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Quantum-like model of brain's functioning: decision making from decoherence

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Abstract

We present a quantum-like model of decision making in games of the Prisoner's Dilemma type. By this model the brain processes information by using representation of mental states in a complex Hilbert space. Driven by the master equation the mental state of a player, say Alice, approaches an equilibrium point in the space of density matrices (representing mental states). This equilibrium state determines Alice's mixed (i.e., probabilistic) strategy. We use a master equation which in quantum physics describes the process of decoherence as the result of interaction with environment. Thus our model is a model of thinking through decoherence of the initially pure mental state. Decoherence is induced by the interaction with memory and the external mental environment. We study (numerically) the dynamics of quantum entropy of Alice's mental state in the process of decision making. We also consider classical entropy corresponding to Alice's choices. We introduce a measure of Alice's diffidence as the difference between classical and quantum entropies of Alice's mental state. We see that (at least in our model example) diffidence decreases (approaching zero) in the process of decision making. Finally, we discuss the problem of neuronal realization of quantum-like dynamics of in the brain; especially roles played by lateral prefrontal cortex or and orbitofrontal cortex.

Keywords: Decision making, dynamics of mental state, quantum-like model, interaction with memory, decoherence, quantum master equation, neuronal paradigm

1. Introduction

The idea that the mathematical formalism of quantum information theory can be used to describe information processes in the brain was elaborated in a series of papers, see [1]–[14]. This approach is based on the fundamental conjecture that the real physical brain developed an ability to represent the probabilistic information in complex linear space, by complex vectors (pure quantum-like mental states) or more generally density operators (mixtures of pure quantum-like mental states).¹ (This paper contains a brief appendix on mathematics of quantum mechanics which may be useful for biologists, see also book [11].)

¹In section 5 we discuss the problem of neuronal realization of quantum-like dynamics of in the brain; especially roles played by lateral prefrontal cortex or and orbitofrontal cortex. We use studies on neuronal correlates observed in the process of decision making [15],

We know well that quantum information processing can be simulated on the classical computer. Therefore our model need not be based on physically quantum brain as, e.g., the models of Homeroff [19], [20] and Penrose [21], [22]. Since the brain has huge computa-

[16]. Of course, these are very preliminary considerations on a possible neuronal realization of the quantum-like dynamics of decision making. However, it is useful to initiate such a discussion as soon as possible to attract attention of neuroscientists and psychologists to the problem of neuronal realization of the quantum-like dynamics of decision making. We also mention that quantum-like probabilistic effects can be found not only in functioning of advanced cognitive systems, but even in cell's biology. For example, we can point to the well known glucose effect on *E. coli* growth; destructive interference of two factors: the presence of lactose and glucose in a *E. coli* cell [17]. It is also well known that a dental epithelial cell grows in a medium as it is (no differentiation). A dental mesenchymal cell grows similarly. However, this two contexts are incompatible, see [18] for statistical data. Their unification induces quantum-like interference.

19 tional resources (a “neural computer” with 300 billions
20 of processing units), the main reason for usage of the
21 quantum representation of information may be not in-
22 creasing the speed of computations. The main distin-
23 guishing feature of the quantum information represen-
24 tation is a possibility to operate with *superpositions of*
25 *states*. We explore this feature of quantum information
26 processing. From the very beginning we want to dis-
27 tance from models of cognition based on quantum phys-
28 ical carriers of information. It seems that it is impossi-
29 ble to make these models consistent with the *neuronal*
30 *paradigm of cognition*. We like to combine quantum
31 information theory with the neuronal paradigm. A pos-
32 sible physical realization of quantum-like processing of
33 information by the brain operating with classical elec-
34 tromagnetic fields was presented in [23].

35 A few years ago J. Busemeyer et al. [2] noticed that
36 quantum-like models of decision making can be used
37 to explain disjunction effect in experiments of cognitive
38 psychology. In particular, he reproduced statistical data
39 from experiments of the Prisoner’s Dilemma (PD) type
40 obtained by Shafir and Tversky [24], [25], see also [26].
41 Busemeyer’s quantum-like model of decision making in
42 PD is a psychological analog of the quantum model for
43 the hydrogen atom. Of course, both models are over-
44 simplified and to be applied to more complicated phe-
45 nomena, they should be essentially modified. However,
46 both express the main distinguishing features of corre-
47 sponding theories (quantum physics and quantum-like
48 cognitive psychology). Therefore in this paper we pro-
49 ceed with PD. Following to tradition of quantum infor-
50 mation theory we call players Alice and Bob, see section
51 2 for details.

52 Our aim is to describe the process of decision mak-
53 ing in the brain; in particular, interaction with mem-
54 ory. Quantum-like models of decision making were pre-
55 sented in a series of papers [1]–[3], [6], [8]–[14].

56 *Dynamical models of decision making* are of the main
57 interest for us. We recall that, in a few papers [1], [2],
58 [12], the process of decision making was described by
59 Schrödinger’s evolution of the mental state. The lat-
60 ter was assumed to be a pure state (mathematically it is
61 represented by a normalized vector of complex Hilbert
62 space). In this paper we shall explore in more detail the
63 idea [14] that decision making has to be represented by a
64 more complicated dynamics which describes the evolu-
65 tion of quantum (or more generally quantum-like state)
66 interacting with an environment. Such dynamics plays
67 an important role in quantum physics. Its fundamen-
68 tal feature is transformation of pure states (described by
69 complex vectors) into mixed states (described by den-
70 sity matrices) – *decoherence*. In our cognitive model

71 memory is an important part of the “mental environ-
72 ment” which induces decoherence of a pure state, Al-
73 ice’s state of mind before starting the process of decision
74 making. In this paper we do not model the process of
75 interaction with memory; as often in quantum informa-
76 tion theory we represent memory (as well as the exter-
77 nal mental environment) as a bath, in our case a “mental
78 bath.” In a future paper we plan to model this process in
79 more detail by using the apparatus of quantum Markov
80 chains, cf. [6].

