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The Origin of Utility: Sexual Selection and Conspicuous Consumption

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Abstract

This paper proposes an explanation for the universal human desire for increasing consumption and the associated propensity to trade survival opportunity off conspicuous consumption. I argue that this desire was moulded in evolutionary times by a mechanism known to biologists as sexual selection, whereby an observable trait — conspicuous consumption in this case — is used by members of one sex to signal their unobservable characteristics valuable to members of the opposite sex. It then shows that the standard economics problem of utility maximisation is formally equivalent to the standard biology problem of the maximisation of individual fitness, the ability to pass genes to future generations, and thus establishes a rigorous theoretical foundation for including conspicuous consumption in the utility function.

JEL Numbers: D01, C73
Keywords: Natural selection, Utility, Darwin, Evolution, Conspicuous Consumption, Veblen, Sexual selection.

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Henry saw his car, a hundred yards away, parked at an angle on the rise of the track, picked out in soft light against a backdrop of birch, flowering heather and thunderous black sky, and felt for the first time a gentle, swooning joy of possession. It is, of course, possible, permissible to love an inanimate object... (Ian McEwan, *Saturday*).

1 Introduction

*Homo Economicus’s* utility function constitutes one of the fundamental building blocks of economics. Its canonical form hinges on two assumptions: that there are trade-offs among the available commodities and that its shape is independent of the budget constraint. The latter implies that choices result from the interaction between income/wealth/resources, which are variable, and preferences, which are fixed. The trade-offs between goods implies that individuals are willing to sacrifice survival enhancing activities, such as the acquisition of nutritious food, of adequate shelter, of health care, to acquire goods with zero or negative survival value like luxury goods, leisure travel, entertainment, and so on. More succinctly, conspicuous consumption for its own sake enhances utility. This standard economics assumption clearly tallies with evidence, but the lack of a theoretical justification for it perplexes other scientists: “Western economics usually assumes that individuals are out to maximise personal gains, but where is the scientific justification for this assumption? And what exactly is ‘personal gain’?” (Trivers 1985, p 1). Trivers’ doubts are spelled out more explicitly by Grafen (1998, p 441): “The formulation of the dynastic utility function in terms of consumption purely for its own sake is inconsistent with the biological viewpoint”. The inconsistency is the apparent lack of any fitness advantage, which any physical or behavioural trait must afford in order to develop and persist in a population.1

In this paper I propose a foundation for the human propensity to trade survival opportunities off conspicuous consumption for its own sake rigorously based on evolutionary arguments, and therefore consistent with the biological viewpoint. The universality of the desire for conspicuous consumption across cultures and continents and the view of evolutionary psychology regarding the speed of adaptations (Barrett et al. 2002 p 12) indicate that this trait

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1Bagwell and Bernheim (1996), Corneo and Jeanne (1997), and Hopkins and Kornienko (2004), among others, have posited that conspicuous consumption of goods such as luxury goods, which are “completely novel in evolutionary terms”, enhances status and that desiring status is evolutionary “hard-wired” to affect directly an individual’s utility (Robson 2001a p 24). This however, still leaves open the question as to why a higher status is desirable, and why individuals would trade survival opportunities off enhanced status.
was hard wired in the brain of early humans prior to their dispersion from Africa, and therefore must have provided evolutionary advantages in the conditions prevailing between one million and 80,000 years ago. I build on the established economics tradition which explains features of human behaviour through evolutionary lenses by looking for fitness advantages of these features. Alchian (1950, pp 213–214) and Friedman (1953) viewed profit maximisation as a selection mechanism for firms. More recently, evolutionary advantages have been suggested for many human traits.\(^2\) My viewpoint, however, differs from most of the literature in that the fitness advantage of the trait considered is not the enhanced survival chances of the individuals with the trait: indeed individuals with a stronger desire for conspicuous consumption for its own sake had a \textit{survival disadvantage}, relative to individuals with a weaker desire. They, however, also had a \textit{reproductive advantage}, which more than offset their survival disadvantage. Thus the trait became established in the human genotype, as the genes linked to the trait became more frequent as generations went by. In the jargon, conspicuous consumption is a signal that causes sexual selection by mate choice. This is an evolutionary mechanism by which individuals of one sex signal their unobservable quality to the opposite sex, and their reproductive success depends on the signal via the mating choice of the individuals of the opposite sex. This mechanism is the driving force for the development of traits which are differentiated by sex and have negative survival value.\(^3\) from the extravagant plumage of pheasants, paradise birds,  

\(^2\)Examples include altruism (Becker 1976, Frank 1987, Bergstrom 1995, Bester and Guth 1998, Eswaran and Kotwal 2004), risk-taking (Robson 1995, Dekel and Scotchmer 1999, Warneryd 2002), experimentation (Robson 2001b), fertility and labour supply (Grafen 1998), preferences in general (Dekel et al. 2007, and the references cited therein), and more specifically, preference for relative consumption (Samuelson 2004), individualistic (Ok and Vega-Redondo 2001) and interdependent preferences (Koçkesen et al. 2000a and 2000b), the rate of intertemporal preferences (Hansson and Stuart 1990, Rogers 1994, Trostel and Taylor 2001), the dependence of utility on the presence of salient unchosen alternatives (Samuelson and Swinkles 2006); intergenerational cultural transmission (Bisin and Verdier 2001) and resource flows (Robson and Kaplan 2003), the demand for grandchildren (Cox and Stark 2005), sibling rivalry (Cox and Fafchamps 2008), and more generally, the structure and development of the family (Bergstrom 1996, Cox 2007), the emergence of trade (Ofek 2001, Seabright 2004, Horan et al 2005), economic growth (Galor and Moav 2002).

\(^3\)It is to the female's advantage to be able to pick the most fit male available for fathering her brood. Unusually fit fathers tend to have unusually fit offspring. One of the functions of courtship would be the advertisement, by a male, of how fit he is. A male whose general health and nutrition enables him to indulge in full development of secondary [not physiologically necessary for reproduction] sexual characters [...] is likely to be reasonably fit genetically [...]
peacocks and many other birds, to the ritual dancing and hopping displays in "leks", to the courtship vocalisations in tigers, deer, crickets, frogs, to the flashing of fireflies, to the complex bowers built and decorated by bowerbirds; to human traits such as the male beard and the female breasts.4

Zahavi (1975) realised that males' signals must be costly, exactly in the sense in which signals are costly in the economics literature (Spence 1973): the higher an individual's quality, the less burdensome it is for him to incur the cost of the signal, and the stronger the signal he will issue to distinguish himself from his lesser rivals in the eye of the females. His explanation of sexual selection was given a solid game theoretic foundation by Grafen (1990a and 1990b).

Consumption for its own sake, conspicuous consumption, I argue here, is precisely such a signal. It is easy to observe and expensive to acquire. It has served, throughout history, as an indicator of an individual's desirability as a mate. Veblen (1899) identified clearly the importance of expensiveness and wastefulness of conspicuous consumption: inexpensive items are not, cannot be effective signals, precisely because their very inexpensiveness makes it possible for everyone to sport them.5 Unlike Veblen, recent economic analysis has had

4See Zahavi and Zahavi (1997) for many more examples, or Andersson (1994, p 10 and Table 6.A, pp 132–142), for a taxonomy of the various mechanisms. Darwin devoted much of the Descent of Man (1871) to it, but, unlike natural selection, sexual selection was rejected for a long time by the scientific community (Anderson 1994, pp 17–19), a consequence, perhaps, of Victorian mental strictures and of Darwin's inability to offer a persuasive explanation of the mechanism through which it might operate (Darwin 1871). Fisher (1930) reprised Darwin's idea, suggesting that sexual selection works through a mechanism called the "runaway process" or the "sexy son hypothesis". This is in the spirit of the herd theory: if all females prefer certain males, then it pays a female with no preference also to choose those males as mates, because her sons will need to attract the current females' daughters, who will inherit their mother's preferences, and will be more likely to do so if they inherit their father's genes. This idea is not fully satisfactory either: in the absence of a cost of acquiring the trait, all males will tend to possess the optimum level as generations go by: the observation of variation across individuals would need to be justified by evolution not having yet completed its course.

5His books are rich in examples. "The chief use of servants is the evidence they afford to the master's ability to pay", rather than helping him in any useful manner (Veblen 1899, p 62). Their cumbersome liveries and unwieldy uniforms are actually designed to prevent them from performing any useful or productive activity. Similarly, skirts persist tenaciously as fashion accessories because, not despite, they "hamper the wearer at every turn and incapacitate her for all useful exertion", thus unmistakably demonstrating that she does not
access to Fisher’s and Zahavi’s insights, and yet has neglected the role of sexual selection as a powerful engine of human evolution.\footnote{For example, in his seminal economic analysis of human evolution, Frank notes how sexual selection traps a species into a prisoner’s dilemma (“peacocks taken as a group would clearly do better if all had smaller tail feathers” Frank 1988 p 23), but otherwise does not link it to human evolution, as other social scientists have done (Diamond 1991 pp 175–180, Miller 2000, Ridley 2003, Buss 2004).}

The signalling model in section 2, closely inspired by Grafen (1990a and 1990b), captures these ideas. It describes a population composed of males and females where the males’ reproductive potential is limited by female choice. Males differ in their value to females, and face a trade-off between wasteful “conspicuous” consumption and unobservable activities which enhance their chance of survival. Females observe males’ conspicuous consumption and choose with whom to mate. In a separating equilibrium, males undertake conspicuous consumption in order to signal their quality to females, and females are more likely to mate with males whose observed consumption is higher. Proposition 2 in Section 3 identifies some conditions on the population and the environment which ensure that a separating equilibrium exists.

The core of the paper is Section 4. Here I show the natural connection between maximisation of fitness and maximisation of a utility function with consumption bundles as arguments. The trade-off between survival and reproduction is mapped one-to-one with the trade-off between “survival activities” and “conspicuous consumption”. This provides an evolutionary foundation to the indifference maps that constitute the basis of the economic analysis of consumer behaviour, suggesting that preferences are not arbitrary, but have evolved in response to our ancestors’ exogenous constraints.

Section 5 discusses some empirical evidence and considers some open economics questions in the light of the ideas of the paper, and the Appendix presents the formal proof of Proposition 2.

