



HAL
open science

Combination of sources of evidence with different discounting factors based on anew dissimilitary measure

Zhun-Ga Liu, Jean Dezert, Quan Pan, Grégoire Mercier

► To cite this version:

Zhun-Ga Liu, Jean Dezert, Quan Pan, Grégoire Mercier. Combination of sources of evidence with different discounting factors based on anew dissimilitary measure. *Decision Support Systems*, 2011, 52 (1), pp.133-141. 10.1016/j.dss.2011.06.002 . hal-00703673

HAL Id: hal-00703673

<https://hal.science/hal-00703673>

Submitted on 7 Jun 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

Combination of sources of evidence with different discounting factors based on a new dissimilarity measure

Zhun-ga Liu ^{a,c,*}, Jean Dezert ^b, Quan Pan ^a, Grégoire Mercier ^c

^a School of Automation, Northwestern Polytechnical University, Xi'an, 710072, PR China

^b Onera, The French Aerospace Lab, F-91761 Palaiseau, France

^c Telecom Bretagne, Technopole Brest-Iroise, 29238, France

The sources of evidence may have different reliability and importance in real applications for decision making. The estimation of the discounting (weighting) factors when the prior knowledge is unknown have been regularly studied until recently. In the past, the determination of the weighting factors focused only on reliability discounting rule and it was mainly dependent on the dissimilarity measure between basic belief assignments (bba's) represented by an evidential distance. Nevertheless, it is very difficult to characterize efficiently the dissimilarity only through an evidential distance. Thus, both a distance and a conflict coefficient based on probabilistic transformations $BetP$ are proposed to characterize the dissimilarity. The distance represents the difference between bba's, whereas the conflict coefficient reveals the divergence degree of the hypotheses that two belief functions strongly support. These two aspects of dissimilarity are complementary in a certain sense, and their fusion is used as the dissimilarity measure. Then, a new estimation method of weighting factors is presented by using the proposed dissimilarity measure. In the evaluation of weight of a source, both its dissimilarity with other sources and their weighting factors are considered. The weighting factors can be applied in the both importance and reliability discounting rules, but the selection of the adapted discounting rule should depend on the actual application. Simple numerical examples are given to illustrate the interest of the proposed approach.

1. Introduction

The theories of evidence [2,16–18], also called theories of belief functions, are widely used in information fusion for decision making [3,8,14] as soon as the information to deal with are uncertain and possibly conflicting and represented by basic belief assignments (bba's). In some real applications, all the sources of evidence to be combined may not have the same reliability, neither the same importance, even if no prior knowledge about the reliability and importance is known. If all the sources are considered equally reliable, the commutative and associative Dempster-Shafer's rule (DS), which has a low computation burden, will generate counter-intuitive results when the sources are highly conflicting as pointed out by Zadeh in [22]. Since few years, many works on the discounting methods for the unreliable sources of evidence have emerged [3,12–14] to solve the problem. The basic idea of the discounting method is that if one source of evidence has the large dissimilarity with the other sources, its reliability should be low. Evidential distance [9] is always used as

the dissimilarity measure in the discounting method. Nevertheless, evidential distance captures only one aspect of the dissimilarity between bba's mainly associated with a distance metric, and it is not enough to characterize the dissimilarity precisely. The determination of the discounting(weighting) factors (reliability or importance factors) in many works mainly lies in the mean of dissimilarity with the other sources without considering the influence of the weight of those other sources. Moreover, these works only focus on the application of the reliability discounting rule, and the notions of importance and reliability have generally been considered as similar until very recently. However, reliability represents ability of the source to provide the correct assessment of the given problem, and importance means somehow the weight of importance granted to the source by the fusion system designer. Therefore, the reliability and the importance of sources are quite distinct notions [19].

The dissimilarity measure between two bba's plays a crucial role in the discounting method, but it is actually difficult to quantify it because several aspects of dissimilarity need to be involved when establishing an efficient and precise measure. In previous works, important research efforts have been done to find good scalar measures to characterize the dissimilarity between two bba's, but all these proposed measures did capture only one aspect of the dissimilarity. From authors opinion, the dissimilarity between two bba's is not only characterized by a well

* Corresponding author at: School of Automation, Northwestern Polytechnical University, Xi'an, 710072, PR China. Tel.: +33 229001007; fax: +33 229001098.

E-mail addresses: liuzhunga@gmail.com (Z. Liu), jean.dezert@onera.fr (J. Dezert), Gregoire.Mercier@telecom-bretagne.eu (G. Mercier).

chosen distance, but also by another aspect which reflects somehow the level of conflict between the bba's. Hence both aspects must count together when defining a good and useful measure of dissimilarity. The evidential distance proposed in [9] is commonly considered as an interesting distance measure, but it is not good enough to capture the difference between bba's in some cases as it will be seen, and its computation burden can become important. The mass of belief committed to the empty set resulting from the conjunctive rule of sources is generally used to measure the degree of conflict [20], but such measure is not always very appropriate, especially for equal and cognitively independent belief functions as shown in [13]. In other words, the mass of belief committed to the empty set cannot efficiently measure the divergence between distinct hypotheses strongly supported by each source. When working in the probabilistic framework, the focal elements are singletons and exclusive, and the degree of the conflict and distance become easier to compute regardless the intrinsic relationship between bba's. The well known probabilistic transformation *BetP*, introduced in [21] is an easy way to approximate any bba into a subjective probability measure and this is the transformation we have adopted in this work. A more efficient (but more complex) transformation can be found in [18].

In this paper, the dissimilarity measure between two bba's is defined from both a distance criterion and their intrinsic level of conflict criterion based on *BetP*. The distance criterion measures the total difference between the bba's, whereas the conflict criterion reveals the degree of divergences between the distinct hypotheses strongly supported by each source. These two criteria are mutually compensable in a certain sense. So the fusion of the distance and the conflict criteria by Hamacher T-conorms fusion rule [7] is used finally as the new scalar dissimilarity measure. We propose also to compute the discounting (weighting) factors of sources from the Perron–Frobenius eigenvalue of the agreement matrix defined from the dissimilarities of bba's. The discounting factors can be applied in the importance discounting rule or in the reliability discounting rule. The interest of our new dissimilarity measure and the determination of discounting factors are illustrated through simple numerical examples. This paper is organized as follows. In Section 2, the distance between bba's is introduced. In Section 3, the measure of intrinsic conflict is presented. In Section 4, the new dissimilarity measure obtained from the fusion of the distance criterion and the conflict criterion is explained. A new method for automatic determination of discounting factors is presented in section 5 and a numerical example is given in Section 6. Section 7 concludes this paper.

2. Distance between bba's

Usually a distance between two bba's is defined to characterize the dissimilarity measure between two sources of evidence. The choice for a well-adapted distance is not easy and many distances have been proposed as shown in [10]. In this paper, we present some commonly used distances including Jusselme's evidential distance d_j [9], Bhattacharyya's distance d_B [15], and the *MaxDiff* distance proposed in Liu.

2.1. Jusselme's distance and Bhattacharyya's distance

• Jusselme's distance

Jusselme's distance d_j [9] is commonly used because it takes into account both the mass and the cardinality of focal elements of each bba's. d_j between $\mathbf{m}_1 = m_1(\cdot)$ and $\mathbf{m}_2 = m_2(\cdot)$ is defined by:

$$d_j(\mathbf{m}_1, \mathbf{m}_2) = \sqrt{\frac{1}{2}(\mathbf{m}_1 - \mathbf{m}_2)' \mathbf{D}(\mathbf{m}_1 - \mathbf{m}_2)} \quad (1)$$

where \mathbf{D} is a $2^{|\Theta|} \times 2^{|\Theta|}$ (conjectured positive) matrix with elements given by $D_{ij} \triangleq \frac{|A_i \cap B_j|}{|A_i \cup B_j|}$, $A_i, B_j \in 2^\Theta$.

