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Importance Measure on Finite Time Horizon and Application to Markovian Multi-state Production Systems

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ABSTRACT: The sensitivity analysis of complex industrial systems aims at identifying, in a multi-unit structure, which components contribute the most to a variation of the performance criterion. In this paper an importance factor, called multi-directional sensitivity measure, defined as the derivative of the performance in the direction of one parameter, in the direction of a group of parameters (failure and repair rates of components for example), or in any direction of the transition rates of a Markovian system is considered. This importance factor proposed for sensitivity analysis of steady state reliability is developed herein for the transient state. It is also extended and applied to the study of the production capacity of multi-state production systems, such as e.g. manufacturing, production lines, which exhibit performances that can settle on different levels depending on the operative conditions of the constitutive components. A simple numerical example is introduced to show why this factor provides an efficient tool to investigate not only the importance of a given component, but also the importance of a class of components, the importance of the maintenance, and, more generally, the effect of the simultaneous change of several design parameters.

1 INTRODUCTION

The sensitivity analysis of complex industrial systems aims at identifying, in a multi-unit structure, which components contribute the most to a variation of the performance criterion. In classical reliability studies (Rausand & Hoyland 2004), many factors are considered (Birnbaum, Fussell-Vesely, Critical importance factors,...) to classify the different elements of a multi-unit system by order of importance. Hence, for example, the decisions for preventive and corrective maintenance, or the monitoring schedule, can be tuned as a function of this classification. Many studies have been done to improve the calculation of these factors, especially when the components can be considered as stochastically independent. In the realistic case of stochastic dependencies existing between some components (shared maintenance resource, cold spare, shared load,...), the definition and the calculation of other criterions is needed and more and more different approaches are proposed. The exact solution for the sensitivity measures for a Markov model relies on the Frank's approach (Frank 1978): the classical set of differential equations is extended to a bigger set of equations including the sensitivity factor equations. However, this approach is computationally burdensome and almost unusable or highly inefficient on a realistic-size systems when the state space dimension is too high. To cope with this problem, some approximate solutions have been

proposed but they are often only applicable on a limited class of systems (e.g. acyclic Markov models with no repair), (Ou & Dugan 2003). Hence, the problem remains widely open.

In the framework of Markov Models, it has been shown in (Do Van et al. 2006) that the perturbation analysis and one of its extension presented in (Cao & Chen 1997) can be very well adapted to reliability or maintenance problems at steady state. The aim of the present paper is to show that the same importance factor can be considered in the transient state. Hence, the sensitivity analysis can be extended from steady state performances to transient state performances. This extension allows, for example, the sensitivity analysis of systems performances on a finite time horizon. The presented importance factor, called multi-directional sensitivity measure (MDSM), corresponds to the derivative of the performance function in the direction of one parameter, in the direction of a group of parameters (failure and repair rates of components for example), or in any direction of the transition rates of the Markov process. More precisely, this importance factor can provide an efficient tool to:

- identify the importance of a given component (parameter), and also the importance of a class of components with respect to the system performance of interest;

- evaluate the effect of the change in any direction of one parameter or a group of design parameters;
- solve maintenance policy optimization and performance improvement problems.

On the other hand, from a practical point of view, many systems such as e.g. manufacturing, production lines, exhibit performance that can settle on different levels (e.g. 100%, 90%, 80%, ... of the nominal capacity), depending on the operative conditions of the constitutive components. These components can be stochastically dependent (Kawauchi & Rausand 2002) and the production capacity has often to be evaluated on a finite-time horizon, and not only at steady state. Many authors have defined importance measures for multi-state systems (Levitin & Lisnianski 1999; Zio & Podofillini 2003; Ramirez-Marquez & Coit 2005) but they mainly focus on universal generating function method and Monte Carlo simulation. In this paper, the multi-directional sensitivity measure, MDSM, can be used to study the sensitivity of the production capacity in the context of Markovian multi-state production systems.

This paper is organized as follows: Section 2 is devoted to the presentation of MDSM for the reliability sensitivity analysis on finite time horizon, and MDSM for the average reliability sensitivity during a given period of time. The link with MDSM of the reliability at steady-state, presented in (Do Van et al. 2006), is also established. Section 3 focuses on the application to multi-state production systems. It is shown how MDSM is used as an appropriate tool for the production capacity sensitivity analysis during a given time period of interest, and also at steady-state. A simple numerical example is introduced in section 4 to illustrate the advantages of the proposed importance measure, MDSM, for both reliability studies and production capacity sensitivity analysis. Finally, Section 5 presents the conclusions drawn from this work.