2 The Model

2.1 The population

need to work (p 171). Corsets and top hats are among his other examples. By the same token, in many animal species, powerful males obtain and protect large territories, much larger than it can be possibly be necessary to provide food and shelter to the family and subordinate individuals (O’Donald 1963, Zahavi and Zahavi 1997, pp 28–29). This is of course an all too accurate description of the behaviour of human ruling classes nowadays and over the entire course of history.
The model is close to Grafen’s seminal paper (1990a). I consider a sexually reproducing population, comprising two sexes, males and females. Their “mating season” is divided into $T > 1$ discrete periods, with matching occurring in the “mating market” during each period. The interpretation of seasons and periods within a season is flexible: the season could be the summer and the periods days; alternatively, the season could be a generation, and each period an oestrus cycle. To ease the presentation, I consider the case $T = 2$; this can naturally be extended to the more complex case of a generic finite $T$ (De Fraja 2006). Offspring are born and reared after the end of the mating season. Within the season, the population dynamics from period to period is governed by survival and mating. In each period, matching is one-to-one: each individual is matched to at most one individual of the other sex. Matching probabilities in period $t$ depend on the population numbers of the two sexes, given by $F_t$ for females and $M_t$ for males. Specifically, a female is matched to a male with probability $q_F(F_t, M_t)$, and a male is matched to a female with probability $q_M(F_t, M_t)$, $t = 1, 2$. A matched pair will mate if the benefit exceeds the cost for both parties. In humans, just as in virtually all sexual species, the opportunity cost of mating differs in the two sexes, and consequently so does parental investment. I capture this asymmetry with the extreme assumption of no paternal investment, but the analysis can be extended to less extreme asymmetry (see below, Section 5.4). Incurring no opportunity cost in mating, a male agrees to mate whenever he is matched with a female, and returns to the mating market in the following period. Therefore males are polygynous, and they may try to choose strategies which allow them to have more than one mate in the season. Polygyny here follows naturally from the assumed extreme lack of paternal investment in offspring, which makes maximising the number of mates the dominant strategy for males. This for simplification and can be replaced with the alternative assumption, as in Robson (1996), that a male share his resources among all his offspring. If a male is believed to have sufficient resources, a female would be willing to share him with another female, in preference to being the sole mate of a male with little resources. Polygyny in this case would be simultaneous, rather than sequential as in my model.\footnote{Another difference is that, in Robson’s set up, resources are observed: for a signalling equilibrium to emerge, they would have to be inferred by females from the signal issued by males.}

Females differ from males in that, to reproduce successfully, maternal in-
vestment is necessary, in the form of pregnancy, lactation and other childcare activities; I assume that a female can have at most one reproductive cycle per season, and that she leaves the market if she mates. In Dixit and Pyndick’s terminology (1994), mating in a period (except the last) kills the option of mating with a higher quality male later in the season. In addition, both males and females may die during the season.

Females differ in their potential for surviving and maintaining fertility during the season. I denote by \( \delta \) a fertile female’s probability of being alive and fertile in the next period. In period \( t \), \( \delta \) is distributed in \([0, 1]\), according to density \( \phi^F_t(\delta) \) and distribution \( \Phi^F_t(\delta) \), with \( \phi^F_t(\delta) = \frac{d \Phi^F_t(\delta)}{d \delta} \). \( F_t \) is the total number of females in period \( t, t = 1, 2 \).

Males differ in a quality, measured by a single dimensional parameter, \( \theta \), which, at the beginning of period \( t \) in the season, is distributed in \([\theta_{\text{min}}, \theta_{\text{max}}]\) \( \subseteq \mathbb{R} \), according to the density \( \phi^M_t(\theta) > 0 \) for \( \theta \in (\theta_{\text{min}}, \theta_{\text{max}}) \), and distribution \( \Phi^M_t(\theta) \), with \( \phi^M_t(\theta) = \frac{d \Phi^M_t(\theta)}{d \theta} \).

\( \Phi^F_t(\theta) \) and \( \Phi^M_t(\theta) \) are exogenously given, whereas \( \Phi^F_t(\theta) \) and \( \Phi^M_t(\theta) \) depend on the matching process. \( \Phi^M_t(\theta) \) is such that the distribution of types below any \( \tilde{\theta} \) in \((\theta_{\text{min}}, \theta_{\text{max}})\) has a monotonic hazard rate: for every \( \tilde{\theta} \in (\theta_{\text{min}}, \theta) \):

\[
\frac{d}{d \theta} \left( \frac{\Phi^M_t(\theta) - \Phi^M_t(\tilde{\theta})}{\phi^M_t(\theta)} \right) < 0, \text{ for } \theta \in (\theta_{\text{min}}, \tilde{\theta}).
\]

The total number of males in period \( t, t = 1, \ldots, T \), is denoted by \( M_t \).

The benefit of mating is measured by a function \( v(\theta) \), satisfying \( v'(\theta) > 0 \), a normalisation, and \( v(\theta_{\text{min}}) > 0 \), mating is always better than not mating: clearly, if this were violated for some \( \theta \) then these types would not mate under any circumstance, and can be removed from the mating process. There is no gain in generality in having differential benefits for males and females. \( v(\theta) \) can be thought of as the expected number of offspring who survive to adulthood and enter the market at the beginning of the next season, even though other interpretations are possible. Following Grafen (1990a), the distribution of types is the same at the beginning of each season, both for females and for males. Sufficient, but not necessary, to ensure this is the absence of today’s adults from the future mating seasons, and lack of correlation between the fathers’ and the sons’ \( \theta \)’s: an individual’s type is drawn at the beginning of the season from a distribution invariant to the father’s type, determined, for example, by the environment. In Grafen (1990a), this assumption is introduced to obtain an equilibrium based on the handicap principle, operating
independently of the Fisher runaway process, mentioned in footnote 4: if the probability density function of the offspring types is independent of the parents’ types, then, by construction, the Fisher effect cannot operate. This of course does not imply absence of natural selection: what is passed on to the next generation is not the genes that determine $\theta$, but rather the genes that determine the link between $\theta$ and the strategy followed by individuals, namely the function $c_1(\theta)$ studied in the paper.

2.2 Males’ strategies.

Males choose\(^8\) two variables, conspicuous consumption $c \in \mathbb{R}_+$, and investment in survival activities, measured by a variable $w \in \mathbb{R}_+$, for example the quality of the diet or the search and adaptation of safe hiding places for the nights: in each period, a male survives with probability $\pi(w) \in (0, 1)$, with $\pi'(w) > 0$ and $\pi''(w) \leq 0$. Conspicuous consumption is wasteful: it has no direct benefit, but has a cost, described by a standard production possibility frontier:

$$f(c, w, \theta) = 0.$$

The values of $c$ and $w$ chosen by a type $\theta$ male satisfy (1). In (1), $f_c(\cdot), f_w(\cdot) > 0$, to capture the trade-off between $c$ and $w$, and $f_{\theta}(\cdot) < 0$, to indicate that higher quality is associated with an expanded production possibility frontier. Females can observe the current value of $c$ only, so they cannot determine directly a male’s quality $\theta$, nor infer it from the observation of $w$. The role of $c$ is therefore to serve as a signal. Formally, males’ strategy set is the set of all measurable mappings from the space of types $[\theta_{\text{min}}, \theta_{\text{max}}]$ to the space of possible signals, in each period (we restrict attention to pure strategies). Males’ strategies are denoted by $\{c_1(\theta), c_2(\theta)\}$. In a separating equilibrium

\(^8\)The term “choose” is typically used in signalling models in economics, but it is also appropriate in the current more biological set-up, where it does not have the implication of conscious decision making which human choice has. Thus the peacock is said to choose the length of his tail, the deer the size of his antlers, the fish the intensity of his coloration, even though these choices are best described as determined by the interaction of the genotype (the DNA instructions) with the environment.

\(^9\)This is therefore unlike Samuelson’s (2004) model where high consumption decreases the probability of survival when the environment is bad, but increases it when it is good. Instead, it is analogous to Robson’s (1996) analysis of risk taking, which also consider polygyny: a risk-averse male may accept slightly unfair bets, which waste resources in expectation, if by doing so he can accumulate wealth and so increase his chance that a second female accepts him as mate even though there are available males with no mates. In Robson’s model, unlike here, wealth is a resource, and so accumulating it is not a wasteful signal.
(which is called a “signalling equilibrium” in the theoretical biology literature), females infer correctly the males’ types from their observed signal, \( c \), and choose the best action given their beliefs, and males choose the best signal, given the females’ actions. Unlike females, males always agree to mate if they are matched to a female.

From (1) it is convenient to define the function \( \omega : \mathbb{R}_+ \times [\theta_{\text{min}}, \theta_{\text{max}}] \rightarrow \mathbb{R}_+ \), which associates to a signal \( c \) and a type \( \theta \) the (maximum) level of \( w \) that individual of type \( \theta \) can acquire who emits signal \( c \): \( \omega(c, \theta) \) is the solution in \( w \) of (1).

It is crucial, for the logic of the handicap principle, that the variable \( w \) cannot be observed by the females. In practice, of course, some resources can be observed, and indeed display of observable survival resources occurs in humans and other species (Yosef 1991). To capture this idea by including \( c \) as an argument of the survival function, writing \( \pi(w, c) \) with \( \pi_c(w, c) > 0 \), would complicate the notation a bit and not alter the substantial analysis, as long as reducing the unobservable resources, \( w \), to increase what is observable, \( c \), also reduces the overall chances of survival: formally, \( \pi_c(w, c) \) can be positive as long as it is less than \( \pi_w(w, c) \frac{f(c, w, \theta)}{f_{\omega}(c, w, \theta)} \). The fundamental feature of the model is the presence of some unobservable aspect of males’ behaviour, measured here by \( w \). Thus the food reserves stored as body fat (which had a positive effect on survival for all but our most recent existence) can be observed by a female, but the time spent in selecting nutritious food or checking whether a food or drink source is safe or contaminated cannot. Recent theories about the evolution of early hominids hold that “carbohydrate-rich roots [...] provided reserve food supplies during hard times [...] and could indeed have been the fallback food that carried our ancestors from dwindling forest into the more open woodland and savanna” (Wrangham and Peterson 1996, p 54). In this vein, a plausible natural interpretation of \( w \) is the time a male spends looking for areas rich in these roots; being a fallback food, they will be needed during lean times, when competition for food is harshest: for this activity to have survival value, these areas must be kept hidden from all other individuals, males and females, and therefore \textit{must remain unobserved}.

\subsection*{2.3 Females’ optimal strategy.}

In each period, a female matched to a male has a very simple action set: either she mates or she does not mate. In the second period, the last, she mates
with any male she is matched with, since there is no value in postponing, and therefore her strategy reduces to the binary choice in period 1. Her choice depends on her survival probability \( \delta \) and on her belief about the type of the male she is matched with. Beliefs in period \( t \) can be described by the function \( c^b_t(\theta) \), the signal expected from a male of type \( \theta \). Formally, let \( V_F^2(\delta) \) be the expected payoff of a female for being in the mating market at the beginning of period \( t \).

**Definition 1** For given beliefs \( c^b_t(\theta) \), define the acceptance function, \( \alpha(\delta) \), as the higher of \( \theta_{\text{min}} \) and of the solution in \( \theta \) to:

\[
v(\theta) = \delta V_F^2,
\]

where:

\[
V_F^2 = q_F(F_2, M_2) \int_{\alpha(\delta)}^{\theta_{\text{max}}} v(\theta) \phi^M_2(\theta) d\theta
\]

Notice that since \( v(\theta) \) is increasing, and \( V_F^2 \) is independent of \( \theta \), there is a unique solution to (2). To see how (3) is derived, note that, in the second period, a female is not matched to a male with probability \( [1 - q_F(F_t, M_t)] \), in which case her payoff is 0, and is matched with probability \( q_F(F_2, M_2) \) and with probability \( \phi^M_2(\theta) \) this male is of type \( \theta \). If \( \theta \geq \alpha(\delta) \), she mates with him, and has payoff \( v(\theta) \), otherwise she postpones, which happens with probability \( \Phi^M_2(\alpha(\delta)) \), and obtains, in the next period, payoff \( \delta V_F^2 \). Putting all of this together (3) is obtained. The acceptance function summarises the strategy followed by females, and can be drawn in Figure 1 using Lemma 1.

**Lemma 1** There exists \( \delta^* > 0 \), such that \( \alpha(\delta) = \theta_{\text{min}} \); there exists \( \bar{\theta} \in [0, 1] \), such that \( \bar{\theta} = v^{-1}(V_F^2) \); the function \( \alpha(\delta) \) is strictly increasing in \( [\delta^*, 1] \).

