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Takeover Contests, Toeholds and Deterrence *

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Abstract

We consider a setting in which two potential buyers, one with a prior toehold and one without, compete in a takeover modeled as an ascending auction with participating costs. The toeholder is more aggressive during the takeover process because she is also a seller of her own shares. The non-toeholder anticipates this extra-aggressiveness of the toeholder. Thus, he is deterred from participating unless he has a high valuation for the target company. This leads to large inefficiency losses. For many configurations, expected target returns are first increasing then decreasing in the size of the toehold.

JEL Classification: D44, G32, G34.

Keywords: takeovers, ascending auctions, toeholds, deterrence.

1 Introduction

In many takeover cases, one bidder owns, prior to the first offer, a toehold in the target company. This is a well-documented fact. Betton and Eckbo (2000), for instance, observed in a sample of 1353 tender offer contests, over the period 1971-1990, that in 36% of the contests, a bidder owns a toehold greater than or equal to 10%. In the sample, the average toehold size is equal to 14.57%.\(^1\) Now, the existence of such a toehold indisputably affects buyers’ motivations in the different outcomes of the takeover. When a toeholder wins the takeover, she needs to buy less shares than a standard bidder since she already owns a fraction of the target company. In a way, winning is less costly for her than it is for a non-toeholder. When she loses a contested takeover, she can sell her shares to the winner. She is no longer a buyer of shares but rather a seller of

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\(^1\)For other empirical data on the presence of toeholds in a takeover context, see, for instance, Bradley, Desai and Kim (1988), Franks and Harris (1989) or Betton et al (2005) which observes a lower prevalence of toeholds.
her own shares. As any other shareholder, she prefers the winning offer to be as high as possible. In both cases, the profit of the toeholder differs from the one of a non-toeholder. Therefore, the toehold affects both her motivation to make a takeover offer and her behavior during a (contested) takeover. We are interested in understanding these two effects of the presence of a toehold and their impact on the outcome of the takeover.

As in a large fraction of the theoretical literature on the takeover issue, we consider the forming of the toehold as an exogenous phenomenon. Owning a toehold in a firm that you want to buy is an advantage during the takeover process. However, buying shares shortly before making a takeover offer may be costly. As a matter of fact, Betton et al (2005), with a very large sample of takeovers, only found 2% of toeholds purchased shortly before making an offer. They explain this observation by the increased resistance of the target firm management facing a "last-minute toehold". Buying a "last-minute toehold" may also raise the shares' prices so much that it is not worth buying it. This explains why, in most cases, firms do not buy toeholds before making a takeover offer. Nevertheless, firms do often own toeholds prior to make a takeover offer. We explain the existence of these toeholds with an extremely simplified two-periods representation. First, firms who have access to a limited amount of money buy shares of several firms of their business sector (for instance, during IPOs or at the time they build partnerships). Later, after several modifications of the structure of the business sector, a firm who bought shares may be interested in buying a company she invested in. Then, firms are precisely in the situation we are interested in. Such situations are often observed in the high-tech business where major companies invest in start-ups that they seldom buy later on or in countries in which the firms' capital is structured by these industrial investments: Germany, Italy, France, Spain, Japan...

We consider a framework with two potential bidders, one with a toehold and one without, in which a takeover is modeled as an ascending auction with independent and private valuations. In this setting, Burkart (1995) and Singh (1998) both observe that the toeholder stays active (i.e. makes counteroffers) for prices strictly higher than her valuation for the target firm. In contrast, the toehold has no effect on the strategy of the other bidder. Thus, the toehold smoothly increases the selling price of shares and deteriorates the efficiency of the takeover process (we shortly introduce this result in section 3).

Compared to this literature that only considers the bidding process, we intend to go upstream and understand the impact of shareholdings both on bidding strategies and on the decision to take part in the takeover process. To do that, we endogenize the participation decision in the takeover contest by taking into account participating costs (which include both opportunity costs and direct expenses paid to bankers, lawyers and so on).

We observe that even with arbitrary low participating costs, the toeholder may deter completely
the non-toeholder from making any takeover offer. The deterrence relies on the extra-aggressiveness of the toeholder. This extra aggressiveness reduces the expected profit of the non-toeholder which becomes lower than the participating cost. Then, in many cases, the non-toeholder prefers not to participate to avoid incurring the participating cost. More precisely, the deterrence phenomenon works as follows. Suppose that a non-toeholder participates in the takeover process with a strictly positive probability when his valuation is above a threshold $v$. Now, the toeholder knows that if the non-toeholder participates in the takeover process, his valuation is, at least, $v$. The toeholder, since she prefers selling her toehold at the highest possible price, will never leave the takeover process for a price below $v$, when the non-toeholder participates in the takeover process. Therefore, a non-toeholder with valuation $v$ incurs a participating cost and does not make any profit during the takeover except when the toeholder does not participate. But, since the toeholder participates with a high probability (because of the toehold, she has extra motivations for taking part in the takeover process), the non-toeholder almost never makes any profit and he is better off never participating. This is true for any value of $v$, hence the deterrence effect. The toehold also has a dramatic effect on the efficiency of the takeover process. The allocation of the target firm is inefficient with a probability higher than $1/2$ because the non-toeholder never obtains the target firm.

We see that the deterrence phenomenon relies on the toeholder aggressiveness and on the high probability for the toeholder to participate in the takeover process. The higher the probability that the toeholder participates in the takeover is, the more she deters the non-toeholder from participating. In the model we introduce, the toeholder participates with a probability high enough so that the non-toeholder is completely deterred. We also consider an extension of the model in which we take into account the existence of a de facto minimum premium required by shareholders to sell their shares. This minimum premium reduces the probability that the toeholder takes part in the takeover process and consequently reduces the deterrence phenomenon. In this context, we show that, for sensitive values of the participating cost and the minimum premium, the deterrence is partial and progressive.

When we take into account the existence of a minimum premium, the impact of a toehold on target returns becomes dual. On the one side, if both bidders participate in the takeover, the toeholder is more aggressive because of her toehold which has a clear-cut positive impact on target

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2Hirshleifer (1995, section 4.5) shows results close to ours. However, his results strongly rely on his perfect information assumption which does not seem to be plausible. In his framework, since the toeholder knows the non-toeholder’s valuation, it is a direct result that she can always bid up to his valuation. Besides, with this assumption, he cannot observe any hump-shaped effect of toehold on target returns. To the best of our knowledge, our paper is the first one showing the dramatic impact of this deterrence effect in an asymmetric information framework.

