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New Results on the Differential Importance Measures of Markovian Systems

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Abstract: This paper presents the development of the differential importance measures (DIM), proposed recently for the use in risk-informed decision-making, in the context of dynamic systems including inter-component, functional dependencies, or more generally, systems described by Markov models at steady state. Several new evaluation formulas of the existing DIMs are provided. The study also proposes an extension one that provides better results than those obtained from these existing measures, and a new calculation formula of the total variation of the system availability provoked by the simultaneous changes of system parameters. A numerical example is introduced to compare the extension proposed and these existing measures. The suggestions for the future research in this area are also presented.

Keywords: Importance Measure, Differential Importance Measure, Parametric Sensitivity Analysis, Markov Process.

1. INTRODUCTION

Reliability importance measures providing information about the importance of components on the system performance (reliability, maintainability, safety, or any performance metrics of interest) have been widely used in reliability studies and risk analysis. They are useful tools to identify design weakness or operation bottlenecks and to suggest optimal modifications for system upgrade. Recently, a new importance measure, called Differential Importance Measure (DIM), has been introduced for use in risk-informed decision-making [1, 2]. The DIM' is defined as a first-order sensitivity measure that ranks the parameters of the risk model according to the fraction of the total change in the risk metric that is due to a small change in the parameters' values, taken one at a time. Since the DIM' does not account for the effects of simultaneous changes of several parameters, a second-order extension of the DIM' , named DIM'' , is considered in [6]. However, DIM'' is limited to pairs of parameters and only applicable when the impacts of changes of more than two parameters are neglected. Hence the high-order effects related to simultaneous changes of three or more parameters can not be quantified. Furthermore, several existing methods to evaluate the DIM' & DIM'' are based on the system structure function that requires the assumptions of stochastic independent components. Hence, in the realistic case of stochastic dependencies existing between some components (shared maintenance resource, cold spare, shared load ...), and/or high-order effects requirements, the problem remains widely open.

The first objective of this paper is to develop the differential importance measures in the context of dynamic systems including inter-component, functional dependencies, or more generally, systems described by Markov models. In such systems, the (un)availability of a component does not depend only on its characteristics but also on other system parameters, and its (un)availability in the system can be different from its (un)availability out of the system, see [4]. In this context, the partial derivatives with respect to the system parameters, rather than to the components' (un)availability, appears to be more relevant and is often preferred for design purposes. Hence, for steady state, one develops the DIMs based on the derivatives with respect to the system parameters. The results show that all high-order derivatives can be evaluated from the first-order one by recursive rules, and therefore, the major difficulties of high-order derivatives are completely solved. This allows us not only to evaluate more accurately the total system performance change provoked by the change of one parameter or a group of parameters, and also to develop the extensions of high-order differential importance measure, named DIM^h , which can provide better results than those obtained from the DIM' and DIM'' . Obviously, the need to resort to information on the high-order effects depends on the magnitude of the change of the parameters' values.

This paper will be organized as follows. The first section is devoted to define the differential importance measure in the context of Markovian systems. The second section focuses on the first-order approach, or more precisely, the definition and the development of DIM^I . The DIM^{II} is presented in the third section. The fourth section presents the high-order approach, or, the proposed extension DIM^h . A numerical example is introduced in the last section to illustrate the advantages of the proposed extended importance measures.

2. DIFFERENTIAL IMPORTANCE MEASURE

Markov processes have been widely used in reliability to study the performance measures (reliability studies, Availability, Maintainability, etc) of many complex dynamical systems including inter-component, functional dependencies (cold spare, shared load, shared resources ...).

Consider a n-components dynamic system described by a Markov model with the transition matrix M and let a row vector π be the vector of steady state probabilities. Chapman-Kolmogorov equations at the steady state can be written as the following:

$$\pi M = 0 \quad (1)$$

The system availability is:

$$A = \sum_{i \in \Omega_O} \pi_i = \pi f,$$

where: Ω_O is a set of operational states, and $f = (f_1, f_2, \dots)^T$ is a column vector associated with the system states, e.g., $f_i = 1$ if system is in operational state i and $f_i = 0$ otherwise.

