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ON VARIANTS OF CM-TRIVIALITY

THOMAS BLOSSIER, AMADOR MARTIN-PIZARRO AND FRANK O. WAGNER

ABSTRACT. We introduce a generalization of CM-triviality relative to a fixed invariant collection of partial types, in analogy to the Canonical Base Property defined by Pillay, Ziegler and Chatzidakis which generalizes one-basedness. We show that, under this condition, a stable field is internal to the family, and a group of finite Lascar rank has a normal nilpotent subgroup such that the quotient is almost internal to the family.

1. INTRODUCTION

Recall that a stable theory is called one-based if the canonical base of any real tuple \bar{a} over an algebraically closed set A is algebraic over \bar{a} . Hrushovski and Pillay have shown [8] that in a stable one-based group, definable sets are finite unions of cosets of $\text{acl}^{\text{eq}}(\emptyset)$ -definable subgroups. Furthermore, the group is abelian-by-finite. In the finite Lascar rank context, the notion of one-basedness agrees with local modularity and also with k -linearity (i.e. the canonical parameter of any uniformly definable family of curves has Lascar rank at most k) for any $k > 0$ (and in particular these notions agree for different k).

Hrushovski, in his *ab initio* strongly minimal set [9], introduced a weaker notion than one-basedness, CM-triviality, of which Baudisch's new uncountably categorical group [2] is a proper example. Pillay [14] showed that neither infinite fields nor bad groups could be interpretable in a CM-trivial stable theory; he concluded that a CM-trivial group of finite Morley rank must be nilpotent-by-finite. In [15] Pillay defined a whole hierarchy of geometrical properties (n -ampleness for $n > 0$), where one-based means non-1-ample, CM-trivial agrees with non-2-ample, and infinite fields are n -ample for all $n > 0$.

In [16], Pillay and Ziegler reproved the function field case of the Mordell-Lang conjecture in all characteristics¹ inspired by Hrushovski's original proof but avoiding the use of Zariski Geometries. Instead, motivated by Campana's work [6] on algebraic coreductions, they showed in particular that the collection of types of finite Morley rank in the theory of differentially closed fields in characteristic 0 has the *Canonical Base Property* (in short, CBP) with respect to the field of constants. That is, given a definable set X of bounded differential degree and Morley degree 1 in a saturated differentially closed field, the field of definition of the constructible

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¹in positive characteristic only for ordinary semiabelian varieties

set determined by X is almost internal to the constant field over a generic realisation of X . Furthermore, they showed the CBP for certain types in the theory of separably closed fields as well as for types of finite rank in the theory of generic difference fields of characteristic 0. The latter result was subsequently generalised by Chatzidakis [7] to all positive characteristics.

Clearly, the CBP is a generalisation of one-basedness, since for the latter the type of the canonical base of the generic type of X is already algebraic over a realisation. For the CBP, we replace algebraicity by almost internality with respect to a fixed invariant collection Σ of partial types. Chatzidakis [7] has shown that for a theory of finite rank, the CBP already implies a strengthening of this notion, called UCBP, introduced by Moosa and Pillay [12] in their study of compact complex spaces.

Kowalski and Pillay [11], generalizing the definability results obtained for one-based theories, showed that a connected group definable in a stable theory with the CBP is central-by-(almost Σ -internal).

The goal of this note is to introduce an analogous generalization of CM-triviality, again replacing algebraicity by almost Σ -internality. In view of the ampleness hierarchy, we call it 2-tightness (and 1-tightness is just the CBP). Similarly to the generalisation from one-based to the CBP, we show that a 2-tight stable field is Σ -internal, and a 2-tight group of finite Lascar rank is nilpotent-by-(almost Σ -internal). Recent work in [4] shows that the inclusion of the CM-trivial theories in the 2-tight ones is proper.

2. ONE-BASEDNESS AND 1-TIGHTNESS

The notions we will present could be generalised to any simple theory, replacing imaginary algebraic closure by bounded closure. However, the role of stability is essential for our main result, Theorem 3.5. We shall thus restrict ourselves to the stable case. In the following we will be working inside a sufficiently saturated model of a stable theory T .

Definition 2.1. The theory T is *one-based* if for every set A and every real tuple c the canonical base $\text{Cb}(c/A)$ is algebraic over c . Equivalently, for every pair of sets $A \subseteq B$ and every real tuple c , the canonical base $\text{Cb}(c/A)$ is algebraic over $\text{Cb}(c/B)$.

