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History in mathematics teaching: current problems and new proposals

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Abstract

In the light of recent works on the use of the history of mathematics in education, this article offers an analysis of the theoretical issues that arise in such studies. After the presentation of some intrinsic difficulties of a good definition of the different fields involved (a discipline, its didactic and its history), some new elements for a didactical analysis (double field hypothesis, double field paradox and its handling) are proposed. Applied on a few examples, these concepts are aimed at being part of a new framework for empirical studies on the use of history of mathematics in education.

Introduction

For decades and all over the world, the usefulness of the history of mathematics in science education has ceased to be a debatable topic. Numerous authors have already promoted a historical approach (for instance, in France, Barbin 1991, 1997a, Dorier 2000). All these authors have pointed out several good reasons that range from the development of a humanist culture to an acute comprehension of scientific results or theories. In most countries, these reflections have led to an integration of elements of history in school official instructions and in teacher training. Thanks to this shift towards the inclusion of history, a new demand has been brought to light. In 2000, the International Commission on Mathematical Instruction (ICMI) world report on the issues of the use of history in mathematics education expresses clearly the feelings of the community: “having history of mathematics as a resource for the teacher is beneficial” (ICMI Study, 2000, p.1). Nonetheless, next to this enthusiastic conclusion, the lack of real research works on this subject has also been mentioned. Around the same time, the didactician J-L. Dorier explains that the specific function of historical elements in the knowledge acquiring process is still pending; according to him, the number of critical studies has not reach a large enough size (J-L. Dorier, 2000). Following these reflections, a first wish was expressed by the community: “on the effectiveness of studying history in the classroom, it seems desirable to collect and to study two kind of materials: 1. To collect experiences of teachers who use history. [...] 2. To collect questionnaires and interviews of teachers and pupils about mathematics.” (ICMI Study, 2000, p.90). Strangely, even though the idea was shared and accepted by many, there was no significant increase in the number of such studies. In a recent article, U.T. Jankvist (2009) renders an account of the latest conclusions by specialists regarding history of mathematics in education (Arcavi – Tzanakis 2000, Siu – Tzanakis 2004). One can summarize this state of the art in terms of two ideas: (1) the literature is full of pertinent uses of historical elements in education but (2) what makes them pertinent rarely exceeds the subjective considerations of the author. Thus, once again, a non-negligible part of the community of educators requests the increase of detailed works on this field. Very recently, in 2011, a few such texts including didactical aspects have been published. Compiled by V.Katz and C.Tzanakis (eds) in the book *Recent Developments on Introducing a Historical Dimension in Mathematics Education*, these articles try to give an answer to the issues specified in the preface: namely, that “there should be more empirical studies on the use of history in the mathematics classroom to get more insight its educational implications.” According to the editors, “in each case, the authors have conducted at least some preliminary research to test the effectiveness of their teaching methods and conclude that the use of history has a positive effect on their students.” (Katz & Tzanakis 2011, viii). Why the first results only appear more than one decade after the first demand? And why, even in the last

productions, R.Chorlay, one of the contributors of a collective book, expresses reservations on his own work and says that “this is neither historical work in the strict sense - in terms of the standards of the community of historians on mathematics - nor directly didactical work, but we think it helps build bridges [...] (in Katz & Tzanakis, 2011, p.64)? At least, one can ask himself if a pertinent study on learning issues of history of mathematics can be or has been really achieved. I think it is possible in multiple ways, but not self-evident. Thus, in the following sections, this article aims to show why didactic studies on the use of historical elements in mathematics education are intrinsically out of the standard theoretical frameworks. Finally, with some examples, a proposed research way is being presented.

Why? How? And some dead ends

One of the starting points proposed for empirical studies (U.T. Jankvist, 2009) consists in a generally admitted distinction between *how* and *why* to use history of mathematics in education. The *why* question can be considered as an operational realization of the reasons mentioned above. For instance, history is a way of better understanding science and its methods, thus the use of historical elements in classrooms allows pupils to develop informed points of view. The *how* question generally deals with the links between the teaching of a subject and its history. The typical example is the elaboration of sessions inspired by the historical evolution of scientific concepts. Both for the *how* and the *why*, it is important to notice that these questionings only intervene in the determination of the aims and teaching materials, that is to say, they are always related to teacher actions in the elaborating phase. *In fine*, even if it can be helpful for the understanding of the implementation of historical elements in a course, the categorization in *how* and *why* gives few tools for an analysis of pupils' learning. Nonetheless, this distinction is a way of making explicit the coexistence of two distinct fields in an activity: the discipline and its history. In such a situation, the questioning on the reasons for a historical approach becomes a way of identifying teaching goals that are specific to history, like acquiring knowledge on an author, a period or the evolution of a concept. It is consequently obvious that the complexity of this interlocking of two fields requires data acquiring. A quantitative rise in empirical studies is a good answer but it raises a new crucial question: what are or can be the aims of such studies?