**Proof** The first two statements are immediate; for the last, total differentiation of (2) yields

\[
\frac{d\alpha(\delta)}{d\delta} = \frac{V_F^2(\delta) + \delta V_F^2(\delta)}{v'(\theta)}> 0
\]

\(^{10}\)Given a prior regarding males’ types \( \phi_M^t(\theta) \), and letting \( c^b_t(\theta) \) be the strategy females believe males will follow in period \( t \), the posterior density function inferred from observing a value \( c \) is such that \( \theta^k \) has zero density if \( \theta^k \notin c_{t-1}^t(c) \), and it is given by

\[
\frac{\phi^t_M(\theta^k)}{\sum_{k=1}^{H} \phi^t_M(\theta^k)},
\]

if \( c_{t-1}^t(c) \) has measure 0 and the image set of \( c_{t-1}^t(c) \) is \( \{\theta^1, ..., \theta^k, ..., \theta^H\} \), and by

\[
\int_{\theta \in c_{t-1}^t(c)} \phi^t_M(\theta) d\theta
\]

if instead \( c_{t-1}^t(c) \) has positive measure.
whenever $\alpha(\delta) > \theta_{\text{min}}$, thus proving the last assertion in the Lemma. It also shows that there can be at most one value $\delta \in [0, 1]$ such that (2) holds, and $v(\theta_{\text{min}}) > 0$ implies the first assertion. Since $v$ is the same in each period, the maximum future payoff cannot exceed $v(\theta_{\text{max}})$, implying the second assertion and establishing the Lemma.

Because $v(\theta)$ is strictly increasing, a female of type $\delta$, matched with a male of type $\theta$, mates with him if $\theta > \alpha(\delta)$, and does not mate if $\theta < \alpha(\delta)$. In words, a female agrees to mate with a male believed to be of type $\theta$, if her payoff for mating, $v(\theta)$, is at least as big as the payoff for not mating. Lemma 1 can be used to represent females’ strategy in the $(\delta, \theta)$-cartesian plane: all females of type $\delta < \delta_0$ mate in the current period with any male type $\theta$, and all females matched with a sufficiently “good” male mate with him (note that, in general, $\bar{\theta}$ is strictly below $\theta_{\text{max}}$). This is summarised in Figure 1: female-male matches such that the combination of types $(\delta, \theta)$ is in the light grey area mate, those in the dark area do not.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{female_choice}
\caption{Female choice in period 1.}
\end{figure}

$1^1$ If $\theta = \alpha(\delta)$ she is indifferent. This is a measure 0 case, and, for definiteness, I assume that she mates with probability 1 in this case.

$1^2$ The model differs from Grafen’s (1990a) where females are all identical, and where males’ strategies are independent of the time in the mating season. Grafen’s model is therefore more directly applicable to traits which cannot be changed readily, such as the plumage or the depth of a frog’s call, mine to activities such as dancing, fighting, singing, bower building, and so on, and of course conspicuous consumption. Certain traits fall in between: a stag’s antlers respond slowly to external conditions. His model and mine share the fundamental asymmetry between forward looking females —whose maximisation strategy involves the exercises of an option, and therefore the forecast of future conditions—, and here-and-now males —for whom the opportunity cost of mating in the present period is lower.
2.4 Population dynamics.

Some individuals are absent from the matching market in the second period: males because of death, and females because of both death and successful mating. These withdrawals depend of course on the strategies followed in period 1. They determine the following dynamics of the distributions of types.

\[ \phi_2^F (\delta) = \frac{F_1}{F_2} \delta \phi_1^F (\delta) \left[ 1 - q_F (F_1, M_1) \left[ 1 - \Phi_1^M (\alpha (\delta)) \right] \right], \quad \forall \delta \in [0, 1], \quad (4) \]

\[ F_2 = F_1 \int_0^1 \delta \phi_1^F (\delta) \left[ 1 - q_F (F_1, M_1) \left[ 1 - \Phi_1^M (\alpha (\delta)) \right] \right] d\delta, \quad (5) \]

\[ \phi_2^M (\theta) = \frac{M_1}{M_2} \pi (\omega (c_1 (\theta), \theta)) \phi_1^M (\theta), \quad \forall \theta \in [\theta_{\text{min}}, \theta_{\text{max}}], \quad (6) \]

\[ M_2 = M_1 \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \pi (\omega (c_1 (\theta), \theta)) \phi_1^M (\theta) d\theta. \quad (7) \]

(4) is the distribution of females in period 2. To see how it is derived, consider females of type \( \delta \). At the beginning of the season, there are \( F_1 \phi_1^F (\delta) \) of them. Of these, a fraction \( \delta \left[ 1 - q_F (F_1, M_1) \right] \) are not matched and survive to the second period. Of the \( F_1 \phi_1^F (\delta) q_F (F_1, M_1) \) who are matched \( 1 - \Phi_1^M (\alpha (\delta)) \) mate and leave the market; of the rest, \( \delta \) survive and \( 1 - \delta \) die. This applies to every \( \delta \), which gives (4). The number of females in the market in the second period, \( F_2 \), is given in (5) by integration of (4). Similarly, but more simply, for males: they leave the market only when they die, which happens with probability \( 1 - \pi (\omega (c_1 (\theta), \theta)) \). Starting with \( M_1 \phi_1^M (\theta) \) males of type \( \theta \), (6) and (7) are obtained.

I do not study the dynamics of the population from season to season, but look instead for a steady state population equilibrium. Offspring inherit from their parents the strategy that links their type with the signal to emit in each period if they are male, and with the acceptance function, again in each period, if they are female. The formal game-theoretic representation of a strategy in a signalling game as a mapping from the set of possible types to the set of admissible actions matches closely the biological definition of allele as a set of instructions specifying the response of the individual hosting that allele to the environment.\(^{13}\) In a population equilibrium, strategies, or alleles, must satisfy two characteristics: firstly, a male’s strategy \( c_1 (\theta) \) must be a best response: it must maximise a male’s season payoff given the distribution of males and

\(^{13}\) The environment includes also the individual’s characteristics: the same allele specifies the individual’s behaviour according to the individual’s sex and characteristics.
females in the population, and assuming that all other males follow \( c_1 (\theta) \) and that all females behave according to \( \alpha (\delta) \) and have consistent beliefs. If this were not the case, then some male type \( \theta \) would prefer to issue a signal other than \( c_1 (\theta) \), and so \( c_1 (\theta) \) could not be part of the equilibrium. Secondly, among the signal functions that are best reply to themselves for given female behaviour, I select the one which gives males the highest payoff: an allele corresponding to strategy \( \tilde{c}_1 (\theta) \) that did not maximise males’ payoff, could not be a population equilibrium, because a population composed of individuals with allele \( \tilde{c}_1 (\theta) \) is invasible by an alternative allele giving higher payoff, which would therefore become more frequent in the population as seasons go by.

3 Equilibrium in the last two periods.

The equilibrium behaviour of both males and females in the second period is very simple: females mate with any male they are matched with. Therefore males do not signal: \( c_2 (\theta) = 0 \). In period 1, instead, females have an opportunity cost of mating and it may be beneficial for them to forgo mating with a male they believe to be of low quality. This makes potentially preferable for males to try to signal to females that they are of good quality, and so increase their probability of being accepted as mate by a female.\(^{14}\)

To lighten notation, with little loss of generality, let \( F_1 = M_1 = 1 \), there is initially the same number of males and females and \( q_F (F_1, M_1) = q_M (F_1, M_1) = 1 \): everyone is matched in the first period. The equilibrium is found by determining simultaneously the males’ optimal strategy, \( c_1 (\theta) \), the number of females, \( F_2 \), and males, \( M_2 \), who are seeking a partner in period 2, the value, for a matched female, of postponing mating to period 2, \( V^F_2 \), and the density of the distribution of females and males in period 2. The variables other than \( c_1 (\theta) \) are immediate from (5), (7), (3), (4), and (6) respectively: (10) is obtained by substituting (12) into (3).

\[
F_2 = \int_0^1 \delta \Phi^M_1 (\alpha_1 (\delta)) \phi^F_1 (\delta) d\delta, \tag{8}
\]

\(^{14}\) A conceptually simple, but algebraically complex induction argument extends the technique given here to the \( T \)-period case. The last period is trivial: as here, females mate with any male and males do not signal. The behaviour in period \( T - 1 \) is found exactly like here, taking as given the distributions of males and females determined by the past history up to period \( T - 2 \). Backward induction takes us recursively back to period 1.
\[ M_2 = \int_{\theta_{\min}}^{\theta_{\max}} \pi (\omega (c_1 (\theta), \theta)) \phi_1^M (\theta) \, d\theta, \] (9)

\[ V_2^F = \frac{qF (F_2, M_2)}{M_2} \int_{\theta_{\min}}^{\theta_{\max}} v (\theta) \pi (\omega (c_1 (\theta), \theta)) \phi_1^M (\theta) \, d\theta, \] (10)

\[ \phi_2^F (\delta) = \frac{1}{F_2} \delta_1 \Phi_1^M (\alpha_1 (\delta)) \phi_1^F (\delta), \] (11)

\[ \phi_2^M (\theta) = \frac{1}{M_2} \pi (\omega (c_1 (\theta), \theta)) \phi_1^M (\theta). \] (12)

\( \delta \), the type of the least impatient female who mates with any available male, is given by

\[ \delta = \frac{v (\theta_{\min})}{V_2^F}. \]

\( \bar{\theta} \), the lowest male type such that every female mates with him, is the solution in \( \theta \) to \( v (\theta) = \delta V_2^F \) for \( \delta = 1 \):

\[ \bar{\theta} = v^{-1} (V_2^F). \]

To state the main result of this section succinctly, define the function:

\[ a (c, w, \theta) = \frac{f_{\theta c} (c, w, \theta)}{f_c (c, w, \theta)} - \left( \frac{\pi'' (w)}{\pi' (w)} + \frac{f_{wc} (c, w, \theta)}{f_w (c, w, \theta)} \right) \frac{f_{\theta} (c, w, \theta)}{f_w (c, w, \theta)}. \]

**Assumption 1** For every \((c, w, \theta) \in C \times W \times [\theta_{\min}, \theta_{\max}]\), let

\[ a (c, w, \theta) + \frac{v' (\theta)}{v (\theta)} > 0, \] (13)

and let \( a_\theta (c, w, \theta) \leq 0 \) and \( a_c (c, w, \theta) > 0 \).

**Proposition 2** If Assumption 1 holds, then there is a population equilibrium where males signal: a male of type \( \theta \) chooses \( c_1 (\theta) \), which is continuous and strictly increasing for \( \theta \in [\theta_{\min}, \bar{\theta}] \), and constant for \( \theta \in (\bar{\theta}, \theta_{\max}] \).

In words, males signal their type, to differentiate themselves from less attractive males: males of type \( \theta \in [\theta_{\min}, \bar{\theta}] \) separate in equilibrium and males of type \( \theta \in (\bar{\theta}, \theta_{\max}] \) all choose the same consumption level as the highest type male.\textsuperscript{15} The proof of this result is in Appendix A. While not readily interpretable, the conditions given in Assumption 1 can well be violated, implying that only some combinations of environmental constraints can give rise to the

\textsuperscript{15} As in Grafen (1990a), there is also a non-separating equilibrium where \( c (\theta) = 0 \) for every \( \theta \in [\theta_{\min}, \theta_{\max}] \): females’ out of equilibrium beliefs can be specified consistently in many ways: for example, suppose that females consider each male equally likely to deviate, and hence associate to any signal \( c > 0 \) type \( \theta \) with density \( \phi_1^{**} (\theta) \). In what follows I concentrate on the separating equilibrium. This itself need not be unique: it may happen that females are choosy enough to compel males to signal a lot, while in another population characterised by identical functional forms and parameters, low discrimination by females and low signalling by males may be the resulting equilibrium.
development of conspicuous consumption as a Zahavian handicap: in different conditions, signalling may not have occurred, or it may have occurred for traits, behavioural or physical other than the desire for conspicuous consumption, just as is the case for other primate species.