In the worst case (when all elements of the power set are focal elements), the computational complexity of this distance can become very important when the cardinality of the frame increases. A main drawback of such distance measure is that it cannot efficiently consider the difference between belief of a single element and of non specific element in some cases as clearly shown in the Example 1.

• Bhattacharyya's distance

Bhattacharyya's distance has been proposed in [15] as:

$$d_B = (1 - \sqrt{\mathbf{m}_1' \sqrt{\mathbf{m}_1}})^p \quad (2)$$

For simplicity, we take $p=1$ here. This distance doesn't take account of the relative specificity of focal elements of each bba. So it cannot efficiently characterize the dissimilarity, especially between the singletons and the ignorance.

Example 1. Let's consider the frame $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ and the following three independent bba's

$$\begin{aligned} \mathbf{m}_1 : & m_1(\theta_1) = m_1(\theta_2) = \dots = m_1(\theta_n) = 1/n \\ \mathbf{m}_2 : & m_2(\Theta) = 1 \\ \mathbf{m}_3 : & m_3(\theta_l) = 1, \text{ for some } l \in \{1, 2, \dots, n\} \end{aligned}$$

According to Eqs. (1) and (2), one gets:

$$\begin{aligned} d_j(\mathbf{m}_1, \mathbf{m}_2) &= d_j(\mathbf{m}_1, \mathbf{m}_3) = \sqrt{\frac{1}{2} \left(1 - \frac{1}{n}\right)} \\ d_B(\mathbf{m}_1, \mathbf{m}_2) &= 1, \quad d_B(\mathbf{m}_1, \mathbf{m}_3) = 1 - \frac{1}{n} \end{aligned}$$

In this example, one sees that \mathbf{m}_3 is absolutely confident in θ_l , but \mathbf{m}_1 and \mathbf{m}_2 can be considered both as very uncertain sources. Actually \mathbf{m}_2 corresponds to the full ignorant source, whereas the Bayesian bba \mathbf{m}_1 has a full randomness, i.e. the maximal entropy. Although the intrinsic nature of uncertainty of \mathbf{m}_1 and of \mathbf{m}_2 is different, from a decision-making point of view, the decision-maker is face to the full uncertainty for taking his/her decision. Intuitively, it is expected that \mathbf{m}_1 is closer to \mathbf{m}_2 than to \mathbf{m}_3 because both sources \mathbf{m}_1 and \mathbf{m}_2 carry uncertainty and they yield to the complete indeterminacy in the decision-making problem. As we see, d_j doesn't characterize well the difference between these two very different cases because \mathbf{m}_1 is at the same distance to \mathbf{m}_2 or to \mathbf{m}_3 . The d_B distance between \mathbf{m}_1 and \mathbf{m}_2 is larger than between \mathbf{m}_1 and \mathbf{m}_3 which is not a good behavior in authors opinions because \mathbf{m}_1 and \mathbf{m}_2 must be considered as quite similar since they represent a full uncertainty decision-making state. From such a very simple example, one sees that d_j and d_B are not well adapted to fully measure the dissimilarity between bba's.

2.2. Probabilistic-based distances

The main problem for evaluating the dissimilarity between two bba's lies in the relationship among their focal elements. Probabilistic transformations allow to approximate any bba into a subjective probability measure based on an underlying frame of discernment whose atomic elements are exhaustive and exclusive.

The probabilistic distance between \mathbf{m}_1 and \mathbf{m}_2 through their approximate subjective probability measures is proposed here. Many transformations exist to approximate a bba into a subjective probability including *BetP*, *DSmP* [5], etc. We concentrate only on the well known and used transformations *BetP* here. Let $m(\cdot)$ be a given bba related with Θ , and the associated *BetP* for any singleton $Y \in \Theta$ is given by [21]

$$BetP(Y) = \sum_{x \in 2^\Theta, Y \subseteq x} \frac{1}{|x|} m(x) \quad (3)$$

where is the cardinality of subset X . For notation convenience, we denote $P_{m_1}(\cdot) \triangleq \text{Bet}P_{m_1}(\cdot)$

- The *MaxDiff* distance

In 2006, Liu proposed *MaxDiff* distance in [11] as

$$\text{MaxDiff}(\mathbf{m}_1, \mathbf{m}_2) = \max_{A \in \Theta} |P_{\mathbf{m}_1}(A) - P_{\mathbf{m}_2}(A)| \quad (4)$$

MaxDiff distance reflects the variation only by the maximal distance between the pignistic probabilities of a pair of the individual element. However, it is not adapted for measuring precisely the total amount of difference between two bba's.

- Minkowski's based distance

In this paper, we propose to use the *DistP* distance based on Minkowski's distance defined as follows: for $t \geq 1$, one takes

$$\text{DistP}_t(m_1, m_2) = \left(\frac{1}{2} \sum_{\substack{\theta_i \in \Theta \\ |\theta_i|=1}} |P_{\mathbf{m}_1}(\theta_i) - P_{\mathbf{m}_2}(\theta_i)|^t \right)^{\frac{1}{t}} \quad (5)$$

The coefficient $\frac{1}{2}$ in (5) allows to have $\text{DistP}_t(\cdot) \in [0, 1]$. The larger t leads to a larger complexity burden. As shown in the example 2, such distance is not recommended when $t > 1$.

Example 2. Let's consider the frame $\Theta = \{\theta_1, \theta_2, \dots, \theta_{2n}\}$ and the following two independent bba's

$$\begin{aligned} \mathbf{m}_1 : m_1(\theta_1) = m_1(\theta_2) = \dots = m_1(\theta_n) = 1/n \\ \mathbf{m}_2 : m_2(\theta_{n+1}) = m_2(\theta_{n+2}) = \dots = m_2(\theta_{2n}) = 1/n \end{aligned}$$

In this example \mathbf{m}_1 and \mathbf{m}_2 totally contradict. The different distance measures between \mathbf{m}_1 and \mathbf{m}_2 are shown in Fig. 1.

The plots for DistP_2 and d_j coincide on this figure since \mathbf{m}_1 and \mathbf{m}_2 are Bayesian bba's. The values of DistP_2 , DistP_3 and *MaxDiff* tend towards 0, meaning that \mathbf{m}_1 and \mathbf{m}_2 are closer and closer with the increase of n , which is obviously abnormal. Only $\text{DistP}_1 = 1$ is constant, and it indicates that \mathbf{m}_1 and \mathbf{m}_2 are completely different. Moreover, the computation burden is lowest when using $t=1$, and that is why we choose to take $t=1$ in this paper. *DistP* characterizes the

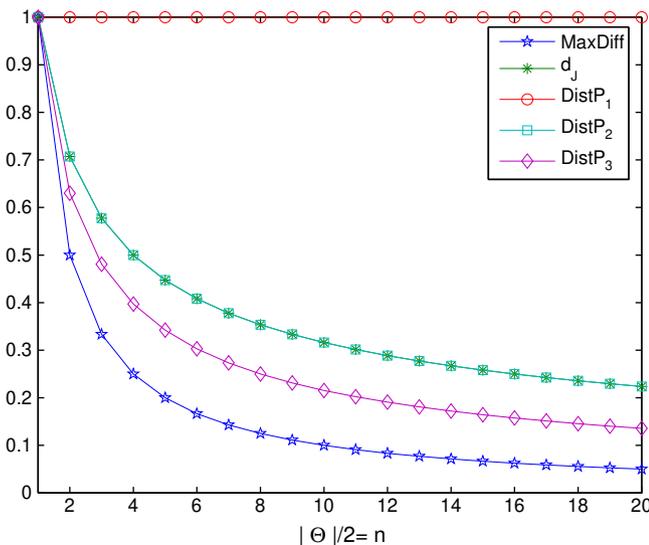


Fig.1. Different distance measures between \mathbf{m}_1 and \mathbf{m}_2 .

dissimilarity by the absolute distance between their associate subjective probabilities.