On this point, a very interesting recent work in Germany (M.Glaubitz, 2011) deals with a comparative study of pupil performance in secondary schools. The purpose of this study was to shed light on “how does the historical teaching affect the students' mathematical skills compared to those who had conventional teaching? How does the historical teaching affect the students' views on mathematics, teaching and their relationship towards it? [and] In what way and to what extent does the historical teaching allow for (meta-)reflection and discussion in the classroom?” In the experiment, a first part of the students have had a typical mathematics course, a second group have had lessons inspired by history of science, mainly in its chronological aspect (the genetic approach). A third group had engaged in a teaching strongly marked by an historical contextualization of mathematics concepts (the hermeneutic approach). In a nut shell, the genetic approach essentially consisted in the insertion of a discovery activity with historical elements at the beginning of the lesson. As for the hermeneutic approach, it was composed of three steps (sometimes repeated), a classical mathematics course, a reading of genuine historical sources, and an important research activity around the contextualization of the historical texts and concepts.

A statistical analysis on six classes (about 175 students) shows that the best performances obtained in the hermeneutic approach, very closely followed by the classical course. The lessons comprising only discovery activities yielded results far below the two others. According to Glaubitz, the poor score of the genetic approach is due to difficulties for children to grasp what is essential in the proposed contents, resulting in turn to a limited stabilization of knowledge. What about the context of this study? The German school system is strongly normalized and one of the main objectives of

Glaubitz's works was to convince parents that a non-traditional education can produce good results in terms of learning. The unofficial purpose of the study was political and under this aspect, it was successful. Glaubitz concludes that the hermeneutic approach is indubitably better than the others as it "could make a notable contribution to humanist, democratic and peace-promoting traditions in education" and the reasons of this success can be found in the good use of history. This last affirmation generally shared by the other members of the community may require some more analysis. The quest for evidence of the effectiveness of teaching enriched with history of mathematics is probably vain. For instance, even the large scale study made by Glaubitz only shows that a good implementation of historical elements produces results only as good as the traditional method. Here appears a well-known didactics result: a bad education leads to poor results and a well thought out one leads to success (whatever it is based on, historical documents or not). In my view, such work mainly raises our awareness to the fact that studying the manner a course is elaborated represents only a part of the job. Centered on the links between knowledge-teacher, this approach may miss other interesting interactions between teacher and pupils, or between pupils and the knowledge at stake. New research ways are open, in particular on classroom practice analysis or, as pointed out in the ICMI study, "a crucial area to explore is the relation between how students achieve understanding in mathematics and the historical construction of mathematics thinking." (ICMI Study, 2000, xvii)

Double field, double field paradox, and naive didactics

Classroom sessions analysis is a traditional methodological tool used in the didactics. On the specific case of history in mathematics education, the research object is thought-worthy and the research field deserves to be well defined. In a didactic study of session comprising historical content, a first idea is to use theoretical frameworks already developed in mathematics didactics themselves. With the help of mathematics didactics, classroom lessons can be analyzed in order to identify, for instance, key moments in pupils activity or to evaluate learning processes. As the theoretical tools in use are suitable to the research object, such an approach is perfectly coherent. No doubt, this analysis will produce interesting results that will add to the evidence of its pertinence to mathematics didactics. The problem consists in the extrapolation of such results to some knowledge on the place of history of mathematics in education. Mathematics didactics can mainly render an account of mathematics learning. The tools developed in this field are suitable to show that a lesson has truly helped pupils to acquire mathematics knowledge but they do not give any information on the role played by the history of mathematics in that lesson. That is to say, mathematics didactics alone do not seem to be the most pertinent tool for an analysis of the potential role of the history in mathematics education. The concepts and methods developed in the mathematics didactics field have to be considered as a part of the available tools. Let us recall some recommendations found in the ICMI report, "there are limits and risks attached to an approach that takes too simplistic a view of the significance of the history in mathematics education." (ICMI Study, 2000, p.64).