Remark 2.2. It was remarked in [8] that if T is one-based, the same conclusion holds for (possibly imaginary) tuples c . As noted in [14], we may assume that A (and B) are small submodels.

Recall some of the results proved by Hrushovski and Pillay [8]:

Fact 2.3. *Let G be an interpretable group in a one-based stable theory. Then:*

- (1) *Every connected definable subgroup H of G is definable over $\text{acl}^{\text{eq}}(\emptyset)$.*
- (2) *The connected component of G is abelian.*

Definition 2.4. The theory T is *CM-trivial* if for every pair of sets $A \subseteq B$ and every real tuple c , if $\text{acl}^{\text{eq}}(Ac) \cap \text{acl}^{\text{eq}}(B) = \text{acl}^{\text{eq}}(A)$ then $\text{Cb}(c/A)$ is algebraic over $\text{Cb}(c/B)$.

Obviously, any one-based theory is CM-trivial.

Remark 2.5. As shown in [14], Remark 2.2 holds for CM-trivial theories as well.

Recall that a *bad group* is a connected group of finite Morley rank which is non-solvable and whose proper connected definable subgroups are all nilpotent. It is unknown whether bad groups exist. A bad group of minimal rank can be taken simple, i.e. with no proper normal subgroups, by dividing out by its finite centre.

Pillay shows in [14] the following:

Fact 2.6. *Let T be a CM-trivial stable theory. Then:*

- (1) *No infinite field can be interpretable.*
- (2) *No bad group can be interpretable.*
- (3) *An interpretable group of finite Morley rank is nilpotent-by-finite.*

For the remainder of this article, we fix an \emptyset -invariant family Σ of partial types.

Definition 2.7. A type p over A is *internal to Σ* , or Σ -*internal*, if for every realisation a of p there is some superset $B \supset A$ with $a \downarrow_A B$, and realisations b_1, \dots, b_r of types in Σ based on B such that a is definable over B, b_1, \dots, b_r . If we replace definable by algebraic, then we say that p is *almost internal to Σ* or *almost Σ -internal*.

Poizat has shown that in a stable theory the parameter set B can be fixed independently of the choice of a .

Definition 2.8. A collection \mathcal{F} of partial types in a theory T is *1-tight* with respect to Σ if for every realisation \bar{c} of a tuple of types in \mathcal{F} over some set A , the type $\text{tp}(\text{Cb}(\bar{c}/A)/\bar{c})$ is almost Σ -internal. We say that T is *1-tight* with respect to Σ if every type in T is (or equivalently, if the family of all types is).

Pillay, Ziegler and Chatzidakis refer to 1-tightness as the *Canonical Base Property*, CBP in short. Our terminology provides for a possible generalisation of non-ampleness for higher values of n .

Remark 2.9. Clearly, any one-based theory is 1-tight with respect to any family Σ .

Lemma 2.10. *The theory T is 1-tight with respect to Σ if and only if for every pair of sets $A \subset B$ and every real tuple c , the type $\text{tp}(\text{Cb}(c/A)/\text{Cb}(c/B))$ is almost Σ -internal.*

So Definition 2.8 generalises either of the two equivalent notions of one-basedness.

Proof. This is similar to [14, Remark 2.1]. One implication is trivial by setting $B = \text{acl}^{\text{eq}}(Ac)$, since then $\text{Cb}(c/B)$ is interalgebraic with c . The other implication follows from the fact that

$$c \downarrow_{\text{Cb}(c/B)} B \text{ yields } c \downarrow_{\text{Cb}(c/B)} \text{Cb}(c/A)$$

and since a non-forking restriction of an almost internal type is again almost internal. \square

Adapting the proof of Fact 2.3, Kowalski and Pillay proved the following [11].

Fact 2.11. *Let G be a type-definable group in a stable theory which is 1-tight with respect to Σ .*

- (1) *Given a connected type-definable subgroup $H \leq G$, the type of its canonical parameter $\text{tp}(\ulcorner H \urcorner)$ is almost Σ -internal.*

(2) If G is connected, then $G/Z(G)$ is almost Σ -internal.

Hrushovski [10] sketched the construction of a theory of finite Morley rank which is not 1-tight. However, if we replace almost internality by analysability, the resulting condition (strong Σ -basedness, or non weak 1- Σ -ampleness in the terminology of [13]) holds in any simple theory if we take Σ to be the family of all non-one-based types [13, Theorem 5.1].

Question. Is every CM-trivial theory of finite rank 1-tight with respect to the family of all non one-based types of rank 1?