3In a related framework, with participating costs but without toeholds, Tan and Yilankaya (2006) also describe equilibria in which bidders may be fully deterred. However, their results only hold with large participating costs and strictly convex distribution function. We consider a concave distribution function and our result holds with small participating costs. We both rely on the effect of participating costs, but, in our case, they are magnified by the aggressiveness due to the presence of the toehold so that we do not need the specific setting they require.
returns. On the other side, precisely because of this extra aggressiveness, the presence of a toehold deters other bidders from participating in the takeover battle (even though the deterrence phenomenon is weaker when we take into account the existence of a minimum premium, it still exists) which reduces target returns. Eventually, there is no general and univocal positive or negative effects of toeholds on shareholders revenue. Expected target returns may be a non monotonic function of the size of the toehold. We show that, in many configurations, they are first increasing then decreasing in the size of the toehold. The non monotonicity obtained in our model may explain the diverging results of empirical studies who intend to identify monotonic effects. As a matter of fact, Betton and Eckbo (2000), Eckbo and Langohr (1989) and Jarell and Poulsen (1989) observe negative impact of toeholds on target returns. However, Franks and Harris observed that toeholds increase target returns and Stulz et al (1990) observed no effect of toeholds on target returns.

Our model also allows to study several other issues related to toeholds. First, we show that some legal dispositions regarding thresholds in buying shares may be related to some aspects of our modeling. Since above some thresholds, toeholds have a negative impact on other shareholders’ expected revenue, it is understandable that shareholders try to prevent the formation of such toeholds through legal dispositions. Besides, we consider the board of a firm which intends to fight the deterrence effect that we identify and explain the motivations for the board to provide help to a white knight when facing an aggressive toeholder. It is a way to alleviate the deterrence phenomenon by reducing the participating cost of the non-toeholder. The deterrence phenomenon also explains how a firm can keep, during a long period, the control of another firm by holding 25 or 30% of its capital without fearing any takeover attempt by a competitor. A toehold of 25% almost completely deters any competitor from making a takeover offer.

Apart from Burkart (1995) and Singh (1998), toeholds in a takeover context have also been considered as a way to alleviate the free rider problem in a single bidder context (à la Grossman and Hart (1980) and Bradley (1980)). If a bidder owns a toehold, she can, at least, make a profit on the shares she owned before the takeover. Thus, the probability of success increases with the size of the toehold and toeholds decrease the amount paid to shareholders. These results first appeared in Shleifer and Vishny (1986), then, with some modifications, in Hirshleifer and Titman (1990) or Chowdry and Jeegadesh (1994).

Neither the Burkart-Singh model which describes a clear positive impact of toeholds on target returns nor the free-rider model which describes a negative effect of toeholds on target returns provides an explanation for the undecided empirical results that we mentioned.

Bulow Huang and Klemperer (1999) have a different approach. They model the takeover as a common value auction. With such a paradigm, the toehold becomes more important because

\[4\text{Goldman and Qian (2005) gives more balanced predictions but they do not seem to be corroborated by empirical data (see Betton et al (2005)).}\]
of effects related to the winner’s curse issue (they show in several related papers that any small asymmetry can have a major effect in a common value framework). Their analysis strongly relies on the common value assumption and does not resist to the introduction of a small private value component. They justify this choice by limiting their analysis to financial bidders without any private motivations for buying the target firm. They observe that an asymmetric distribution of toeholds induces low bids from bidders with small toeholds and high bids from bidders with large toeholds. If toeholds are distributed nearly symmetrically among bidders, they increase expected target returns. In contrast, if they are distributed very asymmetrically, they decrease expected target returns. We believe that the limitation to financial bidders is too restrictive. In most cases, bidders are not purely financial actors, they are strategic (in the Bulow et al. (1999) meaning of the word) and valuations derive from motivations specific to each bidder. This is coherent with Berkovich and Narayanan (1993), Gupta et al. (1997) and Goergen and Reneboog (2002) who all find strong evidence that synergy is the prime motive for mergers and acquisitions. That is why we choose to consider the takeover process with the private value paradigm. Besides, even though we also observe that toeholds may reduce target returns, our analysis differs from theirs on different aspects and the following properties are specific to our model. First, We derive that toeholds deter non-toeholder from making takeover offers. Second, conditional on both bidder’s participating in takeover process, the toehold has a positive impact on target returns. Third, small toeholds may strongly deter the non-toeholder and have a major effect on target returns. Fourth, toeholds may cause major efficiency losses (which cannot be an issue in a common value framework.)

The remainder of the paper is organized as follows. Section 2 presents the model. In section 3, we study a simple representation of the takeover process, no participating cost, no minimum premium. In section 4, we take into account participating costs. In section 5, we consider the impact of the minimum premium required by shareholders. At last, section 6 develops implications of our results and evokes their limits.

2 The model

The takeover process is modeled as an ascending auction where collusion and renegotiation are ruled out. We restrict our attention to a setting with two potential buyers.

We make the following assumptions. (i) The aim of a takeover attempt is to benefit from synergies between the target firm and the bidder. (ii) These synergies are bidder specific and privately known. (iii) To participate in the takeover process, bidders incur a commonly known sunk cost. (iv) Shareholders do not accept to tender if they are not offered a minimum premium.

Formally, we have the following representation: firm A is a potential target for two risk-neutral
bidders, 1 and 2. Each possible bidder \( i \) has a valuation \( v_i \) for the target firm which is private information. It is common knowledge that valuations are independently drawn from a uniform distribution, \( F \), on the interval \([0, 1]\). It is also common knowledge that bidder 1 owns a fraction \( \alpha \) of firm A’s capital with \( 0 < \alpha < \frac{1}{2} \).

The takeover is represented by a two-stages game:

- **Stage 1:** both bidders decide simultaneously if they want to participate in the takeover process.
  Between stage 1 and stage 2, bidders who decided in stage 1 to participate pay a sunk cost \( c \geq 0 \). Participation decisions are observed by all the bidders.