We have considered a 2-period model taking the distribution of types at the beginning of the last-but-one period as given; in a 3-period models it would instead be endogenously determined together with the optimal strategy of period 1. The optimal control problem corresponding to (27) for the 3-period case would be maximised by choice of $c_1(\theta)$, $c_2(\theta)$, $M_2$, $F_2$, $M_3$, $F_3$, $V_2^F$, and $V_3^F$, with appropriate constraints, equivalent to (23) and (24) or derived from (3)-(7), and with the distribution of types at the beginning of period 2 as endogenously determined. The problem would mathematically more cumbersome, but conceptually analogous. Similarly for higher values of $T$. Plausibly, the solution of the model with more than three periods would be such that the interval of values of $\theta$ where the signal function is horizontal is shorter in earlier periods (that is the threshold $\theta_t$ decreases with $t$): this would follow from the fact that a female’s option value of delaying mating decreases as times goes by, since she has fewer opportunities to meet with a high quality male.

4 Utility maximisation.

Living species subject to evolutionary pressure reach a point on the trade-off between individual survival and reproduction which maximises fitness subject to their environmental constraints. Consider for example the males of a species of frogs studied by Ryan et al. (1982). They emit courtship calls, which attract both females, increasing their reproductive chances, and frog eating bats, increasing their chances of death. Louder calls attract more females and more bats. Females benefit by mating with bigger males, and since mating

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16Fitness is a slippery concept in biology, see Dawkins (1982), pp 179–194. In the relatively simple environment I consider here, with basic genetics and no “altruistic” interaction between individuals, individual “inclusive fitness” and the population genetics concept of frequency of the genotype in the population coincide; both correspond to Becker’s use of the concept of “genetic fitness” (1976). In this environment, as shown by Grafen (1998), deriving the equilibrium as the result of a postulated maximisation problem is equivalent to deriving it from population genetics, with no assumption about maximising behaviour. The establishment of rigorous link between these approaches is “formal Darwinism project”, a current line of research in theoretical biology (Grafen 2006a and 2006b).
occurs at night, size cannot be observed by female directly, and corresponds to \( \theta \). The loudness of the courtship call of a male frog is \( c \), and the “preference” for loudness delicately balances the caller’s chances of attracting a female and becoming her mate, and attracting a bat and becoming its meal: the trade-off, however, depends on the male's size. A biologist could therefore, in theory at least, derive frogs’ “preference” for loudness as a function of their size from the observation of the environment where frogs live: in terms of the model described above, knowledge of the functions \( f(c, w, \theta) \) and \( \pi(w) \) gives \( c(\theta) \). Biologists of course would understand clearly that male frogs’ “preference” is in fact moulded by the action of evolutionary pressure, in the sense that the relation between loudness and size and environment is the solution to the problem of maximisation of inclusive fitness. With the exceptions of the works cited in footnote 2, economists do not generally derive human preferences, assuming them instead, without considering whether their assumptions are consistent with an evolutionary viewpoint.

I show in this section that the standard economic problem of maximising utility from consumption subject to a budget constraint, and the biological problem of maximising fitness subject to environmental constraints are formally equivalent. This provides an evolutionary foundation for the existence of a utility function with trade-offs between conspicuous consumption and survival activities.

I consider the two-period case examined in Section 3, using an argument which proceeds in three steps; the extension to the \( T \)-period case is conceptually analogous. In the first step I construct a correspondence between the males’ trade-off between survival and reproduction and their trade-off between consumption goods. I then derive the males’ utility function. This is fully determined by the environment, which includes the behaviour of the females of the species. I require the utility function to satisfy two characteristics routinely assumed in the economics literature. Firstly, that it should be independent of \( \theta \), so that the preferences of individuals with different \( \theta \) can be represented by the same utility function. Secondly, that its maximisation should lead to trade-offs that change continuously with the environment: the indifference curves should be decreasing and convex. In the third and final step I establish the correspondence between the solutions of the two problems.

**Step 1.** Consider an individual facing two independent lotteries. In the first lottery, mating, with probability \( p_m \in [0, 1] \) he mates, obtaining a prize \( v(\theta) \), and with the complement probability, \( 1 - p_m \), he does not mate, which
gives a 0 payoff. The second lottery is survival, which happens with probability $p_s \in [0, 1]$ and gives payoff $q_M (F_2, M_2) v(\theta)$ and again 0 in the event of death. Total expected payoff is therefore:

$$v(\theta) (p_m + q_M (F_2, M_2) p_s).$$

(14)

Consider next the space $[\theta_{\text{min}}, \theta_{\text{max}}] \times W$. A point on this space represents the pair of the individual type $\theta$ and his acquisition of survival assets $w$. Let the probability space $[0, 1]^2$ and the space $[\theta_{\text{min}}, \theta_{\text{max}}] \times W$ be linked by the function $G : [\theta_{\text{min}}, \theta_{\text{max}}] \times W \rightarrow [0, 1]^2$, which associates to a point $(\theta, w)$ the probabilities of mating and of surviving for a male of type $\theta$ who has survival consumption $w$.

$$G : (\theta, w) \mapsto \left( \Phi_f^F \left( \frac{v(\theta)}{V^F_2} \right), \pi(w) \right).$$

The function $G$ depends on the population distribution in the equilibrium, via the parameter $V^F_2$. Notice also that $G$ is one-to-one, and therefore invertible, in $[\theta_{\text{min}}, \theta_{\text{max}}] \times W$.

**Step 2.** Consider the locus

$$I_k = \left\{ p_m, p_s \in [0, 1]^2 | v(\theta) (p_m + q_M (F_2, M_2) p_s) = k \right\}.$$

This is the locus of the points representing combination of probabilities which give the same payoff. These are straight lines with slope $-q_M (F_2, M_2)$, depicted on the RHS diagram in Figure 2, in the unit square $[0, 1]$ in the $(p_m, p_s)$-cartesian space, for various values of $k$.

Consider the space of consumption vectors $C \times W$, where a point $(c, w) \in C \times W$ represents a combination of conspicuous consumption (with no survival value) and survival consumption. The function $G$ can be used to construct indifference curves in the consumption space $(c, w)$ as follows. Start from a locus $I_k$, and apply $G^{-1}$ to each point in $I_k \cap G \left( [\theta_{\text{min}}, \theta_{\text{max}}] \times W \right)$. This is the set of points that gives payoff $k$ and are in the range of $G$, say the points on the dotted line on the RHS of Figure 2. The image of these points under $G^{-1}$ is the locus on the northeast quadrant of the LHS of the diagram: for example, point $(\theta^a, w^a)$ is mapped into point

$$(\theta^a, w^a) = G^{-1} (p_m^a, p_s^a) = G^{-1} \left( \Phi_f^F \left( \frac{v(\theta^a)}{V^F_2} \right), \pi(w^a) \right).$$
Figure 2: Indifference curve and constraint in the \((c, w)\)-cartesian space.

The points on the curve \(G^{-1} (I_k \cap G (\theta_{\min}, \widetilde{\theta} \times W))\) represent combinations of type \(\theta\) and survival consumption \(w\) which give the same payoff. Now simply transfer this curve to the \(C \times W\) space by writing \(c_{1}^{-1} (c)\) instead of \(\theta\):

\[
w = w (c) = \pi^{-1} \left( \frac{k}{v(c_{1}^{-1}(c))} - \Phi \left( \frac{v(c_{1}^{-1}(c))}{\gamma_{2}} \right) \right)
\]

(15) is the indifference curve between \(c\) and \(w\) where the payoff is \(k\). Diagrammatically, the equilibrium signal \(c_{1} (\theta)\) derived in Proposition 2 takes a point \((\theta^{a}, w^{a})\) in the north-east quadrant to the corresponding point in the north-west quadrant. From \(\theta^{a}\), follow the dashed curve clockwise to the south east quadrant, which shows the locus \(c_{1} (\theta)\) derived in the previous section, then to south-west quadrant, which has a \(45^\circ\) line, and determines the abscissa \(c^{a}\) corresponding to the ordinate \(\theta^{a}\) in the north-west quadrant. Repeating the procedure for every point in the set \(I_k \cap G ([\theta_{\min}, \widetilde{\theta}] \times W)\) gives the points \((c^{a}, w^{a})\) which, given the strategies of all members of the population, yield a male the same payoff. The dash-dot line maps another point on the iso-payoff curve. Clearly it is not necessarily the case that these points are feasible or optimal. A standard indifference curve, for goods which are both valuable, the image of the set \(I_k \cap G ([\theta_{\min}, \widetilde{\theta}] \times W)\) must be downward sloping. This is established in the following Lemma.

**Lemma 2** If Assumption 1 holds, then the image of \(I_k \cap G ([\theta_{\min}, \widetilde{\theta}] \times W)\) onto the \((c, w)\)-cartesian space is a decreasing function.
Proof Since Assumption 1 holds, \( c_1(\theta) \) is increasing, and, therefore, it is sufficient to show that the image of \( I_k \cap G \left( [\theta_{\min}, \theta] \times W \right) \) under \( G^{-1} \) onto the \((\theta, w)\)-cartesian space is a decreasing function. To establish this, note that, to be on the \( I_k \) locus, a point \((\theta, w)\) must satisfy
\[
v(\theta) \left( \Phi^F_1 \left( \frac{v(\theta)}{V^2_2} \right) + q_M(F_2, M_2) \pi(w) \right) = k.
\]
Rearrange to get
\[
w = \pi^{-1} \left( \frac{k - \Phi^F_1 \left( \frac{v(\theta)}{V^2_2} \right)}{q_M(F_2, M_2)} \right);
\]
differentiate with respect to \( \theta \):
\[
\frac{dw}{d\theta} = -\pi^{-1}(\theta) \frac{v'(\theta)}{q_M(F_2, M_2)} \left( k \frac{\Phi^F_1 \left( \frac{v(\theta)}{V^2_2} \right)}{V^2_2} \right) < 0.
\]
Note that the indifference curves identify a preference relation on \((c, w)\) in the standard Arrow-Debreu sense, and the utility function is constructed by assigning a numerical value to each indifference class, \( u(c, w) \); it is invariant to strictly monotonic transformation, the standard ordinality requirement of a preference representation. Another standard requirement is that the indifference map should be independent of the budget constraint, or more generally of the resource constraint. In the present set-up the latter is given by \( \theta \), and so this requirement amount to the indifference map being the same for all male’s types \( \theta \in [\theta_{\min}, \theta_{\max}] \). This is the case for the indifference map defined by (15): in other words, individuals choose different combinations of \( c \) and \( w \) not because they have different preferences, but because they have different resources, \( \theta \), at their disposal. As (15) shows, the shape of the utility function is of course determined by the environment, represented here by the functions \( \pi, q_M, q_F, \) and \( v \), and the distributions \( \Phi^F_1(\delta) \) and \( \Phi^M_1(\theta) \): a change in the environment making, say, predators more dangerous may in turn affect the matching probabilities – because individuals become less mobile –, or the frequency of high \( \theta \)-males – who are more likely to survive to adulthood –, and this in turn would alter the indifference curves in (15) and the utility function \( u(c, w) \). Notice that the environment affects the function \( c_1(\theta) \) itself, which in turn enters the expression for the indifference curves in (15). This captures the fact that, as far as each given male is concerned, the function \( c_1(\theta) \) describes the behaviour of the rest of the male population, and is therefore just as much a characteristic of the environment as the functions \( \pi, q_M, q_F, v, \Phi^F_1 \) and \( \Phi^M_1 \).