Now assume that N parameters of the system (e.g., failure and/or repair rates) are simultaneously changed. For example, all of the component in an aircraft would presumably be subjected to many of the same stresses vibration from the engines, shock of landing, irregularities in the power supplied, etc. The total variation of the system availability due to the changes of N parameters can be expanded in McLaurin series as:

$$\begin{aligned} \delta A &= \sum_{i=1}^N \delta x_i \frac{\partial A}{\partial x_i} + \frac{1}{2!} \left(\sum_{i=1}^N \delta x_i \frac{\partial}{\partial x_i} \right)^2 A + \frac{1}{3!} \left(\sum_{i=1}^N \delta x_i \frac{\partial}{\partial x_i} \right)^3 A + \dots \\ &= \delta^I A + \delta^{II} A + \delta^{III} A + \dots \end{aligned} \quad (2)$$

$$\text{where: } \delta^l A = \frac{1}{l!} \left(\sum_{i=1}^N \delta x_i \frac{\partial}{\partial x_i} \right)^l A = \frac{1}{l!} \sum_{i_1=1}^N \sum_{i_2=1}^N \dots \sum_{i_l=1}^N \delta x_{i_1} \delta x_{i_2} \dots \delta x_{i_l} \frac{\partial^l A}{\partial x_{i_1} \partial x_{i_2} \dots \partial x_{i_l}}, \text{ with } l=1, 2, 3, \dots$$

An importance factor measuring the relative contribution of one parameter on the total change of the system availability, called Differential Importance Measure [1], can be defined as

$$DIM(x_i) = \frac{\delta A_{x_i}}{\delta A}, \quad (3)$$

where: δA_{x_i} is the variation of the system availability provoked by a change of parameter x_i (failure or repair rate). Similarly, this variation can be evaluated by

$$\begin{aligned} \delta A_{x_i} &= \delta x_i \frac{\partial A}{\partial x_i} + \frac{1}{2!} \left(\delta x_i \frac{\partial}{\partial x_i} \right)^2 A + \frac{1}{3!} \left(\delta x_i \frac{\partial}{\partial x_i} \right)^3 A + \dots \\ &= \delta^I A_{x_i} + \delta^{II} A_{x_i} + \delta^{III} A_{x_i} + \dots \end{aligned} \quad (4)$$

$$\text{where: } \delta^l A_{x_i} = \frac{1}{l!} \left(\delta x_i \frac{\partial}{\partial x_i} \right)^l A = \frac{1}{l!} (\delta x_i)^l \frac{\partial^l A}{\partial x_i^l}, \text{ with } l=1, 2, 3, \dots$$

3. FIRST - ORDER APPROACH

Assume that δA_{x_i} in equation (4), and δA in equation (2) can be approximated by the first-order differentiations.

$$\delta A_{x_i} \approx \delta^I A_{x_i}, \text{ and } \delta A \approx \delta^I A$$

3.1. Evaluation of $\delta^I A_{x_i}$ and $\delta^I A$

By taking the derivative with respect to a parameter x_i (failure or repair rate) of equation (1), one gets:

$$\frac{\partial \pi}{\partial x_i} M + \pi \frac{\partial M}{\partial x_i} = 0,$$

or,

$$\frac{\partial \pi}{\partial x_i} M = -\pi Q_{x_i}, \quad (5)$$

where $Q_{x_i} = \frac{\partial M}{\partial x_i}$. Since one eigenvalue of M is zero, its regular inverse does not exist. One has to use a generalized inverse, i.e., the group inverse [3], $M^\# = (M - e\pi)^{-1} - e\pi$, with $e = (1, 1, \dots, 1)^T$ and $M^\# M = M M^\# = I - e\pi$. Multiplying both sides on the right with $M^\#$, one gets:

$$\frac{\partial \pi}{\partial x_i} (I - e\pi) = -\pi Q_{x_i} M^\#,$$

or,

$$\frac{\partial \pi}{\partial x_i} - \frac{\partial \pi e}{\partial x_i} \pi = -\pi Q_{x_i} M^\#.$$