- **Stage 2:** bidders who paid the sunk cost compete in an ascending auction defined as follows.
  From an initial value \( R \geq 0 \), the price gradually increases. At any moment, bidders can quit the auction. The auction stops when there is only one bidder left. With two possible bidders, this is equivalent to the moment when the first bidder quits. All the shares of firm A are bought by the remaining bidder at the current price. If both bidders quit for the same price, bidder 2 wins the takeover and buys the shares for the common price. This tie-breaking rule is required for the existence of an equilibrium in cases considered in sections 4 and 5, see the proofs of Proposition 1 and lemma 1 in the appendix (for more details on the link between tie-breaking rules and equilibrium, see Jackson et al (2002)).

Bidder 1 owns, before the takeover process, a fraction \( \alpha \) of the target firm. If she is the winner of the auction, she only buys a fraction \( (1 - \alpha) \) of firm A’s capital. Conversely, if she loses the auction, she sells her toehold to her opponent at the price defined by the auction (the toeholder prefers selling rather than suffering the dilution of her opponent). On the other hand, bidder 2, if he does not win the auction, derives no profit and, if he wins the auction, buys the whole capital of firm A. We can therefore define utility functions as follows.

- If bidder 1 wins the takeover for a price \( p \) : \( U_1 = v_1 - (1 - \alpha)p \) and \( U_2 = 0 \)
- If bidder 2 wins the takeover for a price \( p \) : \( U_1 = \alpha p \) and \( U_2 = v_2 - p \)
- If firm A remains independent : \( U_1 = U_2 = 0 \)

A strategy for bidder \( i \) is a couple \((d_i, b_i)\) with \( d_i : [0, 1] \to \{0, 1\} \) and \( b_i : [0, 1] \to [R, \infty) \). \( d_i(v_i) \) is the participation function of bidder \( i \). If \( d_i(v_i) = 1 \), bidder \( i \) participates in the takeover.

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5 We normalize the Stock Exchange valorization of company A before the takeover process at zero. We choose a uniform distribution for the sake of simplicity. The spirit of our results remains true for a wider range of distribution functions even though the extreme result stated in Proposition 1 does not stand with any distribution function.

6 We rule out the possibility that a company participates with a probability \( q \) with \( 0 < q < 1 \) in order to simplify the exposition of the proofs. Our results would remain unchanged if we allowed buyers to randomize. At the equilibrium, a bidder cannot randomize for an interval of valuations of positive measures.
process when his valuation is \( v_i \). If \( d_i(v_i) = 0 \), bidder \( i \) does not participate in the takeover process when his valuation is \( v_i \). \( b_i \) is a bidding function and \( b_i(v_i) \) is the price for which bidder \( i \), if his valuation is \( v_i \), quits the takeover process if he participates and bidder \( j \) is still active.

In the remainder of the paper, we only consider equilibria with undominated strategies. All the proofs are in the Appendix.

3 Existing results with \( c = 0 \) and \( R = 0 \)

We consider a simplified representation of the takeover with \( c = 0 \) and \( R = 0 \). Participating to the takeover is free of charge (\( c = 0 \)). Stockholders accept to sell their shares for any price over the quotation before the beginning of the takeover process (\( R = 0 \)). We only mention existing results to use them as a benchmark since Burkart (1995) and Singh (1998) already developed a complete analysis of this framework.

**Result 1** When \( c = 0 \) and \( R = 0 \), there is a unique equilibrium with undominated strategy. Bidders always participate. Bidder 2 bids his valuation and bidder 1 bids according to \( b_1 \) defined as

\[
b_1(v_1) = \frac{v_1 + \alpha}{1 + \alpha} = v_1 + \frac{\alpha}{1+\alpha}(1-v_1)
\]

Bidder 2 leaves the takeover process for a price equal to his valuation (standard result in auction theory). Bidder 1 has incentives to quit for a price higher than her valuation in order to increase her revenue conditional on losing the takeover, hence the overbidding, \( \frac{\alpha}{1+\alpha}(1-v_1) \).

We can make three observations regarding this result. First, the takeover process is not efficient since bidder 2 bids his valuation and bidder 1 bids more than her valuation. Second, bidder 2 may be victim of her aggressiveness and win the takeover for a price higher than her valuation (Burkart calls it the ”owner’s curse”). Third, the existence of the toehold has a positive impact on target returns. Bidder 2’s strategy is not affected by the toehold while the price for which bidder 1 quits the takeover increases with the size of her toehold. As far as revenue is concerned, it seems that the impact of the toehold is unambiguously positive. In the following sections, we will see that this result is questionable.

4 Taking participating costs into account

In this section, we take into account the cost to participate in a takeover. The participation decision becomes endogenous. As a result, we observe a dramatic deterrence effect.

In the previous section, we assumed that participating in a takeover is free of charge. This assumption is not credible. Participating in a takeover is costly. It requires the mobilization of the finance direction and the high management which represents an opportunity cost for the firm.
Besides, no major takeover is launched without calling on the expensive services of experts such as consulting firms, bankers, lawyers (when Vodafone bought Mannesman, it payed more than a billion dollars to its lawyers and bankers. It was not even a contested takeover). Whatever the result of the auction is, a bidder participating in the takeover process has to pay the type of expenses that we have mentioned. Thus, these costs are sunk.

We will show that sunk costs dramatically change our analysis of the effects of the toehold. But first, note that the participation decision becomes endogenous because of the existence of these costs. Since participating is costly, the participation decision becomes an issue.

**Proposition 1**  
∀c, α > 0, in any perfect Bayesian equilibrium with undominated strategies the following properties are verified : (i) Player 2 never participates in the takeover when v₂ < 1, (ii) Player 1 participates in the takeover when v₁ > c and never participates in the takeover when v₁ < c.

Bidder 2 never participates in the takeover process. He is fully deterred by bidder 1. This is true whatever the values of α and c are as long as they are strictly positive.

