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Iterative Correction of Intersymbol Interference: Turbo-Equalization

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Abstract. This paper presents a receiving scheme intended to combat the detrimental effects of intersymbol interference for digital transmissions protected by convolutional codes. The receiver performs two successive soft-output decisions, achieved by a symbol detector and a channel decoder, through an iterative process. At each iteration, extrinsic information is extracted from the detection and decoding steps and is then used at the next iteration as in turbo-decoding. From the implementation point of view, the receiver can be structured in a modular way and its performance, in bit error rate terms, is directly related to the number of modules used. Simulation results are presented for transmissions on Gauss and Rayleigh channels. The results obtained show that turbo-equalization manages to overcome multipath effects, totally on Gauss channels, and partially but still satisfactorily on Rayleigh channels.

1. INTRODUCTION

With the growth of mobile radio systems, digital communications have to deal with the problem of transmitting messages over multipath channels. In such cases, channels appear to be frequency-selective and give rise to Doppler effects. Several approaches may be employed in order to overcome channel selectivity. A first possibility consists in equalizing the channel so as to minimize InterSymbol Interference (ISI) at the receiving filter output. Another solution, which we have chosen, takes the channel memory effect into account. In the latter approach, the modulator, the transmission channel and the demodulator are represented by an equivalent discrete-time model that behaves similarly to a convolutional encoder. Symbol detection is based on a Maximum-Likelihood Sequence Estimation (MLSE) and is achieved through the application of the Viterbi algorithm [1]. In the case where the transmission channel is protected by a convolutional encoder and a decoder using the Viterbi algorithm, the detection and decoding modules may be associated in the same way as in turbo-decoding. In order to take the best advantage of the encoding function, the symbol detection has to provide soft outputs and the samples at the output of the equivalent discrete-time channel are processed in an iterative way. This approach is referred to as turbo-equalization in what follows, by analogy with turbo-codes.

2. PRESENTATION OF THE TRANSMISSION CHANNEL

Let us suppose that the binary digits d_k delivered by the source are encoded by a convolutional encoder. The encoded data c_k are reordered by an interleaver, and applied at the input of a Binary Phase Shift Keying (BPSK) modulator. The transmitted signal $e(t)$ is provided by the output of a filter whose impulse response is $h_e(t)$. The signal emitted can be expressed in the form:

$$e(t) = A \sum_k c_k \, h_e(t - kT) \exp j(2\pi f_0 t + \varphi_0) \qquad (1)$$

where A denotes an amplitude, f_0 is the carrier frequency, φ_0 is a constant phase whose value is in $[0, 2\pi]$, and c_k are binary symbols (± 1) transmitted at the rate of one symbol every T seconds.

In the case of a multipath channel, the received signal $Y(t)$, can be written as follows:

$$Y(t) = \sum_{m=0}^{M-1} A_m(t) \sum_k c_k \, h_e(t - \tau_m - kT) \cdot \exp j(2\pi f_0 t + \varphi_0) + B(t) \qquad (2)$$

where $A_m(t)$ are complex-valued independent multiplicative noise processes, gaussian in the case of a Rayleigh-type channel and constant in the case of a Gauss-

type channel. The delays τ_m take into account the different propagation delays on each of the M paths. $B(t)$ is a complex-valued zero mean white gaussian noise with a power bilateral spectral density equal to $2\,N_0$. After demodulation, the samples taken from the output of the receiving matched filter are expressed as:

$$R_n \triangleq R(nT) = \sum_{m=0}^{M-1} A_m(n) \sum_k c_{n-k}\, h_s(kT - \tau_m) + b_n \quad (3)$$

where $A_m(n)$ equals $A_m(nT)$ by definition, b_n denotes the response of the receiving matched filter to the noise $B(t)$, sampled at time nT. $h_s(t)$ is defined by $h_s(t) = h_e(t) \times h_e^*(-t)$ and satisfies the Nyquist criterion. Let:

