production*
19 *Research* 43(17), 3561-3581.
20
21
22
23
24 Le-Duc, T. and De Koster, R. (2007). Travel time estimation and order batching in a 2-block
25 warehouse. *European Journal of Operational Research*, 176, 374-388.
26
27
28
29 Mellema, P.M. and Smith, C.A. (1988) Simulation analysis of narrow-aisle order selection
30 systems, in *Proceedings of the 1988 Winter Simulation Conference*, pp. 597-602.
31
32
33
34 Parikh, P.J. and Meller, R.D. (2008) Selecting between batch and zone order picking strategies in
35 a distribution center. *Transportation Research Part E: Logistics and Transportation*
36 *Review*, 44E(5), 696-719.
37
38
39
40
41 Parikh, P.J. and Meller, R.D. (2009) Estimating picker blocking in wide-aisle order picking
42 systems, *IIE Transactions on Design & Manufacturing*, 41(3), 232-246, 2009.
43
44
45
46 Petersen, C.G. (2002) Considerations in order picking zone configuration. *International Journal*
47 *of Operations & Production Management*, 27, 793-805.
48
49
50
51 Ratliff, H.D. and Rosenthal, A.S. (1983) Order picking in a rectangular warehouse: a solvable
52 case of the traveling salesman problem. *Operations Research*, 31, 507-521.
53
54
55
56 Roodbergen, K.J. (2001) Layout and routing methods for warehouses. PhD Thesis, RSM
57 Erasmus University, the Netherlands.
58
59
60
Roodbergen, K.J. and De Koster, R. (2001a) Routing methods for warehouses with multiple
cross aisles. *International Journal of Production Research*, 39, 1865-1883.

- 1
2
3 Roodbergen, K.J. and De Koster, R. (2001b) Routing order-pickers in a warehouse with a middle
4 aisle. *European Journal of Operational Research*, 133, 32-43.
5
6
7
8 Roodbergen, K. J. and Vis, I. F. A. (2006). A model for warehouse layout. *IIE Transactions*, 38,
9 799-811.
10
11
12 Rosenwein, M.B. (1994) An application of cluster analysis to the problem of locating items
13 within a warehouse. *IIE Transactions*, 26, 101-103.
14
15
16
17 Russell, M.L. and Meller, R.D. (2003) Cost and throughput modeling of manual and automated
18 order fulfillment systems. *IIE Transactions on Design & Manufacturing*, 35(7), 589-603.
19
20
21
22 Tompkins, J.A., White, J.A., Bozer, Y.A., Frazelle, E.H. and Tanchoco, J.M.A. (2003) *Facilities*
23 *Planning*, John Wiley & Sons, NJ.
24
25
26
27 Van Nieuwenhuysse, I. and de Koster, R. (2009), Evaluating order throughput time in 2-Block
28 warehouses with time window batching. *International Journal of Production Economics*,
29 121, 654 - 664.
30
31
32
33
34 Yu, M. and De Koster, R. (2008), Performance approximation and design of pick-and-pass order
35 picking systems. *IIE Transactions*, 40(11), 1054 - 1069
36
37
38
39 Yu, M. and De Koster, R. (2009), The impact of order batching and picking area zoning on order
40 picking system performance. *European Journal of Operational Research*, 198(2), 480-
41 490.
42
43
44
45
46
47
48
49
50
51
52
53
54
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(b)

Figure 2 The impact of different pick list sizes on average throughput time for (a) different order sizes and 3 zones and for (b) different number of zones and order size of 5 items

List of Tables

Table 1 Operational data

Operational data		System parameters	
Average number of items per batch	1000	Number of storage aisles	36
Average number of items per order	1.6		
Max. number of items per route (capacity or pick-list size)	40	Aisle length (L) in seconds	60
Number of order pickers	18	Distance between two consecutive aisles (w_b)	5
Set-up time (t_s) in seconds	180		
Picking time per item ($1/r_{pi}$) in seconds	5	in seconds	
Packing rate (r_{pa}): 8, 3, 1 and 0.5 order(s) per 60 seconds for <u>orders with</u> 1.6, 5, 10, and 20 items <u>on average</u> respectively			

Deleted: order size

Table 2 Possible zoning schemes

Number of zones	Number of storage aisles per zone	Number of order pickers per zone
1	36	18
2	18	9
3	12	6
6	6	3
9	4	2
18	2	1

Table 3 Average throughput time (in minutes)

Order size (items)	Pick-list size	1 zone	2 zones	3 zones	6 zones	9 zones	18 zones	Mean
Small (1.6)	10	<i>152.80</i>	<i>133.96</i>	<i>124.94</i>	<i>132.79</i>	<i>113.54</i>	<i>106.37</i>	127.40
	20	110.62	105.40	103.62	103.06	97.78	97.00	102.91
	30	111.02	110.16	108.14	106.26	100.11	100.68	106.06
	40	116.09	114.71	111.64	110.01	103.66	102.73	109.81
	Mean	122.63	116.06	112.09	113.03	103.77	101.70	111.55
Medium (5)	10	<i>153.04</i>	<i>134.36</i>	<i>127.72</i>	<i>106.48</i>	<i>112.72</i>	<i>99.71</i>	122.34
	20	110.38	98.41	92.31	85.87	86.04	84.75	92.96
	30	98.18	97.22	95.31	88.44	88.79	87.66	92.60
	40	103.21	101.69	98.93	92.25	92.36	89.77	96.37
	Mean	116.20	107.92	103.57	93.26	94.98	90.47	101.07
Large (10)	10	<i>157.47</i>	<i>140.76</i>	<i>134.02</i>	<i>117.77</i>	<i>121.99</i>	<i>120.58</i>	132.10
	20	127.38	127.24	125.04	117.71	122.57	119.52	123.24
	30	132.21	131.62	129.45	120.31	124.31	122.10	126.67
	40	136.91	135.42	133.03	124.79	127.06	122.63	129.97
	Mean	138.49	133.76	130.38	120.14	123.98	121.21	128.00
Very large (20)	10	<i>169.67</i>	<i>157.16</i>	<i>150.27</i>	<i>129.69</i>	<i>144.25</i>	<i>143.02</i>	149.01
	20	127.38	127.24	127.41	121.59	125.14	122.29	125.17
	30	133.21	131.62	129.41	121.89	126.36	123.88	127.73
	40	137.91	135.42	133.03	124.79	128.14	123.93	130.54
	Mean	142.04	137.86	135.03	124.49	130.97	128.28	133.11

* Bold and italic values mean the truncated solutions

Table 4 Average travel time for different storage policies

Storage Policy	Random				Within-aisle class based				Across-aisle class based				
	Pick-list size (q)	10	20	30	40	10	20	30	40	10	20	30	40
2 aisles	35.4	35.4	35.4	35.4	35.3	35.4	35.4	35.4	35.4	35.4	35.4	35.4	35.4
3 aisles	61.6	66.6	68	68.8	55.3	65.3	68.4	69.6	68	70.1	70.4	70.5	70.5
6 aisles	106.3	118.7	119.8	120	90.7	104.6	111.5	116	111.1	119.5	120	120	120

Note: (1) Input values are as follows: aisle length:10.0, centre distance: 2.4, Cross aisles: 0, depot location: left, speed inside and outside aisles: 0.7, additional aisle change time: 0.0, number of replications: 1000.

(2) Zone sizes and percentage of picks for within-aisle and across-aisle class based storage systems are as follows: A items: 10% of the space, 60% of the picks; B items: 30% of the space, 30% of the picks; C items: 60% of the space, 10% of the picks.