Note well that $\pi e = 1$, so $\frac{\partial \pi e}{\partial x_i} = 0$. One obtains finally:

$$I(x_i) = \frac{\partial \pi}{\partial x_i} = -\pi Q_{x_i} M^\#. \quad (6)$$

Consequently,

$$\delta^I A_{x_i} = \delta x_i \frac{\partial A}{\partial x_i} = \delta x_i \frac{\partial \pi}{\partial x_i} f = -\pi \delta x_i Q_{x_i} M^\# f = -\pi Q_{\delta x_i} M^\# f, \quad (7)$$

and,

$$\delta^I A = \sum_{i=1}^N \delta x_i \frac{\partial A}{\partial x_i} = \sum_{i=1}^N \delta^I A_{x_i} = -\pi \sum_{i=1}^N \delta x_i Q_{x_i} M^\# f = -\pi Q_\delta M^\# f, \quad (8)$$

where: $Q_{\delta x_i} = \delta x_i Q_{x_i}$ and $Q_\delta = \sum_{i=1}^N Q_{\delta x_i}$.

Looking at the (7) and (8), it is clear that δA_{x_i} and δA_δ can be determined from the same formula by changing only the matrix $Q_{\delta x_i}$ and Q_δ .

3.2. First-order differential importance measure

Using the results from (7) and (8), the first-order differential importance measure can be written as

$$\text{DIM}^I(x_i) = \frac{\delta^I A_{x_i}}{\delta^I A} = \frac{\pi Q_{\delta x_i} M^\# f}{\pi Q_\delta M^\# f}. \quad (9)$$

Since DIM^I is based on the first-order differentiations, DIM^I is therefore additive. More explicitly, suppose that one is interested in the DIM^I of the subset of parameters x_i, x_j, \dots, x_s . Then,

$$\text{DIM}^I(x_i, x_j, \dots, x_s) = \frac{\delta^I A_{x_i, x_j, \dots, x_s}}{\delta^I A},$$

where: $\delta^I A_{x_i, x_j, \dots, x_s} = \delta^I A_{x_i} + \delta^I A_{x_j} + \dots + \delta^I A_{x_s}$ (see again equation (8)). So finally

$$\text{DIM}^I(x_i, x_j, \dots, x_s) = \text{DIM}^I(x_i) + \text{DIM}^I(x_j) + \dots + \text{DIM}^I(x_s).$$

Note well that $\text{DIM}^I(x_j), \dots, \text{DIM}^I(x_s)$ can be easily evaluated by changing only $Q_{\delta x_i}$ in equation (9) by $Q_{\delta x_j}, \dots, Q_{\delta x_s}$ respectively.

On the other hand, if one needs only the DIM^I of a parameters subset and one does not need DIM^I of each individual, one can evaluate it directly as follows:

$$\text{DIM}^I(x_i, x_j, \dots, x_s) = \frac{\pi Q_{\delta x_i, \delta x_j, \dots, \delta x_s} M^\# f}{\pi Q_\delta M^\# f},$$

where: $Q_{\delta x_i, \delta x_j, \dots, \delta x_s} = Q_{\delta x_i} + Q_{\delta x_j} + \dots + Q_{\delta x_s}$.

4. SECOND - ORDER APPROACH

The DIM^I can be easily evaluated with equation (9), but it does not take into account either the second/higher-order effects or the impacts of the simultaneous changes of several parameters, and it can therefore only be used when the changes of parameters verify the small enough conditions (see [1] for the details). The idea of second-order extension of the DIM^I , named DIM^II , is considered in [6], however, it is only applicable for the independents components systems with the changes due to the components' (un) availability. This section explores the development of DIM^II in the context of Markovian systems and the changes due to the system parameters (failure or/and repair rates).