The intuition is as follows. At the equilibrium, there is a minimum valuation v for which bidder 2 participates in the takeover process. Besides, if he participates in the takeover process, it is a dominant strategy for him to quit when the current price is equal to his valuation. Bidder 1, if she loses a contested takeover, prefers the final price to be as high as possible since, in that case, she sells her shares for this price. As a result, if both bidders participate, bidder 1 never quits the auction for a price below v. Thus, if bidder 1 participates, bidder 2, when he has a valuation of v, does not derive any profit from his participation in the takeover while he has to pay a participation cost c > 0. He would have been better off, not participating. As he can anticipate that such an event will occur, he prefers not to participate. This argument remains true here for any value of v since bidder 1 participates with a high probability, i.e. whenever v₁ ≥ c. As a result, the deterrence phenomenon is complete even if the participating cost and the toehold are small.

By taking into consideration the participating costs, we endogenized the decision to participate in the takeover. It dramatically changes the analysis of the impact of the toehold. Bidder 1 fully deters bidder 2 from participating. The price paid to the remaining shareholders is always zero, i.e. the Stock Exchange quotation before the takeover. The allocation is inefficient with a probability higher than 1/2. These results are true for any strictly positive value of the sunk cost.

They rely on bidder 1’s caring about the price paid even when she loses. She can credibly commit to bid up in case bidder 2 would participate in the auction. This commitment is so strong that it completely deters bidder 2 from making any takeover offer whatever his valuation for the target firm is.
Regarding the robustness of this result, we can observe that the complete deterrence does not rely on the limitation to two competitors. Suppose that we have \( n \geq 2 \) competitors without toehold and one with a toehold. Then, the following strategies are constitutive of an equilibrium: bidder 1, the bidder with a toehold, participates whenever her valuation for the target firm is at least \( c \). If any other bidder participates, bidder 1 quits the takeover process for a price equal to 1. The \( n \) other competitors never participate. All the bidders without toehold are fully deterred.

However, the choice of a uniform function is not neutral. It affects the extent of the deterrence phenomenon. Complete deterrence would not hold with any distribution function. As a matter of fact, for bidder 2 to be fully deterred, we need \( F(c) \) to be low enough. If \( F(c) \) is high, bidder 1 does not participate with a high probability and bidder 2 is not fully deterred. The deterrence phenomenon would exist for any distribution function. But, it would be less pronounced for some other distribution functions. However, we observe that the strategies introduced in proposition 1 that leads to complete deterrence are constitutive of an equilibrium with any distribution \( F \) provided that \( F(c) \leq c \). In order to prove this claim, one should look at the proof of Proposition 1 and see that if the uniform distribution is replaced by any distribution function \( F \) such that \( F(c) \leq c \), the proof remains valid. The necessary condition for the existence of such an equilibrium only concerns the lower part of the distribution. The lower the probability that bidder 1 have a valuation below \( c \), the participating cost, the stronger the deterrence phenomenon is.

Now for distribution functions \( F \) such that \( F(c) > c \), we can show that the deterrence phenomenon does not disappear. There always exists an equilibrium in which the toeholder participates with a strictly higher probability than the non-toeholder if \( \alpha > 0 \) and \( 0 < c < 1 \). The proof of this result is the same as the proof of Proposition 4 in Tan and Yilankaya (2007), \( xF(y) + \alpha y(1 - F(y)) - c = 0 \) replacing equation (19) (\( \bar{b} \) and \( \phi \) are still defined in our context). Nevertheless, we were not able to obtain the necessary conditions for this equilibrium to be unique (the concavity assumption is not sufficient here) so that the question of uniqueness remains open.

The deterrence phenomenon dramatically changes our analysis of the effect of toeholds. Unlike what we tended to infer from the results of the previous section, the impact of a toehold for the remaining stockholders is not always positive. On the contrary, it can be extremely negative if we take into account its influence on the decision to participate in the takeover process or not. Even a relatively small toehold may have a dramatic deterrence effect and an extremely negative impact on the expected revenue of remaining stockholders.
5 Analysis of the takeover with shareholders requiring a minimum premium

In this section, we assume that shareholders may refuse to sell their shares if they are not offered a minimum premium. We obtain more balanced results with such a setting.

Empirical studies show that even when the takeover is not contested, bidders almost always offer a bonus to shareholders\footnote{Several theoretical reasons have been given for justifying this bonus. We may mention the explanation given in Stulz (1988) and Stulz and al (1990) which is compatible with the other assumptions of our model. They consider that shareholders have specific attributes affecting their tendering decisions. Therefore, the supply of shares tendered is an increasing function of the price per share offered by the bidder. This may explain why, in order to be successful, bidders must propose a price for the shares at least equal to $R$, the commonly known minimum premium.}. Integrating these elements in our model is equivalent to the addition of a reserve price. As a first approximation, we consider a unique given reserve price for both bidders. Besides, to rule out trivial cases, we assume that $c + R < 1$ if $c + R \geq 1$, at the equilibrium, bidder 2 never participates while bidder 1 participates if and only if $v_1 > c + (1 - \alpha)R$. We obtain the results stated in the following lemma and proposition.

Lemma 1 For any equilibrium $((d_i^*, b_i^*), (d_2^*, b_2^*))$, there exists a couple $(v_1^*, v_2^*)$ such that for $i = 1, 2$:
- $d_i^*(v_i) = 0$ if $v_i < v_i^*$, $d_i^*(v_i) = 1$ if $v_i > v_i^*$, $b_1^*(v_1) = \max(v_2^*, \frac{v_1 + \alpha}{1 + \alpha})$ and $b_2^*(v_2) = v_2$.
with $v_2^* \geq R + c$ and $v_1^* \geq c$.
Thus, an equilibrium can be defined by a couple $(v_1^*, v_2^*) \in [c, 1] \times [c + R, 1]$.

Proposition 2 $(v_1^*, v_2^*)$ is an equilibrium if and only if one of the following three exclusive sets of conditions holds.