Didactic studies generally enable us to understand teaching situations only relative to the considered discipline. In sessions including historical elements, the simple conjoint presence of a topic and its history is specific enough to create a new situation. This double field invalidates a mono-disciplinary didactic analysis because such a study can only process part of the situation. At school or high school, history of mathematics so appears in link with the considered subject. In this case, history is not a specific teaching topic and, in a way, is not an autonomous discipline. In fact, in several education situations, history of mathematics is not an academic subject (some exceptions exist, in particular, in Denmark). Nonetheless, official instructions sometimes emphasize the fact that pupils have to be aware of epistemological and historical problems. Generally, there is no history of science course but some real teaching goals exist. Teachers have to transmit knowledge

without being able to use traditional tools to do so. Here appears the double field paradox. According to educators themselves, didactics deal mainly with the learning of contents disciplinarily specified¹ (Reuter 2007, 17). This definition has two consequences. The first is, as it is not a school subject, it seems not to be pertinent to try to speak about didactics of the history of mathematics at school. The second consequence of the classical definition of didactics refers to the trial of a multi-disciplinary approach of the double field hypothesis. In sessions involving historical material, the presence of two different fields could conduce to the use of works on didactics of inter-disciplinarity. This topic is defined (Lenoir & Sauvé, 1998) by the creation of an interaction between two or more disciplines at a pedagogical level. This leads to the creation of complementary or cooperation links that are helpful in pupil learning process². No doubt, this definition takes into account the richness of the relationships which exist between two disciplines. Many users of history of mathematics in education will certainly agree with the ideas of inter-fecundity noticed above. Unfortunately, this enthusiasm cannot hide the fact that didactics of inter-disciplinarity are foremost didactics in general and concern only school subjects. The history of mathematics is not a school topic and it is doubtful that such a theoretical framework will produce good results.

History of mathematics and mathematics education

Returning to the issue of a good definition of theoretical frameworks involving historical elements suitable for empirical studies on teaching situations, the question becomes: what are the possible interactions between didactics and *in situ* teacher practice? Didacticians elaborate analysis frameworks comprising many concepts organized in a model. This model is then used for the study of teaching situations. From school to university, mathematics is a teaching object. Whatever the level, some mathematics contents are transmitted. The question of the choice and the nature of this knowledge do not matter for the following reflection. What I would like to underline is that learning purposes are varied, more or less complex, and this richness creates teaching situations very different from one another. The analysis of key elements in this learning belongs to didactics in which adapted concepts and models will be elaborated. History of mathematics in education, daughter of the university discipline, is in the same situation. In term of time or notions, history of mathematics contents are as varied as pure mathematics contents. Thus, course aims can go from acquiring knowledge on specific period or author to the development of an individual epistemological thinking. As mentioned previously, the only difference consists in lack of explicit definition jointed to a large part of subjectivity. Relationships between firstly, mathematics – as teaching objects— and secondly, the history of mathematics – as teaching object— can be conceived in a parallel way (figure 1). Like in mathematics education, the purposes of didactics dealing with history of mathematics must be the analysis of pertinent elements of such learning.

1 Les didactiques « s'intéressent principalement aux apprentissages de contenus spécifiés disciplinairement », Y. Reuter (ed.), *Dictionnaire des concepts fondamentaux des didactiques*, de Boeck, 2007, 17.

2 L'interdisciplinarité se définit comme « la mise en relation de deux ou plusieurs disciplines scolaires qui s'exerce à la fois aux niveaux curriculaire, didactique et pédagogique et qui conduit à l'établissement de liens de complémentarité ou de coopération, d'interpénétrations ou d'actions réciproques entre elles sous divers aspects (finalités, objets d'étude, concepts et notions, démarches d'apprentissages, habiletés techniques, etc...), en vue de favoriser l'intégration des processus d'apprentissage et des savoirs chez les élèves. », Lenoir & Sauvé, De l'interdisciplinarité scolaire à l'interdisciplinarité dans la formation à l'enseignement: un état de la question, *Revue française de pédagogie*, 1998, n°124, 121-153.