$$\Gamma_k(n) = \sum_{m=0}^{M-1} A_m(n)\, h_s(kT - \tau_m) \quad (4)$$

and let us suppose that the ISI is limited to $(L_1 + L_2)$ symbols. Eq. (3) may be written in the form:

$$R_n = \sum_{k=-L_2}^{L_1} \Gamma_k(n)\, c_{n-k} + b_n \quad (5)$$

Quantities $\Gamma_k(n)$ are expressed as a linear combination of the multiplicative noises $A_m(n)$, therefore, they are gaussian in the case of a Rayleigh-type channel and constant in the case of a Gauss-type channel. Consequently, the set of modules made up of the modulator, the transmission channel and the demodulator can be represented by an equivalent discrete-time channel (Fig. 1).

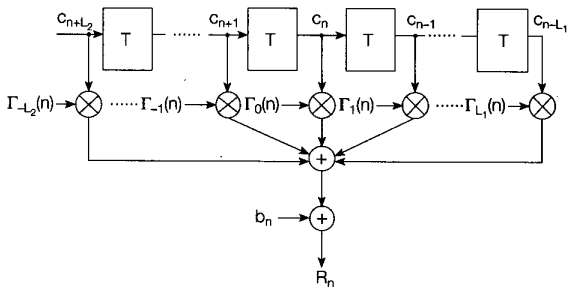

Fig. 1 - Equivalent discrete-time model of a channel with intersymbol interference.

After a change of variable k, eq. (5) may also be expressed in the form:

$$R_n = \sum_{k=0}^{L_1+L_2} \Gamma_{k-L_2}(n)\, c_{n+L_2-k} + b_n \quad (6)$$

If we denote $S_n = (c_{n+L_2}, \ldots, c_{n-L_1+1})$ the state of the equivalent discrete-time channel at time nT, sample R_n depends on the channel state S_{n-1} and on the symbol c_{n+L_2}. Therefore, the equivalent discrete-time channel

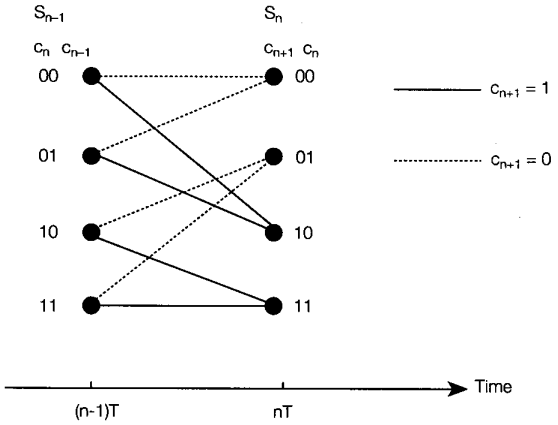

Fig. 2 - Trellis diagram for $L_1 = L_2 = 1$.

can be modeled as a Markov chain and its behavior can be represented by a trellis diagram (Fig. 2).

3. PRINCIPLE OF TURBO-EQUALIZATION

In order to use a soft-input channel decoder, the symbol detector has to provide information about the reliability of the symbols estimated. This information may be obtained by using a Soft-Output Viterbi Algorithm (SOVA) [2 - 4], that associates an estimation of the Logarithm of its Likelihood Ratio (LLR), $\Lambda_1(c_n)$, to each symbol c_n detected:

$$\Lambda_1(c_n) = \log \frac{\Pr\{c_n = +1 | \boldsymbol{R}\}}{\Pr\{c_n = -1 | \boldsymbol{R}\}} \quad (7)$$

where $\boldsymbol{R}$ denotes the vector of samples that constitutes the observation. After deinterleaving, the SOVA decoder provides a new LLR value of c_k, $\Lambda_2(c_k)$, that may be derived by analogy with the calculations made in [5] and expressed in the form:

$$\Lambda_2(c_k) = \Lambda_1(c_k) + z_k \quad (8)$$

where z_k is the extrinsic information associated with symbol c_k and provided by the channel decoder (Fig. 3). In fact, the extrinsic information z_k is another estimation of the LLR of symbol c_k conditioned on the decoding step:

$$z_k = \log \frac{\Pr\{c_k = +1 | \, decoding\}}{\Pr\{c_k = -1 | \, decoding\}} \quad (9)$$

Hence, z_k may be used through a feedback loop by the symbol detector, after interleaving. This is the basis of turbo-equalization principle.