81 In quantum physics interaction of a quantum sys-
82 tem with a bath is described by a quantum version of
83 the master equation. Quantum Markovian dynamics
84 given by the Gorini-Kossakowski-Sudarshan-Lindblad
85 (GKSL) equation, see e.g. [27] for detail, is the most
86 popular approximation of quantum dynamics in the
87 presence of interaction with a bath. We remind shortly
88 the origin of the GKSL-dynamics. The starting point is
89 that the state of a composite system, a quantum system s
90 combined with a bath, is a pure quantum state, complex
91 vector Ψ , which evolution is described by Schrödinger’s
92 equation. This is an evolution in a Hilbert space of the
93 huge dimension (since a bath has so many degrees of
94 freedom). The existence of the Schrödinger dynam-
95 ics in the huge Hilbert space has a merely theoretical
96 value. Observers are interested in the dynamics of the
97 state ϕ_s of the quantum system s . The next fundamental
98 assumption in derivation of the GKSL-equation is the
99 Markovness of the evolution, the absence of long term
100 memory effects. It is assumed that interaction with the
101 bath destroys such effects. Thus, the GKSL-evolution
102 is Markovian evolution. Finally, we point to the condi-
103 tion of the factorizability of the initial state of a com-
104 posite system (a quantum system coupled with a bath),
105 $\Psi = \phi_s \otimes \phi_{\text{bath}}$, where \otimes is the sign of the tensor prod-
106 uct. Physically factorization is equivalent to the absence
107 of correlations (at the beginning of evolution; later they
108 are induced by the interaction term of Hamiltonian –
109 the generator of evolution). One of distinguishing fea-
110 tures of the evolution under the mentioned assumptions
111 is the existence of one or a few *equilibrium points*. The
112 state of the quantum system s stabilizes to one of such
113 points in the process of evolution; a pure initial state, a
114 complex vector ψ_s , is transformed into a mixed state, a
115 density matrix $\rho_s(t)$. In contrast to the GKSL-evolution,
116 the Schrödinger evolution does not induce stabilization;
117 any solution different from an eigenvector of Hamil-
118 tonian will oscillate for ever. Another property of the
119 Schrödinger dynamics is that it always transfers a pure
120 state into a pure state, i.e., a vector into a vector. And
121 we want to obtain mixed states, diagonalized in the ba-
122 sis corresponding to the decision operator. The GKSL-

123 evolution gives such a possibility.

124 In the process of decision making the brain selects a
 125 pure mental state describing possible decisions of the
 126 problem under consideration and drives this state –
 127 denote it by ϕ_A (superposition of possible decisions of
 128 Alice in her game with Bob). The state of the mental
 129 environment is represented by another complex vector,
 130 say ϕ_B . In general, this is a huge mental state represent-
 131 ing all superpositions in memory and even permanent
 132 supply of superpositions created by the brain through
 133 its interaction with the environment. However, if Alice
 134 is concentrated on her strategy of play with Bob,
 135 we can restrict ϕ_B to Alice’s mental image of the possible
 136 actions of Bob. In reality ϕ_B belongs to complex
 137 Hilbert space of a large dimension. Therefore the standard
 138 assumption used in the derivation of the GKSL-
 139 equation is fulfilled. (In this paper, we consider a toy
 140 model in which ϕ_B is two dimensional, representing super-
 141 position of possible actions of Bob created in Alice’s
 142 brain. Thus formally one of the most important assump-
 143 tions of derivation of the GKSL-equation is not fulfilled.
 144 However, more detailed analysis shows that, in fact, in
 145 quantum physics the dimension of a bath is not crucial.
 146 The crucial property of a bath is that it is very stable
 147 to fluctuations in the quantum system s interacting with
 148 it. This assumption is fulfilled if Alice’s image of possible
 149 actions of Bob is sufficiently stable with respect
 150 to fluctuations of the state of her possible actions.) The
 151 assumption of Markovness of the mental state evolution
 152 in decision making is natural. To proceed quickly to
 153 a decision, Alice must ignore the history of her reflec-
 154 tions on possible actions with respect to Bob.² An input
 155 from (long-term) memory or mental environment de-
 156 stroys (working) memory of her reflections. (Working
 157 memory does not preserve a long chain of Alice’s reflec-
 158 tions.) Finally, we can assume that the initial composite
 159 state is factorized, i.e., correlations between Alice’s im-
 160 age of Bob and her possible actions are created in the
 161 process of decision making. Under these assumptions
 162 we can model the process of decision making by using
 163 the GKSL-equation.

164 The mental state representing possible actions of Alice
 165 stabilizes to one of equilibrium points of the GKSL-
 166 dynamics. (In the mathematical model stabilization is
 167 achieved only in the limit $t \rightarrow \infty$. However, in reality
 168 the brain cannot wait too long. We can assume the
 169 presence (in the brain) of a threshold ϵ which is used
 170 to terminate the process of stabilization of the mental

²Such reflections are processed in her working memory. So, we discuss Markovness of working memory. Of course, in our model long-term memory is not ignored; it is a part of the mental bath.

171 state to a point of equilibrium, see also Remark 3.) A
 172 model equation considered in this paper has a single
 173 equilibrium point. Thus Alice elaborates the unique so-
 174 lution (which depends only on the mental environment,
 175 in particular, memory). However, in general the GKSL-
 176 equation can have a few different equilibrium points. In
 177 such a case depending on the initial state of mind Alice
 178 can obtain different solutions of the same problem. Such
 179 equations with a richer structure of equilibrium points
 180 will be studied in one of coming papers.

181 2. Prisoner’s Dilemma

182 In the paper of [14], we designed a quantum-
 183 like model for decision-making process in two-player
 184 games. This section explains briefly how a player in our
 185 model decides his own action, cf. also Busemeyer et al.
 186 [2].

187 2.1. Pay-off Table of Two-player Game

188 Let us consider a two-player game with two strate-
 189 gies. We name two players Alice (“A”) and Bob (“B”).
 190 Two strategies which A and B can choose are de-
 191 noted by “0” and “1”. The following table shows pay-
 192 offs assigned to possible four consequences of “ $0_A 0_B$ ”,
 “ $0_A 1_B$ ”, “ $1_A 0_B$ ” and “ $1_A 1_B$ ”. Here, a , b , c and d denote

$A \setminus B$	0_B	1_B
0_A	$(a \setminus a)$	$(b \setminus c)$
1_A	$(c \setminus b)$	$(d \setminus d)$

193 the values of pay-offs.

194 For example, a game of prisoner’s dilemma (PD) type
 is characterized by the relation of

$$c > a > d > b.$$

195 For the player A, his pay-off will be a or c if the player
 196 B chose “0” and b or d if the player B chose “1”. In
 197 the both cases, from the relations of $c > a$ and $d >$
 198 b , he can obtain larger pay-offs if he choose 1. The
 199 situation is same for the player B. Conventional game
 200 theory concludes that in PD game a “rational” player,
 201 who wants to maximize his own payoff, always chooses
 202 “1”.

203 However, the above discussion does not explain com-
 204 pletely the process of decision-making in real player’s
 205 mind. Actually, as seen in statistical data in some ex-
 206 periments on so called disjunction effect [24]–[26] real
 207 players frequently behave “irrationally”. Our model
 208 is an attempt to describe such real player’s behaviors
 209 by some mathematical formalism. Our model is a

210 “quantum-like model” which is derived from basic con- 259
 211 cepts of quantum mechanics, but not an expansion of 260
 212 conventional game theory.