Now assume that δA_{x_i} in equation (4) and δA in equation (2) can be approximated by the sum of the first and second-order differentiations, e.i.

$$\delta A_{x_i} \approx \delta^I A_{x_i} + \delta^{II} A_{x_i}, \text{ and } \delta A \approx \delta^I A + \delta^{II} A.$$

4.1. Evaluation of $\delta^{II} A_{x_i}$ and $\delta^{II} A$

By taking the derivative with respect to a parameter x_j of equation (5):

$$\frac{\partial^2 \pi}{\partial x_i \partial x_j} M + \frac{\partial \pi}{\partial x_i} \frac{\partial M}{\partial x_j} = -\frac{\partial \pi}{\partial x_j} Q_{x_i},$$

or,

$$\frac{\partial^2 \pi}{\partial x_i \partial x_j} M = -\frac{\partial \pi}{\partial x_i} Q_{x_j} - \frac{\partial \pi}{\partial x_j} Q_{x_i}. \quad (10)$$

Using the group inverse $M^\#$, and noting that $MM^\# = I - e\pi$, $\pi e = 1$, one gets:

$$I(x_i, x_j) = \frac{\partial^2 \pi}{\partial x_i \partial x_j} = -\left(\frac{\partial \pi}{\partial x_i} Q_{x_j} + \frac{\partial \pi}{\partial x_j} Q_{x_i}\right) M^\#. \quad (11)$$

So:

$$\begin{aligned} \sum_{i=1}^N \sum_{j=1}^N \delta x_i \delta x_j \frac{\partial^2 \pi}{\partial x_i \partial x_j} &= -\left(\sum_{i=1}^N \delta x_i \frac{\partial \pi}{\partial x_i} \sum_{j=1}^N \delta x_j Q_{x_j} + \sum_{j=1}^N \delta x_j \frac{\partial \pi}{\partial x_j} \sum_{i=1}^N \delta x_i Q_{x_i} \right) M^\# \\ &= -2 \sum_{i=1}^N \delta x_i \frac{\partial \pi}{\partial x_i} \sum_{j=1}^N \delta x_j Q_{x_j} = -2 \sum_{i=1}^N \delta x_i \frac{\partial \pi}{\partial x_i} Q_\delta M^\#. \end{aligned} \quad (12)$$

Finally, by using (6) and (11), one obtains:

$$\delta'' A = \pi \sum_{i=1}^N \delta x_i Q_{x_i} M^\# Q_\delta M^\# f = \pi (Q_\delta M^\#)^2 f. \quad (13)$$

So,

$$\delta'' A_{x_i} = \pi (Q_{\delta x_i} M^\#)^2 f. \quad (14)$$

4.2. Second-order differential importance measure

By using (7), (8), (13) and (14), the second-order differential importance measure can be written as:

$$\text{DIM}''(x_i) = \frac{\delta' A_{x_i} + \delta'' A_{x_i}}{\delta' A + \delta'' A} = \frac{-\pi Q_{\delta x_i} M^\# f + \pi (Q_{\delta x_i} M^\#)^2 f}{-\pi Q_\delta M^\# f + \pi (Q_\delta M^\#)^2 f} = \frac{\pi Q_{\delta x_i} M^\# (I - Q_{\delta x_i} M^\#) f}{\pi Q_\delta M^\# (I - Q_\delta M^\#) f}. \quad (15)$$

Note well that DIM'' is no longer additive since DIM'' related to a subset of parameters accounts the impacts of simultaneous changes of pairs of parameters, but DIM'' related to a individual one does not. The DIM'' of the subset of parameters can however be expressed as the following:

$$\text{DIM}''(x_i, x_j, \dots, x_s) = \frac{\delta' A_{x_i x_j \dots x_s} + \delta'' A_{x_i x_j \dots x_s}}{\delta' A + \delta'' A} = \frac{\pi Q_{\delta x_i, \delta x_j, \dots, \delta x_s} M^\# (I - Q_{\delta x_i, \delta x_j, \dots, \delta x_s} M^\#) f}{\pi Q_\delta M^\# (I - Q_\delta M^\#) f}.$$

It is obvious that the results obtained from DIM'' are more precise than those provided by DIM' due to the second-order differentiation effects. However, the DIM'' is only applicable when the higher-order effects and the impacts of the changes of more than two parameters are neglected. From a practical point of view, this assumption is not always reliable. Hence, the next section is devoted to the development of the high-order approach which quantifies the higher-order effects and the impacts of simultaneous changes of three or more parameters.