NOTATION LIST

A	transition rates matrix of Markov models
A^\sharp	group inverse of A
Q	directional perturbation matrix
$P(t)$	column vector of state probabilities at time t
$\tilde{P}(t)$	column vector of average state probabilities vector during a given period time $[0, t]$
π	column vector of steady-state probabilities
$R(t)$	system reliability (availability) at time t
$\bar{R}(t)$	average reliability (availability) during a given period time $[0, t]$
R	system availability at infinite time
X	row vector of state production capacities
$S(t)$	system production capacity at time t
$\bar{S}(t)$	average production capacity during a given period time $[0, t]$

S	system production capacity at infinite time
λ, μ	failure and repair rate of one unit
$I_Q^R(t), I_Q^S(t)$	sensitivity of $R(t)$ and $S(t)$ in the direction Q
$I_\lambda^R(t), I_\lambda^S(t)$	sensitivity of $R(t)$ and $S(t)$ w.r.t the parameter λ
\bar{I}_Q^R, \bar{I}_Q^S	sensitivity of $\bar{R}(t)$ and $\bar{S}(t)$ in the direction Q
I_Q^R, I_Q^S	sensitivity of R and S in the direction Q

2 IMPORTANCE MEASURE ON FINITE TIME HORIZON

Markov processes have been widely used to analyse and assess the performances (reliability, availability, maintainability, production capacity, etc...) of many complex dynamical systems with inter-component and functional dependencies (cold spare, shared load, shared resources, ...). This section explores the application of MDSM in reliability studies of Markovian systems on a finite time horizon (transient state), and also a link with MDSM at steady state presented in (Do Van et al. 2006).

Consider a dynamic system described by a Markov model and let the column vector $P(t)$ be the vector of state probabilities, and P_0 be the initial state probabilities vector. The system of the first order Chapman-Kolmogorov equations applied to homogeneous Markovian process (without additional dynamical variables) is:

$$\frac{dP(t)}{dt} = AP(t). \quad (1)$$

The solution of (1) can be expressed as:

$$P(t) = e^{At}P_0 = F_A(t)P_0, \quad (2)$$

where $F_A(t) = e^{At}$ is the exponential matrix.

A perturbation on one parameter or a group of parameters of the system is equivalent to a perturbation in the transition rates matrix A . It leads to a perturbed transition matrix:

$$A_\delta = A + \delta Q,$$

where δ is a small real number and Q is a directional matrix in which a 0 indicates that the parameter at the same place in A is not perturbed and a number α different from 0 indicates that the parameter at the same place in A is perturbed of an amount $\alpha\delta$. The only condition on the structure of Q to ensure that the matrix A_δ is also a transition matrix is the sum of its columns equals 0. As an example, two units C_1 and C_2 in a parallel structure with constant failure rates λ_1, λ_2 and constant repair rates μ_1, μ_2 are considered. Each component can be running or failed (failed states are noted \bar{C}_1 and \bar{C}_2). The Markov graphs of this system with two types of perturbation are sketched

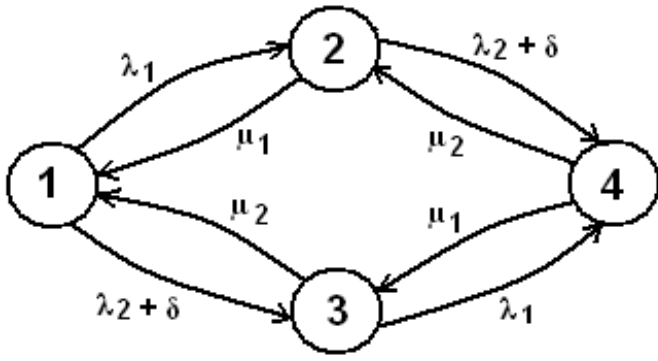


Figure 1. Perturbation on λ_2 .

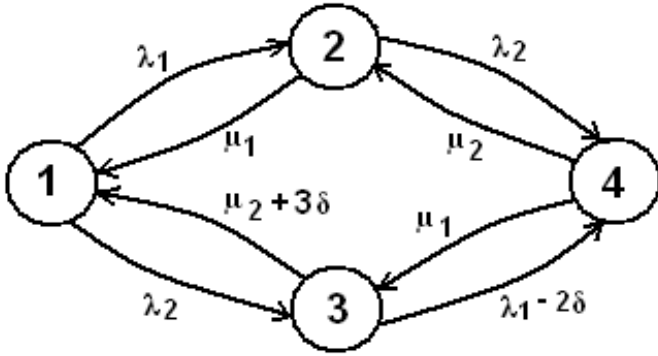


Figure 2. Perturbation on state 3.

in Figures 1 and 2: one perturbation on one specific parameter in Figure 1 (on λ_2), and the other on the probability of being in one specific state in Figure 2 (state number 3). These perturbations correspond to two different directional matrixes Q_1, Q_2 :

$$Q_1 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -2 & 0 \end{pmatrix}.$$

The variations in the transition rates matrix affect the transient solution $P(t)$ that becomes $P_\delta(t)$ (with the same initial condition $P_\delta(0) = P(0)$); $P_\delta(t)$ verifies:

$$\frac{dP_\delta(t)}{dt} = A_\delta P_\delta(t). \quad (3)$$

The derivative of $P(t)$ in the direction of Q is defined as:

$$\frac{dP(t)}{dQ} = \lim_{\delta \rightarrow 0} \frac{P_\delta(t) - P(t)}{\delta}. \quad (4)$$

This is the key quantity that is used to define the MDSM in the following.