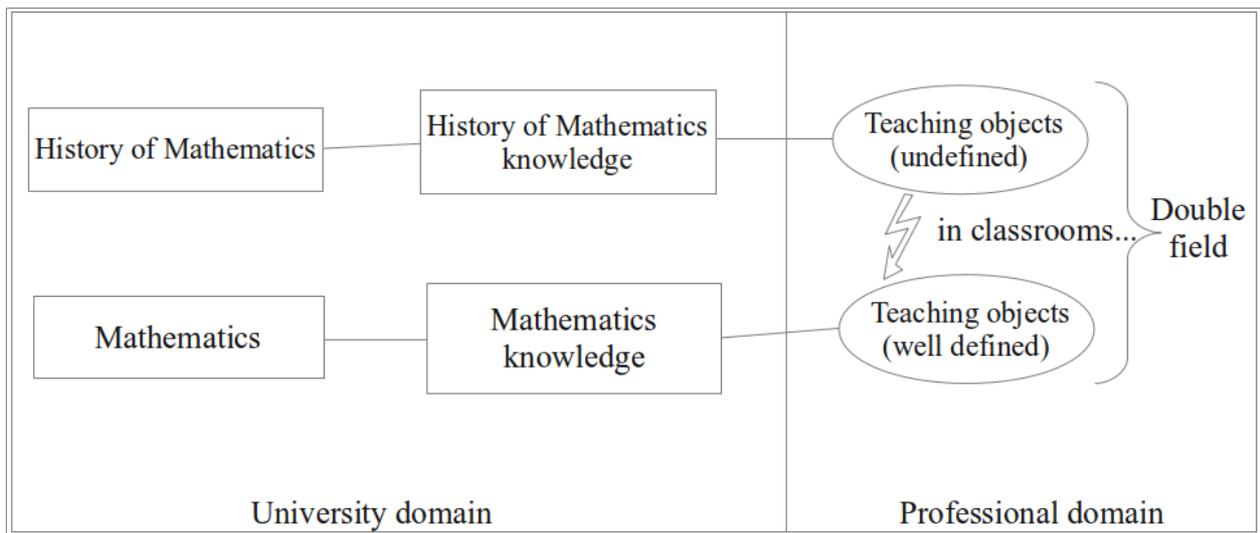


Figure 1

In most countries, teachers have been assigned the function of inviting pupils to an epistemological reflection and many of them do not feel at ease with this new mission (double field paradox). History of mathematics, in its academic aspect, is not suitable to a school instruction. But the concepts, the sources, and the activities can go through a pedagogical filter which gives educators the opportunity to appropriate them as tools for session building. Whereby, a classroom situation is at the interface of a discipline, history of this discipline, and many other elements the teacher can decide to integrate in his course. The teacher is not a researcher; he is a professional in education in school knowledge and know-how. Like mathematics teaching skills, competences in the history of mathematics implementation can be regarded as use of savant contents that have their own domain. No doubt, in the same way that it is preferable to learn enough mathematics to be able to teach them, teacher engagement in sessions involving historical elements without having any education on those topics could be hazardous. In France, and in most countries, the involvement of historians of mathematics community in the recent institutional changes may have certainly prevented us from this pitfall, but strangely, for many teachers, the double field paradox remains...

The tool / teaching goal duality: understanding the paradox

In order to show how the elements previously presented open a way to elaborate a framework for empirical studies, I would like to come back to a second distinction generally accepted in the use of history in mathematics education. In the first paragraphs of this article, the classical separation between *how* and *why* of the use of history has been a reminder. This distinction is often completed with a dichotomy between the history of mathematics *as a tool*, and its use *as a teaching goal* (for instance Barbin & al. (ed), 2008). The international community of historians of mathematics involved in teacher training have recently taken the two notions, history *as a tool* and history *as a goal*, as an entry for empirical studies (Jankvist, 2010). In their most recent paper, Jankvist and Kjeldsen explain that “the word goal must be understood in the sense that it is considered a goal to show the students something about the historical development of mathematics” and the tool notion appears when “history is used as an assisting means, or an aid, in the teaching and learning of mathematics.” (Jankvist & Kjeldsen, 2011) The consideration of two different uses of history of mathematics is judicious but it should be handle with care in order to prevent the risk of falling into mono-disciplinary studies that are not, according to what I have tried to explain, the most pertinent for science sessions involving history. One of the difficulties in the use of history in mathematics education consists in the co-presence of two knowledge fields, one being a school topic and the other not. A teacher who is proposing such a session is in a situation in which he strives to reach multiple learning purposes, some disciplinary, others epistemological (or historical). In order to

analyze in a rich didactical manner and articulate what is at stake in such situations, a new point of view is necessary.