5. HIGH - ORDER APPROACH

The impact of h th-order differentiations ($h=3,4,\dots$) can be expressed as the following:

$$\delta^h A_{x_i} = \frac{1}{h!} \left(\delta x_i \frac{\partial}{\partial x_i} \right)^h A = \frac{1}{h!} (\delta x_i)^h \frac{\partial^h \pi}{\partial^h x_i} f,$$

and,

$$\delta^h A = \frac{1}{h!} \left(\sum_{i=1}^N \delta x_i \frac{\partial}{\partial x_i} \right)^h A = \frac{1}{h!} \sum_{i_1=1}^N \sum_{i_2=1}^N \dots \sum_{i_h=1}^N \delta x_{i_1} \delta x_{i_2} \dots \delta x_{i_h} \frac{\partial^h \pi}{\partial x_{i_1} \partial x_{i_2} \dots \partial x_{i_h}} f.$$

5.1. Evaluation of $\delta^h A_{x_i}$ and $\delta^h A$

By taking the derivative with respect to a parameter x_k of equation (10), one obtains the following equation:

$$\frac{\partial^3 \pi}{\partial x_i \partial x_j \partial x_k} M + \frac{\partial^2 \pi}{\partial x_i \partial x_j} \frac{\partial M}{\partial x_k} = -\frac{\partial^2 \pi}{\partial x_i \partial x_k} Q_{x_j} - \frac{\partial^2 \pi}{\partial x_j \partial x_k} Q_{x_i},$$

or,

$$\frac{\partial^3 \pi}{\partial x_i \partial x_j \partial x_k} M = -\frac{\partial^2 \pi}{\partial x_i \partial x_j} Q_{x_k} - \frac{\partial^2 \pi}{\partial x_i \partial x_k} Q_{x_j} - \frac{\partial^2 \pi}{\partial x_j \partial x_k} Q_{x_i}.$$

Using the group inverse $M^\#$ with $MM^\# = I - e\pi$ and $\pi e = 1$. One gets:

$$\frac{\partial^3 \pi}{\partial x_i \partial x_j \partial x_k} = -\left(\frac{\partial^2 \pi}{\partial x_i \partial x_j} Q_{x_k} + \frac{\partial^2 \pi}{\partial x_i \partial x_k} Q_{x_j} + \frac{\partial^2 \pi}{\partial x_j \partial x_k} Q_{x_i} \right) M^\#.$$

So:

$$\begin{aligned} \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \delta x_i \delta x_j \delta x_k \frac{\partial^3 \pi}{\partial x_i \partial x_j \partial x_k} = & - \left(\sum_{i=1}^N \sum_{j=1}^N \delta x_i \delta x_j \frac{\partial^2 \pi}{\partial x_i \partial x_j} \sum_{k=1}^N \delta x_k Q_{x_k} + \sum_{i=1}^N \sum_{k=1}^N \delta x_i \delta x_k \frac{\partial^2 \pi}{\partial x_i \partial x_k} \sum_{j=1}^N \delta x_j Q_{x_j} + \right. \\ & \left. + \sum_{j=1}^N \sum_{k=1}^N \delta x_j \delta x_k \frac{\partial^2 \pi}{\partial x_j \partial x_k} \sum_{i=1}^N \delta x_i Q_{x_i} \right) M^\# = -3 \sum_{i=1}^N \sum_{j=1}^N \delta x_i \delta x_j \frac{\partial^2 \pi}{\partial x_i \partial x_j} Q_\delta M^\#. \end{aligned} \quad (16)$$

From (6), (12) and (16), the following equation can be shown:

$$\delta^{\text{III}} A = \frac{1}{3!} \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \delta x_i \delta x_j \delta x_k \frac{\partial^3 \pi}{\partial x_i \partial x_j \partial x_k} f = -\pi(Q_\delta M^\#)^3 f.$$

Similarly, by taking the h th-order derivatives, one obtains finally:

$$\delta^h A = (-1)^h \pi(Q_\delta M^\#)^h f, \text{ with } h = 3, 4, 5, \dots \quad (17)$$

Looking at this equation, it is clear that the h th-order effects can be easily evaluated and it is based only on the first-order derivative, or more precisely, based on the group inverse $M^\#$.