- $v_2^*(v_1^* - (1 - \alpha)R) + \alpha(v_2^* - R)(1 - v_2^*) = c$, $v_1^*(v_2^* - R) = c$ and $v_2^* > \frac{v_1^* + \alpha}{1 + \alpha}$
- $v_2^*(2\alpha R - R) + \frac{1 - \alpha}{2}(v_2^*)^2 + \frac{(v_2^* + \alpha)^2}{2 + 2\alpha} - \alpha R = c$, $v_1^*(v_2^* - R) = c$ and $v_2^* \leq \frac{v_1^* + \alpha}{1 + \alpha}$
- $(v_1^*, v_2^*) = (c + (1 - \alpha)R, 1)$ and $(1 - \alpha)(1 + R) \leq c$

Complete deterrence is no longer the general rule. It only occurs for extreme values of $c$ and $\alpha$. The intuition is as follows. If bidder 1 wants to deter bidder 2 from participating, she has to participate in the takeover with a high probability. However, if bidder 2 is completely deterred, bidder 1 only derives a positive profit from her participation if $v_1 - (1 - \alpha)R \geq c$. Thus, if $v_1 < (1 - \alpha)R + c$, she is strictly better off not participating. If $R$ is high enough, bidder 1 does not participate with a non negligible probability and if bidder 2 has a high valuation, he derives a strictly positive expected revenue from a participation in the takeover. For high values of $v_2$, bidder 2 is better off participating in the takeover battle. The deterrence phenomenon becomes more progressive and a function of the size of $c$, $R$ and $\alpha$. Let us consider their effects on the deterrence phenomenon.
The participating cost, \(c\). As in the standard case, it has a negative impact on bidders’ participation. For bidder 2, this is reinforced by the presence of the toehold. Since bidder 1 is more aggressive, the expected profit of bidder 2 is lower and he is more likely to make a profit lower than the participating cost.

The minimum premium, \(R\). As in the standard case, it deters bidders with valuations below it from participating in the takeover. The higher \(R\) is, the lower is the probability for bidder 1 to participate. Thus, bidder 2 is less deterred by bidder 1. This may more than counterbalance the (standard) negative impact of the minimum premium on his incentives to participate. Eventually, bidder 2’s probability to participate may be locally increasing in \(R\).

The toehold, \(\alpha\). It has a non ambiguous effect on participation decisions. For higher values of \(\alpha\), bidder 1 participates more often in the takeover process. Because of the deterrence effect, the higher \(\alpha\), the lower the probability for bidder 2 to participate.

A rise in \(\alpha\) has a clear-cut effect on participation decision. This is not the case as far as target returns are concerned. A rise in \(\alpha\) has both a positive and a negative effect on target returns. It raises the probability that bidder 1 takes part in the takeover process and increases the value of her bid but it also deters bidder 2 from participating for a higher range of valuations. Expected target returns are not a monotonic function of the size of the toehold. Depending on the values of the parameters, the effect of an increase of \(\alpha\) may be positive or negative. Even if we fix \(c\) and \(R\), this effect may vary locally as the following example shows it.

**Example 1** We consider the following setting: \(c = 0.02\) and \(R = 0.15\). Figure 1 represents shareholders’ expected target returns depending on \(\alpha\). Expected target returns are first increasing then decreasing in the size of the toehold.

For technical reasons, we are unable to obtain the necessary conditions for the expected revenue of the shareholders to follow the hump shaped pattern identified in Example 1. Nevertheless, as illustrated by figures 2 and 3, this shape can be observed for many other values of \(c\) and \(R\). The observation of these graphics (and others, by the author) provides an intuition regarding the conditions for which this property holds. It holds when \(R\) is not too high and \(c\) is not too small. As a matter of fact, if \(R\) is large, the main issue is whether any bidder will participate. Deterrence of bidder 2 by bidder 1 is a second order effect. If \(c\) is very small, the deterrence effect becomes weaker since bidder 2 participates even if his expected profit is low. Thus, the major consequence of the toehold is bidder 1’s overbidding. Estimating the true values of \(c\) and \(R\) is beyond the scope of this paper. Nevertheless, considering the motivations that we gave for the existence of the minimum premium, intermediary values of \(R\) seem to be appropriate. We also explained why we believe that
participating costs are non negligible. Therefore, it seems reasonable to consider that \( c \) and \( R \) could be such that toeholds have a non monotonic effect on expected target returns.

These results may explain the difficulties for empirical studies to find a clear-cut effect of toeholds on the final price that we mentioned in the introduction. We showed that target returns may be a non monotonic function of the size of the toehold. These empirical studies only consider the possible existence of a positive or a negative effect. These diverging results could be explained by the different toeholds’ distribution of their samples.

6 Applications and predictions

In this section, we apply our results to several aspects of the questions of the control and the taking of control of firms, propose testable predictions and suggest limits to the model.

6.1 The optimal size of a toehold

Let us consider the minimum premium, \( R \) and the participating cost, \( c \) as fixed, elements of the market structure which is common knowledge. We assume that these parameters are such that expected target returns follow a hump-shaped function of the toehold size. It is then possible to compute an optimal toehold \( \alpha^* \), \( \alpha^* \) being such that the expected revenue of the remaining stockholders is maximized when a possible buyer owns a toehold of size \( \alpha^* \).

For \( \alpha < \alpha^* \), the deterrence phenomenon is weak. However, as \( \alpha \) is small, bidder 1 does not bid much more aggressively than a bidder without toehold. Then, the expected price of the remaining shares is not maximized. For \( \alpha > \alpha^* \), the deterrence effect prevails. Other possible bidders stay out of the takeover process too often. The expected revenue of the remaining stockholders tends to decrease with the size of the toehold.

A board of administrators concerned about the revenue of all its stockholders should make sure that no possible purchaser buys progressively a toehold substantially higher than \( \alpha^* \). Otherwise, this potential buyer would deter too much other competitors from trying to buy the firm. Eventually, this would have a negative effect on other shareholders’ expected revenue.

Some legal dispositions seem to be related to this observation. For example, in many legislations, any firm who buys shares of another firm, beyond some well-defined thresholds, must officially declare its intentions. The firm may even be forced to make a takeover offer. One of the aims of these measures is precisely to protect the remaining shareholders from a too powerful toeholder who would deter other potential buyers from making takeover offers.

\(^8\)Remember that, with our normalization, \( c \) should not be compared to the final price paid to the shareholders but to the premium paid to them, or more precisely, to the distribution interval of this premium.
6.2 How to fight the deterrence effect

Suppose that a bidder yet holds a fraction $\alpha$ of firm A such that the deterrence phenomenon exceeds the positive impact of the toehold. What can the board of firm A do in order to reduce the deterrence effect?