In a previous exploratory work (de Vittori & Loeuille, 2009), the comparative study of secondary teacher training sessions led to highlighting key moments, when trainees have really entered in the intended learning. A typology has been elaborated in which five task entry modalities have been pointed out: technical, philosophical, linguistic, practical and dramatic (related to plays or role playing games). Let us first recall some important points in the technical modality. This type of task entry appears when a pure technical work is done following a historical document. For instance, in the aforementioned study, trainees were exposed to a geometry demonstration which enabled them to understand Galileo's arguments (*Il Saggiatore*, passage on Earth-comets distance, sub-lunar or supra-lunar objects and discussion on their nature). By redoing the geometric proof, the trainees were naturally engaged in a reflection on the necessity of such mathematical arguments. Thus, they started to question the scientific context during Galileo's time and in that way entered in the contextualization learning which was the purpose of that session. In this example, the work was exclusively based on mathematics contents, nonetheless this modality can be extended to any science topic as soon as students activity goes only by already acquired scientific knowledge. Clearly, in a session on epistemology and history of science, a scientific discipline can become a tool for a teaching goal. At school, this situation exists too. A typical instance can be found when a secondary mathematics teacher proposes pupils to do a geometrical drawing inspired by Euclid's *Elements* (de Vittori & Loeuille, 2009). That way, pupils can smoothly enter a historico-epistemological thinking through the questioning on the reasons of such mathematical construction or proof. Like at the university level, in a classroom situation, mathematics can be used as a tool for a work on historical topics.

During a mathematics course involving history, whether it takes place at school or in university, one of the preferred documents is the text. Intrinsically this type of historical source requires a work on language. Even if it is carefully chosen, for its simplicity for example, the reading of an ancient text goes with explanations about some words or about the style. The quest for answers to these grammatical or semantic questions is one of the possible learning entries (E.Barbin uses the word *dialogism*, that is to say a kind of discussion between authors from the past and contemporary reader; see also Katz & Tzanakis, eds, 2011, chapter 2). For instance, always in the secondary school session previously mentioned, the teacher proposes to the pupils an activity based on a short quotation of a 17th century text. In this document, some words had been written in old French (e.g. *quarré* instead of *carré*) and the style differed from our (e.g. a perpendicular is *falling* on ...). Pupils' questioning on the spelling of the words has led to a recall of some geometry definitions and has enabled entering in the learning contents; in this case, a lesson on polygons. Thus, in this situation, history is a tool, a means of reaching mathematics learning goals.

Both examples below suggest a way of analyzing the links between mathematics and its history in a classroom situation. As noticed previously, history as a tool and history as a goal can appear here as two elements of a unique session (figure 2). What is interesting here is that mathematics is both a tool and a goal.

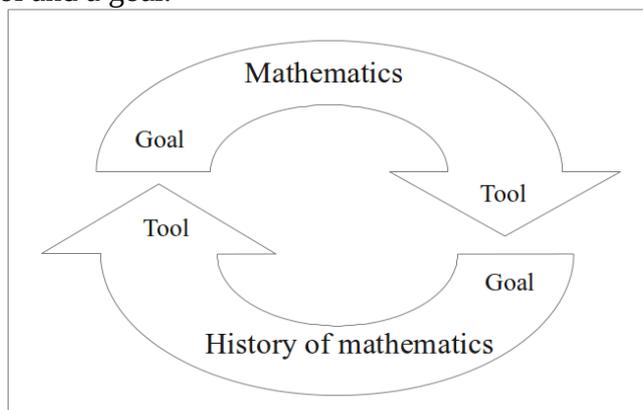


Figure 2

A circulation is evident. This dynamic relationship brings to the fore the fact that the classroom lesson is a place where links between history and mathematics can be reconstructed. To my mind, here lies a deep cause of the double field paradox, namely that the history of mathematics and mathematics have been separated by the academic research process. In his classroom, a teacher who combines science and its history constantly goes from the disciplinary domain to the historical domain, then from the historical domain to the disciplinary one, and so on and so forth. In this way, he is reassembling what was and should be a whole. The way the teacher manages this circulation and all the pedagogical artifices he uses in order to avoid the double field paradox are research topics suitable for empirical studies that transcend traditional academic borders.