To evaluate the LLR of symbol c_{n-L_1}, the Viterbi algorithm used in the detector has to calculate a metric at

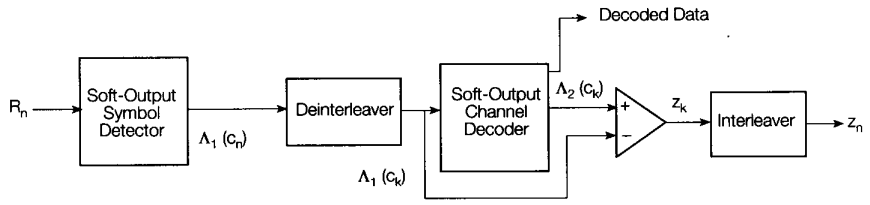

Fig. 3 - Extrinsic information deriving scheme.

time nT for every branch in the trellis. This metric may be written in the form [1]:

$$\lambda_n^i = \left| R_n - r_n^i \right|^2 - 2\sigma_b^2 \log \Pr\left\{c_{n-L_1} = i\right\} \qquad i = \pm 1 \quad (10)$$

where:

$$r_n^i = \sum_{k=0}^{L_1+L_2-1} \hat{\Gamma}_{k-L_2}(n) \cdot c_{n+L_2-k} + \hat{\Gamma}_{L_1}(n) \cdot i \qquad i = \pm 1 \quad (11)$$

and $\hat{\Gamma}_{k-L_2}(n)$, $0 \le k \le L_1 + L_2$, represents an estimation of quantity $\Gamma_{k-L_2}(n)$ and σ_b^2 denotes the variance of noise b_n, that is $\sigma_b^2 = E[|b_n|^2]$.

The *a priori* probabilities $\Pr\{c_{n-L_1} = i\}$ used in relation (10) may be estimated from the extrinsic information z_{n-L_1}, if we assume that

$$z_{n-L_1} \approx \log \frac{\Pr\left\{c_{n-L_1} = +1\right\}}{\Pr\left\{c_{n-L_1} = -1\right\}} \qquad (12)$$

Then, the following expressions can be derived from (12):

$$\Pr\left\{c_{n-L_1} = +1\right\} \approx \frac{\exp z_{n-L_1}}{1 + \exp z_{n-L_1}} \qquad (13a)$$

$$\Pr\left\{c_{n-L_1} = -1\right\} \approx \frac{1}{1 + \exp z_{n-L_1}} \qquad (13b)$$

Using (13 a), (13 b) and (10), metrics λ_n^i are equal to:

$$\lambda_n^{+1} = \left| R_n - r_n^{+1} \right|^2 - \gamma z_{n-L_1} \qquad (14a)$$

$$\lambda_n^{-1} = \left| R_n - r_n^{-1} \right|^2 \qquad (14b)$$

Note that the common term $\log(1 + \exp z_{n-L_1})$ has been suppressed in eqs. (14 a) and (14 b). Coefficient γ is a weight introduced to take into account variance σ_b^2 and the fact that the extrinsic information is only an estimation of the *a priori* probability. Its value depends on the signal-to-noise ratio, that is to say the reliability of the extrinsic information.