Lemma 1. Let $\mathbf{m}_1, \mathbf{m}_2$ be two bba's defined on 2^Θ . The probabilistic-based distance $\text{DistP}_t(\mathbf{m}_1, \mathbf{m}_2) \in [0, 1]$.

- If $\mathbf{m}_1 = \mathbf{m}_2$, then $\text{DistP}(\mathbf{m}_1, \mathbf{m}_2) = 0$, but its reciprocal is not true.
- If $\text{DistP}(\mathbf{m}_1, \mathbf{m}_2) = 1$, then \mathbf{m}_1 and \mathbf{m}_2 totally contradict and therefore there is no compatible elements supported by the both bba's, and its reciprocal is true.

In the example 1, one has $\text{DistP}(\mathbf{m}_1, \mathbf{m}_2) = 0$ and $\text{DistP}(\mathbf{m}_1, \mathbf{m}_3) = \text{DistP}(\mathbf{m}_2, \mathbf{m}_3) = (n-1)/n$, which indicates the distance between \mathbf{m}_1 and \mathbf{m}_2 is much smaller than that of \mathbf{m}_1 and \mathbf{m}_3 .

In the dissimilarity measure, the degree of the divergence between the distinct hypotheses strongly supported by each source must play an important role. Unfortunately, *DistP* is unable to reveal such divergence which makes it not a good candidate for a good dissimilarity measure if we use it as sole criteria.

Lemma 2. Even if the distance/dissimilarity measure is high, it is possible in some cases that the two bba's strongly support the same hypothesis. Reciprocally, if the distance measure is low, it is possible in some cases that the bba's commit the most of their masses of belief to different incompatible elements of the frame.

This lemma is illustrated through the next Example 3.

Example 3. Let's consider the frame $\Theta = \{\theta_1, \theta_2, \theta_3\}$ with Shafer's model and the following three independent bba's

$$\begin{aligned} \mathbf{m}_1 : m_1(\theta_1) = 0.5, m_1(\theta_2) = 0.3, m_1(\theta_3) = m_1(\Theta) = 0.1 \\ \mathbf{m}_2 : m_2(\theta_1) = 0.8, m_2(\theta_3) = 0.2 \\ \mathbf{m}_3 : m_3(\theta_1) = 0.3, m_3(\theta_2) = 0.5, m_3(\Theta) = 0.2 \end{aligned}$$

The various distances between these bba's are:

$$\begin{aligned} d_j(\mathbf{m}_1, \mathbf{m}_2) = 0.3109, & d_j(\mathbf{m}_1, \mathbf{m}_3) = 0.2160 \\ \text{MaxDiff}(\mathbf{m}_1, \mathbf{m}_2) = 0.2667, & \text{MaxDiff}(\mathbf{m}_1, \mathbf{m}_3) = 0.2333 \\ \text{DistP}(\mathbf{m}_1, \mathbf{m}_2) = 0.3333, & \text{DistP}(\mathbf{m}_1, \mathbf{m}_3) = 0.2333. \end{aligned}$$

Although \mathbf{m}_1 and \mathbf{m}_2 strongly support the same hypothesis θ_1 , whereas \mathbf{m}_3 strongly supports θ_2 , the dissimilarity between \mathbf{m}_1 and \mathbf{m}_2 is larger than that between \mathbf{m}_1 and \mathbf{m}_3 according to the distance measures. We see that the divergence between the hypotheses strongly supported by each source is not taken into account efficiently with *DistP* and other distance measures. This simple example shows that such distance measures are not good enough to properly characterize the dissimilarity between bba's. Therefore, we propose to use another criterion to reflect more efficiently the degree of divergence/conflict among the belief functions. This new criterion will be a complementary criterion of the probabilistic-based distance measure *DistP*(...) and help to define a new refined and efficient dissimilarity measure.

3. Intrinsic conflict of belief functions

As in [11], a qualitative definition of conflict between two beliefs in the context of DST is given.

Definition 1. A conflict between two beliefs can be interpreted qualitatively as the fact that one source strongly supports one hypothesis and the other strongly supports another hypothesis, and the two hypotheses are not compatible (their intersection is empty).

This definition is intuitively consistent, and it will be adopted here. According this definition, the conflict mainly comes from pairs of incompatible hypotheses which are separately strongly supported by two different sources of evidence. So the extent of conflict should

be reflected by the conflicting belief of the pair of incompatible hypotheses strongly supported by their sources. Let $m_1(\cdot)$ and $m_2(\cdot)$ be two independent bba's over Θ . Their degree of conflict, as defined by Shafer in [17] and interpreted by Smets, is given by

$$m_{12}(\emptyset) \triangleq \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1)m_2(X_2) \quad (6)$$

$m_{\oplus}(\emptyset) \equiv m_{12}(\emptyset)$ is generally used to evaluate the level of conflict [20] between the two sources of evidence. Nevertheless, $m_{\oplus}(\emptyset)$ is the sum of all the masses of belief committed to the empty set through the conjunctive rule of combination. However, such measure is not very appropriate to really characterize the conflict between bba's, particularly in case of two equal bba's as already reported in several published works [13,11].

Actually, one needs to pay more attention to the hypothesis which gets the most credibility in the bba's. If two sources of evidence commit the most plausibility to compatible or same elements, we argue that they are consistent in the element they strongly support, and they do not contradict with each other. Otherwise, they are considered in conflict. In order to overcome the limitation of $m_{\oplus}(\emptyset)$ as the traditional measure of the conflict, a new measure of level of conflict, called *conflict coefficient* is proposed using probabilistic-based transformations and based on the definition 1.

Definition 2 (conflict coefficient). Let \mathbf{m}_1 and \mathbf{m}_2 be two bba's on 2^Θ . Their associated subjective probabilities are $P_{\mathbf{m}_i}(\cdot)$, $i=1, 2$. The Conflict coefficient denoted by $ConfP$ is defined by

$$ConfP(\mathbf{m}_1, \mathbf{m}_2) = \begin{cases} 0, & \text{if } X_{max}^{\mathbf{m}_1} \cap X_{max}^{\mathbf{m}_2} \neq \emptyset \\ P_{\mathbf{m}_1}(X_{max}^{\mathbf{m}_1})P_{\mathbf{m}_2}(X_{max}^{\mathbf{m}_2}), & \text{otherwise.} \end{cases} \quad (7)$$

where $X_{max}^{\mathbf{m}_i} = \underset{x \in 2^\Theta}{argmax} P_{\mathbf{m}_i}(x)$, $i=1, 2$

The conflict coefficient is defined in using the maximal approximate subjective probability of the bba's. If two sources of evidence distribute most of their mass of belief to compatible elements, there is no conflict between the two sources in such conditions. Otherwise, the amount of conflict will be represented by the product of the pair of maximal subjective probability from different sources.

Lemma 3. Let \mathbf{m}_1 and \mathbf{m}_2 be two independent bba's on 2^Θ . $m_{\oplus}^{12}(\emptyset) \in (0, 1)$, even if $ConfP(\mathbf{m}_1, \mathbf{m}_2) = 0$. Also, $\mathbf{m}_{\oplus}^{12}(\emptyset) = 1$, if $ConfP(\mathbf{m}_1, \mathbf{m}_2) = 1$.