Teacher and pupils in classroom: an example of reconstruction

Below is part of a discussion between a teacher and one of his students. The situation takes place at the end of a session during which the pupils (about 11 years old) had to redo some old Indian constructions. Following indications given on an exercise sheet (with some historical elements like names, dates, and pictures), in small groups in the playground, the students drew a square with a single piece chalk and a piece of string. After an introductory text which gives some information on the kind of problems the old Indian mathematician were interested in, the exercise sheet comprises a question on the squaring the circle problem (“Les mathématiciens grecs ont aussi étudié cette dernière situation, à la règle non graduée et au compas. Comment a-t-on appelé par la suite ce problème ?”).

Teacher: “Have you answered the question there? Lora, have you answered the question on the problem? What’s it called ?” (*Vous avez répondu à la question là ? Lora tu as répondu à la question sur le problème, ça s’appelle ?*)

Student: “Squaring the circle” (*La quadrature du cercle.*)

Teacher: “Squaring the circle; what’s that?” (*La quadrature du cercle, c’est quoi cette histoire là ?*)

Student: “It’s when there is a circle and when one has to find a square which has the same area.” (*C’est quand il y avait un cercle et quand il fallait trouver le carré qui avait la même aire que ce cercle là.*)

Teacher: “Yes, ok and what was the problem?” (*Oui d’accord et alors quel problème s’est posé ?*)

Student: “Well...” (*Bin...*)

Teacher: “Have they managed to do it?” (*Est-ce qu’ils ont réussi à faire ça ?*)

Student: “Well, no because they didn’t yet know the pi number.” (*Bin non parce qu’ils ne connaissaient pas encore le nombre pi.*)

Teacher: “And that it’s once one knew well the pi number that one realized that it was not possible, that’s it?” (*Et c’est quand on a su bien connaître le nombre pi qu’on s’est rendu compte que c’était pas possible, c’est ça ?*)

Student: “Yes.” (*Oui.*)

Teacher: “There you are, then.” (*Voilà.*)

The discussion happened in the presence of six pupils, the three members of the first group rejoined by another group that had finished their work. The young girl the teacher is speaking to is one of his best students, the other pupils, in particular the second group, are students of average abilities. The beginning of the dialogue seems to be simply checking the right answer to a question, but after a few sentences, the teacher gets into another role. The question on squaring the circle becomes a way for him to give reconstruction elements. The square construction, the circle, pi number, are linked in both the history of mathematics and mathematics. For such questions, a separation between the historical part and the mathematical part impoverishes the meaning. In this case, following mainly his own sensitivity, the teacher feels the need for a reconstruction. He does not give much information, and in fact, he does not give anything that was not already known by the pupils, but he

engages a process. It is interesting to notice that these first exchanges are followed by an open discussion on Archimedes and his work. The discussion involves all the six pupils who are making links between other courses (history and geography, the never ending screw), their own culture (Eureka!, Archimedes' principle). Sometimes, the teacher intervenes when mistakes are made. Here is an example of a historical one.

Student: He [Archimedes] wrote a book named Elements. (*Il a écrit un livre qui s'appelle Elements.*)

Teacher: Oh, you're mistaking between another Greek mathematician. (*Ah, vous confondez avec un autre mathématicien grec.*)

Student: Oh no, that was Euclid. (*Ah nan, c'était Euclide.*)

Teacher: Very well, can you tell me anything about Euclid? (*Très bien, tu peux m'en parlé d'Euclide?*)

Student: He has... taken up again point by point all the mathematics. (*Il a ... il a repris point par point toutes les mathématiques.*)

Teacher: Waow. Yeah. (*Oh dis donc, ouais.*)

Student: And he wrote Elements. (*Et il a écrit Elements.*)

Teacher: The Elements... what was it? (*Les Elements... c'était quoi en fait cette histoire d'Elements?*)