According to the terms and results of our model, the board should try to change the minimum premium or to reduce participating costs. Affecting the value of the minimum premium is difficult. Thus, an adequate policy for the board would be to minimize the participating costs. It could, for instance, destroy existing poison-pills. However, it is doubtful that the high management have the right incentives to do so. If a takeover succeeds, whoever the winner is, members of the high management are likely to lose their jobs. Therefore, they prefer maintaining the probability of a successful takeover at a low level.

In contrast, the high management could make the following statement: “If this stockholder sets off a takeover process, we will help any other competitor who could be interested in a takeover. We will reduce his participating cost through any kind of alliance or by providing the help of shareholder-friends”. This reminds of the white knight searching process. The management has the right incentives to do so. Once a bidder first announced his takeover attempt, the high management has incentives to find another possible bidder. There is a common belief that the white knight will be more likely than the other bidder to keep the high management, or a fraction of it, in place, out of gratitude. This strategy is also in the stockholders interest, as it increases the expected final price as long as it does not deter too much bidder 1 from making an offer (since bidder 1 has more reasons than bidder 2 to participate, she is unlikely to be seriously deterred). In the model, this behavior is equivalent to the creation of differentiated entry costs, $c_1$ and $c_2$ with $c_2 < c_1$. With a lower $c_2$, bidder 2 takes part in the takeover process for lower values of $v_2^*$. This kind of attitude is often observed although, in numerous cases, the board attempts are fruitless.

6.3 How to control safely a firm with less than half of its shares

Let us consider the results through a different angle. The toeholder point of view. We often observe that a firm has, de facto, the control of another firm without owning 50% of its capital and without even trying to buy these 50%. Our model could provide an explanation for this kind of situation.

$R$ and $c$ are given, parts of the market structure with $c$ not extremely low and $R$ not extremely high. Then, there exists an $\tilde{\alpha} < \frac{1}{2}$ such that if a firm owns a fraction $\tilde{\alpha}$ of firm A, it almost

\footnote{To illustrate this point, let us consider a setting close to the one we introduced in example 1: $R = 0.15$, $\alpha = 0.15$. If $c_1 = c_2 = 0.02$, we have $v_1^* \simeq 0.14621$ and $v_2^* \simeq 0.28679$ and if $c_1 = 0.02$ and $c_2 = 0.01$, we have $v_1^* \simeq 0.17535$ and $v_2^* \simeq 0.20703$. We see that the deterrence phenomenon still exists with $c_1 < c_2$, however lowering $c_2$ has an obvious positive effect on bidder 2’s participation decision which is much larger than its negative effect on bidder 1’s participation decision.}
completely deters any other competitor from attempting to make a takeover offer. Besides, if shareholdings are scattered, as a major shareholder, the firm owning the toehold $\alpha$ has the effective control of firm A without the need to buy the majority of firm A’s capital. Therefore, it may be useless while costly to buy the remaining necessary shares to own the 50%. As long as having the effective control is the main objective, buying the majority of the shares is almost superfluous.

6.4 Limits

We chose to restrain our study to a specific distribution function, the uniform distribution function. We already mentioned the impact of that choice. With a distribution function which puts more weight on lower valuations, the deterrence phenomenon would be stronger. With distribution functions which puts more weight on intermediary and high valuations the deterrence phenomenon would be weaker.

We considered cases in which only one possible buyer owns a toehold. A more general model would represent situations in which both bidders may own a fraction of the target company. In such a case, results would crucially depend on the distribution of toeholds. Bidders may own toeholds of different sizes. If one bidder owned a large toehold and the other a small one, we would obtain results close to the one we have presented. However, even without a minimum premium, bidder 1, the bidder with the highest toehold, would not completely deter bidder 2 from participating. Bidder 2 would participate with a positive probability, would it only be to rise the final price. Since he owns a toehold, if he loses the takeover process, he also prefers the final price to be high. Now, if bidders had toeholds of almost identical size, we would obtain extremely different results. The deterrence phenomenon that we highlighted would almost disappear. There would not be any more a favored bidder who deters a less favored bidder. In that case, toeholds have a positive effect on the final price and efficiency (see intuitions of these results in Engelbrecht-Wiggans (1994), Maasland and Onderstal (2007) and Ettinger (2002)).

The assumption that bidders perfectly know their valuations for the target company at the time they decide to participate and incur the sunk cost is also a key element of the model. If bidders were to discover their valuations after deciding to take part in the takeover and to pay the cost, the equilibrium would be different. Nevertheless, for some configurations, we could still observe a non monotonic effect of toeholds on target returns. However, such a representation seems less accurate than the one we considered.

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10The equilibrium of this game is as follows. For $c$ below a threshold $c$, both bidders always participate in the takeover process. For intermediary values of $c$, between $c$ and $c$, bidder 1 always participate and bidder 2 never participates. For $c$ higher than $c$, no bidder ever participates. $c$ is decreasing in $a$ and $\tau$ is increasing in $a$. Besides, bidder 2’s participation decisions is not informative. Thus, bidder 1 submits $b_1(v_1)$ ($b_1$ being the bidding function that we defined in section 3 if her valuation is $v_1$.

11
7 Appendix

7.1 Proof of proposition 1

First, we show that if \(((d_1, b_1), (d_2, b_2))\) is an equilibrium, then for \(\forall i = 1, 2, \exists \hat{v}_i\) such that \(d_i(v) = 0\) if \(v < \hat{v}_i\) and \(d_i(v) = 1\) if \(v > \hat{v}_i\).

We define \(EU_i(\hat{v}_i, \hat{b}_i)\) as the expected utility of bidder \(i\) with valuation \(\hat{v}\) and bidding \(\hat{b}\), if he participates in the takeover process.

Suppose that \(\exists i, \hat{v}_i, \hat{b}_i\) such that \(\hat{v}_i < \hat{b}_i\), \(d_i(\hat{v}_i) = 1\) and \(d_i(\hat{b}_i) = 0\). As \(d_i(\hat{v}_i) = 1\), the expected revenue of bidder \(i\) with valuation \(\hat{v}_i\) if he participates in the takeover must be at least \(c\), otherwise he would be strictly better off not participating. We write it \(EU_i(\hat{v}_i, \hat{b}_i(\hat{v}_i)) \geq c\). If bidding \(b_i(\hat{v}_i)\), bidder \(i\) wins the auction with a strictly positive probability, then \(EU_i(\hat{v}_i, b_i(\hat{v}_i)) > EU_i(\hat{v}_i, \hat{b}_i(\hat{v}_i))\).