213 **Remark 1.** (Rationality) The conventional decision 262
 214 making theory is based on the notion of rationality 263
 215 which is closely related to the *Savage sure thing prin-* 264
 216 *ciple* (SSP) [29] (one of the basic principles of mod- 265
 217 ern economics) having a large impact to cognitive psy- 266
 218 chology and cognitive science, see [12], [11] for popu- 267
 219 lar presentations for biologists and psychologists. This 268
 220 principle matches well our intuition. This is a gen- 269
 221 eral principle of *consistency of our decisions*. If, e.g., 270
 222 Noboru plans to buy some shares under the condition 271
 223 that an event A happens and at the same time he plans 272
 224 to buy these shares even under the condition that A does 273
 225 not happen, then it seems natural for him to buy them 274
 226 even if there is no information whether A happened or 275
 227 not. This principle of consistency of decision making 276
 228 is related to the very foundation of classical probabil- 277
 229 ity theory, *the formula of total probability*, see [2], [12], 278
 230 [11], [9] for details. The formula of total probability 279
 231 is the basis of the Bayesian approach to decision mak- 280
 232 ing. Therefore violation of SPP (which is equivalent 281
 233 to violation of the formula of total probability) implies 282
 234 impossibility to use the Bayesian scheme of decision 283
 235 making. In game theory the approach developed by von 284
 236 Neumann and Morgenstern [30] is also closely related 285
 237 to SSP. In particular, the use of the mini-max principle 286
 238 and the Nash equilibrium are characteristic for “rational 287
 239 players”, rational in the sense of SSP. We remark 288
 240 that SSP and the von Neumann-Morgenstern [30] ap- 289
 241 proach to game theory had been criticized from the first 290
 242 days of their creation. Many bright thinkers pointed 291
 243 to inadequacy of SSP-rationality to real brain’s func- 292
 244 tioning.³ However, a better mathematical model had 293
 245 not been created. In particular, we can point to a re- 294
 246 cent detailed analysis of SSP-rationality [31]. The au- 295
 247 thors of this paper rightly couple SSP-rationality with 296
 248 the Bayesian approach to decision making. They criti- 297
 249 cized the Bayesian scheme of decision making pointing 298
 250 to numerous experiments in cognitive science and psy- 299
 251 chology demonstrating inapplicability of this scheme. 300
 252 However, the conclusion of the authors of [31] – it is im- 301
 253 possible to create a general formal mathematical model 302
 254 of decision making – is not acceptable for us. We gen- 303
 255 eralize the conventional Bayesian scheme of decision 304
 256 making by operating with a priori probabilistic ampli-
 257 tudes, instead of Bayesian a priori probabilities. Thus,
 258 opposite to [31], we claim that, in spite of violations of

³Thus the critique of SSP-rationality is not the main aim of our paper.

SSP, the notion of rationality can be formalized.

Remark 2. (Quantum games) Theory of quantum 261
 262 games is an important part of quantum information the- 263
 264 ory, see, e.g., [32], [33]. In such a game theory quan- 265
 266 tum carriers of information, e.g., photons, are used as 267
 268 a source of randomness. Quantum game theory is typi- 269
 270 cally coupled to quantum *physics*. In contrast, we con- 271
 272 sider games played by “classical” macroscopic players. 273
 274 We do not assume that any really quantum source of ran- 275
 276 domness is involved in a game.⁴ Therefore, opposite to 277
 278 the traditional quantum game theory, we do not assume 279
 280 that Alice and Bob e.g. share entangled photons. This 281
 282 is the basic *interpretational difference* between “physi- 283
 284 cal quantum game theory” and our quantum-like game 285
 286 theory. This difference in interpretations plays a cru- 287
 288 cial role in application of quantum mathematics to real 288
 289 games. Opposite to e.g. [33], we consider the real PD 289
 290 and not a quantum PD-game which is based on sharing 290
 291 of entangled quantum systems by “prisoners.” (Experts 291
 292 in “physical quantum game theory” typically stress its 292
 293 role in future world of quantum technologies.) Our 293
 294 quantum-like game approach has violation of the for- 294
 295 mula of total probability as the point of departure. So, 295
 296 we do not speculate what kind of gambling would be 296
 297 popular after invention of quantum computers and com- 297
 298 munication. Finally, we point to an interesting pub- 298
 299 lication of Cheon and Tsutsui [34] which may serve 299
 300 as a bridge between the traditional theory of quantum 300
 301 games and our quantum-like game theory. They rightly 301
 302 stressed the role of non-factorizability of probabilities 302
 303 in generation of nonclassical strategies. The condition 303
 304 of non-factorizability can be formulated without the us-
 age of the formalism of quantum mechanics. There
 is no need to couple this condition to entanglement of
 states of quantum carriers of information. Therefore the
 Cheon-Tsutsui approach can be considered as quantum-
 like. We now turn to the mathematical structures of
 quantum and quantum-like game theories. If one for-
 get about interpretation differences⁵, then quantum
 and quantum-like games are based on the same mathemat-
 ics – the mathematical formalism of quantum mechan-
 ics. The main problem of the “ordinary quantum game
 theory” is that it considers physical quantum systems as
 sources of randomness. Therefore all actions on these
 systems which are permitted by quantum mechanics can
 be used in a quantum game. All possible unitary trans-

⁴Although we do not reject completely physical quantum models of brain’s functioning, e.g., based on entanglement in brain microtubules, we proceed without coupling to such models.

⁵I guess that for adherents of the conventional interpretation of quantum mechanics it would be impossible.

305 formations are in usage as game strategies. Roughly 348
 306 speaking in this framework it is possible to obtain ev-
 307 erything. In contrast, quantum-like games under con-
 308 sideration are coupled to brain's functioning (at least to
 309 psychological functions). This stimulate *dynamical ap-*
 310 *proach* to decision making, i.e., selection of strategies
 311 through quantum-like mental dynamics, see Busemeyer
 312 et al. [2], [12], Khrennikov [1], [11]. In [14] we made
 313 a new step: in the process of decision making the final
 314 state is determined not by a unitary evolution, but by a
 315 decoherence process described by the quantum master
 316 equation. This is an important extension of the mathe-
 317 matical formalism of quantum game theory. One of the 349
 318 main distinguishing features of the "quantum(-like) de- 350
 319 coherence game theory" is non-invertibility of actions 351
 320 of players. Different initial states can produce the same 352
 321 output. This feature of our model matches well infor- 353
 322 mation processing by (macroscopic) biological systems. 354
 323 (Unitarity is not present in the macroworld. Therefore 355
 324 any model based on application of the quantum formal- 356
 325 ism to macrosystems has to contain a mechanism of vio- 357
 326 lation of linearity; decoherence is the most natural one.)