3. DEFINABLE GROUPS IN 2-TIGHT THEORIES

In this section, we will introduce a weaker notion than CM-triviality, called 2-tightness, analogous to the generalisation from one-based to 1-tight. We will show that stable 2-tight fields and 2-tight simple groups of finite Lascar rank are Σ -internal. More generally, we show that a 2-tight group of finite Lascar rank is nilpotent-by-(almost Σ -internal).

Definition 3.1. The theory T is called *2-tight* with respect to Σ if for every pair of sets $A \subset B$ and every real tuple c , if $\text{acl}^{\text{eq}}(Ac) \cap \text{acl}^{\text{eq}}(B) = \text{acl}^{\text{eq}}(A)$, then $\text{tp}(\text{Cb}(c/A)/\text{Cb}(c/B))$ is almost Σ -internal.

Remark 3.2. Every CM-trivial theory is 2-tight with respect to any family Σ . By Lemma 2.10, a 1-tight theory with respect to Σ is 2-tight with respect to Σ .

As in Remark 2.2, we may assume that A and B in the definition are small submodels. If T is 2-tight, the property also holds for imaginary tuples c .

Definition 3.3. A type p is *foreign* to Σ if every non-forking extension of p over B is orthogonal to every extension over B of any type in Σ .

We will repeatedly use the following connection between being foreign and internal for stable groups [18, Theorem 3.1.1 and Corollary 3.1.2]

Fact 3.4. (1) If the generic type of a stable type-definable group G is not foreign to Σ , then there is a relatively definable normal subgroup N of infinite index such that G/N is Σ -internal.
 (2) A definably simple stable group or a stable division ring G whose generic type is not foreign to Σ must be Σ -internal. In particular, if its generic type is almost Σ -internal then G is Σ -internal.

Inspired by Fact 2.6 we show the following.

Theorem 3.5. Let T be stable and 2-tight with respect to Σ .

- (1) An interpretable field K is Σ -internal.
- (2) An interpretable simple non-abelian group of finite Lascar rank whose proper connected subgroups are nilpotent is Σ -internal.
- (3) An interpretable group G of finite Lascar rank is nilpotent-by-(almost Σ -internal).

Proof. (1) We adapt the proof of [14, Proposition 3.2]. Let K be our interpretable field. Consider a generic plane P given by $ax + by + c = z$, where a, b, c are generic independent. Take a line ℓ contained in P given by $y = \lambda x + \mu$ such that λ, μ are generic independent over a, b, c . Take a point $p = (x, y, z)$ in $\ell \subset P$ generic over a, b, c, λ, μ .

Pillay has shown that $\text{acl}^{\text{eq}}(Ap) \cap B = A$, where $A = \text{acl}^{\text{eq}}(\ulcorner P \urcorner)$ is the imaginary algebraic closure of the canonical parameter of the plane and $B = \text{acl}^{\text{eq}}(\ulcorner P \urcorner \ulcorner \ell \urcorner)$ that of the parameters of the plane and the line. Furthermore, he has also shown that $\text{Cb}(p/A) = \ulcorner P \urcorner$ and $\text{Cb}(p/B) = \ulcorner \ell \urcorner$. By 2-tightness, $\text{tp}(\ulcorner P \urcorner / \ulcorner \ell \urcorner)$ is almost Σ -internal. Note that the line ℓ is given by the equations

$$\begin{aligned} y &= \lambda x + \mu \\ z &= (a + b\lambda)x + (c + b\mu). \end{aligned}$$

Since a, b, c, λ, μ are generic independent, so are $a, b\lambda, c + b\mu, \lambda, \mu$ and therefore also

$$a, b\lambda + a, c + b\mu, \lambda, \mu.$$

Now $a \in \text{dcl}(\ulcorner P \urcorner)$ and $\ulcorner \ell \urcorner \in \text{dcl}(a + b\lambda, c + b\mu, \lambda, \mu)$; as $\text{tp}(\ulcorner P \urcorner / \ulcorner \ell \urcorner)$ is almost Σ -internal, so is $\text{tp}(a/a + b\lambda, c + b\mu, \lambda, \mu)$. Since $a \perp a + b\lambda, c + b\mu, \lambda, \mu$, we obtain that $\text{tp}(a)$ is almost Σ -internal. Thus K must be Σ -internal by Fact 3.4