Proof. The former part of the Lemma 3 occurs in many cases, especially when considering two equal bba's. The later part can be proved easily. If $ConfP(\mathbf{m}_1, \mathbf{m}_2) = 1$, it means \mathbf{m}_1 and \mathbf{m}_2 assign the mass to totally different focal elements. Therefore, one gets $m_{\oplus}^{12}(\emptyset) = 1$.

This lemma implies that $m_{\oplus}(\emptyset)$ is not very efficient when the bba's are not in conflict, and $m_{\oplus}(\emptyset)$ is similar to $ConfP$ in case of highly conflicting situations. The conflict coefficient reflects well the divergence of incompatible hypotheses that two sources of evidence commit most of their belief to. However, it ignores the other elements of bba's. This is shown in the next example.

Example 4. Let's consider the frame $\Theta = \{\theta_1, \theta_2\}$ with Shafer's model and the following three independent bba's

$$\begin{aligned} \mathbf{m}_1 : & m_1(\theta_1) = 1 \\ \mathbf{m}_2 : & m_2(\Theta) = 1 \\ \mathbf{m}_3 : & m_3(\theta_1) = 0.9, m_3(\Theta) = 0.1 \end{aligned}$$

\mathbf{m}_1 and \mathbf{m}_3 are much closer than \mathbf{m}_1 and \mathbf{m}_2 , since \mathbf{m}_1 and \mathbf{m}_3 distribute most of their mass of belief to the same hypothesis θ_1 ,

whereas \mathbf{m}_2 is fully ignorant (i.e. \mathbf{m}_2 is the vacuous belief assignment). Nevertheless, from the formula (6) and (7), one gets

$$ConfP(\mathbf{m}_1, \mathbf{m}_2) = ConfP(\mathbf{m}_1, \mathbf{m}_3) = 0$$

$$m_{\oplus}^{12}(\emptyset) = m_{\oplus}^{13}(\emptyset) = 0.$$

So in such case, we cannot make a distinction between \mathbf{m}_1 and \mathbf{m}_2 , and between \mathbf{m}_1 and \mathbf{m}_3 at all only from these conflict measures.

If the proposed probabilistic-based distance is used in this example, one gets

$$\begin{cases} DistP(\mathbf{m}_1, \mathbf{m}_2) = 0.5, \\ DistP(\mathbf{m}_1, \mathbf{m}_3) = 0.05 \end{cases}$$

Naturally, the dissimilarity between \mathbf{m}_1 and \mathbf{m}_2 is quite larger than between \mathbf{m}_1 and \mathbf{m}_3 according to the probabilistic-based distance measure.

Actually, the probabilistic-based distance $DistP$ and the conflict coefficient $ConfP$ are complementary and they separately capture different aspects of the dissimilarity of bba's. If the dissimilarity is only characterized by $DistP$, it cannot show whether the two sources of evidence conflict or not in the hypothesis they strongly support. Reciprocally, if $ConfP$ is considered as the unique criterion to measure the dissimilarity, the level of the difference between the two bba's is not taken into account. Taking into account both criteria $DistP$ and $ConfP$ in the elaboration of a new measure of dissimilarity seems therefore a natural way to improve existing measures of dissimilarity in order to capture two of its main aspects.

4. A new dissimilarity measure

In this section, we propose a new dissimilarity mixing both $DistP$ and $ConfP$ to characterize more efficiently the dissimilarity between two bba's. The new dissimilarity measure, denoted $DismP$, will be defined by the fusion of $DistP$ and $ConfP$ complementary measures/criteria. The fusion rule $f(\cdot)$ we propose to use to define $DismP$ must satisfy the two following important properties:

- (1) Commutativity $f(x, y) = f(y, x)$;
- (2) Monotonicity $f(x, y) \leq f(x', y) \leq f(x', y') \leq f(1, 1) = 1$, if $x \leq x', y' \leq y'$;

Hence, the dissimilarity measure $dismP$ obtained from the fusion $DistP$ and $ConfP$ should be no less than any one of them. Also, $DismP$ cannot be larger than the sum of $DistP$ and $ConfP$, nor 1, which corresponds to the following inequality constraints: $\max\{x, y\} \leq f(x, y) \leq \min\{1, (x + y)\}$.

Hamacher T-conorm fusion rule [7], denoted $T(\cdot)$, satisfies these constraints and that's why we take $f(\cdot) = T(\cdot)$. Therefore, we finally define $DismP(\dots)$ by

$$\begin{aligned} DismP(\mathbf{m}_1, \mathbf{m}_2) &\triangleq T(DistP(\mathbf{m}_1, \mathbf{m}_2), ConfP(\mathbf{m}_1, \mathbf{m}_2)) \\ &= \frac{DistP(\mathbf{m}_1, \mathbf{m}_2) + ConfP(\mathbf{m}_1, \mathbf{m}_2)}{1 + DistP(\mathbf{m}_1, \mathbf{m}_2)ConfP(\mathbf{m}_1, \mathbf{m}_2)} \end{aligned} \quad (8)$$

The dissimilarity measure is useful in many domains [10], for instance for belief functions approximation algorithms [1], for defining the agreement between sources of evidence as a basis for discounting factors [6,3], for combination rules parameters estimation [4,23], as well as for selecting an adapted combination rule [12], etc. In this paper, we are interested in using it for the automatic determination of discounting factors of the sources of evidence.

For a comparison between the different dissimilarity measures, we now use an example drawn from [9] to show the behavior of d_j , d_B , $Maxdiff$, $m(\emptyset)$, $DistP$, $ConfP$, and $DismP$.

Example 5. Let Θ be a frame of discernment with element θ_1, θ_2 , etc. For notation conciseness, we use 1, 2, etc. to denote θ_1, θ_2 , etc., and the notation $m(\theta_i \cup \theta_j)$ is also replaced by $m(i, j)$ in the sequel. The two bba's are defined as follows:

$$\begin{aligned} \mathbf{m}_1 : \quad & m_1(2, 3, 4) = 0.05, m_1(7) = 0.05 \\ & m_1(\emptyset) = 0.1, m_1(A) = 0.8 \\ \mathbf{m}_2 : \quad & m_2(1, 2, 3, 4, 5) = 1 \end{aligned}$$

A is the subset of Θ . We consider twenty cases where subset A is progressively augmented by including a new element in it. In other words, for case $i = 1, 2, 3, \dots, 20, A_i = \{1, 2, \dots, i\}$.

The comparison of the aforementioned dissimilarity measures between \mathbf{m}_1 and \mathbf{m}_2 for the 20 cases is graphically illustrated in Fig. 2.

As we can see, d_j and $DistP$ present a similar behavior in this example, but the computation of $DistP$ is easier. In this example $DistP$ mainly depends on $DistP$ since $ConfP$ is small here, and this explains why plots of $DistP$ and $DistP$ are very close. $MaxDiff$ becomes very small from case 2, and its value even decreases from case 8 to 13, which is abnormal since we expect a growth of the dissimilarity measure. d_B always indicates that \mathbf{m}_1 and \mathbf{m}_2 are totally different but in case 5, and it cannot distinguish the variation among these cases. In case 5, $A = \{1, 2, 3, 4, 5\}$ is the same with the only focal element in $athbfm_2$, which leads d_B to decrease substantially in this case. $ConfP$ implies that both bba's commit the most plausibility to the compatible element from the cases 1 to 6, but \mathbf{m}_1 begins to distribute its total belief to another element which is different from the element strongly supported by \mathbf{m}_2 from case 7. The divergence degree, reflecting the strong support of sources in different hypotheses, is becoming lower and lower. Indeed, since \mathbf{m}_1 becomes more and more uncertain, all singletons get small probability gain through the probabilistic transformation. Nevertheless, the conflicting mass of belief $m(\emptyset)$ keeps a low value around 0.05 and is not representative of the divergence between the sources.