By using (8), (13), and (17), the total variation of the system availability provoked by the changes of N parameters at the same time, δA , can be approximated by the following equation:

$$\delta A \approx -\pi Q_\delta M^\# f + \pi(Q_\delta M^\#)^2 f + \dots + (-1)^h \pi(Q_\delta M^\#)^h f. \quad (18)$$

And the variation of the system availability provoked by the change of one parameter can be:

$$\delta A_{x_i} \approx -\pi Q_{\delta x_i} M^\# f + \pi(Q_{\delta x_i} M^\#)^2 f + \dots + (-1)^h \pi(Q_{\delta x_i} M^\#)^h f. \quad (19)$$

5.2. High-order differential importance measure

From (18) and (19), the high-order differential importance measure can be written as:

$$\text{DIM}^h(x_i) = \frac{-\pi Q_{\delta x_i} M^\# f + \pi(Q_{\delta x_i} M^\#)^2 f + \dots + (-1)^h \pi(Q_{\delta x_i} M^\#)^h f}{-\pi Q_\delta M^\# f + \pi(Q_\delta M^\#)^2 f + \dots + (-1)^h \pi(Q_\delta M^\#)^h f}. \quad (20)$$

$\text{DIM}^h(x_i, x_j, \dots, x_s)$ for simultaneous changes of several parameters can be evaluated with equation (20) by only the taking $Q_{\delta x_i \delta x_j \dots \delta x_s}$ instead of $Q_{\delta x_i}$.

Note well that DIMs can be evaluated by the finite differences method, the changes of the system configuration may however be required for each evaluation of the availability variation. The interest of these approaches is precisely to be able to evaluate DIMs of any parameter or any group of parameters by changing only the corresponding matrix Q without additional calculations.

6. A NUMERICAL EXAMPLE

The purpose of this section is to show how the DIMs can be used in reliability sensitivity analysis and to compare DIM^I , DIM^{II} with proposed extension DIM^h through a simple example sketched in Figure 1. Unit 4 (C4) is in cold redundancy with unit 3 (C3). As soon as C3 is repaired, C4 stops. This system includes also a shared load between unit 1 (C1) and branch of unit 2 (C2), C3 and C4. The failure of C1 increases therefore from λ_1 to $\bar{\lambda}_1$ when the branch (C2, C3 and C4) is failed. Table 1 gives the values of failure rates $\lambda_i / \bar{\lambda}_i$, and the repair rates μ_i ($i = 1, \dots, 4$).

Assume that all components' failure rates are changed simultaneously. For example, all of the component in an aircraft would presumably be subjected to many of the same stresses vibration from the engines, shock of landing, irregularities in the power supplied, etc.

Figure 1: System structure

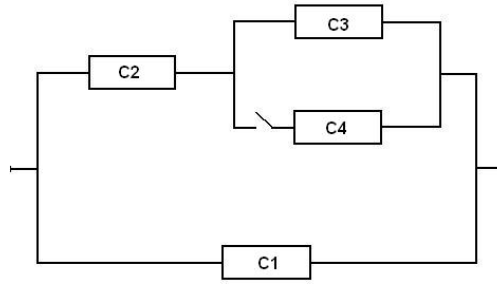


Table 1: Components' failure/repair rate

Unit	λ_i	μ_i	$\bar{\lambda}_i$
C1	2×10^{-3}	1×10^{-3}	2.5×10^{-3}
C2	1×10^{-3}	9×10^{-3}	-
C3	2×10^{-3}	2.9×10^{-3}	-
C4	3×10^{-3}	6×10^{-3}	-

To illustrate the application of DIMs, a uniform percentage change ($\frac{\delta\lambda_i}{\lambda_i} = \frac{\delta\lambda_j}{\lambda_j} = \omega$, with $i, j = 1, \dots, 4$) is considered, and more precisely, three scenarios of changes are proposed: $\omega = 1\%$, $\omega = 7\%$ and $\omega = 10\%$ in order to show the need of the high order approach.