Step 3. The standard consumer’s problem is
\[
\max_{c, w} u(c, w), \quad \text{s.t.:} \quad f(c, w, \theta) = 0. \tag{16}
\]
We can transfer the points on the constraint by repeating the procedure described in Step 2 in the opposite direction. This maps the locus \( f(c, w, \theta) = 0 \) from the space \( C \times W \) to the space of lotteries \([0, 1]^2\). Take a value of \( \theta \in [\theta_{\min}, c^{-1}(c_{\text{max}})] \), say \( \theta^a \). This determines a constraint \( f(c, w, \theta^a) \), depicted in the north-west diagram as the thick black line, and mapped into the north-east diagram as the set of points \( c_1^{-1}(c), \omega(c_1^{-1}(c), \theta^a) \) (not shown).

Applying \( G \) to this set, and the thick curve in the RHS of the diagram is obtained. This is the set of points satisfying:

\[
(p_m, p_s) = G(c_1^{-1}(c), \omega(c_1^{-1}(c), \theta^a)) = \left( \Phi_1^F \left( \frac{v(c_1^{-1}(c))}{V_2} \right), \pi(\omega(c_1^{-1}(c), \theta^a)) \right).
\]

(17)

Consider the problem of choosing the optimal trade-off between the survival and the reproduction lottery.

\[
\max_{p_m, p_s} \nu(\theta) \left( p_m + \frac{F_2}{M_2} p_s \right), \quad \text{s.t.: } (17).
\]

(18)

In order to have a well defined utility maximisation problem (and a well defined fitness maximisation biology problem), the constraint must lie below the indifference curve. On the RHS diagram, this is simply the requirement that the image of the constraint be concave, as depicted. The relevant condition is given in Proposition 3.

**Proposition 3** If

\[
\left( \frac{f_{cc}(\cdot)}{f_{c}(\cdot)} - \frac{f_{cm}(\cdot)}{f_{m}(\cdot)} - \frac{\pi''(\cdot)}{\pi'(\cdot)} \frac{f_{c}(\cdot)}{f_{m}(\cdot)} \right) - \left( \frac{\phi_1^{F'}(\cdot)}{\phi_1^F(\cdot)} \right) v' - 1(\cdot) + \frac{v''(\cdot)}{v'(\cdot)} < 0,
\]

(19)

then there is an internal solution to problem (18), which is mapped by \( G^{-1} \) into the solution to problem (16).

**Proof** An interior solution exists if \( \frac{dp_s}{dp_m} < 0 \). (17) is a locus parameterised by \( c \). To derive its slope, take the derivative of both \( p_s \) and \( p_m \) with respect to \( c \).

\[
\frac{dp_s}{dp_m} = \frac{\pi'(\omega(c_1^{-1}(c), \theta^a) \omega_c(c_1^{-1}(c), \theta^a))}{\phi_1^F \left( \frac{v(c_1^{-1}(c))}{V_2} \right)}.
\]

(20)

Invert the first component of (17) \( c = c_1 \left( v^{-1} \left( V_2 \Phi_1^F^{-1}(p_m) \right) \right) \) and substitute this value into (20):

\[
\frac{dp_s}{dp_m} = V_2 \frac{\pi' \left( \omega \left( v^{-1} \left( V_2 \Phi_1^F^{-1}(p_m) \right), \theta^a \right) \omega_c \left( v^{-1} \left( V_2 \Phi_1^F^{-1}(p_m) \right), \theta^a \right) \right)}{\phi_1^F \left( \Phi_1^F^{-1}(p_m) \right) \frac{v' \left( v^{-1} \left( V_2 \Phi_1^F^{-1}(p_m) \right) \right)}{V_2 \Phi_1^F^{-1}(p_m)}}.
\]

17By construction, it touches the indifference curve at point \((c_1(\theta), w)\).
Differentiate the above with respect to \( p_m \) and re-arrange to obtain:

\[
\frac{d^2 p_s}{dp_m^2} = \frac{\pi' (\cdot) \omega_c (\cdot) v^{-1} (\cdot) V_2 F \phi_0^T (p_m) \phi_0^T (\cdot)}{\phi_0^T (\cdot) v' (\cdot)} \left\{ \left[ \frac{\pi'' (\cdot)}{\pi(\cdot)} \omega_c (\cdot) \right] + \frac{\omega_c (\cdot)}{\omega_c (\cdot)} \left[ \frac{\phi_c'' (\cdot)}{\phi_c'' (\cdot)} v' (\cdot) + \phi_c' (\cdot) \right] \right\}.
\]

(21)

**Lemma 3** \( \omega_c (\cdot) = -\frac{f_c (\cdot)}{f_w (\cdot)} < 0 \), \( \omega_\theta (\cdot) = -\frac{f_\theta (\cdot)}{f_w (\cdot)} > 0 \), and

\[
\omega_{c\theta} (\cdot) = -\frac{f_{c\theta} (\cdot)}{f_w (\cdot)^2} f_w (\cdot) - f_{wc} (\cdot) f_\theta (\cdot).
\]

**Proof** Start from total differentiation of \( f (c, w, \theta) = 0 \), to get \( f_c (\cdot) dc + f_w (\cdot) dw + f_\theta (\cdot) d\theta = 0 \). And \( \omega_c = \frac{dw}{dc} \), and so on. ■

To complete the proof of the Proposition, simply apply Lemma 3 to (21) to determine that \( \frac{d^2 p_s}{dp_m^2} \) is negative if and only if the condition given in the statement of the Proposition is satisfied. ■

As before, the point of these conditions is not so much their plausibility or interpretability, but rather the fact that they could be violated or satisfied, implying that whether a given species’ biology problem is equivalent to a standard utility maximisation problem depends in general on exogenous circumstances. If our ancestors were in a signalling equilibrium, and if (19) held at that equilibrium, then their behaviour was equivalent to the maximisation of the utility function which includes conspicuous consumption as an argument attributed to *Homo Economicus*. Without (19), lack of convexity might have taken the solution to a corner of the feasible set, such as points on the vertical axis in the LHS diagram in Figure 2, where \( c = 0 \): conspicuous consumption does not occur.

The procedure described applies to the region where the function \( c_1 (\theta) \) is invertible; a slightly more complex argument along similar lines can be used to show that the indifference curves in the \((c, w)\)-cartesian space would be horizontal for values of \( c \) higher than \( c_1 (\theta) \). But also note, as argued at the end of Section 3, that the “flat” bit of the signal function \( c_1 (\theta) \) becomes shorter as the number of periods in the season increases, and so therefore does the range of values of \( c \) were the indifference curves are horizontal.

As a final remark, note that this set-up provides the natural conditions for the emergence of trade: two individuals with different \( \theta \)s could both move to a higher indifference curve by exchanging some \( c \) for some \( w \), whenever the slope of their resource constraints are different at their respective pre-trade tangency point. Analysing fully this possibility would require the study of
the way females take into account the possibility that some of the $c$ displayed by males be acquired through exchange, and hence adjust their inference of a male’s types $\theta$ from his observed signal $c$.

5 Concluding remarks

This paper views a utility function with trade-offs between consumption for its own sake and survival activities as hard wired by evolutionary pressure exerted by sexual selection. It therefore links Grafen’s (1990a and 1990b) analysis of the role of sexual selection in shaping the genome with the work, by Robson and others, on the evolutionary foundations of utility functions (Robson 2001a, and the references cited in footnote 2). The hard-wiring mechanism is discussed briefly in Section 5.1, and the paper ends with some empirical evidence, necessarily somewhat heuristic in nature, and with the interpretation in the light of the ideas proposed here of some aspects of human behaviour that economists have typically found difficult to reconcile with the standard optimising model.

5.1 Is the utility function hard-wired?

The analysis takes the utility function with conspicuous consumption as an argument as hard-wired in the human brain,\(^{18}\) just as, say, the capacity to learn a language, and addresses the question: How did this utility function become hard-wired? Fearing snakes, throwing projectiles accurately, liking sweet foods all became hard-wired because individuals carrying the genes that determine these behaviours were more likely to pass these genes to future generations. The biologist’s distinction between proximate and ultimate cause is useful here: the proximate cause of us liking sugar is the network of messages from the taste buds to the brain stimulated by the chemical composition of some components of sugar which creates a pleasurable reaction, well understood by biochemists. But the ultimate cause is the fact that, over countless generations, individuals who liked sugar had a survival advantage, well understood by biologists, over those who did not. In this perspective we can also easily understand why we love our own children, even in the absence of any personal survival advantage.

\(^{18}\)This is also the view held by Postlewaite (1998) in a paper that hints at some of the ideas presented here, see pp 781–782. He considers social status as “instrumental in determining ultimate consumption levels” (p 785, the marriage model in Cole et al. 1992 is built on this view).
of this trait: genes that make individuals love, nurture and protect their own children are more likely to populate future generations. But why consumption for its own sake? Where was the evolutionary benefit of coveting goods? The *proximate* cause of our desire to consume is of course the pleasurable sensation deriving from the possession and display of beautiful, comfortable, rare, precious and luxurious goods, eloquently described by Ian McEwan (2005, p 75–76), and powerfully exploited by Western explorers conquering aboriginals with the offer of glass beads and other conspicuous items. I argue here that the *ultimate* cause is sexual selection: individuals who had a stronger desire for consumption for its own sake were more likely to be chosen as mates by members of the opposite sex and hence more likely to pass on to future generations the genes inducing them to desire goods. I stress the unconscious nature of the link: among social animals, “the motivation of a male chimpanzee who challenges another’s rank is not that he foresees more matings or better food or a longer life”, but a hard-wired and hence simpler and deeper urge to dominate his peers (Wrangham and Peterson 1996, p 199). Similarly for humans, the desire for conspicuous consumption, the love for our children, the pleasure from sugar are all designed by evolution to be ends in themselves, to operate independently of whether there is an immediate survival or reproductive reward: the starving and the obese both enjoy food, adoptive parents love their children as much as biological parents, and humans, young, old, and homosexual, all desire consumption for its own sake.