5. Discounting factors of sources of evidence

In this section, we propose a new method for determining the discounting (weighting) factors of the sources based on the dissimilarity measure. The derivation of the weights of the sources is based on the underlying (and usually well-adopted) principle that the *Truth lies in the majority opinion*. When one has a set of n

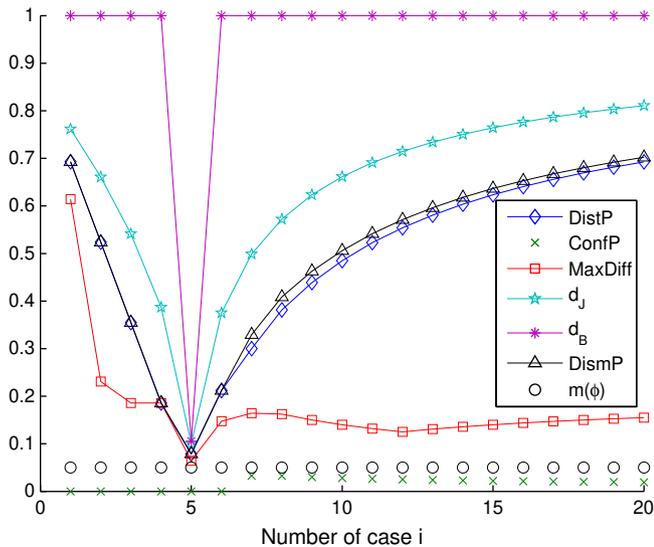


Fig.2. Different dissimilarity measures between \mathbf{m}_1 and \mathbf{m}_2 .

sources of evidence to combine, the scalar dissimilarity measure $DistP$ between each pair of sources must be obtained by (8) at first, and the mutual support degree among these sources will be given by:

$$sup(m_i, m_j) = (1 - DistP(m_i, m_j))^q \quad (9)$$

For simplicity, one suggests to take $q = 1$. The mutually support degree $n \times n$ matrix is then defined by

$$\mathbf{S} = \begin{bmatrix} 1 & sup_{12} & \dots & sup_{1n} \\ sup_{21} & 1 & \dots & sup_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ sup_{n1} & sup_{n2} & \dots & 1 \end{bmatrix} \quad (10)$$

where $sup_{ij} \triangleq sup(m_i, m_j)$.

The weight of each source of evidence is denoted by $w_i, i = 1, 2, \dots, n$. We argue that the weighting factors should be relative, not only with the support degree gained from the other sources, but also with the weights of those other sources. The problem of joint estimation of all weighting factors consists in solving for $i = 1, 2, 3, \dots, n$,

$$\lambda w_i = w_1 sup_{1i} + w_2 sup_{2i} + \dots + w_n sup_{ni} \quad (11)$$

or more concisely written as

$$\lambda \mathbf{w} = \mathbf{S} \mathbf{w} \quad (12)$$

where $\mathbf{w} \triangleq [w_1, w_2, \dots, w_n]'$ and λ is the proportion coefficient. The Perron–Frobenius vector (the eigen vector associated to the maximal positive eigen value) of \mathbf{S} is used as the credibility factor, that is $\lambda_{max} \cdot \mathbf{w} = \mathbf{S} \cdot \mathbf{w}$.

The source with the largest weighting factor is considered as totally reliable and important, and there is no need to revise it. The other sources are discounted with the factor as

$$w'_i = w_i / \max(w) < 1 \quad (13)$$

w'_i is called the *relative weighting factor* of the source i .

In the discounting process, the reliability discounting rule and importance discounting rule must be selected by the system designer according to his/her application. The reliability and importance represent two distinct notions. Reliability represents ability of the source to provide the correct assessment of the given problem, whereas importance means somehow the weight of importance granted to the source.

• Reliability discounting rule

Two kinds of approaches, including the average bba's method [3,14] and Shafer's discounted bba's method [17], have been proposed for the combination of unreliable sources of evidence. The average bba's method [14] combines iteratively (sequentially) the arithmetic mean of all the bba's. Deng et al. in [3] modified this method to compute the weighted average bba's by taking into account the evidential distance d_j , but we know that d_j is no good enough in all cases to precisely measure the dissimilarity. Moreover, in the average bba's method, all the independent sources of evidence are represented by the same average of bba's, and the distinctness of the different sources is not taken into account efficiently. The combination results are nothing but the iterative combination of the average bba's. So this method is too sensitive to the weighting factors. Since the determination of reliability factors is not robust enough, this can lead to wrong decision. The classical Shafer's discounting method [17] distributes the discounted mass to the ignorance according to the corresponding reliability factor, and all the discounted bba's remain distinct and independent. Shafer's

discounting method seems more reasonable, and that's why it is applied here. We recall briefly how it is applied:

$$\begin{cases} m'(A) = \alpha \cdot m(A), & A \neq \emptyset \\ m'(\emptyset) = 1 - \sum_{A \in 2^\Theta} m'(A) \end{cases} \quad (14)$$

where α is the reliability (discounting) factor of $m(\cdot)$. The discounted mass is committed to the ignorance $m'(\emptyset)$.

- Importance discounting rule

The recent importance discounting method proposed in [19] is defined by:

$$\begin{cases} m'(A) = \beta \cdot m(A), & A \neq \emptyset \\ m'(\emptyset) = 1 - \sum_{A \in 2^\Theta} m'(A) \end{cases} \quad (15)$$

where β is the importance (discounting) factor of $m(\cdot)$. This importance discounting rule allows to have $m(\emptyset) \geq 0$, and preserves the specificity of the primary information since all focal elements are discounted with same importance factor.

If we assume that all the sources of evidence are reliable but they don't share the same importance, the importance discounting rule can be applied. Otherwise, the reliability discounting rule is selected. One can take either $\alpha = w_i$ for the reliability discounting, or $\beta = w_i$ for the importance discounting, depending on the rule we prefer to apply in the given application under consideration. If the importance discounting rule is applied, DS rule defined in [17] will be useless since it will not respond to the discounting of sources towards the empty set (see proof in [19]), and PCR5₀ can be used as an efficient combination rule instead. The fusion result m_{PCR5_0} will be normalized by redistributing the mass of belief committed to the empty set to the other focal elements and proportionally to their masses as it is shown in the examples given in [19]. If the reliability discounting rule is chosen, all the combination rules can be used, since such discounting doesn't commit a strictly positive mass of belief to the empty set.

6. Numerical examples

The proposed approach provides a new alternative to combine uncertain sources of evidence with different reliability/importance without a priori knowledge on the sources. It can be well adapted for the fusion of highly conflicting sources of information for decision-making support. The sources which are highly conflicting with the majority of other sources will be automatically assigned with a very low reliability/importance factor thanks to the new dissimilarity measure in order to decrease their bad influence in the fusion process. Two simple illustrative examples are presented in this section to show the interest of our new approach with respect to other methods.

The context of these examples could correspond to an automatic target identification system using some independent sensors where the signals arising from these sensors are supposed to have been processed into bba's by some given methods. The construction of bba's is application dependent and is out of the scope of this paper. Here, we assume no prior knowledge about reliability, nor importance about the sources of evidence.

Example 6 Bayesian bba's. In this example, we want to show how the proposed method works for the decision making from Bayesian bba's. Let's consider three simple Bayesian bba's over the frame $\Theta = \{\theta_1, \theta_2, \theta_3\}$ as in Table 1. It is assumed that sources of evidence No.3 can provide possibly two similar bba's denoted m_{3A} and m_{3B} , and let's see how the small difference affects the fusion results.