Table 2: DIMs & important component's ranking, cas $\omega = 1\%$

DIMs	$C1(\lambda_1, \bar{\lambda}_1)$	$C2(\lambda_2)$	$C3(\lambda_3)$	$C4(\lambda_4)$	Order
DIM^I	0.222047	0.209410	0.210678	0.357866	$C4 > C1 > C3 > C2$
DIM^{II}	0.220546	0.209234	0.209637	0.357326	$C4 > C1 > C3 > C2$
DIM^{III}	0.220560	0.209238	0.209645	0.357333	$C4 > C1 > C3 > C2$
DIM^{VI}	0.220560	0.209238	0.209645	0.357333	$C4 > C1 > C3 > C2$

Table 2 represents the results of DIM^I , DIM^{II} , DIM^{III} and DIM^{VI} for the case $\omega = 1\%$. According to these importance measures, the components importance rankings could be drawn. The results show that DIM^I , DIM^{II} , DIM^{III} and DIM^{VI} can provide the same ranking. The most important component is C4 and C2 is the less important one.

Table 3: DIMs & important component's ranking, cas $\omega = 7\%$

DIMs	$C1(\lambda_1, \bar{\lambda}_1)$	$C2(\lambda_2)$	$C3(\lambda_3)$	$C4(\lambda_4)$	Order
DIM^I	0.222047	0.209410	0.210678	0.357866	$C4 > C1 > C3 > C2$
DIM^{II}	0.211533	0.208183	0.203384	0.354088	$C4 > C1 > C2 > C3$
DIM^{III}	0.212220	0.208364	0.203818	0.354426	$C4 > C1 > C2 > C3$
DIM^{VI}	0.212189	0.208356	0.203801	0.354412	$C4 > C1 > C2 > C3$

Looking at the results presented in Table 3 and Table 4, the DIM^I of all components' failure rates remains unchanged that is why the component importance ranking based on DIM^I always holds. However, the DIM^{II} , DIM^{III} , and DIM^{VI} 's values change leading to different rankings. For the case $\omega = 7\%$, the DIM^{II} , DIM^{III} , and DIM^{VI} provide the same importance ranking. Note however that

this ranking is not the same as the one obtained by DIM^I , this means DIM^I may not be used for this case.

Table 4: DIMs & important component's ranking, cas $\omega = 10\%$

DIMs	$C1(\lambda_1, \bar{\lambda}_1)$	$C2(\lambda_2)$	$C3(\lambda_3)$	$C4(\lambda_4)$	Order
DIM^I	0.222047	0.209410	0.210678	0.357866	$C4 > C1 > C3 > C2$
DIM^{II}	0.207021	0.207656	0.200254	0.352466	$C4 > C2 > C1 > C3$
DIM^{III}	0.208418	0.208025	0.201136	0.353156	$C4 > C1 > C2 > C3$
DIM^{VI}	0.208329	0.208003	0.201088	0.353116	$C4 > C1 > C2 > C3$

For the case $\omega = 10\%$, the importance rankings based on DIM^I and on DIM^{II} are not the same and both of them are different from the one provided by DIM^{III} or DIM^{VI} (see Table 4). This means that DIM^{III} can provide a more precise importance ranking than DIM^I and DIM^{II} .

It is therefore clear that the high-order approach would provide better results than those obtained from the first and second-order approach. Obviously, the need to resort to information on the high-order effects depends on the magnitude of the change of parameters' values.

7. CONCLUSIONS

In this work, the differential importance measures applied traditionally in the context of systems with independent components described by static boolean models is extended to the dynamic systems including inter-components, functional dependencies, or more generally, systems described by Markov models. In the cases of steady state, our developments allow: (i) several new computation formulas for DIM^I , DIM^{II} ; (ii) the definition of DIM^h which provides better results than those obtained from DIM^I and DIM^{II} ; (iii) the more accurate evaluation of the total system performance change provoked by the simultaneous changes of system parameters at the same time.

Our further research focuses on the bounds of McLaurin series to find the minimal h for which DIM^h can provide the true importance ranking, and on the estimation method to evaluate the DIMs from the operating feedback data. At last more detailed applications of these measures to decision-making in reliability engineering, e.g. to the optimization of maintenance policies, are under study.

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