5.2 Resources and signalling.

The Murdock and White (1969) dataset contains information on around 200 “pre-industrial” human societies, where conditions and behaviours are more likely to resemble those prevailing at the time our psychological traits took shape. The necessarily approximate nature of this dataset, and the relative lack of economic variables restrict its use in economics, and while certainly unsuitable for a rigorous test of the conditions in Assumption 1, which depend on the shape of the constraint \( f(c, w, \theta) \), the survival function \( \pi(w) \), and the benefit function \( v(\theta) \) in a non-linear way, it allows nevertheless a simple test of the link between resources, \( \theta \), and signalling, \( c \). In my model, if resources are barely sufficient for survival, then signalling is prohibitively expensive, and hence unlikely to emerge as a viable equilibrium strategy. I account for the interaction between female choice and the benefit of signalling via a stylised
model where the exogenously given society resources affect the degree of polygyny (more resources allow a more unequal distribution of resources and hence of females), and where signalling is beneficial to the extent that females exert choice. Polygyny, *ceteris paribus*, increases the benefit of signalling. To estimate this, I construct indices of how polygynous a society is (the variable “polygyny” in the equations below), and of how close it is to subsistence (the variable “resources”, containing, among others, the development and the reliance on agriculture, the quality of the environment, including diseases, the extent of female contribution to subsistence, the frequency of famines and starvation). Signalling (“signal”) is measured by the presence of activities, such as mining, woodcarving, hunting large game, musical instruments and so on, which are not necessary for survival, and whether these activities are carried out by men or by women; “female choice” measures how necessary a woman’s agreement is for her to be married or re-married. A two stage least square estimation of simultaneous equations, to account for the endogeneity of polygyny, gives the following results (161 observations are used, *t*-statistics are in brackets, and the Pseudo $R^2$ are 0.09 and 0.20):

\[
\text{polygyny} = 0.337 + 0.338 \text{ resources} - 0.101 \text{ pop. density} \\
(2.64) \quad (3.84) \quad (2.56) \\
\text{signal} = 1.59 + 0.205 \text{ polygyny} + 0.136 \text{ female choice} - 0.265 \log(\text{population}) \\
(5.63) \quad (1.82) \quad (2.63) \quad (4.69)
\]

The coefficients in these equations have the predicted signs: more resources entail more polygyny, and more female choice increases the likelihood that signalling is taking place in the society. Coefficients are significant at the 5% level with the exception of the polygyny index in the second equation, which is significant at 10%. Population size and density also influence the endogenous variables; the use of different control variables does not alter the qualitative nature of the estimation, indicating, within the limits of the dataset, a certain robustness of the results, even though other interpretations are clearly feasible.

5.3 Conspicuous consumption in the Pleistocene.

Kohn and Mithen’s (1999) theory that handaxes were the product of sexual selection also suggests support for my paper. Handaxes are very intriguing archaeological finds: manufactured for over one million years, they are frequently

\(^{19}\text{Details are available on request or at www.le.ac.uk/economics/gdf4/curres.htm.}\)
found unused in very large hoards in individual sites, perfected well beyond the necessity of use, their very symmetry and size making many of them inappropriate for any practical purpose such as throwing or butchering. All these features induce Kohn and Mithen to reject the “survival” justification of such a persistent oddity, and to opt instead for a sexual selection explanation: the ability to knap and handle symmetric and polished handaxes was used as reliable indicator of a potential mate’s quality by those of the opposite sex, and thus conferred a reproductive advantage which outweighed the survival costs involved in their production, the time diverted from feeding or hunting while looking for materials and knapping, the risk of injury to hands and eyes, and so on.20

Viewing sexual selection as a powerful engine for evolution can help biologists explain the development of certain traits which would provide little or no survival value unless fully developed. A classic example is the evolution from reptiles’ scales to birds’ feathers: light and flimsy scales have lower survival value than either solid scales or fully formed feathers, so it is difficult to explain what drove evolution from the former to the latter, given that evolution does not plan for the long term. According to some avian biologists (eg Cowen and Lipps 2000), sexual selection can indicate a possible route. If the flimsiness and lightness of an individual’s scales serve as a costly signal (since flimsy and light scales are less useful as a defence mechanism) reptiles with flimsier and lighter scales, would visibly signal their unobservable characteristics. Eventually this sexually selected trait also proved to have survival advantages, and become further established and developed.21 If we translate this argument to humans, we can address one of the Darwin’s puzzles of human evolution, the development of mental abilities with little or no survival value at the time they evolved. In the spirit of Miller (2000), my paper can be interpreted to

20 It is also worth noting that the unobserved characteristic of interest to females which is signalled is single dimensional, the contribution to fitness of offspring. Therefore, a single dimensional signal is sufficient, and this might also explain the exclusive reliance on handaxes, which appear to have been the only item which was manufactured on the planet for a very long period of time: the explosion in tools use and variety occurred around 50,000 years ago.

21 If, however, some scales evolved gradually to become signals to advertise certain traits, such as elegance of movement or certain jumps, then the scales may increase in size by the process of signal selection, handicapping their bearer’s gait or jump, attesting to the reliability of the signal. Such scales can attain extravagant dimensions. The extravagant scales may then turn out to be of help in gliding. Once gliding becomes an important adaptation, the extravagant scales can evolve into utilitarian feathers by utilitarian selection” (Zahavi 2007).
suggest that dexterity, manufacturing, understanding of the relative merits of
different materials, knowledge of the sources of such materials, mining, and so
on, all developed as a means of impressing the members of the opposite sex
and influencing their mating choices, and were continually improved by sexual
selection, until the time where the level of development reached by these traits
would also have a survival value, in manufacturing sharper spears, in selecting
and mining suitable materials and so on.\textsuperscript{22}

The arbitrariness of sexually selected traits may also prompt the question
of possible alternative routes that human evolution might have taken. One
conceivable example is information gathering and processing: what if, at the
dawn of the human race, our ancestors had considered the most desirable
mates those with the best ability to recognise remember and classify features
of the environment such as leaves in trees, animal footprints, birds flights,
star configurations, weather patterns, rather than those with the most con-
spicuous consumption? Clearly a signal of this nature would soon acquire a
large potential survival value, and might lead to many imaginable different
human evolutionary paths beginning there. The exploration of this possibility
is however best left to science fiction writers.

5.4 Sex differences and female signalling

In the paper only females choose and only males signal. This is a logical con-
sequence of the unrealistic assumption that the opportunity cost of mating
is zero for male and strictly positive for females. This makes the algebraic
treatment convenient and is realistic for some species. However, it clearly is
the case that human males do make parental investment and therefore incur
a cost in terms of foregone mating opportunities. This implies that males too
would prefer not to mate if they believe that a female they have met is of
sufficiently low quality; for example, the benefit of mating to a male could
depend on the value of $\delta$ of the female he is matched with, given that her
survival is likely to be helpful to the weaning of the offspring. The probability
of a male agreeing to mate with a first period match would be strictly less
than one, and in particular would depend on his beliefs regarding his poten-
tial mate’s $\delta$. If $\delta$ is not perfectly observable, then an equilibrium may exist
where females signal their own fertility through a costly signal, conspicuous

\textsuperscript{22}Other traits, such as the aesthetic sense or the ability to judge the symmetry of a man-
ufact also developed, but did not have a survival value. Miller (2000) extends this argument
to include the development of many specifically human activities such as art, music, conver-
sation, humour and so on.
consumption, or a different trait such as hair length or breast size (Zahavi and Zahavi 1997). Ironing out the details of the equilibrium would require modifying appropriately the probabilities of being in the market in the second period, which for males would now also depend on their acceptance strategy, endogenising appropriately the females' survival probability, determining the additional variables $V_2^M (\theta)$ (a male's value of going to the next period) and $c_F^T (\delta)$ (females' optimal signalling), and deriving the incentive compatibility constraint for females.

Even with the adaptation of the model along the above lines, given the asymmetry in reproductive opportunities of males and females, and the nature of the qualities that are valued by members of the other sex, it would still be the case that individuals of different sexes signal their qualities in different ways, in line with the evolutionary biologists' view that sexually selected traits differ in the two sexes. The idea proposed in this paper receives therefore an indirect confirmation from the differences observed in the general attitude of men and women towards consumption. Few nowadays question the evolutionary psychologists' view that there are profound differences between the sexes (eg Buss 1994 and 2004): not only in the attitudes towards casual and extramarital sex, which has a direct explanation in terms of differential parental investment and justifies the assumption in the model that men and women have different attitudes towards multiple mating in the season, but also in the attitudes towards wealth and resources. Examples range from warfare, which is typically waged by men and is almost universally caused by the quest for more territory and resources, to the observations that men value earnings and possessions more than women do, that they are more systematic collectors than women, and that men with more resources have more sexual partners (Kanazawa 2003), and more attractive partners, to Buss's team's results, who systematically analysed lonely hearts columns and conducted surveys in 37 different societies, amply demonstrating how women prefer wealth in men, when men prefer youth in women (Buss et al. 1990). The combined force of these and many other examples overwhelmingly supports the claim that "men seek wealth because they know it attracts women" (Ridley 2003, p 54). In order to perform this function, wealth must of course be displayed: in our forebears' environment, conspicuous consumption was one of the few effective ways to display one's wealth.

---

23 Both in human and in chimpanzees, the only other species in which it is known that bands of males set out to attack other males from a different group (Buss 2004, pp 280ff).
5.5 Altruism

Most people leave tips in restaurants which they will never visit again. By the same token, overwhelming experimental evidence indicates that subjects playing ultimatum or dictator games do not take advantage of other players who are complete strangers and will remain so at the conclusion of the experiment. Many of the explanations for this \textit{prima facie} irrational behaviour are based on some form of maladaptation (see Samuelson 2005, pp 96–100). Seabright (2004, pp 61–62) suggests that a tendency not to take advantage of short term opportunities evolved at a time when essentially all interactions happened within a group. There has therefore been no opportunity for a Darwinian evolution of the ability to distinguish between members of the group, co-operation towards whom does have a long term evolutionary advantage, and strangers, who will not be met again, and there is no such advantage. Similarly, we should not “be surprised if the physiological and psychological mechanisms that have evolved to sustain equilibria in repeated games should somehow be triggered inappropriately in one-shot situations”, such as anonymous experimental ultimatum games (Binmore 1994, p 183). Frank’s view (1987), on the other hand, is not based on maladaptation: altruistic individuals enjoyed an evolutionary advantage, as they could be trusted in cooperative ventures, which exceeded the short term cost of altruistic acts. According to Frank, emotions have evolved both as a signal and as a commitment device: feeling guilty if we cheat someone, be they strangers or friends, reduces the utility of selfishly taking advantage of short term opportunities.

In addition to the survival advantages which “genes for altruism” may confer to their human bearers, this paper suggests that there may also have been strong reproductive benefits accruing to individuals who behaved altruistically: consumption is meant to be displayed to convert it into mating opportunities, and, in an environment where long-lasting physical commodities are rare, “acts of altruism” may serve as a substitute for durable stores of value. By increasing $c$, an act of altruism serves as a signal that one is accumulating enough resources that the cost of “wasting” some on strangers is negligible. A recent example is the potlatch, a ceremonial feast in some Native American population in which chiefs ostentatiously destroy wealth (similar events occur in New Guinea, and among the Maori, the Koha, the Kula, the Moka): “the potlatch consists of goods that are perishable or vulnerable; the prestige that it buys is a good that is durable and portable” (Ridley 1986, p 122).
This viewpoint may shed light on some altruistic acts which are inconsistent with the maladaptation explanation: for example, in restaurants, men tip better than women, and men accompanied by women tip better than men alone and than men accompanied by other men (Miller 2000, p 326); similarly, men are more likely to give to street beggars if they are walking with a woman (Stark 1992). The interpretation that men try to impress women with their altruism runs into the obvious difficulty that from a woman’s survival perspective what matters is her partner’s altruism towards her; acts of altruism towards strangers are in fact bad news, because they indicate a propensity to profligacy which would divert resources away from her and their offspring. If, however, as contended here, altruism is a form of consumption, then it acts as a costly signal of a man’s unobservable qualities, and it make perfect sense that it should be practised more when it is more likely to be observed by females.

This argument, incidentally, is in line with the conclusion that Zahavi and his associates have drawn following their decades long observation of the Arabian babblers (Turdiodes squamiceps), small birds living in groups in the Israeli deserts. These birds are altruistic: they share “chick care”, they put their lives at greater risk by acting as sentinels for the group and mobbing predators, they share food with non-relatives. A repeated game justification based on Trivers’ (1985) concept of reciprocal altruism would predict that these birds would try to cheat when they can get away undetected, reaping benefits without paying the costs. In fact, they do nothing of the sort: they compete to perform the apparently altruistic acts: dominant birds, upon seeing a subordinate trying to act as sentinel, will attack and drive off the subordinate, taking over the sentinel role; they also try forcibly to stuff food down the throats of reluctant non-relatives. Zahavi and Zahavi believe that the Arabian babblers are using these ‘altruistic’ acts as handicaps to display their fitness, thereby attaining better reproductive prospects within the group. “The helper benefits from the act of helping, and the benefits to others are incidental” (Zahavi and Zahavi 1997, p 134, my emphasis).