Table 1
Three bba's to be combined.

	\mathbf{m}_1	\mathbf{m}_2	\mathbf{m}_{3A}	\mathbf{m}_{3B}
θ_1	0	0.6	0.75	0.7
θ_2	0.9	0.25	0.15	0.2
θ_3	0.1	0.15	0.1	0.1

After the determination of the discounting factors of each source, both the reliability discounting rule and the importance discounting rule will be applied and analyzed separately.

Dempster-Shafer's rule (DS) provides a good compromise between the specificity of the result and the computation burden but this rule can lead to very counter-intuitive results in case of high conflicting situation. Applying reliability discounting technique (when used judiciously with proper discounting factors) can indeed decrease the degree of conflict between the bba's by committing the discounted the mass to ignorance. In this example, we compare DS rule with PCR5 rule for combining discounted bba's when reliability discounting is used. We use only PCR5₀ fusion rule when the importance discounting is used because DS rule is not responding to such kind of importance discounting. The bba's will be fused sequentially using PCR5₀ as explained in details in [19]. The results obtained for this example are shown in the Tables 2 and 3. In the first row of the Tables 2 and 3, \mathbf{m}_1^i corresponds to the (sequential) fusion of sources $\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_i$. This corresponds to $\mathbf{m}_1^i = \Delta \mathbf{m}_1 \oplus \mathbf{m}_2 \oplus \dots \oplus \mathbf{m}_i$, $\mathbf{m}_1^{3A} = \Delta \mathbf{m}_1 \oplus \mathbf{m}_2 \oplus \mathbf{m}_{3A}$, and $\mathbf{m}_1^{3B} = \Delta \mathbf{m}_1 \oplus \mathbf{m}_2 \oplus \mathbf{m}_{3B}$, where \oplus denotes either DS, PCR5 or PCR5₀. The first column of the Tables 2 and 3 describes the method used for combining the sources of evidence and the underlying measure of dissimilarity used to automatically derive the discounting factors of each source. For example, average bba's & DS" indicates that the arithmetic mean of all the bba's are iteratively combined by DS rule as in [14]. " \bar{d}_j & DS" means that the averaged distance \bar{d}_j has been used to compute the (reliability) discounting factors and that DS rule has been used to combine discounted sources, and so on.

Analysis of the results. We can see that the bba's \mathbf{m}_2 and \mathbf{m}_{3A} or \mathbf{m}_{3B} commit most belief on θ_1 , and \mathbf{m}_{3A} is very close to \mathbf{m}_{3B} , but \mathbf{m}_1 , which distributes the largest belief to θ_2 , highly conflicts with \mathbf{m}_2 and $\mathbf{m}_{3A/B}$. Thus, \mathbf{m}_1 won't be considered so reliable or important as the other ones according to our assumed underlying principle.

For decision-making purpose, the fusion results presented in Tables 2 and 3 show that \mathbf{m}_1^{3A} or \mathbf{m}_1^{3B} are very similar except with the average bba's method because both \mathbf{m}_{3A} and \mathbf{m}_{3B} commit the largest mass of belief to θ_1 , and the difference between the different methods is quite small. However, \mathbf{m}_1^{3A} obtained with the average bba's method consider that θ_1 is most likely to be true, whereas \mathbf{m}_1^{3B} believes that θ_2 should correspond to the truth, and therefore they lead to opposite conclusion for decision-making support. This indicates that the average bba's method is not robust enough and it is very risky for the decision-making support in such cases because all the sources of evidence are considered equally in the average bba's method.

Table 2
Combination results using reliability discounting rule.

	$m_1^{3A}(\theta_1)$	$m_1^{3B}(\theta_1)$	$m_1^{3A}(\theta_2)$	$m_1^{3B}(\theta_2)$	$m_1^{3A}(\theta_3)$	$m_1^{3B}(\theta_3)$
No discount & DS	0	0	0.9574	0.9677	0.0426	0.0323
No discount & PCR5	0.5489	0.5118	0.4259	0.4628	0.0252	0.0253
Average bba's & DS	0.5235	0.4674	0.4674	0.5235	0.0091	0.0091
\bar{d}_j & DS	0.6498	0.6078	0.3267	0.3722	0.0235	0.0201
d_j & DS	0.7264	0.6823	0.2502	0.2968	0.0234	0.0209
Disp & DS	0.8332	0.7958	0.1454	0.1829	0.0214	0.0213

The maximum of the belief of the element in the combination results is labeled by bold data in Table 2, and it is similar in the following tables.

Table 3

Combination results using importance discounting rule.

	$m_1^{3A}(\theta_1)$	$m_1^{3B}(\theta_1)$	$m_1^{3A}(\theta_2)$	$m_1^{3B}(\theta_2)$	$m_1^{3A}(\theta_3)$	$m_1^{3B}(\theta_3)$
\bar{d}_j & PCR5 ₀	0.6233	0.5936	0.3486	0.3780	0.0280	0.0284
d_j & PCR5 ₀	0.6730	0.6392	0.2979	0.3316	0.0291	0.0292
DismP & PCR5 ₀	0.8154	0.7815	0.1522	0.1860	0.0324	0.0325

Table 4

Five bba's to be combined.

	\mathbf{m}_1	\mathbf{m}_2	\mathbf{m}_3	\mathbf{m}_4	\mathbf{m}_5
θ_1	0.8	0.4	0	0.3	0.45
θ_2	0.1	0.2	0.95	0.2	0.1
θ_3	0	0.1	0.05	0.25	0
$\{\theta_1, \theta_2\}$	0	0.3	0	0.2	0
$\{\theta_2, \theta_3\}$	0	0	0	0	0.15
Θ	0.1	0	0	0.05	0.3

The fusion results obtained with the different methods are shown in Tables 5 and 6.

DS rule provides the unreasonable result that θ_1 is impossible to happen, which is illogical since there are two sources among three that consider θ_1 as being most possibly true. Once the discounting approach by \bar{d}_j , d_j , or *DismP* is applied before the fusion of DS, we get the largest mass of belief to θ_1 as expected. The results of PCR5 suggests that θ_1 takes the most mass of belief, and the results become more specific and more efficient for decision-making when the importance discounting method is additionally applied before the fusion of PCR5 as shown in Table 3. Moreover, we can see that if *DismP* is used as the criterion of dissimilarity coupled with the proposed method of reliability/importance weight determination, it can produce the most specific results for decision-making support. This example illustrates that our proposed method can work well with Bayesian bba's even in high conflicting cases.

Example 7. Now let's consider another set of five normalized bba's with imprecise focal elements over the frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3\}$ as given in Table 4. In this example \mathbf{m}_3 is a Bayesian bba, whereas all other bba's are non-Bayesian.

Table 5

Combination results using reliability discounting rule.