2.1 Transient state

Let $Z(t) = P_\delta(t) - P(t)$. From Equations 1 and 3, $Z(t)$ verifies:

$$\frac{dZ(t)}{dt} = A_\delta Z(t) + (A_\delta - A)P(t). \quad (5)$$

With the initial condition $Z(t_0) = P_\delta(t_0) - P(t_0)$, the solution of (5) is:

$$Z(t) = e^{A_\delta(t-t_0)} Z(t_0) + \int_{t_0}^t e^{A_\delta(t-s)} (A_\delta - A)P(s) ds.$$

Chose $t_0 = 0$, so $Z(t_0) = 0$, hence $Z(t)$ can be expressed as:

$$Z(t) = \int_0^t e^{A_\delta(t-s)} (A_\delta - A)P(s) ds,$$

or,

$$Z(t) = \int_0^t F_{A_\delta}(t-s) (A_\delta - A)P(s) ds. \quad (6)$$

Replacing $P_\delta(t) - P(t)$ in (4) by $Z(t)$ and using (6), the derivative of $P(t)$ in the direction of Q can be expressed as:

$$\frac{dP(t)}{dQ} = \lim_{\delta \rightarrow 0} \frac{1}{\delta} \int_0^t F_{A_\delta}(t-s) (A_\delta - A)P(s) ds.$$

$$\frac{dP(t)}{dQ} = \lim_{\delta \rightarrow 0} \int_0^t F_{A_\delta}(t-s) \frac{A_\delta - A}{\delta} P(s) ds.$$

Using $Q = (A_\delta - A)/\delta$,

$$\frac{dP(t)}{dQ} = \int_0^t (\lim_{\delta \rightarrow 0} F_{A_\delta}(t-s)) QP(s) ds.$$

$$\frac{dP(t)}{dQ} = \int_0^t F_A(t-s) QP(s) ds. \quad (7)$$

Using (2), Equation 7 can be written as:

$$\frac{dP(t)}{dQ} = \int_0^t F_A(t-s) QF_A(s) P_0 ds. \quad (8)$$

The reliability (availability) of the system is defined as:

$$R(t) = \sum_{i \in \Omega_O} P_i(t) = fP(t),$$

where Ω_O is a set of operational states, and $f = (f_1, f_2, \dots, f_n)$ is a row vector associated with the performance of the system in each state. For reliability models, $f_i = 1$ if system is operational in state i and $f_i = 0$ otherwise. The sensitivity of the system reliability $R(t)$ in the direction of interest Q (i.e the MDSM of $R(t)$) is defined as:

$$I_Q^R(t) = \frac{dR(t)}{dQ} = f \frac{dP(t)}{dQ}.$$

Using (8), $I_Q^R(t)$ can be expressed as:

$$I_Q^R(t) = \int_0^t f F_A(t-s) QF_A(s) P_0 ds. \quad (9)$$

$I_Q^R(t)$ may be evaluated by a numerical integration method or directly by making a suitable expansion of matrix exponentials by using, for example, the uniformisation method (Neuts 1995).

Equation 9 allows the evaluation of the system reliability sensitivity in any direction of interest at time t in the transient state.

2.2 Average on a finite time horizon

As mentioned earlier, the sensitivity analysis of the average reliability during a given period of time can be considered as a performance metrics of interest for reliability studies.

By taking integrals for a given period of time $[0, t]$, the following differential equation can be derived from Equation 5:

$$\int_0^t \frac{dZ(s)}{ds} ds = A_\delta \int_0^t Z(s) ds + (A_\delta - A) \int_0^t P(s) ds.$$

So:

$$Z(t) - Z(0) = A_\delta \int_0^t Z(s) ds + (A_\delta - A) \int_0^t P(s) ds. \quad (10)$$

Let us define:

- $\tilde{P}(t) = \int_0^t P(s) ds.$
- $\tilde{Z}(t) = \tilde{P}_\delta(t) - \tilde{P}(t) = \int_0^t Z(s) ds.$