4. ITERATIVE IMPLEMENTATION OF TURBO-EQUALIZATION

The different processing stages in the turbo-equalizer present a non-zero internal delay, so turbo-equalizing can only be implemented in an iterative way. At each iteration, a new value of extrinsic information is calculated and used by the symbol detector at the next iteration. Therefore, the turbo-equalizer can be implemented in a modular pipelined structure, where each module is associated with one iteration. Then, performance in Bit Error Rate (*BER*) terms is a function of the number of chained modules. An example of turbo-equalization implementation is illustrated in Fig. 4 in the case of a 4-stage process. The rank q module, $1 \le q \le 4$, has two inputs and three outputs. Input R^q receives the samples from the receiving matching filter after a delay equal to the latency of the $(q-1)$ previous modules. Input z^q represents the extrinsic information of the previous iteration provided by the rank $(q-1)$ module. Output R^{q+1} is equal to input R^q delayed of the latency of the module, and z^{q+1} is the extrinsic information provided by the current iteration. These outputs are unused for the last module and do not appear on the figure. Output D^q provides the decoded data and is only used at the last module output.

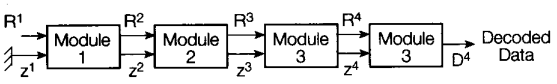

Fig. 4 - Modular pipelined structure of a turbo-equalizer for a 4-iteration process.

When extrinsic information is used by the symbol detector, it can be proved that, at iteration q, the LLR of symbol c_n, $\Lambda_1^q(c_n)$, may be expressed as:

$$\Lambda_1^q(c_n) = \tilde{\Lambda}_1^q(c_n) + \gamma^q z_n^{q-1} \qquad (15)$$

where $\tilde{\Lambda}_1^q$ is a term depending on the samples of observation $\boldsymbol{R}$ and on z_n^{q-1}, $k \ne n$, and z_n^{q-1} denotes the extrinsic information of symbol c_n determined at iteration $q-1$. If we apply the same approach as in turbo-decoding, the quantity $\gamma^q z_n^{q-1}$ provided by the channel decoder at the previous iteration has to be subtracted from $\Lambda_1^q(c_n)$, as illustrated in Fig. 5. Hence, after deinterleaving, the channel decoder input is in fact equal to:

$$\tilde{\Lambda}_1^q(c_k) = \Lambda_1^q(c_k)\Big|_{z_k^{q-1}=0} \qquad (16)$$

At the channel decoder output, the extrinsic information z_k^q may also be written as follows, using (8):

$$z_k^q = \Lambda_2^q(c_k)\Big|_{\tilde{\Lambda}_1^q(c_k)=0} \qquad (17)$$

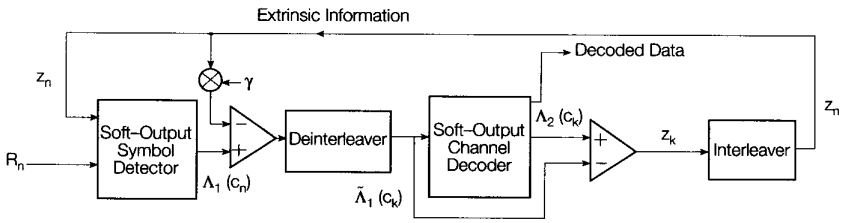

Fig. 5 - Principle of turbo-equalization (under zero internal delay assumption).

5. SIMULATION RESULTS

Performance of this device has been evaluated for a rate $R = 1/2$ recursive systematic encoder with constraint length $K = 5$, and generators $G_1 = 23$, $G_2 = 35$. Bits were interleaved in a non uniform matrix whose dimensions are 64 by 64. The modulation used was a BPSK modulation, with a Nyquist filter whose transfer function $H_s(f)$ was a raised cosine with a rolloff $\alpha = 1$, on both gaussian and Rayleigh channels.