327 2.2. Complex Hilbert Space Representation of Player's 360 328 Mind

329 We present a model for the decision-making process
 330 in two-players games.⁶ We focus on player A's mind. In
 331 principle, the player A is not informed of which action
 332 the player B chooses. The player A will be conscious
 333 of two potentials of B's action simultaneously, and then
 334 he can not deny either of these potentials. In our model,
 335 this indeterminacy the player A holds is described by
 336 the following quantum superposition

$$|\phi_B\rangle = \alpha|0_B\rangle + \beta|1_B\rangle \in \mathbf{C}^2. \quad (1)$$

337 The values of α and β are related to degrees of con-
 338 sciousness to B's actions (so to say, these are complex
 339 probabilistic amplitudes of A's intentions that B may
 340 make decisions 0 or 1, respectively). We call this com-
 341 plex vector $|\phi_B\rangle$ a *prediction state vector*. (In accordance
 342 with the formalism of quantum mechanics, we assume
 343 $|\alpha|^2 + |\beta|^2 = 1$.)

344 The player A who is getting to choose the action "0"
 345 will be conscious of two consequences of " 0_A0_B " and
 346 " 0_A1_B " with probability amplitudes of α and β . This
 347 situation is described with a vector from $\mathbf{C}^2 \otimes \mathbf{C}^2$ given

by

$$\begin{aligned} |\Phi_{0_A}\rangle &= \alpha|0_A0_B\rangle + \beta|0_A1_B\rangle \\ &= |0_A\rangle \otimes |\phi_B\rangle \end{aligned} \quad (2)$$

Similarly,

$$|\Phi_{1_A}\rangle = |1_A\rangle \otimes |\phi_B\rangle, \quad (3)$$

is given for the situation such that A is getting to choose
 "1". By using these state vectors $|\Phi_{0_A}\rangle$ and $|\Phi_{1_A}\rangle$, we
 define the following vector:

$$|\Psi_A\rangle = x|\Phi_{0_A}\rangle + y|\Phi_{1_A}\rangle \in \mathbf{C}^2 \otimes \mathbf{C}^2, \quad (4)$$

($|x|^2 + |y|^2 = 1$). This state vector describes A's intentions
 to act. We call it a *mental state vector* (of A's intentions
 to act).

The player A's brain in this mental state chooses his
 own action probabilistically. His decision is described
 as "quantum measurements" of projectors correspond-
 ing to the vectors $|\Phi_{0_A}\rangle$ or $|\Phi_{1_A}\rangle$ on the state $|\Psi_A\rangle$. (Prob-
 abilities of "0" and "1" are given by $P(0_A) = P_{0_A} = |x|^2$
 and $P(1_A) = P_{1_A} = |y|^2$.)

In our model, the decision-making process is de-
 scribed as a dynamics changing $|x|^2$ and $|y|^2$, and its dy-
 namics has an *equilibrium solution*. Such stabilization
 of the mental state explains the following psychological
 activity in the player's mind: The player has two psy-
 chological tendencies, the one to choose 0 and the one
 to choose 1. Degrees of these two opposite tendencies
 change in his mind, and they become stable with bal-
 ancing. (Fluctuations die and the definite probabilistic
 picture of the situation is created in A's mind).

The most simple dynamics of the stabilization of
 probabilities, the equations like chemical equilibration
 can be used in cognitive modelling:

$$\begin{aligned} \frac{d}{dt}P_{0_A} &= -kP_{0_A} + \tilde{k}P_{1_A}, \\ \frac{d}{dt}P_{1_A} &= kP_{0_A} - \tilde{k}P_{1_A}. \end{aligned} \quad (5)$$

371 The parameter $k(\tilde{k})$ corresponds to the velocity of the
 372 reaction from 0_A to 1_A (from 1_A to 0_A), and in the equi-
 373 librium state, the probabilities P_{0_A} and P_{1_A} are given as

$$P_{0_A}^E = \frac{\tilde{k}}{k + \tilde{k}}, \quad P_{1_A}^E = \frac{k}{k + \tilde{k}}. \quad (6)$$

374 The differential equations (5) are not described as a
 375 quantum dynamics. In fact, they give only a part of the
 376 complete system of quantum dynamical equations, the
 377 dynamics of the diagonal terms of the density matrix.
 378 We shall complete the system (5) and obtain a quantum
 379 dynamics; a system of differential equation for elements
 380 of the density matrix of the mental state, (12).

⁶At the first stage it repeats the model of Busemeyer et al [2]. How-
 ever, we restrict consideration to factorized initial state. This factor-
 ization will provide a possibility to apply decoherence dynamics.

2.3. Amplitudes of Velocities of Random Transitions in Alice's Mind

As we have seen from Eq. (5), player's tendency to choose 1 or 0 is proportional to the velocity k or \tilde{k} , and these parameters determine the stability solution (6). The choice of k and \tilde{k} is a very important issue in our model. We assume, their values are determined through comparison of possible consequences, $0_A 0_B$, $0_A 1_B$, $1_A 0_B$ and $1_A 1_B$. The player in our model will consider the following four kinds of comparisons:

$$\begin{aligned} 0_A 0_B &\stackrel{k_1}{\rightleftharpoons} 1_A 0_B, & 0_A 1_B &\stackrel{k_2}{\rightleftharpoons} 1_A 1_B, \\ &\tilde{k}_1 & &\tilde{k}_2 \\ 0_A 1_B &\stackrel{k_3}{\rightleftharpoons} 1_A 0_B, & 0_A 0_B &\stackrel{k_4}{\rightleftharpoons} 1_A 1_B. \end{aligned} \quad (7)$$

These comparisons are represented like the conditions of chemical equilibrium, each of which is specified by the reaction velocities, k_i and \tilde{k}_i .