Note that $d\tilde{Z}(t)/dt = Z(t)$ and $Z(0) = 0$, so the differential equation 10 can be written as:

$$\frac{d\tilde{Z}(t)}{dt} = A_\delta \tilde{Z}(t) + (A_\delta - A) \tilde{P}(t), \quad (11)$$

whose the solution is:

$$\tilde{Z}(t) = \int_0^t F_{A_\delta}(t-s)(A_\delta - A) \tilde{P}(s) ds.$$

The derivative of $\tilde{P}(t)$ in the direction Q is expressed as:

$$\frac{d\tilde{P}(t)}{dQ} = \lim_{\delta \rightarrow 0} \frac{\tilde{Z}(t)}{\delta} = \lim_{\delta \rightarrow 0} \int_0^t F_{A_\delta}(t-s) Q \tilde{P}(s) ds,$$

or,

$$\frac{d\tilde{P}(t)}{dQ} = \int_0^t F_A(t-s) Q \tilde{P}(s) ds. \quad (12)$$

The average reliability during a given period $[0, t]$:

$$\bar{R}(t) = \frac{1}{t} \int_0^t R(s) ds = \frac{1}{t} \int_0^t f P(s) ds = \frac{1}{t} f \tilde{P}(t).$$

The sensitivity of $\bar{R}(t)$ in the direction Q (i.e the MDSM of $\bar{R}(t)$) is finally:

$$\bar{I}_Q^R = \frac{d\bar{R}(t)}{dQ} = \frac{1}{t} f \frac{d\tilde{P}(t)}{dQ} = \frac{1}{t} f \int_0^t F_A(t-s) Q \tilde{P}(s) ds. \quad (13)$$

This equation allows the calculation of the average reliability sensitivity during a given period $[0, t]$ in any direction of interest Q .

2.3 Link with the steady-state

If the system is repairable, then when t tends towards infinity, the system reaches a steady state behavior, so $\lim_{t \rightarrow \infty} \{dP(t)/dt\} = 0$. Let $\pi = (\pi_1, \pi_2, \dots, \pi_n)'$ be the column vector representing the steady state probabilities ($\pi = \lim_{t \rightarrow \infty} P(t)$), and let $Z_\pi = \lim_{t \rightarrow \infty} Z(t)$, then Equation 5 becomes:

$$A_\delta Z_\pi + (A_\delta - A)\pi = 0,$$

or

$$-A_\delta \frac{Z_\pi}{\delta} = Q\pi. \quad (14)$$

Since matrix A_δ is not invertible, the generalized inverse (or group inverse) $A_\delta^\# = (A_\delta - \pi_\delta e)^{-1} - \pi_\delta e$, with $e = (1, 1, \dots)$, has to be used to solve Equation 14 for Z_π , see (Meyer 1975) for details. Using the relations $A_\delta^\# A_\delta = I - \pi_\delta e$ and $e\pi = e\pi_\delta = 1$, it follows that:

$$\frac{Z_\pi}{\delta} = -A_\delta^\# Q\pi.$$

The derivative of π in the direction of Q can be defined as:

$$\frac{d\pi}{dQ} = \lim_{\delta \rightarrow 0} \frac{Z_\pi}{\delta} = -\lim_{\delta \rightarrow 0} A_\delta^\# Q\pi.$$

Since $A_\delta^\#$ is continuous, i.e., $\lim_{\delta \rightarrow 0} A_\delta^\# = A^\# = (A - \pi e)^{-1} - \pi e$ (Cao & Chen 1997), $d\pi/dQ$ can be expressed as:

$$\frac{d\pi}{dQ} = -A^\# Q\pi. \quad (15)$$

Let us note $R = \lim_{t \rightarrow \infty} R(t) = f\pi$, the system availability at infinite time (steady state). Hence the sensitivity of R in the direction Q (i.e the MDSM of R) can be written as:

$$I_Q^R = \frac{dR}{dQ} = -f A^\# Q\pi. \quad (16)$$

The exact solution is obtained by calculating the group inverse. An estimate solution has been proposed by Cao in (Cao & Chen 1997): $G = f A^\#$, called potential vector, can be estimated directly from a single sample path observation. This method seems to be very powerful for Markov sensitivity analysis and Markov decision-making problems and it is used to study the reliability sensitivity analysis for steady-state systems in (Do Van et al. 2006).

3 APPLICATION TO MULTI-STATE PRODUCTION SYSTEMS

For multi-state production systems, for example, manufacturing, production line, power generation,

the performance output of interest is not only the reliability (availability) but also the production capacity. This section explores, in the framework of Markovian multi-state production systems, how the production capacity is evaluated and how MDSM is extended to study the production capacity sensitivity.

Assume that a unique production (or treatment) capacity X_i corresponds to each state i and let $P_i(t)$ be the probability of being in state i at time t . The production capacity at time t is then:

$$S(t) = \sum_{i \in \Omega} P_i(t) X_i,$$

where Ω is the state space of the production system. Another formulation of $S(t)$ is

$$S(t) = XP(t), \quad (17)$$

where $X = (X_1, X_2, \dots, X_m)$ is a row vector representing state production capacities. Note finally that X_i may depend not only on the production capacity of the components, but also on the system structure.