5.6 The endowment effect

Consider, to end, the “endowment effect” (Thaler 1980): people require more to be separated from an object they own than they are willing to pay for the same object when they do not own it. The experimental evidence, which is inconsistent with traditional explanations based on transaction costs or income effects, has been interpreted in support of the theory of reference dependent
utility (Kahneman et al. 1990). A different, not necessarily alternative, explanation is suggested by the analysis of this paper. To the extent that possession of physical goods is a signal, utility should be increased by the possession of visible goods more than by the right to receive the same goods, because, while today this right is represented by money, with a high certainty of conversion, in the Pleistocene, when our utility function took shape, the conversion of promises into goods was likely to be less than certain. Having a claim to a good was not the same as possessing the good itself. In a variant of the experiment which seems designed to test this idea, instead of exchanging money for goods, experimental subjects traded money for tokens that represented a claim to those same goods. In this case, there was no observed endowment effect: subjects had the same trade-off to receive and to give up tokens. This clearly tallies with the ideas of this paper, once it is noted that tokens and money have the same signal value, which is lower than the signal value displayed by physical possession of the goods.

Appendix A: Existence of a separating equilibrium.

In this technical section I prove Proposition 2. I do so by looking for a signal as a function of type $c_1(\theta)$ which is a best reply to itself and which is such that there is no alternative strategy (allele) that gives a higher payoff, and that can therefore invade a population made entirely of individuals following $c_1(\theta)$. In addition, females themselves optimise and make correct inferences about males’ types from the signals they issue. Mathematically, this corresponds to a mechanism design problem, and the revelation principle ensures the solution can be found by maximisation of the payoff function subject to the incentive compatibility constraint, which imposes the same constraint as the requirement that a strategy be a best response to itself.

Suppose to begin that all males follow strategy $c_1(\theta)$. Given the females’ optimal strategy, and given that females correctly infer a male’s type from the observation of the signal $c_1(\theta)$, consider the options open to a male matched with a female. If he has type $\theta \in [\theta_{\text{min}}, \theta]$, he mates if and only if the female has type $\delta = \alpha^{-1}(\theta)$ or less (see Figure 1), and so he mates with probability $\Phi_F(\alpha^{-1}(\theta))$. From (2), $\alpha^{-1}(\theta) = \frac{\Phi_F(\theta)}{V_F}$. If he has type $\theta$ or above, he will mate with probability 1. This gives the probability of mating for a male of type $\theta$. 

29
matched to a female as:

\[
\begin{cases}
\Phi^F_1 \left( \frac{v(\theta)}{V_2^s} \right) & \text{for } \theta \in [\theta_{\min}, \overline{\theta}] \\
1 & \text{for } \theta \in [\overline{\theta}, \theta_{\max}] 
\end{cases}
\]

The (season) payoff to a male of type \( \theta \) is therefore:

\[
U(\theta) = \begin{cases}
v(\theta) \Phi^F_1 \left( \frac{v(\theta)}{V_2^s} \right) + q_M(F_2, M_2) v(\theta) \pi(\omega(c_1(\theta), \theta)) & \text{for } \theta \in [\theta_{\min}, \overline{\theta}] \\
v(\theta) + q_M(F_2, M_2) v(\theta) \pi(\omega(c_1(\theta), \theta)) & \text{for } \theta \in [\overline{\theta}, \theta_{\max}] 
\end{cases}
\]

(22)

I next determine the condition that ensures the optimality of the signal emitted by a male, that is, for every type \( \theta \in [\theta_{\min}, \theta_{\max}] \), the signal that maximise the season expected payoff of a male with type \( \theta \). This is done by showing that there cannot be profitable deviations from a candidate strategy. To do so, since issuing a signal outside the image of \([\theta_{\min}, \theta_{\max}]\) under \( c_1 \) is a strategy open to males, requires that females’ belief be defined for these signals. One system of beliefs that satisfies the consistency requirement of sequential equilibrium (Kreps and Wilson 1982) is the following. Let \( C_1 \subseteq \mathbb{R}_+ \) be the image of \([\theta_{\min}, \theta_{\max}]\) under \( c_1 \). I define a female’s beliefs if she observes a signal \( \hat{c} \) which does not belong to \( C_1 \) as follows: if \( \hat{c} < \min C_1 \), then \( c_1^{-1} \left( \hat{c} \right) = \theta_{\min} \) with probability 1, if \( \hat{c} > \max C_1 \), then \( c_1^{-1} \left( \hat{c} \right) = \theta_{\max} \) with probability 1. If \( C_1 \) is not an interval, then \( c_1^{-1} \left( \hat{c} \right) = \inf \{ \theta \in [\theta_{\min}, \theta_{\max}] \mid c_1(\theta) > \hat{c} \} \). In words, if a female see a signal lower (higher) than the lowest (highest) expected signal, then she assume that the issuer of that signal is the lowest (highest) type. If she sees a signal in a “hole” in the image of \([\theta_{\min}, \theta_{\max}]\) (for this to happen, \( c_1(\theta) \) must be discontinuous) she infers that issuer of that signal is the lowest possible type above the “hole”.

**Lemma 4** Let \( c_1(\theta) \) be a male’s optimal signal. \( c_1(\theta) \) is continuous and it satisfies \( c_1(\theta_{\min}) = 0 \):

\[
\dot{U}(\theta) = v'(\theta) \Phi^F_1 \left( \frac{v(\theta)}{V_2^s} \right) + q_M(F_2, M_2) \left[ v(\theta) \pi'(\omega(c_1(\theta), \theta)) \omega_\theta(c_1(\theta), \theta) + v'(\theta) \pi(\omega(c_1(\theta), \theta)) \right],
\]

and

\[
\frac{dc_1(\theta)}{d\theta} > 0,
\]

(24)

for \( \theta \in [\theta_{\min}, \overline{\theta}] \). For \( \theta \in [\overline{\theta}, \theta_{\max}] \), \( c_1(\theta) \) is constant and equal to \( c_1(\overline{\theta}) \).
Proof Begin by noting that the optimal signal \( c_1 (\theta) \) must be increasing: \( \theta_a > \theta_b \) implies \( c_1 (\theta_a) \geq c_1 (\theta_b) \). If \( c_1 (\theta_a) < c_1 (\theta_b) \) then a male of type \( \theta_b \) would improve his payoff by choosing signal \( c_1 (\theta_a) \): he would incur a lower cost and receive a higher benefit.

Clearly, since \( c_1 (\theta_{\text{min}}) = 0 \) there cannot be a signal “below” the image. A deviation “above” the image, say \( c_0 > \max \{ c_1 (\theta_{\text{min}}, \theta_{\text{max}}) \} \) cannot be profitable, since a male would incur lower cost at no loss of benefit by choosing \( c_1 (\theta_{\text{max}}) \) instead of \( c_0 \). Next we show that the image of \( [\theta_{\text{min}}, \theta_{\text{max}}] \) is an interval, and so \( c_1 \) is continuous. By contradiction, let \( c_1 \) be discontinuous at a point \( \theta_0 \in (\theta_{\text{min}}, \theta_{\text{max}}) \):

\[
\lim_{\theta \to \theta_0^+} c_1 (\theta) < \lim_{\theta \to \theta_0^-} c_1 (\theta).
\]

Then there exists \( \varepsilon > 0 \) such that a male of type \( \theta_0 + \varepsilon \) would benefit by choosing signal \( c_1 (\theta_0 - \varepsilon) \): since \( \phi^F (\theta) \) and the acceptance function \( \alpha (\theta) \) are continuous, he would have an infinitesimal reduction in benefit and a discrete reduction in cost.

Therefore, if there is a profitable deviation, it must be to a point in the image of \( [\theta_{\text{min}}, \theta_{\text{max}}] \). Suppose a male has true type \( \theta \) but behaves as if he had type \( \hat{\theta} \): to do so, he needs to emit signal \( c_1 (\hat{\theta}) \): this will induce a female to mate with him with probability \( \Phi_F^F \left( \frac{v(\theta)}{V^2} \right) \) if \( \hat{\theta} < \bar{\theta} \), and probability 1 if \( \hat{\theta} > \bar{\theta} \); but it will also change his probability of survival to \( \pi (\omega(c_1 (\hat{\theta}), \theta)) \). His (season) payoff for choosing signal \( c_1 (\hat{\theta}) \) would therefore be:

\[
\varphi (\theta, \hat{\theta}) = \begin{cases} 
  v (\theta) \left[ \Phi_F^F \left( \frac{v(\theta)}{V^2} \right) + q_M (F_2, M_2) \pi (\omega(c_1 (\hat{\theta}), \theta)) \right] & \text{for } \theta \in [\theta_{\text{min}}, \bar{\theta}] \\
  v (\theta) \left[ 1 + q_M (F_2, M_2) \pi (\omega(c_1 (\hat{\theta}), \theta)) \right] & \text{for } \theta \in [\bar{\theta}, \theta_{\text{max}}].
\end{cases}
\]

A male will choose the “best” possible value of \( \hat{\theta} \): the value of \( \hat{\theta} \) such that \( \frac{\partial \varphi (\theta, \hat{\theta})}{\partial \hat{\theta}} = 0 \):

\[
\begin{align*}
  & \begin{cases} 
    v(\theta) \Phi_F^F \left( \frac{v(\theta)}{V^2} \right) \varphi' (\hat{\theta}) + q_M (\cdot) \varphi (\cdot) \pi' (\omega(c_1 (\hat{\theta}), \theta)) \omega_c (c_1 (\hat{\theta}), \theta) c'_1 (\hat{\theta}) = 0 & \text{for } \theta \in [\theta_{\text{min}}, \bar{\theta}] \\
    q_M (\cdot) \pi' (\omega(c_1 (\hat{\theta}), \theta), \theta) \omega_c (c_1 (\hat{\theta}), \theta) c'_1 (\hat{\theta}) = 0 & \text{for } \theta \in [\bar{\theta}, \theta_{\text{max}}]
  \end{cases} \\
  \text{(25)}
\end{align*}
\]

By the second line, \( c_1 (\theta) \) is constant for \( \theta \in [\bar{\theta}, \theta_{\text{max}}] \), yielding utility:

\[
U (\theta) = v (\theta) \left[ 1 + q_M (F_2, M_2) \pi (\omega(c (\bar{\theta}), \theta)) \right] & \text{for } \theta \in [\bar{\theta}, \theta_{\text{max}}].
\]

By continuity of \( c_1 (\theta) \), the value of the constant value of the signal is \( c_1 (\bar{\theta}) \). Consider now \( \theta \in [\theta_{\text{min}}, \bar{\theta}] \). Differentiate (22) with respect to \( \theta \):

\[
\dot{U} (\theta) = v (\theta) \left[ \varphi' (\theta) \varphi^F \left( \frac{v(\theta)}{V^2} \right) + q_M (\cdot) \pi (\omega(c_1 (\theta), \theta)) \dot{\omega}_c (c_1 (\theta), \theta) c'_1 (\theta) \right] \\
+ v' (\theta) \left[ \Phi_F^F \left( \frac{v(\theta)}{V^2} \right) + q_M (\cdot) \pi (\omega(c_1 (\theta), \theta)) \right] + v (\theta) q_M (\cdot) \pi (\omega(c_1 (\theta), \theta)) \omega_0 (c_1 (\theta), \theta).
\]