Suppose \(i = 2\) and \(EU_2(\hat{v}_i, b_i(\hat{v}_i)) \geq c\), bidder 2 bidding \(b_i(\hat{v}_i)\) must win the takeover with a strictly positive probability as otherwise his utility, gross of the payment of the sunk cost, is equal to zero. If \(i = 1\), because it is a dominated strategy for bidder 2 with a valuation below \(c\) to participate, if bidder 1 participates in the takeover, she wins with a strictly positive probability. Then, \(\forall i = 1, 2\), if bidder \(i\) participates, he wins with a strictly positive probability. Therefore, \(\forall i = 1, 2, EU_i(\hat{v}_i, b_i(\hat{v}_i)) > EU_i(\hat{v}_i, \hat{b}_i(\hat{v}_i)) \geq c\) and if bidder \(i\) has a valuation \(\hat{v}_i\), he is strictly better off participating. Hence, \(d_i(\hat{v}_i) = 0\) is impossible.

Second, it is a standard result that \(b_2(\hat{v}_2) = \hat{v}_2\) is a weakly dominant strategy. The inference that bidder 2 could make on bidder 1’s valuation because bidder 1 participates does not alter this result. And we chose to restrain to equilibria with undominated strategies.

Suppose that \(0 < \hat{v}_2 < 1\) (\(\hat{v}_2 = 0\) is impossible for trivial reasons).

Bidder 1 can infer information from bidder 2’s decision to participate. When bidder 2 participates in the takeover, \(\hat{v}_2 \geq \hat{v}_2\). Besides, if bidder 1 loses the auction, her utility is strictly increasing in the price paid by bidder 2 so that, at the equilibrium, if both bidders participate in the auction, bidder 1 cannot bid strictly less than \(\hat{v}_2\). Therefore, bidder 2, if his valuation is \(\hat{v}_2\), only makes a profit when bidder 1 does not participate. At the equilibrium, we must have: \(F(\hat{v}_1)\hat{v}_2 \geq c \Leftrightarrow \hat{v}_1\hat{v}_2 \geq c\). Besides \(\hat{v}_1\hat{v}_2 > c\) is not possible otherwise there would exist a \(\hat{v}_2 < \hat{v}_2\) such that bidder 2 with valuation \(\hat{v}_2\) could make a strictly positive profit participating in the takeover. Therefore we obtain: \(\hat{v}_1\hat{v}_2 = c\).

Now consider bidder 1. Suppose that when she participates in the takeover, she leaves when the price is equal to \(\hat{v}_2\), her expected utility when her valuation is equal to \(\hat{v}_1\) is: \(\hat{v}_1\hat{v}_2 + \alpha(1 - \hat{v}_2)\hat{v}_2 - c\) (the first part of the expression corresponds to what she gets when bidder 2 does not participate which happens with probability \(\hat{v}_2\) because of the uniform distribution function and the second part corresponds to what she gets when bidder 2 participates). Now, we showed that \(\hat{v}_1\hat{v}_2 = c\). Therefore, the expected utility of player 1 when her valuation is equal to \(\hat{v}_1\) and he submits \(\hat{v}_2\) is equal to \(\alpha(1 - \hat{v}_2)\hat{v}_2 > 0\). Bidder 1 with valuation \(\hat{v}_1\) is strictly better off if she participates than if she does not which means that \(\hat{v}_1\) must be equal to \(0\). Bidder 1 always participates in the takeover and bidding less than \(\hat{v}_2\) is a dominated strategy for her. Therefore, bidder 2 with valuation \(\hat{v}_2\) cannot win the takeover and make a strictly positive profit so that his expected if he participates is \(-c\). We found a contradiction which means that \(\hat{v}_2 \neq 1\) is impossible.

Now, if an equilibrium exists, it must be such that \(\hat{v}_2 = 1\) and \(b_2(\hat{v}_2) = \hat{v}_2\). Bidder 1’s best response is \(\hat{v}_1 = c\).
We showed that any perfect bayesian equilibrium must be such that \((\hat{v}_1, \hat{v}_2) = (c, 1)\). The last necessary step is to show that such a perfect bayesian equilibrium does exist. We propose the following equilibrium: bidder 1 participates in the auction if and only if \(v_1 \geq c\) and bidder 2 participates in the takeover if and only if \(v_2 = 1\). Whenever the takeover is contested, bidder 1 submits 1 and bidder 2 submits \(v_2\). One can easily check that this is an equilibrium (with bidder 1 believing that if bidder 2 participates in the auction \(v_2 = 1\)). We need the tie-breaking rule that we mentioned page 5. With a standard tie-breaking rule, this would not be an equilibrium since when \(v_2 = 1\), bidder 1 would have to pay with probability \(1/2\) a price much higher than \(v_1\). Q.E.D.

7.2 Proof of Lemma 1

First, we need to show that if \(((d_1, b_1), (d_2, b_2))\) is an equilibrium, then \(\forall i = 1, 2, \exists \hat{v}_i\) such that \(d_i(v) = 0\) if \(v < \hat{v}_i\) and \(d_i(v) = 1\) if \(v > \hat{v}_i\). The proof is the same as the one we exhibited in the second paragraph of the proof of proposition 1.

Second, bidder 2’s dominant strategy, if he participates, is still to bid his valuation.

Now, bidder 1 faces an optimization problem close to the one considered in section 3, the only difference is that at the equilibrium, if bidder 1 observes that bidder 2 participates, she knows that bidder 2’s valuation is in the interval \([v_2^*, 1]\). Then bidder 1 maximizes the following expression, choosing a \(b_1 \geq v_2^*\), since bidding less than \(v_2^*\) is a strictly dominated strategy (again, we need the tie-breaking rule that we mentioned page 5)

\[
v_2^*(v_1 - (1 - \alpha)R) + (b_1 - v_2^*)(v_1 - (1 - \alpha)(b_1 + v_2^*/2)) + (1 - b_1)ab_1
\]

\[
= v_2^*(v_1 - (1 - \alpha)R) - v_2^*v_1 + (1 - \alpha)v_2^*/2 + (v_1 - (1 - \alpha)b_1/2)b_1 + \alpha(1 - b_1)b_1
\]

(1)

The first part of the second expression does not depend on the choice of \(b_1\) and the second part is maximized when \(b_1 = \frac{v_1 + \alpha}{1 + \alpha}\).