Consider that each component has 2 states: failed state and running state. When a component is failed, its production capacity value is zero. When a component is running, its production capacity can depend on the state of others and can have different levels; herein, the assumption of two levels is considered. Hence, more precisely, the production capacity of component i can be equal to:

- 0 if component i is failed;
- y_i^N if component i has no operational dependence with other components, i.e. the failures of other components do not affect its production capacity;
- y_i^D if component i exhibits operational dependences with other components, i.e. its production volume is affected by the failures of other components.

The system production capacity of each system state is calculated by dividing the system into subsystems and basic subsystems which are series or parallel structures.

Let $X_k^{struct.}$ represents the production capacity of a structure (subsystem or basic subsystem) with n units that is in state k ($k = 1, 2, \dots, m$).

Let Y_i^k represents the production capacity level of component i when the system (or the structure) is in a state k . So Y_i^k can equal 0, y_i^N , or y_i^D , and one gets:

- for a series structure:

$$X_k^{series-struct.} = \min(Y_1^k, Y_2^k, \dots, Y_n^k).$$

- for a parallel structure:

$$X_k^{parallel-struct.} = \sum_{i=1}^n Y_i^k.$$

Using (8) and (17), the directional sensitivity (or directional derivative) of the production capacity in the direction of Q (i.e the MDSM of $S(t)$) at time t is written as:

$$I_Q^S(t) = \frac{dS(t)}{dQ} = X \int_0^t F_A(t-s) Q F A(s) P_0 ds. \quad (18)$$

The average production capacity during a given period $[0, t]$ is defined as:

$$\bar{S}(t) = \frac{1}{t} \int_0^t X P(s) ds,$$

or,

$$\bar{S}(t) = \frac{X}{t} \tilde{P}(t). \quad (19)$$

Using (12) and (19), the sensitivity of the average production capacity during a given period in the direction of interest Q (i.e the MDSM of $\bar{S}(t)$) can be expressed as:

$$\bar{I}_Q^S = \frac{d\bar{S}(t)}{dQ} = \frac{X}{t} \int_0^t F_A(t-s) Q \tilde{P}(s) ds. \quad (20)$$

If the system reaches a steady state, then when t tends towards infinity, $\lim_{t \rightarrow \infty} \bar{S}(t) = \lim_{t \rightarrow \infty} S(t) = X\pi$. Let $S = X\pi$, S is called the system production capacity at steady state. So using (15), the sensitivity of the production capacity at steady-state in the direction Q (i.e the MDSM of S) can be written as:

$$I_Q^S = \frac{dS}{dQ} = X \frac{d\pi}{dQ} = -X A^\# Q \pi. \quad (21)$$

The multi-directional sensitivity measure, MDSM, can be used in multi-state production systems to evaluate the variation of production capacity at time t (or for a given period) when one or a group of parameters change of value at the same time. It turns out to be useful also to find the importance rule of one or even of a group of parameters for the system production capacity.

4 NUMERICAL EXAMPLE

The purpose of this section is to show how the MDSM can be used in reliability sensitivity analysis and in production capacity analysis through a simple example. Both reliability and production capacity criterions are considered for the transient state of Markovian system, for its steady state and also for a given time period of interest.

Figure 3 represents a part of a production line with 4 units divided into 2 groups:

- Group A: units C_1 and C_2 are treatment units, their production capacities are 50 (for normal operation state), 0 (for failed state). When C_2 is failed, the production capacity of unit C_1 increases by 20% (for the simplicity, all capacities can be normalized, they actually represent a given amount of products per hour(hr))
- Group B: units C_3 and C_4 are identical package units, C_4 is in cold redundancy with C_3 . As soon as C_3 is repaired, C_4 is stopped. The production capacity values of C_3 and C_4 are 100 and 0 corresponding to the running state and the failed state respectively (there are no degraded conditions for them).

The nominal production capacity of the system is 100 products/hr.

The corresponding Markov process and the production capacity distribution of each state are sketched in Figure 4. Table 1 gives the values of failure rates λ_i ($\bar{\lambda}_i$ for failure of shared load case, when C_2 is failed and C_1 is functioning, for example), the repair rates μ_i , ($i = 1, \dots, 4$), and also the production capacities.

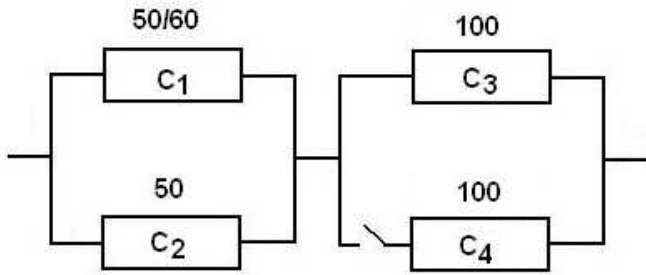


Figure 3. A part of production line.