	\mathbf{m}_1^2	\mathbf{m}_1^3	\mathbf{m}_1^4	\mathbf{m}_1^5
No discount & DS	$\mathbf{m}(\theta_1) = \mathbf{0.8451}$ $m(\theta_2) = 0.0986$ $m(\theta_3) = 0.0140$ $m(\theta_1, \theta_2) = 0.0423$	$\mathbf{m}(\theta_2) = \mathbf{0.9948}$ $m(\theta_3) = 0.0052$	$\mathbf{m}(\theta_2) = \mathbf{0.9965}$ $m(\theta_3) = 0.0035$	$\mathbf{m}(\theta_2) = \mathbf{0.9971}$ $m(\theta_3) = 0.0029$
No discount & PCR5	$\mathbf{m}(\theta_1) = \mathbf{0.8311}$ $m(\theta_2) = 0.1150$ $m(\theta_3) = 0.0239$ $m(\theta_1, \theta_2) = 0.03$	$\mathbf{m}(\theta_1) = \mathbf{0.4076}$ $m(\theta_2) = 0.5850$ $m(\theta_3) = 0.0068$ $m(\theta_1, \theta_2) = 0.0006$	$\mathbf{m}(\theta_1) = \mathbf{0.4037}$ $m(\theta_2) = 0.5100$ $m(\theta_3) = 0.0848$ $m(\theta_1, \theta_2) = 0.0015$	$\mathbf{m}(\theta_1) = \mathbf{0.5196}$ $m(\theta_2) = 0.4154$ $m(\theta_3) = 0.0482$ $m(\theta_1, \theta_2) = 0.0004$ $m(\theta_2, \theta_3) = 0.0164$
\bar{d}_j & DS	$\mathbf{m}(\theta_1) = \mathbf{0.7611}$ $m(\theta_2) = 0.1177$ $m(\theta_3) = 0.0303$ $m(\theta_1, \theta_2) = 0.0909$	$\mathbf{m}(\theta_1) = \mathbf{0.5705}$ $m(\theta_2) = 0.3367$ $m(\theta_3) = 0.0246$ $m(\theta_1, \theta_2) = 0.0682$	$\mathbf{m}(\theta_1) = \mathbf{0.6361}$ $m(\theta_2) = 0.3159$ $m(\theta_3) = 0.0144$ $m(\theta_1, \theta_2) = 0.0336$	$\mathbf{m}(\theta_1) = \mathbf{0.7086}$ $m(\theta_2) = 0.2662$ $m(\theta_3) = 0.0097$ $m(\theta_1, \theta_2) = 0.0155$
d_j & DS	$\mathbf{m}(\theta_1) = \mathbf{0.7659}$ $m(\theta_2) = 0.1166$ $m(\theta_3) = 0.0294$ $m(\theta_1, \theta_2) = 0.0881$	$\mathbf{m}(\theta_1) = \mathbf{0.6239}$ $m(\theta_2) = 0.2791$ $m(\theta_3) = 0.0252$ $m(\theta_1, \theta_2) = 0.0718$	$\mathbf{m}(\theta_1) = \mathbf{0.6858}$ $m(\theta_2) = 0.2645$ $m(\theta_3) = 0.0146$ $m(\theta_1, \theta_2) = 0.0351$	$\mathbf{m}(\theta_1) = \mathbf{0.7528}$ $m(\theta_2) = 0.2217$ $m(\theta_3) = 0.0096$ $m(\theta_1, \theta_2) = 0.0159$
DismP & DS	$\mathbf{m}(\theta_1) = \mathbf{0.7503}$ $m(\theta_2) = 0.1196$ $m(\theta_3) = 0.0319$ $m(\theta_1, \theta_2) = 0.0957$ $m(\Theta) = 0.0025$	$\mathbf{m}(\theta_1) = \mathbf{0.7157}$ $m(\theta_2) = 0.1598$ $m(\theta_3) = 0.0308$ $m(\theta_1, \theta_2) = 0.0913$ $m(\Theta) = 0.0024$	$\mathbf{m}(\theta_1) = \mathbf{0.7670}$ $m(\theta_2) = 0.1655$ $m(\theta_3) = 0.0194$ $m(\theta_1, \theta_2) = 0.0477$ $m(\Theta) = 0.0004$	$\mathbf{m}(\theta_1) = \mathbf{0.8254}$ $m(\theta_2) = 0.1424$ $m(\theta_3) = 0.0120$ $m(\theta_1, \theta_2) = 0.0198$ $m(\theta_2, \theta_3) = 0.0002$ $m(\Theta) = 0.0002$

Analysis of the results. From the Table 4, one sees that the bba's \mathbf{m}_1 , \mathbf{m}_2 , \mathbf{m}_4 and \mathbf{m}_5 assign most of their belief to θ_1 , but \mathbf{m}_3 oppositely commits its largest mass of belief to θ_2 . Therefore \mathbf{m}_3 is considered as the least reliable or unimportant source based on the aforementioned underlying principle, and it can be considered as a noisy source (outlier).

From the Tables 5 and 6 and after the combination of all the sources, one sees that the DS rule (without discounting process) concludes that the hypothesis θ_1 is very unlikely to happen whereas θ_2 is almost sure to happen. Such result is unreasonable since the majority sources assign most of their belief to θ_1 , but only one source distributes its largest mass of belief to θ_2 . Such unexpected behavior shows that DS rule is risky to use to combine sources of evidence in a high conflicting situation. The result of PCR5 (with no discounting of bba's) indicates that θ_1 has a higher mass of belief than θ_2 (as expected) after the sequential fusion of the five sources, even if θ_2 has got a bigger mass than θ_1 after some intermediate steps of the sequential fusion process. This behavior of PCR5 rule may cause troubles for fast decision-making support (in the case we don't want to wait to process all the sources). To avoid such problem due to the non-associativity property of PCR rules, it is better to combine the sources altogether in a unique and global fusion step. Once importance (or reliability) discounting method is applied, *thbfm*₃ becomes strongly discounted because of its largest dissimilarity with the other sources. One sees that the proposed dissimilarity measure coupled with the automatic discounting factors determination generates a more specific result than using the method proposed in [3,13] which was only based on the mean of evidential distance d_j to determine the reliability factor. Therefore, our new approach improves the decision-making support. If d_j is used instead of *titDismP* as the bba's distance in our method for automatic discounting factors determination, one still gets better specific results than those obtained with the method presented in [3,13]. These results show the effectiveness and the interest of this new method for the estimation of discounting factors of the sources of evidence.

The interest of the new method proposed in this work lies in the elaboration of a more efficient dissimilarity measure which can be used for the determination of the discounting factors of the sources involved in the fusion process. The new dissimilarity measure is obtained from the T-conorm fusion of two components: a distance

Table 6
Combination results using importance discounting rule.

	m_1^2	m_1^3	m_1^4	m_1^5
\bar{d}_j & PCR5 _o	$m(\theta_1) = \mathbf{0.7897}$ $m(\theta_2) = 0.1237$ $m(\theta_3) = 0.0292$ $m(\theta_1, \theta_2) = 0.0574$	$m(\theta_1) = \mathbf{0.5848}$ $m(\theta_2) = 0.4046$ $m(\theta_3) = 0.0058$ $m(\theta_1, \theta_2) = 0.0048$	$m(\theta_1) = \mathbf{0.5668}$ $m(\theta_2) = 0.3475$ $m(\theta_3) = 0.0834$ $m(\theta_1, \theta_2) = 0.0023$	$m(\theta_1) = \mathbf{0.6583}$ $m(\theta_2) = 0.2763$ $m(\theta_3) = 0.0480$ $m(\theta_1, \theta_2) = 0.0007$ $m(\theta_2, \theta_3) = 0.0167$
d_j & PCR5 _o	$m(\theta_1) = \mathbf{0.7915}$ $m(\theta_2) = 0.1233$ $m(\theta_3) = 0.0290$ $m(\theta_1, \theta_2) = 0.0562$	$m(\theta_1) = \mathbf{0.6342}$ $m(\theta_2) = 0.3556$ $m(\theta_3) = 0.0053$ $m(\theta_1, \theta_2) = 0.0049$	$m(\theta_1) = \mathbf{0.6119}$ $m(\theta_2) = 0.3038$ $m(\theta_3) = 0.0820$ $m(\theta_1, \theta_2) = 0.0023$	$m(\theta_1) = \mathbf{0.6968}$ $m(\theta_2) = 0.2384$ $m(\theta_3) = 0.0470$ $m(\theta_1, \theta_2) = 0.0007$ $m(\theta_2, \theta_3) = 0.0171$
DismP & PCR5 _o	$m(\theta_1) = \mathbf{0.7865}$ $m(\theta_2) = 0.1240$ $m(\theta_3) = 0.0293$ $m(\theta_1, \theta_2) = 0.0595$ $m(\theta) = 0.0006$	$m(\theta_1) = \mathbf{0.8389}$ $m(\theta_2) = 0.1498$ $m(\theta_3) = 0.0040$ $m(\theta_1, \theta_2) = 0.0073$	$m(\theta_1) = \mathbf{0.7995}$ $m(\theta_2) = 0.1300$ $m(\theta_3) = 0.0680$ $m(\theta_1, \theta_2) = 0.0025$	$m(\theta_1) = \mathbf{0.8448}$ $m(\theta_2) = 0.0982$ $m(\theta_3) = 0.0374$ $m(\theta_1, \theta_2) = 0.0008$ $m(\theta_2, \theta_3) = 0.0188$