(2) Let G be our group, which we may assume infinite. Since G contains an infinite definable abelian subgroup, there is in G a maximal non-trivial type-definable connected nilpotent subgroup T . We first observe that T intersects trivially any other maximal type-definable connected nilpotent subgroup S : Let $I = T \cap S$ be the intersection and assume S is such that I is non-trivial of maximal rank possible. Since both T and S are connected, the indices of I in T and in S are both infinite. Hence so is the index of I in $N_T(I)$ because T is nilpotent [17, Proposition 1.12]. Simplicity of G yields that $N_G(I) \neq G$. Therefore, $N_G(I)^0$ is nilpotent and contained in some maximal type-definable connected nilpotent subgroup S_1 . But then $I < N_T(I)^0 \leq S_1 \cap T$ yields an intersection of larger rank, whence $S_1 = T$ by maximality. Now $I < N_S(I)^0 \leq S_1 \cap S = T \cap S$ again provides an intersection of larger rank, a contradiction. In particular, T intersects all its distinct conjugates trivially. Moreover, as $N_G(T)^0$ must also be nilpotent, maximality of T implies that T has bounded index in its normalizer. So

$$U\left(\bigcup_{g \in G} T^g\right) = U(G/N_G(T)) + U(T) = U(G/T) + U(T) = U(G)$$

and hence every generic element of G is contained in some conjugate of T .

We can now use the proof of [19, Lemma 2.5], which generalizes [14, Lemma 3.4 and Proposition 3.5], taking \mathfrak{B} to be the family of G -conjugates of T , and Q the family of all types of rank 1 (so Q -closure cl_Q equals imaginary algebraic closure). It is shown in [19] that for a, b, c in G generic independent and a generic conjugate S of T over a, b, c , we have

$$\text{acl}^{\text{eq}}(b, c, A) \cap B = A,$$

where $A = \text{acl}^{\text{eq}}(\ulcorner (b, c) \cdot G_a \urcorner)$ is the imaginary algebraic closure of the canonical parameter of the coset determined by (b, c) of the subgroup

$$G_a = \{(g, g^a) \mid g \in G\},$$

and $B = \text{acl}^{\text{eq}}(\ulcorner P \urcorner \ulcorner l \urcorner)$, where l is the coset determined by (b, c) of the subgroup

$$S_a = \{(s, s^a) \mid s \in S\}.$$

Moreover, $\text{Cb}(b, c/A) = \ulcorner (b, c) \cdot G_a \urcorner$ and $\text{Cb}(b, c/B) = \ulcorner (b, c) \cdot S_a \urcorner$. By 2-tightness, $\text{tp}(\text{Cb}(b, c/A) / \text{Cb}(b, c/B))$ is almost Σ -internal.

Since G is simple non-abelian, its center is trivial, so a is definable over $\ulcorner G_a \urcorner$, which in turn is definable over $\ulcorner (b, c) \cdot G_a \urcorner = \text{Cb}(b, c/A)$. Furthermore, $\ulcorner (b, c) \cdot S_a \urcorner$ is definable over $\text{acl}^{\text{eq}}(b, c, \ulcorner S \urcorner, \ulcorner a \cdot Z(S) \urcorner)$. Hence $\text{tp}(a/b, c, \ulcorner S \urcorner, \ulcorner a \cdot Z(S) \urcorner)$ is almost Σ -internal. As b, c are generic independent from a and the parameters of S , we have

$$\begin{array}{ccc} b, c & \downarrow & a, \\ & \ulcorner S \urcorner, \ulcorner a \cdot Z(S) \urcorner & \end{array}$$

so the non-forking restriction $\text{tp}(a/\ulcorner S \urcorner, \ulcorner a \cdot Z(S) \urcorner)$ is almost Σ -internal as well.

Now, since a is generic over $\ulcorner S \urcorner$, it is generic in $aZ(S)$ over $\ulcorner a \cdot Z(S) \urcorner, \ulcorner S \urcorner$ by stability. So the generic type of $aZ(S)$ is almost Σ -internal, and so is the generic type of $Z(S)$. Since T (and hence S) is nilpotent infinite, so is its center [17, Proposition 1.10]. As a simple group of finite Lascar rank is unidimensional, the generic type of G cannot be foreign to $Z(S)$, nor to Σ . Hence G is Σ -internal by Fact 3.4.

(3) Suppose first that G is simple and consider $H \leq G$ connected non-solvable infinite of minimal rank. Then either H has a simple quotient which is as described in (2) and hence Σ -internal, or H has a proper solvable connected subgroup which is not nilpotent. In this case, we conclude that H has an interpretable section defining an infinite field, which is also Σ -internal by (1). In both cases, unidimensionality yields that G is not foreign to Σ and by Fact 3.4, the group G is Σ -internal as well.