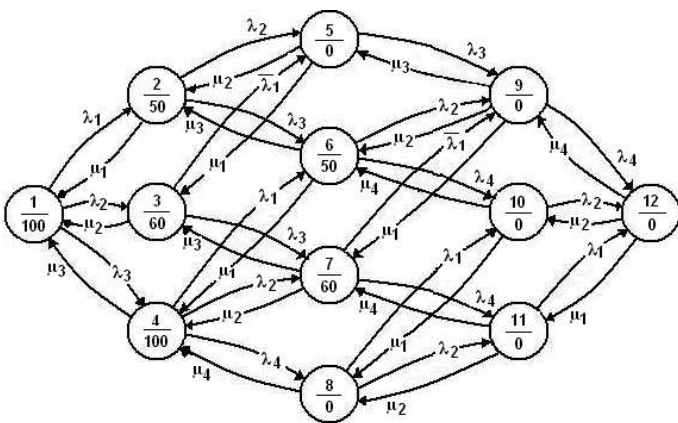


Figure 4. Markov process & production capacities distribution.

Table 1. Transition rates & production capacities.

Units	λ_i	μ_i	$\bar{\lambda}_i$	Production capacity
C_1	$4.5e-4$	$4e-3$	$1e-3$	0/50/60
C_2	$4.5e-4$	$4e-3$	-	0/50
C_3, C_4	$6.0e-4$	$3e-3$	-	0/100

Consider first the system availability $R(t)$ and the system production capacity $S(t)$. Their behaviours *vs* time are shown in Figure 5. After about 3000hr the asymptotic behaviour is reached. Their average values during a period of one year ($t_s = 8760hr$) are $\bar{R}(t_s) = 0.95$, and $\bar{S}(t_s) = 87.73\%$ respectively.

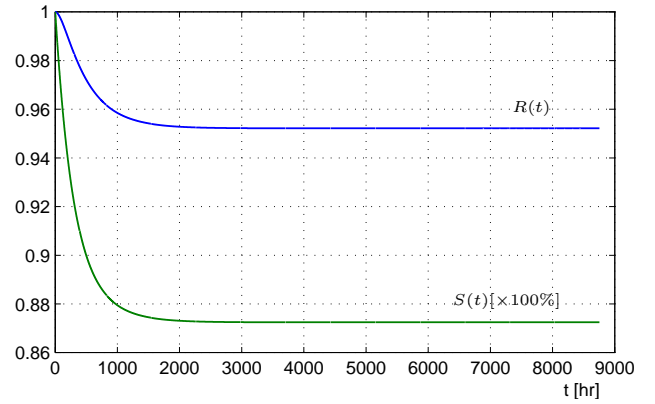


Figure 5. System availability & production capacity.

4.1 Availability sensitivity to one parameter

Consider now the proposed importance measure, MDSM, for the system availability analysis. Many directions of sensitivity can be proposed. First some specific directions Q_{λ_i} are considered to study the sensitivity of system availability w.r.t the parameter of interest λ_i (failure rates, for example). They are noted: $I_{\lambda_i}^R(t) = I_{Q_{\lambda_i}}^R(t)$, $\bar{I}_{\lambda_i}^R = \bar{I}_{Q_{\lambda_i}}^R$ et $I_{\lambda_i}^R = I_{Q_{\lambda_i}}^R$, (for $i = 1, \dots, 4$). The numerical values are obtained by numerical integration of Equations 9, 13 and 16.

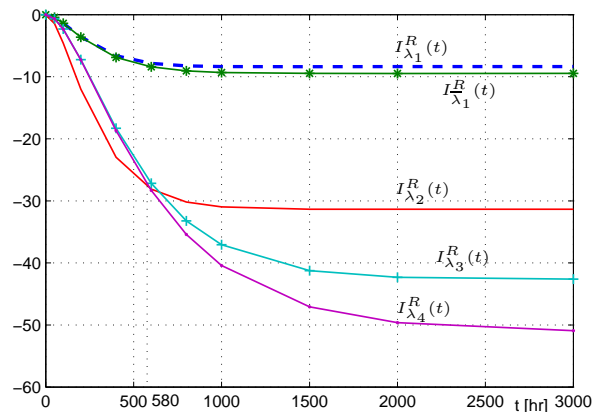


Figure 6. Availability sensitivities to failure rates.

The behavior *vs* time of the system availability sensitivities w.r.t the failure rates are shown in Figure 6. It is clear that an increase of a failure rate leads to a decrease of the system availability. The sensitivity of the system availability to C_1 is shared between the sensitivity to $\bar{\lambda}_1$ and to λ_1 , hence the impact of failure rate of C_1 on the system availability sensitivity is: $I_{\bar{\lambda}_1}^R(t) + I_{\lambda_1}^R(t)$. According to the impact of components' failure rate on the system availability,

Table 2. Average availability & production capacity sensitivity analysis.