measure and a new measure of the conflict between two bba's. This new measure is larger than its two components. The sources of evidence in high conflict with the majority of other sources get a bigger dissimilarity measure than the averaged evidential distance. The discounting (weighting) factors of the sources are computed based on the dissimilarities of the sources taken altogether. The small group of highly conflicting sources always gets small weights, whereas the majority of normal sources (i.e. the sources in agreement) get large weights. The few highly conflicting sources generally get lower weighting factors in our new method of determination of weights than in the arithmetic average method used in [3,13]. Therefore the majority of non (or low) conflicting sources play a more important role in the fusion (as intuitively expected for a rational and good behavior). Such new method can provide interesting results and valuable help for automatic or semi-automatic decision-making support systems. The choice of the two discounting rules (importance versus reliability) is left to the users according to their own purposes.

7. Conclusions

In this paper, a new combination approach of sources of evidence with different discounting (weighting) factors has been proposed based on a new dissimilarity measure between bba's. We have shown through simple examples that the notion of dissimilarity includes at least two aspects represented by the difference between bba's and also by their level of conflict. After analyzing the limitation of the classical dissimilarity measures, a new dissimilarity measure mixing the probabilistic-based distances with the degree of conflict was developed. In this paper the *BetP* transformation was used in the definition of the distance between two bba's to measure the difference between two bba's. A new conflict coefficient was also introduced to overcome the limitations of the classical degree of conflict represented traditionally by the mass committed to the empty set through the conjunctive rule. This new conflict coefficient allows to measure more efficiently the divergence between distinct hypotheses strongly supported by each source of evidence. The distance and conflict measures characterize two different aspects of the dissimilarity between bba's. A new method for the automatic determination of weighting factors of the sources has been also presented when no prior knowledge is given about the reliability or the importance of the sources. All the weighting factors are computed jointly from a global optimization problem based on the dissimilarities among bba's, and this makes the evaluation of weighting factors more precise and reasonable. The weighting factors can be applied with the reliability discounting method or with the importance discounting method as well. The numerical examples presented in this paper illustrate clearly the potential interest of this new approach for applications dealing with evidential reasoning for decision-making

support. The extension of this approach can naturally be done in the DSMT (Dezert-Smarandache Theory) framework as well, and using DSMP transformation instead of classical Pignistic transformation. This is left for future investigations and out of the scope of this paper.

Acknowledgements

The authors want to thank anonymous reviewers for their remarks which helped us to improve the quality of this paper. This work has been partially supported by the China Natural Science Foundation (No. 61075029) and PhD Thesis Innovation Fund from Northwestern Polytechnical University (No. cx201015).

References

- [1] F. Cuzzolin, A geometric approach to the theory of evidence IEEE transactions on systems, Man, and Cybernetics - Part C : Applications and Reviews 38 (4) (2008) 522–534.
- [2] A.P. Dempster, A generalization of Bayesian inference, Journal of the Royal Statistical Society, Series B 30 (1968) 205–247.
- [3] Y. Deng, W.K. Shi, Z.F. Zhu, Q. Liu, Combining belief functions based on Distance of evidence, Decision Support Systems 38 (3) (2004) 489–493.
- [4] T. Denoeux, Conjunctive and disjunctive combination of belief functions induced by nondistinct bodies of evidence, Artificial Intelligence 172 (2008) 234–264.
- [5] J. Dezert, F. Smarandache, A new probabilistic transformation of belief mass assignment, in Proceedings of Fusion 2008 Conference, Cologne, Germany, July, 2008.
- [6] M.C. Florea, E. Bosse, A.L. Josselme, Metrics, distances and dissimilarity measures within Dempster-Shafer theory to characterize sources' reliability, Proceeding of Cognitive Systems with Interactive Sensors Conference (COGIS '09), 2009.
- [7] H. Hamacher, Über logische Aggregationen nicht-binär explizierter Entscheidungskriterien, Rita G. Fischer Verlag, Frankfurt, Germany, 1978.
- [8] J.Y. Jaffray, Linear utility theory for belief functions, Operations Research Letters 8 (1989) 107–112.
- [9] A.L. Josselme, D. Grenier, E. Bossé, A new distance between two bodies of evidence, Information Fusion 2 (1) (2001) 91–101.
- [10] A.L. Josselme, P. Maupin, On some properties of distances in evidence theory, in Proceeding of workshop on the theory of belief functions, Brest, France, April, 2010.
- [11] W. Liu, Analyzing the degree of conflict among belief functions, Artificial Intelligence 170 (11) (2006) 909–924.
- [12] Z. Liu, Q. Pan, Y. Cheng, J. Dezert, Sequential adaptive combination of unreliable sources of evidence, in Proceeding of workshop on the theory of belief functions, Brest, France, April, 2010.
- [13] A. Martin, A.L. Josselme, C. Osswald, Conflict measure for the discounting operation on belief functions, in Proceeding of Fusion 2008 Conference, Cologne, Germany, July, 2008.
- [14] C.K. Murphy, Combining belief functions when evidence conflicts, Decision Support Systems 29 (1) (2000) 1–9.
- [15] B. Ristic, P. Smets, The TBM global distance measure for the association of uncertain combat ID declarations, Information Fusion 7 (3) (2006) 276–284.
- [16] G. Shafer, J. Pearl (Eds.), Readings in uncertain reasoning, Morgan Kaufmann, 1990.
- [17] G. Shafer, A mathematical theory of evidence, Princeton University Press, Princeton, New Jersey, 1976.
- [18] F. Smarandache, J. Dezert, Advances and Applications of DSMT for Information Fusion, Vol. 1–3, American Research Press, Rehoboth, 2004–2009, available at <http://fs.gallup.unm.edu/DSMT.htm>.

- [19] F. Smarandache, J. Dezert, J.-M. Tacnet, Fusion of sources of evidence with different importances and reliabilities, in Proceedings of Fusion 2010 Conference, Edinburgh, UK, July, 2010.
- [20] P. Smets, Analyzing the combination of conflicting belief functions, *Information Fusion* 8 (4) (2007) 387–412.
- [21] P. Smets, Decision making in the TBM: the necessity of the pignistic transformation, *International Journal of Approximate Reasoning* 38 (2) (2005) 133–147.
- [22] L.A. Zadeh, A simple view of the Dempster–Shafer theory of evidence and its implication for the rule of combination, *AI magazine* 7 (2) (1986) 85–90.
- [23] L.M. Zouhal, T. Denoeux, An evidence-theoretic k-NN rule with parameter optimization, *IEEE Transactions on Systems, Man and Cybernetics – Part C* 28 (2) (1998) 263–271.