The velocities k and \tilde{k} should have the forms reflecting effects of the four comparisons and interferences between them. In order to define these velocities in the appropriate forms, we introduce complex numbers μ and $\tilde{\mu}$, which determine k and \tilde{k} by

$$k = |\mu|^2, \quad \tilde{k} = |\tilde{\mu}|^2,$$

and define these μ and $\tilde{\mu}$ (see [14] for details) as

$$\begin{aligned} \mu &\equiv |\alpha|^2 \mu_1 + |\beta|^2 \mu_2 + \alpha \beta^* \mu_3 + \alpha^* \beta \mu_4 \\ \tilde{\mu} &\equiv |\alpha|^2 \tilde{\mu}_1 + |\beta|^2 \tilde{\mu}_2 + \alpha^* \beta \tilde{\mu}_3 + \alpha \beta^* \tilde{\mu}_4. \end{aligned} \quad (8)$$

Here $\mu_{i=1,2,3,4}$ and $\tilde{\mu}_{i=1,2,3,4}$ are complex numbers satisfying $|\mu_i|^2 = k_i$, $|\tilde{\mu}_i|^2 = \tilde{k}_i$ for given k_i and \tilde{k}_i . We point out that already here we introduced *complex amplitudes*. These are amplitudes of velocities of fluctuations (in Alice's brain) between various possibilities. Thus, although the system of equations (5) looks as purely classical dynamics, the quantum-like structure of decision making is already incorporated in it, through these complex amplitudes. The system of equations (5) depends on the prediction state, i.e., the state of possible Bob's actions which was created in Alice's brain. This is an example of so called *adaptive dynamics* [28]. In principle, adaptive dynamics in combination with complex amplitudes of velocities is sufficient to reproduce statistical data obtained in experiments on the disjunction effect [24], [25], [26]. As was mentioned, this dynamics can be represented as a quantum-like dynamics of density operator, (12). The use of the quantum-like form

for dynamical equations simplifies essentially the formalism and provides a possibility to use standard mathematical methods of quantum mechanics and quantum information theory. However, this is not the main reason to apply the quantum-like representation of dynamical equations. The main reason is that we proceed under the basic assumption that the brain (as a macroscopic neuronal system) really uses the vector representation of probabilities.

2.4. Quantum-like Model of Irrational Behavior

The parameters (k_i, \tilde{k}_i) introduced in the previous subsection specify the player's four kinds of comparisons, see Eq. (7). It is natural that these comparisons depend on a given game, namely its pay-off table like $(a, b, c$

$A \setminus B$	0_B	1_B
0_A	$(a \setminus a)$	$(b \setminus c)$
1_A	$(c \setminus b)$	$(d \setminus d)$

and d in the above table mean values of pay-offs.) The most simple relation of pay-offs and parameters (k_i, \tilde{k}_i) can be obtained via depending on magnitude relation between values of pay-off. In the case of PD-type game, the relation of pay-offs is $c > a > d > b$, and then, k_i and \tilde{k}_i are given as

$$\begin{aligned} k_1 = 1, \quad k_2 = 1, \quad k_3 = 1, \quad k_4 = 0, \\ \tilde{k}_1 = 0, \quad \tilde{k}_2 = 0, \quad \tilde{k}_3 = 0, \quad \tilde{k}_4 = 1. \end{aligned} \quad (9)$$

Such setting is simple, but not real. The real player's decision-making will depend on differences between pay-offs, not only magnitude relations. That is, the following setting will be more realistic

$$\begin{aligned} k_1 = f_1(|a - c|), \quad k_2 = f_2(|b - d|), \quad k_3 = f_3(|b - c|), \quad k_4 = 0; \\ \tilde{k}_1 = 0, \quad \tilde{k}_2 = 0, \quad \tilde{k}_3 = 0, \quad \tilde{k}_4 = \tilde{f}_4(|a - d|). \end{aligned} \quad (10)$$

The functions $f_i(x)$ are assumed to be monotone increasing functions.

Under the settings of k_i and \tilde{k}_i of (9) or (10), the probability $P_{0_A}^E$ of Eq. (6) is non-zero as a result. Thus, our model explains that the player A generally has potential to make the "irrational" choice of 0 in the PD-game. The reason for this result is that the parameter of \tilde{k}_4 is non-zero. The \tilde{k}_4 represents the degree of tendency to choose 0 which occurs from the comparison between consequences of $0_A 0_B$ and $1_A 1_B$. It should be noted that such comparison is not considered in classical game theory.

3. Dynamics of Alice's Mental State, Density Matrix

In our model, the dynamics of probabilities corresponding to the mental state is specified by the differential equations Eq. (5):

$$\begin{aligned}\frac{d}{dt}P_{0_A} &= -kP_{0_A} + \tilde{k}P_{1_A}, \\ \frac{d}{dt}P_{1_A} &= kP_{0_A} - \tilde{k}P_{1_A}.\end{aligned}$$

Here, $P_{0_A} = |x|^2$ and $P_{1_A} = |y|^2$. (x and $y \in \mathbf{C}$ are coefficients of the mental state $|\Psi\rangle = x|0_A\rangle \otimes |\phi_B\rangle + y|1_A\rangle \otimes |\phi_B\rangle = x|\Phi_{0_A}\rangle + y|\Phi_{1_A}\rangle$, where $|\phi_B\rangle = \alpha|0_B\rangle + \beta|1_B\rangle$, $\alpha, \beta \in \mathbf{C}$ is the prediction vector defined in Eq. (1). The parameters k and \tilde{k} are defined in (8).

In this section, we complete the dynamics Eq. (5) to a quantum dynamics of the density matrix $\rho = |\psi\rangle\langle\psi|$,

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}. \quad (11)$$

corresponding to the mental state $|\psi\rangle = x|0_A\rangle + y|1_A\rangle$ describing the superposition of A's decisions. Details of derivation (based on discrete quantum Markov chains) can be found in [14]. Here we present the final dynamical equation:

$$\begin{aligned}\frac{d}{dt}\rho_{00}(t) &= -|\mu_0|^2\rho_{00} + |\mu_1|^2\rho_{11}, \\ \frac{d}{dt}\rho_{01}(t) &= -\frac{1}{2}\rho_{01} + \mu_0^*\mu_1\rho_{10}, \\ \frac{d}{dt}\rho_{10}(t) &= -\frac{1}{2}\rho_{10} + \mu_0\mu_1^*\rho_{01}, \\ \frac{d}{dt}\rho_{11}(t) &= |\mu_0|^2\rho_{00} - |\mu_1|^2\rho_{11},\end{aligned} \quad (12)$$

where

$$|\mu_0|^2 = \frac{|\mu|^2}{|\mu|^2 + |\tilde{\mu}|^2}, \quad |\mu_1|^2 = \frac{|\tilde{\mu}|^2}{|\mu|^2 + |\tilde{\mu}|^2}.$$

It is clear that the equations for diagonal parts correspond to the equations (5) essentially.⁷

⁷We state again that the dynamics (12) is an oversimplified version of the GKSL-equation. In general, the equations for the diagonal terms contain the off-diagonal terms and vice versa. Surprisingly enough, already the equation (12) describes the process of decision making which is essentially different from the classical Markovian decision making. We shall see that the presence of the off-diagonal terms decreases the entropy of the mental state. In some way off-diagonal terms monitor the interaction with memory, the mental bath, in the process of decision making. In the quantum-like regime the brain uses less memory in the process of decision making, i.e., it performs a more optimal selection of information for decision making.