Units		Value	Order		Value	Order
C_1	$\bar{T}_{\lambda_1}^R$	-8.11	4	$\bar{T}_{\lambda_1}^S$	-8627.99	1
	$\bar{T}_{\lambda_1}^R$	-9.03		$\bar{T}_{\lambda_1}^S$	-908.40	
C_2	$\bar{T}_{\lambda_2}^R$	-30.25	3	$\bar{T}_{\lambda_2}^S$	-8916.69	2
C_3	$\bar{T}_{\lambda_3}^R$	-39.91	2	$\bar{T}_{\lambda_3}^S$	-3657.71	4
C_4	$\bar{T}_{\lambda_4}^R$	-47.57	1	$\bar{T}_{\lambda_4}^S$	-4323.26	3

in Figure 6, C_2 is the most critical component during a period $[0, 580hr]$. From $t = 580hr$ to infinite time, $|(I_{\lambda_1}^R(t) + I_{\lambda_1}^S(t))| < |I_{\lambda_2}^R(t)| < |I_{\lambda_3}^R(t)| < |I_{\lambda_4}^R(t)|$, the most critical component is C_4 , and the components importance ranking is: $C_1 < C_2 < C_3 < C_4$. This order can be explained intuitively from the system structure: C_1 and C_2 are in a parallel group (group C), their repair rates are the same ($\mu_1 = \mu_2$), but the failure rate of C_1 is shared between λ_1 and $\bar{\lambda}_1$ ($\bar{\lambda}_1 > \lambda_1 = \lambda_2$), hence the availability of C_2 is higher than that of C_1 , consequently the system availability is more sensitive to C_2 than to C_1 . Furthermore, C_3 is more important than C_2 and less critical than C_4 because C_1 & C_2 are in a parallel structure, and C_4 is in cold spare with C_3 . When C_3 is functioning, C_4 is in standby so the impact of C_3 on the system availability behavior is more important than C_1 and C_2 . When C_4 is running, C_3 is already failed, so a failure of C_4 leads a failure of the whole system, and consequently C_4 is more important than C_3 . This order is also true for the average values during a period of one year presented in Table 2 and also for the availability sensitivity at steady state, in Table 3.

4.2 Production capacity sensitivity to one parameter

Consider now the application of MDSM for system productivity analysis. As in the previous section, many directions of interest can be proposed for the production capacity sensitivity study. To illustrate the advantage of MDSM in the production capacity analysis, the same mentioned directions are used to study the sensitivity of system production capacity w.r.t the failure rates, note also: $I_{\lambda_i}^S(t) = I_{Q_{\lambda_i}}^S(t)$, $\bar{T}_{\lambda_i}^S = \bar{T}_{Q_{\lambda_i}}^S$ and $I_{\lambda_i}^S = I_{Q_{\lambda_i}}^S$ (for $i = 1, \dots, 4$). The results are obtained by numerical integration of Equations 18, 20 and 21.

In Figure 7, the importance factors $I_{\lambda_i}^S(t)$ ($i = 1, \dots, 4$) are sketched. The results show that an increase of a failure rate leads to a decrease of the system productivity. The order of their importances in the first short period $[0, 650hr]$ is: $|I_{\lambda_1}^S(t)| < |I_{\lambda_3}^S(t)| < |I_{\lambda_4}^S(t)| < |I_{\lambda_2}^S(t)| < |I_{\lambda_1}^S(t)|$, and from $t = 650hr$ to infinity, it is: $|I_{\lambda_1}^S(t)| < |I_{\lambda_3}^S(t)| < |I_{\lambda_4}^S(t)| < |I_{\lambda_1}^S(t)| < |I_{\lambda_2}^S(t)|$. It is clear that the value of sum $|I_{\lambda_1}^S(t) +$

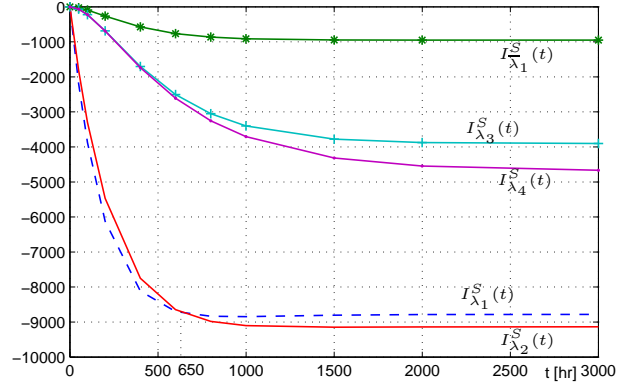


Figure 7. Production capacity sensitivity to failure rates.

$I_{\lambda_1}^S(t)$ is always higher than $|I_{\lambda_2}^S(t)|$. According to the impact of components' failure rate on the system production capacity, C_1 is the most critical component, and the components importance ranking is: $C_3 < C_4 < C_2 < C_1$. This order is maintained in Table 2, with the average productivity sensitivity during a period of one year, and also in Table 3, with the productivity sensitivity at infinite time horizon.