Let now G be an arbitrary group of finite Lascar rank in a 2-tight theory. Clearly we may assume that G is infinite and connected. By [3], there is a series of definable G -invariant subgroups

$$G = G_0 \triangleright G_1 \triangleright G_2 \triangleright \cdots \triangleright G_n = \{1\}$$

such that all quotients $Q_i = G_i/G_{i+1}$ for $i < n$ are either finite, abelian, or simple. In fact, applying repeatedly Fact 3.4 with respect to a single type of Lascar rank 1 which is either a completion of a type in Σ or foreign to Σ , we may refine the series and assume that every abelian quotient is unidimensional, and either Σ -internal or foreign to Σ . In the latter case, no definable section of the quotient is Σ -internal. Clearly all the finite quotients are Σ -internal; by the previous paragraph, the simple quotients are Σ -internal as well.

Let C be the centraliser in G of all Σ -internal quotients. If Q_i is abelian, then trivially $(C \cap G_i)' \leq C \cap G_{i+1}$; otherwise C centralises Q_i by definition, so again $(C \cap G_i)' \leq C \cap G_{i+1}$. Thus C is solvable.

Suppose that C^0 is not nilpotent. Then there is a definable section H of C^0 with a normal subgroup $K \triangleleft H$, such that K is the additive group of an infinite interpretable field and H/K is isomorphic to an infinite multiplicative subgroup $T \leq K^\times$, such that the multiplicative action of T on K^+ comes from the action of H/K on K by conjugation. By [17, Lemme 3.4] (or [18, Proposition 3.7.7] for the case of finite Lascar rank), the subgroup T generates K additively. It follows that K has no non-trivial H -invariant subgroup: If $K_0 \leq K$ is H -invariant and $0 \neq a \in K_0$, then $a^{-1}K_0$ is also H -invariant and contains T . As it is closed under addition, $a^{-1}K_0 \geq \langle T \rangle^+ = K$, whence $K_0 = K$. Note that K is Σ -internal by part (1).

Let \tilde{K} be the preimage of K in C , and let $i < n$ be minimal such that $\tilde{K} \leq G_i$. Since $\tilde{K} \cap G_{i+1}$ is normal in \tilde{K} , it induces an H -invariant subgroup of K , which

must be trivial. Therefore K embeds definably into Q_i , which must be Σ -internal as K is. But Q_i is centralised by C , so K is centralised by H . But this means that the multiplicative action of T on K^+ is trivial, a contradiction.

We conclude by showing that G/C^0 is almost Σ -internal. Stability yields that $C_G(Q_i) = C_G(\bar{q}_i)$ for some finite tuple \bar{q}_i in Q_i . Hence any element $gC_G(Q_i)$ in $G/C_G(Q_i)$ is definable over the tuple $\bar{q}_i\bar{q}_i^g \in Q_i$. Therefore $G/C_G(Q_i)$ is Q_i -internal, and thus Σ -internal. Now G/C embeds definably into the product

$$\prod \{G/C_G(Q_i) : Q_i \text{ } \Sigma\text{-internal}\}$$

via the map

$$gC \mapsto (gC_G(Q_i) : Q_i \text{ } \Sigma\text{-internal}).$$

Therefore G/C is Σ -internal, and G/C^0 is almost Σ -internal. \square

Remark 3.6. If Σ contains all non one-based types of rank 1, [13, Theorem 6.6] implies in particular that a group G of finite Morley rank has a nilpotent subgroup N such that G/N is almost Σ -internal, without any hypothesis of 2-tightness. Hence part (3) above only provides additional information if Σ does not contain all non one-based types of rank 1. Note that if Σ contains all 2-ample types, then any simple theory is *weakly* 2-tight with respect to Σ by [13, Theorem 5.1], where we replace almost internality by analysability in the definition.

Question. What can one say about groups of infinite rank, or even merely stable ones? For instance, in a 2-tight stable group, can one find a nilpotent normal subgroup such that the quotient is almost Σ -internal?

Remark 3.7. The theory of the free pseudospace [3] is neither CM-trivial nor 1-tight with respect to the family of rank 1 types. However it is 2-tight with respect to the family of rank 1 types [4]. Unfortunately, this theory is trivial and therefore no infinite group is interpretable. Thus it provides little insight into the significance of the above theorem.

Any almost strongly minimal theory is 1-tight and thus 2-tight with respect to the family of rank 1 types. However, this example is orthogonal to the previous one. We ignore if there is an example of a non-trivial 2-tight non 1-tight theory which does not decompose into a trivial and a 1-tight part.

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