In this dynamics, any initial state of $\rho(0)$ approaches the unique equilibrium state ρ_E ;

$$\rho_E = \begin{pmatrix} |\mu_0|^2 & 0 \\ 0 & |\mu_1|^2 \end{pmatrix}, \quad (13)$$

see [14] for calculations.

The presence of nondiagonal terms in $\rho(t)$ for finite t does not play any role in the asymptotic limit $t \rightarrow \infty$. However, it modifies essentially the information structure of the evolving mental state. We shall explain this point in more detail.

4. Dynamics of Entropy in the Process of Decision Making

As usual in quantum information theory, we can consider von Neumann entropy, see, e.g., [27], of the quantum (mixed) state $\rho(t)$ and study its dynamics, $S(t) = -\text{Tr}\rho(t)\ln\rho(t)$. This dynamics depends nontrivially on the initial state $\rho(t_0)$ as well as amplitudes of velocities of fluctuations between various possibilities in Alice's brain. In our model Alice's initial state of mind is always pure, this is

$$|\phi_A\rangle = x_0|0_A\rangle + y_0|1_A\rangle, \quad (14)$$

$|x_0|^2 + |y_0|^2 = 1$. The state $|\Psi_A\rangle$, see (4), can be written as

$$|\Psi_A\rangle = |\phi_A\rangle \otimes |\phi_B\rangle. \quad (15)$$

(We state again that the predictions state $|\phi_B\rangle$ providing the quantum-like representation of Alice's image of possible Bob's actions has been already used to determine dynamics of $\rho(t)$, see (8).) The density matrix $\rho(t_0)$ corresponding to the pure state $|\phi_A\rangle$ has the form

$$\rho(t_0) = \begin{pmatrix} |x_0|^2 & x_0y_0^* \\ x_0^*y_0 & |y_0|^2 \end{pmatrix}, \quad (16)$$

We can always assume that x_0 is a real number, $0 \leq x_0 \leq 1$, and

$$y_0 = \sqrt{1 - x_0^2} e^{i\theta_{A0}}.$$

We shall not play with all parameters involved in complex amplitudes μ_0 and μ_1 . We just select μ_0 as a real parameter and

$$\mu_1 = \sqrt{1 - \mu_0} e^{i\theta_\mu}.$$

Figure 1 contains graphs of quantum entropy $S_1(t), S_2(t), S_3(t)$ corresponding to three different choices of parameters:

S_1 , solid line : $x_0 = 0.33, \mu_0 = 0.85, \theta_{A0} = 0.1, \theta_\mu = 0.2$;

S_2 , dashed : $x_0 = 0.315, \mu_0 = 0.625, \theta_{A0} = 0.1, \theta_\mu = 0.2$; ⁵¹³
 S_3 , dotted : $x_0 = 0.99, \mu_0 = 0.925, \theta_{A0} = 0.1, \theta_\mu = 0.2$. ⁵¹⁴

(So, phases are the same for all graphs, we play only with amplitudes.) These graphs show that behavior of quantum entropy depends essentially on parameters, i.e., the initial state of Alice's mind and velocities of fluctuations between various possibilities.

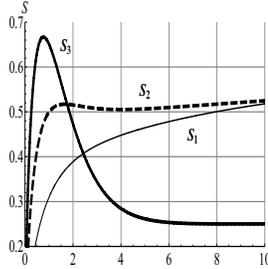


Figure 1: Dynamics of quantum entropy

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There are two main forms of graphs:

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1). Entropy monotonically increases by stabilizing to a constant value; the first stage of the process of decision making is characterized by very quick increasing of entropy, then entropy increases very slowly; for large t , it varies negligibly, see the graph of S_1 .

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We see that the first period of decision making is characterized by attraction of a large amount of new information (mainly from memory, but from the external environment as well); entropy increases very quickly. Then, when the processing of this information is more or less finished, entropy stabilizes (becoming a constant) by approaching the equilibrium point.

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2). Entropy increases very quickly by approaching a local maximum, then it decreases and, finally, stabilizes. Typically, on the way to stabilization it has a local minimum and then it increases again.

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In both cases, at the initial stage of decision making entropy increases as the result of interaction with "environment" (in particular, memory), then (for some clusters of parameters) it decreases and uncertainty is minimized, see S_2, S_3 , but then (before the final step of decision making) it increases again. Sometimes this final increasing of entropy is negligibly small, see the graph of S_3 ; sometimes entropy increases visibly, see S_2 .

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The dynamics of quantum entropy represents the dynamics of uncertainty in Alice's decision state, entropy of choices between strategies. However, it does not describe the dynamics of Alice's diffidence to make the decision at this point t . We are looking for a quantity

which can be considered as a measure of Alice's diffidence.

First of all we introduce classical entropy corresponding to the diagonal terms in the density matrix $\rho(t)$. This is entropy for Alice's choice between two possible decisions if she were ignoring the off-diagonal terms. The presence of these terms is a consequence the quantum-like representation of information in the brain. The complete mental state of Alice is a pure state (of a huge dimension); the density matrix of Alice's decision state is extracted from this state. The quantum-like dynamics of this density matrix induces not only dynamics of the diagonal terms, but even off-diagonal ones. We set

$$SCL(t) = -(\rho_{00}(t) \ln \rho_{00}(t) + \rho_{11}(t) \ln \rho_{11}(t)),$$

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the corresponding graphs are represented on Figure 2.

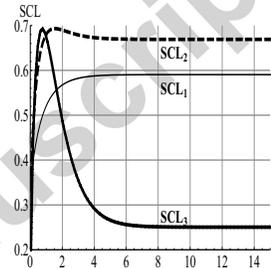


Figure 2: Dynamics of classical entropy

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We see that shapes of the graphs for classical entropy are similar to shapes of corresponding graphs for quantum entropy. However (!), quantum entropy is always less than classical entropy. Thus the quantum-like processing of information induces less uncertainty. In some way, the quantum-like representation in e.g. memory provides a possibility to use a smaller database to make a decision. We can speculate that these off-diagonal terms are responsible for Alice's diffidence.

Let us introduce the diffidence-quantity

$$D(t) = SCL(t) - S(t).$$

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It is positive and, as we see from the graphs on Figure 3, diffidence decreases and it approaches zero when Alice's decision state approaches the point of equilibrium.