For both channels, $M = 5$ independent paths were considered, each with a mean power $P_m = E[|A_m(n)|^2]$, so that the total mean power was normalized: $\sum_{m=0}^{M-1} P_m = 1$. The delays τ_m were chosen as multiples of T ($\tau_m = mT$, $\Gamma_k(n) = A_k(n)$ since $h_s[(k-m)T] = \delta_{k-m,0}$). The coefficients for the gaussian channel were chosen equal to:

$$\Gamma_0(n) = \sqrt{0.45}, \Gamma_1(n) = \sqrt{0.25}, \Gamma_2(n) = \sqrt{0.15}, \Gamma_3(n) = \sqrt{0.10}, \Gamma_4(n) = \sqrt{0.05}$$

On the Rayleigh channel considered, the five paths had equal mean power ($P_i = 1/M$, $\forall i \in [1, M]$). A parameter BT, which is the product of the Doppler bandwidth and the symbol duration, fixes the variation velocity of the channel: the smaller BT is, the more slowly the channel parameters vary during a time interval symbol.

The discrete-time equivalent channel was modeled by a 16-state trellis, and the symbole detector was working on the SOVA algorithm presented in [4]. The channel coefficients $\Gamma_k(n)$ were supposed perfectly known. After deinterleaving, the soft estimations provided by the SOVA detector were used by the decoder which also worked on a 16-state trellis and the SOVA algorithm. The extrinsic information extracted from the decoder was used by the symbol detector according to the principle depicted in Fig. 5.

The BER was computed as a function of signal to noise ratio E_b/N_0, where E_b is the mean energy received per information bit d_k and N_0 is the noise power bilateral spectral density. The signal to noise ratio E_b/N_0 may be expressed as:

$$\frac{E_b}{N_0} = \frac{\sum_{m=0}^{M-1} P_m}{\sigma_b^2} \tag{18}$$

Results are presented in Figs. 6, 7 and 8. On a gaussian channel (Fig. 6), the gain at the first iteration, compared to the classical Viterbi detector which provides a binary decision, is 1.7 dB for a BER of 10^{-5}. At the second iteration, the gain is 2.2 dB better. After the fifth iteration, the total gain is 5.2 dB. Finally, the turbo-equalizer manages to completely compensate for the degradation due to the interference generated by the multipath effects after six iterations. Then, the BER is the same as on a non selective gaussian channel (without intersymbol interference).

Two types of Rayleigh channels with $BT = 0.1$ (Fig. 7) and $BT = 0.001$ (Fig. 8) were examined. In both of these cases, the gain after the first iteration is about 2.2 dB, that is to say better than that on the gaussian channel. This can be explained by the fact that the system takes advantage of the diversity created by the multiple paths. The global gain after the third iteration remains inferior to that on the gaussian channel. The limit that has been considered is the BPSK modulation with encoding and without interference on a gaussian channel. This limit is not completely achieved: the third iteration is 0.8 dB from this limit for $BT = 0.1$ and 2 dB for $BT = 0.001$. A degradation is noted in the case of $BT = 0.001$ compared to the case of $BT = 0.1$, this is due to the use of the same 64 by 64 interleaving matrix in both cases. Such dimensions were chosen to simulate a realistic receiver from the implementation point of view. When $BT = 0.001$, the channel parameters

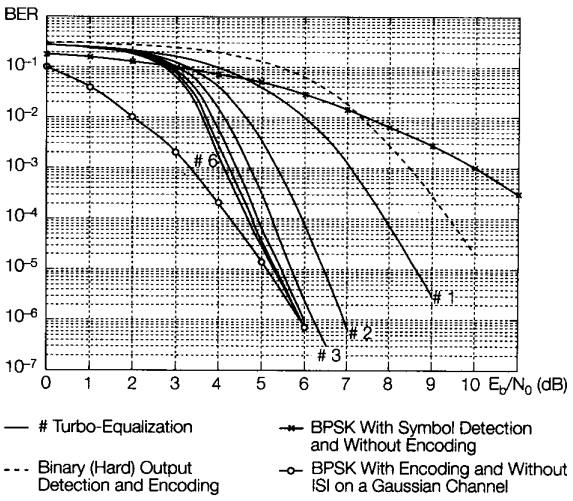

Fig. 6 - Performance of turbo-equalization over a gaussian channel (convolutional encoding with $K = 5$).

