

# Supplementary material for 'Matter wave scattering on an amplitude-modulated optical lattice'

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In this supplementary material, we provide additional details concerning interband transitions driven by the modulation of an optical lattice in the limit of a vanishing depth optical potential.

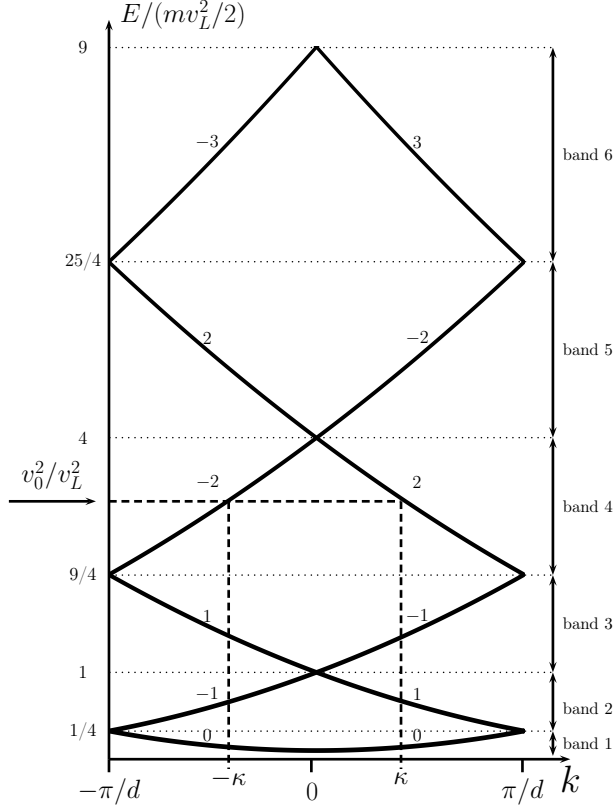


FIG. 1: The band diagram is constructed by the superposition of parabolic energy spectra centered around all reciprocal points  $E_n(k) = \hbar^2(k - nk_L)^2/2m$ . For each band we have indicated the index  $n$  of the parabola from which the corresponding branches have been constructed.

The interaction of a propagating matter wave packet with a localized optical lattice of finite size and modulated amplitude yields to the reflection of some classes of velocities depending on the frequency of modulation.

Let us consider for sake of simplicity an optical lattice of vanishing depth. The incident velocity  $v_0$  dictates the band in which the incoming atom enters. For instance, if  $v_L < v_0 < 3v_L/2$  the atom will be on band 3 (see Fig. 1). As in the main article,  $v_L = \hbar/md$  where  $d$  denotes the spacing between lattice sites.

In the limit of a vanishing lattice depth, the band structure can be constructed by the superposition of parabolic en-

ergy spectra centered around all reciprocal points  $E_n(k) = \hbar^2(k - nk_L)^2/2m$ , where  $n$  is an integer and  $k_L = 2\pi/d$ . For instance, bands 2 and 3 are constructed from the parabolas centered at  $\pm\hbar k_L$ .

In the limit of a small amplitude modulation, the modulation drives interband transitions. In the main article, we provide some examples of mechanisms that explain the reflection of specific class of velocities depending on the frequency. Here, we give some insight on the class of velocities concerned by a specific frequency modulation  $\nu$  in the limit of a vanishing depth optical potential.

In Fig. 2, we have represented schematically the experimentally observed depletion lines (dashed and dotted lines) in velocity space (see Fig. 1 of the main article). For each depletion lines and in the different velocity domains, we have also indicated two numbers. The first one corresponds to the band on which atoms lies according to their incident energy  $E = mv_0^2/2$ . The second number corresponds to the band on which atoms are resonantly transferred by the modulation.

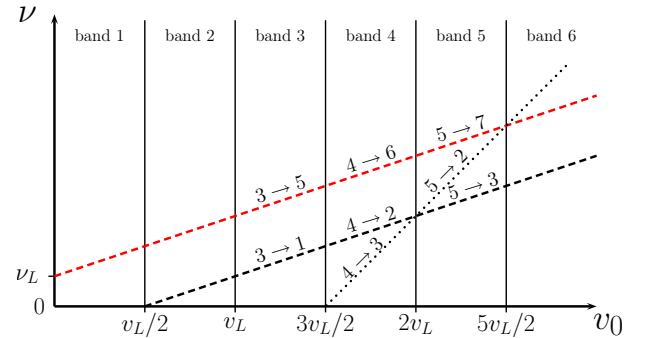


FIG. 2: Schematics of the different depletion lines observed in Fig. 1 of the article along with the numbers of the bands involved in the transition driven by the frequency of the amplitude modulation. With the experimental parameters,  $v_L \simeq 7.1$  mm/s.

The one Floquet photon transition frequencies that promote an atom from parabola  $n$  to  $n'$  is given by

$$\pm\nu_{n \rightarrow n'} = (E_n - E_{n'})/\hbar. \quad (1)$$

The sign  $+$  ( $-$ ) corresponds to a transition to a lower (upper) band. By energy conservation, we have

$$\frac{1}{2}mv_0^2 = E_n(k). \quad (2)$$

Combining Eqs. (1) and (2), we get

$$\pm\nu_{n\rightarrow n'} = \frac{E_n - E_{n'}}{h} = -(n - n')^2\nu_L + \frac{n' - n}{d}v_0. \quad (3)$$

As a first example, let us consider the dotted depletion line (see Fig. 2). In the incident velocity domain  $3v_L/2 < v_0 < 2v_L$ , it corresponds to a resonant transition between bands 4 and 3. According to Fig. 1 we have *a priori* two possibilities either a transition from parabola  $n = 2$  to parabola  $n' = -1$  or a transition from parabola  $n = -2$  to parabola  $n' = 1$  occurring at  $\pm\kappa$ . The slope of branch  $n = -2$  is positive which thus corresponds to a propagation from left to right (positive

group velocity). The states of the incoming packet with positive velocity in the range of energy that corresponds to band 4 will thus be projected onto the states that corresponds to the branch  $n = -2$ . From Eq. (3), the equation for the dotted line is  $\nu = -9\nu_L + 3v_0/d$ .

Such an interpretation can be made for all depletion lines. The lower dashed line corresponds successively to the inter-band transitions  $3 \rightarrow 1$ ,  $4 \rightarrow 2$ ,  $5 \rightarrow 3$ . From Eq. (3), we deduce its equation  $\nu = -\nu_L + v_0/d$ . Similarly we find for the upper (red) dashed line  $\nu = \nu_L + v_0/d$ . The offset between the two dashed lines is thus equal to  $2\nu_L \simeq 10.8$  kHz.