Remark 3. As it was rightly pointed by a referee of this paper, "if a decision is made when the difference reaches a threshold, then the decision time is deterministic. This does not explain why is there a distribution of decision times across from the same person." We can speculate that the diffidence threshold ϵ_{DF} is not rigidly determined by brain's architecture of a concrete person,

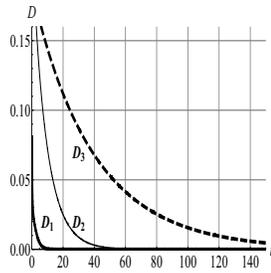


Figure 3: Dynamics of diffidence

535 but it can fluctuate depending on context. It is natural
 536 to assume that ϵ_{DF} has the Gaussian distribution. In this
 537 model decision times are random, since the difference
 538 threshold is random.⁸

539 5. On experimental verification of dynamics of men- 540 tal quantum-like state; coupling to the brain ar- 541 chitecture

542 One of the referees of this paper asked a important
 543 question:

544 “How to measure or observe the dynamics of quan-
 545 tum entropy and difference during decision making in
 546 PD, in neurobiological, behavioral or psychological ex-
 547 periments?”

548 In principle, the formalism of quantum information
 549 theory provides a possibility to reconstruct a density
 550 matrix on the basis of statistical data obtained through
 551 a series of measurements of a family of incompatible
 552 observables – *quantum tomography*. Since observations
 553 are incompatible, we are not able to perform them on
 554 the same biological system: human person, animal, nor
 555 even insect. Therefore the state reconstruction (at each
 556 instance of time) will demand a large statistical ensem-
 557 ble of biological systems participating in experiments.
 558 In theory the family of incompatible observables depend
 559 on continuous parameters; in reality we discretize these
 560 parameters. However, to obtain a good approximation,
 561 the discretization has to be sufficiently fine. Thus even
 562 for one instance of time the experiment demands a huge
 563 group of participants. Moreover, members of such a
 564 group should behave homogeneously. It is not clear
 565 whether it would be possible to perform experiments on
 566 monitoring of mental state dynamics in PD experiments
 567 with people. We speculate that it may be possible to
 568 design PD-type games for simpler biological systems:

⁸A similar problem was discussed in [12].

569 animals? may be even insects? In the later case we have
 570 a better chance to create samples which behave homo-
 571 geneously. This is an interesting problem addressed to
 572 psychologists and cognitive scientists.

573 However, it is too early to concentrate studies on the
 574 aforementioned problem. The following fundamental
 575 problem is essentially more important. In conventional
 576 quantum tomography we have to be able to measure lin-
 577 ear combinations of incompatible observables. For ex-
 578 ample, let \hat{p} and \hat{q} denote the momentum and position
 579 operators, respectively. We also have to measure rota-
 580 tions of the type $\hat{z}_\theta = \cos \theta \hat{q} + \sin \theta \hat{p}$, $\theta \in [0, 2\pi)$. In quan-
 581 tum physics such measurement can be reduced to rota-
 582 tion of the state. And nowadays the technology of quan-
 583 tum state manipulation is well developed, so there is no
 584 problem to perform a state rotation.⁹ We have to de-
 585 velop a similar technology of mental state manipulation
 586 in biology. At the moment it is too early to speculate
 587 about a possibility of realization of this exciting project.
 588 In biology the situation is more complicated than in
 589 physics: there is no (or at least it is not yet known) cou-
 590 pling between the structure of space and mental states.
 591 For example the polarization vector is still meaningful
 592 in quantum physics¹⁰. Therefore we can use the space-
 593 representation of classical physics to prepare quantum
 594 systems in special states of polarization.

595 Another important question is about the *neuronal ar-*
 596 *chitecture* of quantum-like dynamics of decision mak-
 597 ing. By our model the dynamics described by the quan-
 598 tum master equation is realized on neuronal level. It de-
 599 scribes interaction of the mental state encoding possible
 600 decisions with mental environment. As was remarked in
 601 [16]: “An optimal action, namely an action that is most likely
 602 to yield the most desirable outcome, often changes accord-
 603 ing to the state of the animals environment. Therefore, if a sensory
 604 stimulus informs the animal of a change in the state of its en-
 605 vironment, the animal needs to store such information until it
 606 produces an appropriate action or until this information can be
 607 combined with another stimulus to determine the new state of
 608 the environment.” In that paper it was pointed to the *lat-*

⁹We point to the tremendous development of quantum technolo-
 gies of state manipulation during the last ten years. This development
 was stimulated by large interest to quantum computing and cryptog-
 raphy. At the very beginning of quantum mechanics there was a huge
 gap between theoretical consideration involving all possible quantum
 states and possibilities for experimental realization of these states.

¹⁰The situation is more delicate. In quantum physics the polariza-
 tion cannot be defined in the same way as in classical electrodynamics.
 If the polarization vector were well defined, it would serve as a hidden
 variable. As a consequence of Bell’s theorem, the latter does not exist.
 (This viewpoint has not been commonly accepted, see e.g. [11]). In
 any event the operator of projection of polarization to any direction is
 well defined.

609 *eral prefrontal cortex* as responsible for the state repre- 651
 610 sentation of the mental environment. We may speculate 652
 611 that the linear operator of the quantum master equation, 653
 612 see appendix, is stored in this domain of brain. (In the 654
 613 real situation this operator is time dependent.) 655

614 In [15], [16] it was also remarked that lesions which 656
 615 encompass the *orbitofrontal cortex* in human patients as 657
 616 well as psychiatric conditions as substance abuse and 658
 617 frontotemporal dementia may crucially change the abil- 659
 618 ity to represent the *statistical interdependence* between 660
 619 the values of alternative options [16]. Knowledge of 661
 620 such interdependence might allow the decision makers 662
 621 to switch to a better choice immediately, as soon as 663
 622 they experience the reduction of values from a partic- 664
 623 ular choice. We may speculate that the quantum-like 665
 624 representation of the mental state (by a density matrix) 666
 625 is performed in the orbitofrontal cortex. The density 667
 626 matrix describes correlations between alternative op- 668
 627 tions; it seems that lesions in the *orbitofrontal cortex* 669
 628 and aforementioned psychiatric conditions destroy (at 670
 629 least partially) the density matrix. 671

630 **Conclusive remarks.** We elaborated a quantum-like 672
 631 model of decision making based on interaction between 673
 632 the decision-state, i.e., the state carrying probabilities 674
 633 for possible choices, and the state of memory (or more 675
 634 general mental environment). We can speculate that 676
 635 decoherence may be a key element of transition from 677
 636 quantum unconsciousness (in which mind is represented 678
 637 by vector superpositions) to classical consciousness (in 679
 638 which the mind is represented by classical probabilistic 680
 639 mixtures).¹¹