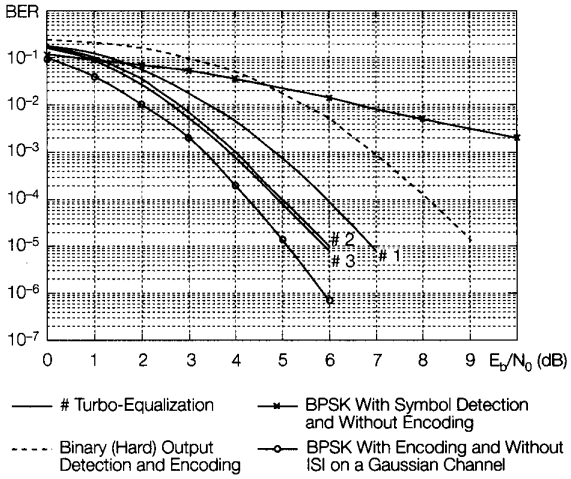

Fig. 7 - Performance of turbo-equalization over a Rayleigh channel with $BT = 0.1$ (convolutional encoding with $K = 5$).

vary so slowly that, when fading occurs, it can last so long that it can affect a number of symbols which is approximately the size of the interleaving matrix. In such a case, a 64 by 64 interleaving is less effective. Therefore, it is necessary to adapt the interleaver dimensions according to the variation speed of the channel.

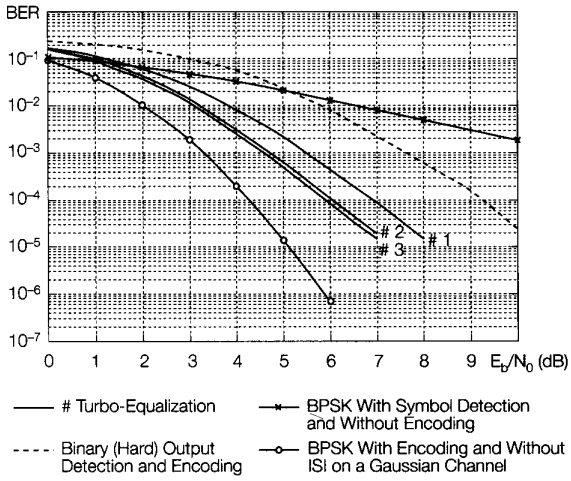

Fig. 8 - Performance of turbo-equalization over a Rayleigh channel with $BT = 0.001$ (convolutional encoding with $K = 5$).

6. CONCLUSION

In this paper a new method of combating the detrimental effects of intersymbol interference on multipath channels has been presented. It is based on the association of two successive soft-output decisions, achieved by a symbol detector and a channel decoder, in an iterative process. At each iteration a new piece of information, called extrinsic information, is calculated and used at the next iteration. By analogy with turbo-codes, this receiving scheme has been called turbo-equalization.

The receiver is made up of identical pipelined elementary modules, and the rank q elementary module uses data information coming from the demodulator and extrinsic information provided by the rank $(q - 1)$ module in order to take decisions. The performance of turbo-equalizing is directly related to the number of iterations.

The results presented in this paper have been obtained from simulations of Gauss and Rayleigh channels. In the case of a gaussian channel, the compensation for interference is complete and the behavior of a Gauss channel without multipath effects is reached after a few iterations. In the case of a Rayleigh channel, compensation is only partially achieved. However, this is still satisfactory because the bit error rate is close to that obtained on a gaussian channel without intersymbol interference. It is to be noted that these results were obtained in the case where the channel coefficients are supposed known. In practice, these coefficients have to be estimated, and the bit error rate is therefore degraded. However, further simulations are currently being processed in these conditions to show that the iterative process is also able to correct the degradation due to estimation, and turbo-equalization seems to provide results of great interest. Finally, the results obtained clearly show that turbo-equalization is an effective method of overcoming multipath effects in digital transmissions.

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