Student: Well, all the maths. (*Ben, tout les maths.*)

Teacher: That is to say? What did he do in fact? One can say that his book is what? A kind of... (*C'est-à-dire? Il a fait quoi en fait ? On peut considérer que son ouvrage c'est quoi? Une sorte de...*)

Student: Textbook. (*De manuel.*)

Teacher: A mathematics textbook. Good. (*De manuel de math. Bon.*)

This session was recorded in 2011. In 2007, an interview of the same teacher was done. Speaking about his thoughts on the implementation of history in mathematics education, the teacher insisted on the fact that mathematics teaching is historically unstructured. According to him, contrary to education on other topics, the mathematics courses are jumping through history, going from Antiquity to Middle-Age, then back to 18th, ... (*"Alors que nous on va passer de l'Antiquité au Moyen-Age, on va aller au 18e siècle, on va revenir à l'Antiquité, il n'y a pas quelque chose de progressif, on a vite fait de s'y perdre. C'est pour ça que les enfants ne savent plus qui a fait quoi et quand, et ne situent pas dans le temps, et ça pose des problèmes de repères."*) This situation creates trouble in the pupils' mind and one of the teacher's roles can be the reestablishment of the historico-epistemological links (*"comprendre que les choses sont arrivées petit à petit, parce qu'il y avait des besoins, ça répondait à des besoins concrets, petit à petit on a développé une science autour de tout ça, et puis la science a pu évoluer par elle-même et pour elle-même, ça ça me semble intéressant de leur faire comprendre."*)

No doubt that this aspect of the teaching situation is deeply based on epistemological views of the teacher. One can find some studies on this topic. What I would only like to underline with this short example is that a session comprising historical elements must be conceived as a complex whole. Both mathematics and the history of mathematics are aspects of a teaching situation that could be treated as a whole. Any dichotomized approach (history/mathematics, goal/tool, how/why) should stay a first step of a study complemented by the giving of synchronic elements that take into account the unity necessarily at stake in a classroom.

Conclusion: on didactical studies

Teaching history of mathematics or teaching with/through history of mathematics? At school, the question is relatively clear: history of mathematics is not a school topic. The situation seems to be different when a specific teaching exists. In France for instance, recent institutional changes have

led to the creation of courses on epistemology and the history of science in many curricula. These courses have been largely developed in the teacher training sector. The history of mathematics already existed in curriculum aimed to researchers' education but its massive presence in pre-professional courses is rather new. The situation is similar in many countries and except in a few situations, history of mathematics is a fully-fledged discipline mainly at university. Thus and as it has already been noticed by some authors (Guedj & al. 2007, Reforehst 2006), didactics of history of science might be considered at this level. Nonetheless, one might remember that the context is really not the same. The History of mathematics can be taught as a "research" topic. In this case, even if it takes place in a professional curriculum, the course is based on methods and contents close to the courses for future researchers. A didactical study at this level is interesting but it is not related to classroom situations. If didactics of history of mathematics is defined there, its aims will be very different from those of a second one centered on classroom activities.

Sometimes mentioned by educators, another manner of introducing history in curriculum consists in a pre-professional course dedicated to teachers. In this case, the contents are chosen in order to give teacher elements on topics suitable to schools. Some examples of activities may even be given. What is the reference field of such a course? Part of it is dealing with history, but this field is not considered for itself, it is viewed as an element of an acquiring process of professional skills. Another part is related to classroom and mathematics education. Generally, everything is based on the teacher's experience (or on some reported experiments) and, in a way, the teacher is probably sharing his own reconstruction skill. In such a professional-to-professional relationship, designing a research field is not easy and may even be useless. More changing and more mixed than the university sector, this professional sector comes with specific questions on which a joint work with all the different actors (teachers, researchers, ...) can be engaged. Empirical studies are undoubtedly possible. Will they be relevant to works on didactics of mathematics (if one thinks that history is an indivisible part of mathematics) or didactics of the history of mathematics (if university level studies raise enough specificity of this field)? I do not know yet, but one thing is certain: as the relationships between a science, its didactics and its history are complex and much richer than those expressed in this short article, whatever its name is, it's at its best that didactics studies on the history of mathematics in mathematics education stay in touch with all the related disciplines (mathematics, didactics of mathematics and history of mathematics) and all the recent research results.

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