640 6. Appendix: Mathematical formalism of quantum 681 641 mechanics 682

642 The mathematical formalism of quantum mechanics 683
 643 describes states of systems and observables, see book 684
 644 [11] for a simple representation of quantum mathemat- 685
 645 ics for biologists and psychologists. (The quantum- 686
 646 like approach is based on the observation that “quantum 687
 647 mathematics” can be applied outside quantum physics.) 688

648 The basic mathematical structure of quantum mech- 689
 649 anics is a complex Hilbert space: a linear space over 690
 650 complex numbers H (i.e., it is possible to form linear 691

combinations of vectors with complex coefficients) en-
 651 dowed with a Hermitian bilinear form mapping a pair
 652 of vectors $\psi_1, \psi_2 \in H$ into a complex number denoted
 653 $\langle \psi_1 | \psi_2 \rangle$ (we use Dirac’s notation which is typical for
 654 quantum information theory). We shall be interested
 655 in complex vectors normalized by one, i.e., $\psi \in H$
 656 such that $\|\psi\|^2 = \langle \psi | \psi \rangle = 1$. Such vectors encode so
 657 called *pure states* of quantum systems. Normalization
 658 by one is crucial for the probabilistic interpretation of
 659 pure states. Observables (e.g., the energy-observable or
 660 the position observable) are encoded by self-adjoint op-
 661 erators.

The theory is especially simple in the finite dimen-
 662 sional case (which is typically considered in quantum
 663 information theory). Here $H = \mathbf{C}^n$ is the Cartesian
 664 product of n -copies of the set of complex numbers \mathbf{C} .
 665 Hence, a pure state $\phi = (z_1, \dots, z_n)$, where $\|\psi\|^2 =$
 666 $|z_1|^2 + \dots + |z_n|^2 = 1$. Observables are given (in an or-
 667 thonormal basis) by Hermitian matrices $A = (a_{ij})$; here
 668 $\bar{a}_{ij} = a_{ji}$. In the Dirac notation the matrix elements are
 669 written as $\langle \phi_1 | A | \phi_2 \rangle$. Coordinates of a state vector are
 670 interpreted as probabilities (Born’s interpretation of a
 671 pure quantum states [11]). One can say that the quan-
 672 tum formalism provides a geometric representation of
 673 probabilities which is based on the Euclidean distance.

Dynamics of a pure state is described by the
 674 Schrödinger equation

$$675 i \frac{d\psi}{dt}(t) = \mathcal{H}\psi(t), \psi(0) = \psi_0, \quad (17)$$

676 where the operator \mathcal{H} is the generator of evolution
 677 (Hamiltonian, the operator of energy). We remark that
 678 Schrödinger dynamics is deterministic. By fixing the
 679 initial state we can find the state of a quantum system
 680 at any instance of time. The fundamental problem of
 681 quantum foundations is that this state does not deter-
 682 mine values of observables. Measurements induce ran-
 683 domness.

We remark that each pure state ψ determines a self-
 684 adjoint operator, the projector onto this state; $\rho \equiv |\psi\rangle\langle\psi|$
 685 (the last symbol is simply the Dirac notation): $\rho\phi =$
 686 $\langle\phi|\psi\rangle\psi$. We recall the basic properties of ρ_ψ :

- 687 a) it is positively defined, i.e., $\langle\phi|\rho|\phi\rangle \geq 0$ for any ϕ ;
- 688 b) it is Hermitian;
- 689 c) its trace (the sum of diagonal elements) equals to
 690 one.

The Schrödinger dynamics for pure states (vectors)
 691 can be rewritten as dynamics for corresponding opera-
 692 tors:

$$693 i \frac{d\rho}{dt}(t) = [\mathcal{H}, \rho(t)], \rho(0) = \rho_0, \quad (18)$$

¹¹Thus, mind is always probabilistic, but it has different representa-
 694 tions in consciousness and unconsciousness. The process of interac-
 695 tion of the decision-state with memory can be described by the same
 696 model as decoherence of a pure state of a quantum system interact-
 697 ing with an environment, a bath; memory is a kind of mental bath for
 698 decision states.

692 where $[\mathcal{H}, \rho] = \mathcal{H}\rho - \rho\mathcal{H}$ is the commutator of opera- 722
693 tors.

Consider now a statistical mixture (in the classical 723
sense) of a few projection operators ρ_i corresponding 724
to pure states ψ_i with weights $p_i \geq 0$, $\sum p_i = 1$, 725

$$\rho = p_1\rho_1 + \dots + p_n\rho_n. \quad (19)$$

694 Each operator of this form satisfies conditions a)–c) and 728
695 vice versa. Denote the class of all operators with prop- 729
696 erties a)–c) by the symbol $\mathcal{D}(H)$. This is the space of 730
697 states of quantum systems. Its elements (called *density* 731
698 *operators*) can be interpreted as statistical mixtures of 732
699 pure states. In general a density operator can be rep- 733
700 resented in the form (19) in many ways. There is one 734
701 special expansion corresponding to eigenvectors of ρ . 735
702 The density operator corresponding to a pure state can 736
703 be characterized in the following way: in the basis of 737
704 eigenvectors, its matrix has only one nonzero element 738
705 (equal to one), i.e., up to a permutation of eigenvectors: 739

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (20)$$

706 where the blocks of zeros have the corresponding sizes. 747
707 However, this takes place only in the basis of eigenvec- 748
708 tors, cf. (16). 749

Dynamics of a quantum state is described by the 751
equation (18), *von Neumann equation*. This dynamical 752
model can be used only in the absence of interaction of 753
a quantum system with an environment, a bath. If such 754
interaction is essential (so a system cannot be consid- 755
ered as isolated), von Neumann dynamics (18) has to be 756
modified and additional terms have to be included in it. 757
The basic postulate of quantum theory is that the state 758
dynamics is *linear*. Therefore modified dynamics has 759
the form 760

$$i\frac{d\rho}{dt}(t) = [\mathcal{H}, \rho(t)] + L(\rho), \rho(0) = \rho_0, \quad (21)$$

709 where L is a linear operator. This operator has to be 766
710 chosen in such a way that starting with $\rho_0 \in \mathcal{D}(H)$, we 767
711 shall obtain a trajectory $t \rightarrow \rho(t)$ in $\mathcal{D}(H)$. The corre- 768
712 sponding conditions on linear systems were formulated 769
713 by Gorini, Kossakowski, Sudarshan, and Lindblad, see, 770
714 e.g., [27] for details. 771

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721 tober 2010). 782

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>A novel model of quantum-like processing of mental information is presented.

>This model matches the neuronal paradigm of cognitive science.

>Classical electromagnetic signals provide the basis for quantum-like representation.

>Signals are induced by joint activity of neurons.

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