Now, we have to consider two cases:

- If \(v_2^* \leq \frac{v_1 + \alpha}{1 + \alpha}\), \(b_1 = \frac{v_1 + \alpha}{1 + \alpha}\) is the solution of our maximization program.

- If \(v_2^* > \frac{v_1 + \alpha}{1 + \alpha}\), then \(\forall b_1 \geq v_2^*\), the derivative of the expression (2) is negative and the optimal bid for bidder 1 is \(v_2^*\).

7.3 Proof of proposition 2

Let us first eliminate corner solutions.

Suppose that \(v_1^* = 0\). As bidder 1 bids at least \(v_2^*\), bidder 2 is fully deterred and participates with probability zero. Then bidder 1 is strictly better off not participating if \(v_1 < c + (1 - \alpha)R\). Hence, \(v_1^* = 0\) cannot be part of an equilibrium. Besides, \(v_2^* = 0\) cannot be part of an equilibrium since participating in the takeover is costly.

Suppose that \(v_1^* = 1\). Bidder 2’s best response is \(v_2^* = c + R\). For bidder 1 not to participate with valuation \(v_1 < 1\), it must be the case that she does not derive a strictly positive profit if she participates
when \( v_1 = 1 \). Then

\[
-c + (c + R)(1 - (1 - \alpha)R) + (1 - c - R)(1 - (1 - \alpha)\frac{1 + c + R}{2}) \leq 0
\]

equivalent to

\[
-R^2 + 1 - 2c + c^2 + \alpha + \alpha R - \alpha c^2 \leq 0
\]

\( \alpha + \alpha R - \alpha c^2 > 0 \) and \( -R^2 + 1 - 2c + c^2 = (1 - c - R)(1 - c + R) > 0 \). As a result, the inequation (4) cannot be verified and \((v_1^*, v_2^*) = (1, c + R)\) cannot be an equilibrium.

Then any equilibrium must satisfy one of the following conditions:

- \( 0 < v_1^* < 1 \) and \( v_2^* = 1 \),
- \( 0 < v_1^*, v_2^* < 1 \) and \( v_2^* > \frac{v_1^* + \alpha}{1 + \alpha} \),
- \( 0 < v_1^*, v_2^* < 1 \) and \( v_2^* \leq \frac{v_1^* + \alpha}{1 + \alpha} \),

which we study separately.

- \( 0 < v_1^* < 1 \) and \( v_2^* = 1 \)

Necessary and sufficient conditions for such a \((v_1^*, v_2^*)\) to be an equilibrium are as follows. Bidder 1 is indifferent between participating and not participating when her valuation is \( v_1^* \). Bidder 2, if his valuation is 1, is not strictly better off participating than non-participating. From lemma 5.1, we derive that bidder 1, whenever she participates, bids 1 so that bidder 2 makes a profit only when bidder 1 does not participate. These are equivalent to

\[
v_1^* = c + (1 - \alpha)R \quad \text{and} \quad -c + v_1^*(1 - R) \leq 0
\]

Replacing, in the second equation, \( v_1^* \) by its expression in the first equation, we obtain

\[
v_1^* = c + (1 - \alpha)R \quad \text{and} \quad (1 - \alpha)(1 - R) \leq c
\]

- \( 0 < v_1^*, v_2^* < 1 \) and \( v_2^* > \frac{v_1^* + \alpha}{1 + \alpha} \)

A sufficient and necessary condition for such a \((v_1^*, v_2^*)\) to be an equilibrium is that for \( i = 1, 2 \), bidder \( i \) is indifferent between participating and not participating if \( v_i = v_i^* \). This is equivalent to the following conditions\(^{11}\) for bidder 1 and bidder 2 respectively

\[
v_2^*(v_1^* - (1 - \alpha)R) + \alpha(1 - v_2^*)v_2^* - c = \alpha(1 - v_2^*)R
\]

\[
v_1^*(v_2^* - R) - c = 0
\]

Formula (5) is equivalent to

\[
v_2^*(v_1^* - (1 - \alpha)R) + \alpha(v_2^* - R)(1 - v_2^*) - c = 0
\]

- \( 0 < v_1^*, v_2^* < 1 \) and \( v_2^* \leq \frac{v_1^* + \alpha}{1 + \alpha} \)

We can apply the arguments we have just mentioned. As \( v_2^* \leq \frac{v_1^* + \alpha}{1 + \alpha} \), in \( v_1^* \), bidder 1 does not bid \( v_2^* \). Then, the equivalent of expressions (6) and (5) are respectively:

\(^{11}\)From lemma 1, we know the bidding function of bidder 1 and 2 if they participate.
\[ v_1^*(v_2^* - R) - c = 0 \]

\[ v_2^*(v_1^* - (1 - \alpha)R) + (v_1^* + \frac{\alpha}{1 + \alpha} - v_2^*)(v_1^* - (\frac{1 - \alpha}{2})(v_1^* + \frac{\alpha}{1 + \alpha} + v_2^*)) + \alpha(1 - \frac{v_1^* + \alpha}{1 + \alpha}, \frac{v_1^* + \alpha}{1 + \alpha} - c) = \alpha(1 - v_2^*)R \]

The second equation is equivalent to

\[ v_2^*(2\alpha R - R) + \frac{1 - \alpha}{2}(v_2^*)^2 + \frac{(v_1^* + \alpha)^2}{2 + 2\alpha} - \alpha R = c \]

Q.E.D.
References


Figure 1: Expected target returns depending $\alpha$, for $c = 0.02$ and $R = 0.15$. 
Figure 2: Expected target returns depending $\alpha$ and $c$, for $R = 0.15$. 
Figure 3: Expected target returns depending $\alpha$ and $R$, for $c = 0.05$. 