This ranking is not the same as the components importance ranking according to the system availability sensitivity in the previous section since the supply of production for the group $B(C_3$ or $C_4)$ depends on the state of group $A(C_1$ & $C_2)$, hence the impact of C_1 & C_2 on the system production capacity is more important than C_3 & C_4 ("bottleneck effect"). Moreover, the production capacity of C_1 is higher than that of C_2 .

Table 3. Availability & production capacity sensitivity analysis at steady state.

Units		Value	Order		Value	Order
C_1	$I_{\lambda_1}^R$	-8.37	4	$I_{\lambda_1}^S$	-8780.36	1
	$I_{\lambda_1}^R$	-9.47		$I_{\lambda_1}^S$	-950.99	
C_2	$I_{\lambda_2}^R$	-31.34	3	$I_{\lambda_2}^S$	-9133.92	2
C_3	$I_{\lambda_3}^R$	-42.60	2	$I_{\lambda_3}^S$	-3902.43	4
C_4	$I_{\lambda_4}^R$	-51.12	1	$I_{\lambda_4}^S$	-4683.41	3

4.3 Availability & production capacity sensitivity to a group of parameters

In this section, composite directions of sensitivity are considered, i.e. the perturbation is not limited to a single specific state or parameter. Instead, perturbations on a group of parameters, or more generally, perturbations on a group of transition rates are studied. The sensitivity in these directions can help to identify the importance of components group. In Table 4, the direction denoted $A(\bar{\lambda}_1, \lambda_1, \lambda_2)$ indicates that the failure rates of C_1 and C_2 are simultaneously perturbed of the same amount. Similarly, the direction denoted $B(\lambda_3, \lambda_4)$ corresponds to the simultaneous perturbation of the failure rates of C_3 and C_4 by the same amount. Hence, the derivative of the system performance in one of these direc-

Table 4. Sensitivity analysis to group of components

Direction	Availability		Production capacity	
	$\frac{d\bar{R}(t_s)}{dQ}$	$\frac{dR}{dQ}$	$\frac{d\bar{S}(t_s)}{dQ}$	$\frac{dS}{dQ}$
$A(\bar{\lambda}_1, \lambda_1, \lambda_2)$	-47.39	-49.18	-18453.08	-18865.27
$B(\lambda_3, \lambda_4)$	-87.48	-93.87	-7980.97	-8585.84
$(\bar{\lambda}_1, 2.73\mu_2)$	0.008	0.35	1965.45	2085.70
$(\lambda_4, 6.21\mu_3)$	-0.03	1.81	-0.39	170.05

tions gives an importance measure the effect of the corresponding group of components on the system performance. According to these measures on the system availability, the groups/components importance ranking is: $C_3 < A(C_1, C_2) < C_4 < B(C_3, C_4)$. The groups/components importance measures with respect to the system productivity can also be derived and gives the following ranking: $B(C_3, C_4) < C_2 < C_1 < A(C_1, C_2)$.

When one parameter of the system is changed (increased failure rate, components degradation, for example), the system performances (availability, productivity) deteriorates. This variation can be compensated completely or partially if at the same time, other parameters of the systems (repair rates, for example) can be perturbed to compensate for this change in performance. This action can be performed by choosing a suitable direction of perturbation Q . In Table 4, two directions of perturbation are proposed to keep the system availability or/and the system productivity at the same level in the case of a degradation components C_1 and C_4 . The direction $(\lambda_i, \alpha\mu_j)$ indicates that if λ_i (for $i = 1, 4$) is increased by an amount δ , then at the same time, μ_j is perturbed of an amount $\alpha\delta$. A sensitivity close to zero in a direction of the form $(\lambda_i, \alpha\mu_j)$ indicates that the change on μ_j almost balances the effect of the change on λ_i . From a practical point of view, this can be seen as a mean to tune the maintenance parameters, such that a perturbation on the failure rate has no impact on the system availability or/and system productivity. Maintenance policies parameters can then be optimally tuned in this way and the optimal solution can also depends on other criterions (maintenance cost, for example).

5 CONCLUSION

The multi-directional sensitivity measure, MDSM, can be used to investigate the performance sensitivity of dynamic systems in any direction of one parameter, or in any direction of a group of parameters, and, more generally, the effect of the simultaneous change of several design parameters. This factor can be extended to the multi-state production systems whose performance output is not only the system availability (or reliability) but also its production capacity. The sensitivity of both performance outputs are studied in the transient state, during a given period of time and at steady state. On the basis of the results of the sen-

sitivity analysis in the different directions of interest, the most critical component, the group of most critical components can be identified. The maintenance policies parameters can be also tuned to keep a constant reliability (availability) or/and productivity level in presence of components degradation, etc...

Our futur research work focuses on the direction sensitivity optimisation for maintenance policy parameters, and on the development of methods to estimate our proposed measure, MDSM, with the operating feedback data in the transient state.

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