Incentive compatibility implies \( \frac{\partial \varphi (\theta, \hat{\theta})}{\partial \hat{\theta}} \bigg|_{\hat{\theta} = \hat{\theta}} = 0 \), and so, by (25), the first term in the above vanishes, thus establishing (23). Consider (24): \( c_1 (\theta) \) must be strictly
monotonic, in order for the females to be able to “invert” it and infer $\theta$ from $c$, and I showed early that it is increasing. Finally, for $\theta \in [\bar{\theta}, \theta_{\max}]$, the last part of the statement, notice that there is no point in increasing the signal beyond the level that induces every female to mate, and males of higher type than $\bar{\theta}$ will therefore not separate. ■

We can now solve the problem with optimal control techniques, (Leonard and van Long 1992, LvL hereafter). The state variable is $U(\theta)$, and $c_1(\theta)$ is the control variable. $U(\bar{\theta})$ is free, and the lower boundary condition is given by the requirement that the lowest type issues a 0 signal, obtaining payoff:

$$U(\theta_{\min}) = v(\theta_{\min}) \Phi^F \left( \frac{v(\theta_{\min})}{V_{\theta}^2} \right) + q_M(F_2, M_2) v(\theta_{\min}) \pi(\omega(0, \theta_{\min})).$$

The problem can therefore be stated formally as an optimal control problem, with a free terminal “time” $\bar{\theta}$, a scrap value function, — the second integral in the maximand (27) — (LvL pp 244 ff), and control parameters $M_2, F_2, V_{\theta}^2$ (LvL pp 253 ff):

$$\max_{c_1(\theta), M_2, F_2, V_{\theta}^2} \int_{\theta_{\min}}^{\bar{\theta}} U(\theta) \phi^M_1(\theta) \, d\theta + \int_{\theta_{\min}}^{\theta_{\max}} v(\theta) \left[ 1 + q_M(F_2, M_2) \pi(\omega(c(\bar{\theta}), \theta)) \right] \phi^M_1(\theta) \, d\theta,$$

s.t: (9), (8), (10), (23) and (24).

**Proof of Proposition 2.** To apply optimal control solution methods, the integral constraints (9) and (10) need to be replaced by auxiliary constraints and state variables (LvL p 191). Write (10) as:

$$\int_{\theta_{\min}}^{\bar{\theta}} v(\theta) \pi(\omega(c_1(\theta), \theta)) \phi^M_1(\theta) \, d\theta = \frac{M_2 V_{\theta}^2}{q_M(F_2, M_2)} - \int_{\theta_{\min}}^{\theta_{\max}} v(\theta) \pi(\omega(c(\bar{\theta}), \theta)) \phi^M_1(\theta) \, d\theta,$$

and replace it with:

$$k(\theta) = v(\theta) \pi(\omega(c_1(\theta), \theta)) \phi^M_1(\theta),$$

$$k(\theta_{\min}) = 0, \quad k(\bar{\theta}) = \frac{M_2 V_{\theta}^2}{q_M(F_2, M_2)} - \int_{\theta_{\min}}^{\theta_{\max}} v(\theta) \pi(\omega(c(\bar{\theta}), \theta)) \phi^M_1(\theta) \, d\theta.$$

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24 Grafen (1990a) determines the equilibrium using functional analysis, and Bergstrom et al. (2002) propose an approach based on vector field analysis. As far as I am aware, there are no approaches using optimal control.

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32
Similarly for (9), which is replaced by:

\[
\dot{h}(\theta) = \pi^2(c_1(\theta), \theta) \phi_1^M(\theta),
\]

\[
h(\theta_{\text{min}}) = 0, \quad h(\bar{\theta}) = M_2 - \int_0^{\theta_{\text{max}}} \pi^2(c_1(\theta), \theta) \phi_1^M(\theta) \, d\theta. \tag{29}
\]

The Lagrangean associated to problem (27) can now be written as:

\[
\mathcal{L} = U(\theta) \phi_1^M(\theta) \, d\theta + \\
+ \mu(\theta) \left\{ v^1(\theta) \Phi_1^M \left( \frac{v(\theta)}{\mu(\theta)} \right) + q_M(F_2, M_2) \left[ v(\theta) \pi^2(c_1(\theta), \theta) \omega_\theta(c_1(\theta), \theta) \\
+ v(\theta) \pi \omega(c_1(\theta), \theta) \right] \right\} + \xi(\theta) v(\theta) \pi \omega(c_1(\theta), \theta) \phi_1^M(\theta) \\
+ \zeta(\theta) \pi^2(c_1(\theta), \theta) \phi_1^M(\theta) + \lambda \left[ F_2 - \int_0^1 \delta \Phi_1^M \left( \max \left\{ v^{-1}(\delta V_F^1), \theta_{\text{min}} \right\} \right) \phi_1^F(\delta) \, d\delta \right],
\]

where \( \xi(\theta) \) and \( \zeta(\theta) \) are the Pontryagin multipliers associated to constraint (23), (28) and (29), and \( \lambda \) the Lagrange multiplier associated to constraint (8) (LvL Theorem 7.1.1, p 222). To determine the costate variable \( \mu(\theta) \), differentiate \( \mathcal{L} \) with respect to \( U(\theta) \).

\[-\frac{\partial \mathcal{L}}{\partial U(\theta)} = \dot{\mu}(\theta) = -\phi_1^M(\theta). \]

Solving the above with the boundary conditions \( \mu(\bar{\theta}) = 0 \) (because \( U(\bar{\theta}) \) is free), and \( \mu(\theta_{\text{min}}) \) free (LvL Theorem 7.1.1, p 222), gives:

\[
\mu(\theta) = \Phi_1^M(\bar{\theta}) - \Phi_1^M(\theta). \]

The first order conditions for the transformed integral constraints are \(-\frac{\partial \mathcal{L}}{\partial c_1(\theta)} = \dot{\xi}(\theta) = 0 \) and \(-\frac{\partial \mathcal{L}}{\partial c_1^1(\theta)} = \dot{\zeta}(\theta) = 0 \), implying that \( \xi \) and \( \zeta \) are constant. Notice also that they are both positive: they increase the value of the Lagrangean (30), and can be interpreted as the shadow prices (LvL p 152ff) of \( M_2 \) and \( V_F^2 \), an increase in both of which increase males' payoff. Now, \( c_1(\theta) \), which must satisfy the condition \(-\frac{\partial \mathcal{L}}{\partial c_1(\theta)} = 0 \). Expanding it and re-arranging gives,

\[
v(\theta) \left[ \pi''(\cdot) \omega_\theta(\cdot) \omega_\epsilon(\cdot) + \pi'(\cdot) \omega_\theta(c_1(\theta), \theta) \right] + \pi'(\theta) \omega_\theta(c_1(\theta), \theta) \left( v'(\theta) + \frac{\delta \Phi_1^M(\theta) (v(\theta) + \xi)}{\Phi_1^M(\theta) - \Phi_1^1(\theta) q_M(F_2, M_2)} \right) = 0,
\]

\[
\pi''(\cdot) \omega_\theta(c_1(\theta), \theta) + \frac{\phi_1^M(\theta)}{\pi'(\cdot)} \omega_\theta(c_1(\theta), \theta) + \frac{v'(\theta)}{v(\theta)} + \frac{\phi_1^1(\theta)}{q_M(F_2, M_2)} \left( \xi + \frac{\zeta(\theta)}{v(\theta)} \right) = 0. \tag{31}
\]

Now use Lemma 3 to replace the terms in \( \omega(\cdot) \), and write (31) as:

\[
a(c_1(\theta), w, \theta) + \left( \frac{v'(\theta)}{v(\theta)} + \frac{\phi_1^M(\theta)}{q_M(F_2, M_2)} \left( \xi + \frac{\zeta(\theta)}{v(\theta)} \right) \right) = 0. \tag{32}
\]

Now notice that, since \( a(c_1(\cdot), w, \theta) \) can be obtained from (32) as a function where \( F_2, M_2, V_F^2 \), and \( \bar{\theta} \), and the multipliers \( \xi \) and \( \zeta \) are parameters. This can be
substituted for \( c_1(\theta) \) in the appropriate constraints and first order conditions, and solved. This is conceptually simple, but algebraically complex, and I do not do it here. In order for the expression \( c_1(\theta) \) derived from (32) to be a feasible solution, it is also necessary that it is strictly increasing in \( [\theta_{\text{min}}, \theta] \) (see (24)). The derivative with respect to \( \theta \) of the term in the large brackets in (32) is:

\[
\frac{v''(\theta)}{v(\theta)} - \left( \frac{v'(\theta)}{v(\theta)} \right)^2 + \frac{d}{d\theta} \left( \frac{\phi^{M}_{1}(\theta)}{\phi^{M}_{1}(\theta)} - \phi^{\mu}_{1}(\theta) \right) \left( \xi + \frac{\zeta}{v(\theta)} \right) = -\frac{\phi^{M}_{1}(\theta)}{\Phi^{M}_{1}(\theta)} q_M(F_2, M_2) \tag{33}
\]

Rewrite (32) as

\[
\xi + \frac{\zeta}{v(\theta)} = -\frac{q_M(F_2, M_2) \left(a(c, w, \theta) + \frac{v'(\theta)}{v(\theta)}\right)}{\phi^{M}_{1}(\theta)},
\]

and so (33) can be written as:

\[
A = \frac{v''(\theta)}{v(\theta)} - \left( \frac{v'(\theta)}{v(\theta)} \right)^2 - \frac{d}{d\theta} \left( \frac{\phi^{M}_{1}(\theta)}{\phi^{M}_{1}(\theta)} - \phi^{\mu}_{1}(\theta) \right) \left[ a(c, w, \theta) + \frac{v'(\theta)}{v(\theta)} \right] \tag{34}
\]

- \frac{\phi^{M}_{1}(\theta)}{\Phi^{M}_{1}(\theta)} - \frac{\phi^{\mu}_{1}(\theta)}{\phi^{M}_{1}(\theta)} \frac{\zeta v'(\theta)}{q_M(F_2, M_2) v(\theta)^2}.

If (13) holds, then the term in the square bracket is positive and so the third term is negative. The other terms are all negative: recall that

\[
\frac{d}{d\theta} \left( \frac{\Phi^{M}_{1}(\theta)}{\phi^{M}_{1}(\theta)} - \Phi^{\mu}_{1}(\theta) \right) < 0, \text{ for every } \theta \in (\theta_{\text{min}}, \theta_{\text{max}})
\]

and so the derivative of the reciprocal is positive, \( v'(\theta) > 0, \phi''(\theta) < 0, \) and \( \zeta, \xi > 0 \). Therefore \( A \) is itself negative. Next totally differentiate (32) with respect to \( \theta \) and \( c \):

\[
a_c(c, w, \theta) dc + [a(\theta) + A] d\theta = 0,
\]

and so

\[
\frac{dc_1}{d\theta} = -\frac{a(\theta) + A}{a_c(c, w, \theta)}.
\]

By the assumptions in the statement, \( A \) is negative, see (34), \( a(\theta) < 0 \) and \( a_c(c, w, \theta) > 0 \), and therefore \( \frac{dc_1}{d\theta} > 0 \), which ends the proof. \( \blacksquare \)
References


Figure 1: Female choice in period 1

\[
\theta = v^{-1}(V_2^F)
\]
Figure 2: Indifference curve and constraint in the $(c, w)$ space.