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# On Convergence of Non-Monotone Series. A Letter.

J.I. Pillay

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## Abstract

We hope to bring to a close the analyses of the convergence of real valued series of the form  $\sum f(n)|n \in N$  for differentiable functions, via the introduction of a theorem as a necessary condition for convergence of such series regardless of the nature of  $f$ .

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## Introduction

The difficulties surrounding the analysis of fourier series for the purpose of establishing everywhere convergent functions has been expressed in a recent paper by SV. Konyagin[1]. Many techniques used in the establishment of convergence are specific to monotone convergent series, as such convergence is difficult to establish in Fourier series. We introduce a new technique for use in such cases.

### 1. Convergence

#### Theorem 1.1.

The series  $\sum_1^{\infty} f(n)|n \in N$  is convergent if and only if the integral

$$\lim_{h \rightarrow \infty} \int_1^h x f'(x) dx$$

converges.

#### Proof

A necessary condition for a series of the form  $\sum_1^{\infty} f(n)$  to be convergent is that

$\lim_{h \rightarrow \infty} f(h) = 0$ .<sup>1</sup> Using the property stated, we have that  $\int_{\infty}^c f'(x) dx = f(c)$ . From

this, we may re write  $\sum_1^{\infty} f(n)$  as  $\sum_{c=1}^{\infty} \int_c^{\infty} f'(x) dx$ . From the summation, it is easy to

see that over every interval of one starting at one, the integrals are  $nA(n)$  where  $n$  is the  $n^{th}$  interval of length one beginning at one and  $A(n) = \int_n^{n+1} f'(x) dx$ .

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<sup>1</sup>For an oscillating nonconvergent series, we may subtract  $k$  from the integrals that follow in our proof, where  $k$  is the limit of the function in concern.

We note now that  $\{n \cap x\} = N$ , thus  $\sum_{\forall n} nA(n)$  is approximately  $\int_1^{\infty} x f'(x) dx$ . The difference between the two is smaller than  $\int_1^{\infty} f'(x) dx$ , since the sum of the difference between the upper and lower bound integrals  $\sum_{\forall n} \int_n^{n+1} f'(x) dx$  and  $\sum_{\forall n} \int_{n+1}^{n+2} f'(x) dx$ , is :  $\int_1^{\infty} f'(x) dx$ . Finally since  $\int_1^{\infty} f'(x) dx = f(1)$ , this integral is always convergent and as such will not influence the behaviour of  $\int_1^{\infty} x f'(x) dx$ .

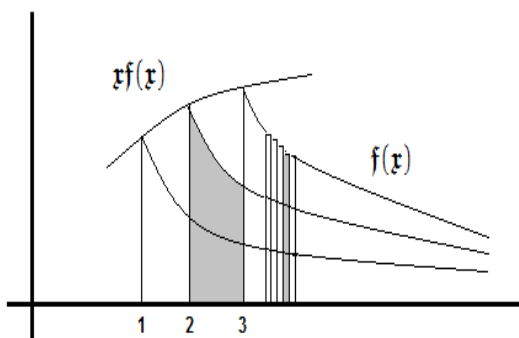


Figure 1:

# References.

- [1] SV. Konyagin. Almost everywhere convergence and divergence of Fourier series. *Proceedings of the international congress of mathematics, Madrid, Spain.* (2006).