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# Two-phase flow: structure, upscaling, and consequences for macroscopic transport properties

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## Abstract

In disordered porous media, two-phase flow of immiscible fluids (biphasic flow) is organized in patterns that sometimes exhibit fractal geometries over a range of length scales, depending on the capillary, gravitational and viscous forces at play. These forces, as well as the boundary conditions, also determine whether the flow leads to the appearance of fingering pathways, i.e., unstable flow, or not. We present here a short review of these aspects, focusing on drainage and summarizing when these flows are expected to be stable or not, what fractal dimensions can be expected, and in which range of scales. We base our review on experimental studies performed in two-dimensional Hele-Shaw cells, or addressing three dimensional porous media by use of several imaging techniques. We first present configurations in which solely capillary forces and gravity play a role. Next, we review configurations in which capillarity and viscosity are the main forces at play. Eventually, we examine how the microscopic geometry of the fluid clusters affects the macroscopic transport properties. An example of such an upscaling is illustrated in detail: For air invasion in a mono-layer glass-bead cell, the fractal dimension of the flow structures and the associated scale-ranges, are shown to depend on the displacement velocity. This controls the relationship between saturation and the pressure difference between the two phases at the macroscopic scale. We provide in this case expressions for dynamic capillary pressure and residual fluid phase saturations.

# 1 Introduction

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The physics of two phase flows in porous media is a complex and rich topic, with obvious applications to the hydraulics of the vadoze zone, be it water infiltration, its evaporation, or the transport of Dense Non-aqueous Phase Liquids (DNAPL) down to the aquifers (Dridi et al., 2009). Hydrogeologists and soil scientists aim at relating volumetric flow, pressure head, and water content at the Darcy scale, which is a meso-scale above which the medium and the flow are described by continuous mathematical fields. They also need to predict the front displacement of the injected fluids, its localizing or non localizing character, and the fluid mass distribution behind it. The basic laws of multiphase flows treated at mesoscopic scale as a continuum require a closure of simultaneous flow according to Darcy’s law. A key point of this closure is a functional relation between the capillary pressure and (water) saturation in the form of retention curves; another key point in the dependence of the relative permeabilities on saturation.

The physics community has been mostly concerned with characterizing and understanding flow structures/patterns from the pore scale and up. These structures and processes have a major impact on the retention curves (see e.g. review by Blunt (2001)). Notably, viscous fingering may have strong influence on retention curves, resulting in dynamic saturation–pressure curves in porous media as we will show in section 4 below.

These flow structures can vary from compact to ramified and fractal (Lenormand et al., 1988; 1989; Måløy et al., 1985; Méheust et al., 2002; Sandnes et al., 2011; Holtzman and Juanes, 2010). One major issue is to simplify this complexity by keeping just enough information to describe the relevant physics at the relevant scale for the flow considered, without discarding

54 essential information. For example, the simple invasion percolation model is sufficient to model  
55 the flow structure obtained under slow drainage conditions.

56         The fact that such simple models can describe simple features of complex systems arises  
57 from the property of universality in critical states: many physical dynamic systems, governed by  
58 a competition of simple forces with a disorder in thresholds, are in the vicinity of so-called  
59 critical points in statistical physics, as discussed e.g. by Domb (1996) or Feder (1988): these are  
60 characterized by scale invariance over some ranges, fractal dimensions, and by critical exponents  
61 precising how characteristic lengths that limit the fractal ranges depend on the system size or  
62 driving speed. An interesting aspect of such critical points is that the corresponding exponents  
63 and fractal dimensions do not depend on the small scale details of the system, but are controlled  
64 by how the systems are invariant under some form of upscaling. Consequently many systems  
65 differing at small scale are characterized by the same critical exponents: they are said to belong  
66 to the same universality class. This allows to describe complex systems using sometimes simple  
67 computer models.

68         The porous body of a piece of soil or rock consists of pores and fracture networks of  
69 different length scales and shapes, whose permeability presents large spatial variations. These  
70 structures can be correlated at large scale (see e.g. Brown, 1995; Zimmerman and Bodvarsson,  
71 1996; Méheust and Schmittbuhl 2003, Neuville et al. 2010a) or present a finite correlation length  
72 (Neuville et al. 2010b). The variations in permeability results in flow channeling (see e.g.  
73 Brown, 1987; 1995, Drazer and Koplik, 2002, Méheust and Schmittbuhl 2000, 2001, 2003,  
74 Neuville et al., 2010a, 2011a, 2011b, 2011c) and a potential permeability anisotropy (Méheust  
75 and Schmittbuhl 2000, 2001). In general the soil/rock is a dynamic medium where the porosity  
76 can be modified by the fluids involved due to chemical reactions and desorption/adsorption

77 mechanisms (Szymczak and Ladd, 2011), in addition to the fluid pressure and the mechanical  
78 stress acting on the porous medium (Johnsen et al., 2006; Goren et al., 2010; 2011). The  
79 chemical composition and nano/micro structure of the rock further decides the wetting properties  
80 of the fluids which is crucial for the capillary front advancement in two-phase flow. For  
81 example, when a fluid with high viscosity is displaced in a porous medium by a fluid with a  
82 lower viscosity, the displacing fluid tends to channel through the paths of lower flow resistance,  
83 thereby forming pronounced fingers. The physical properties of the fluids play an important  
84 practical role on natural flows: e.g., in soil and groundwater, the identification of pollution  
85 sources is difficult due to the fact that organic pollutants can rapidly migrate down to the bottom  
86 of the aquifer and/or along paths different from the water (Benremita and Schäfer, 2003).

87 In addition, when the porous medium is deformable, branching structures can be observed with  
88 transitions to fracturing of the porous medium (Lemaire et al., 1993; Cheng et al., 2008; Sandnes  
89 et al., 2011; Holtzman and Juanes, 2010; Chevalier et al., 2009; Johnsen et al., 2006; Varas et al.,  
90 2011) or formation of fingers, channels or bubbles in it (Johnsen et al., 2006; 2007; 2008; Kong  
91 et al., 2011; Vinningland et al., 2007a; 2007b, 2010, 2011; Niebling et al., 2010a; 2010b).

92 In this review we will address the detailed structure and dynamics of two phase flow in  
93 fixed and disordered porous media based on pore scale experiments. We will limit the discussion  
94 mostly to drainage, i.e. to situations where a non wetting fluid displaces a wetting one - even  
95 though imbibition, where a wetting fluid invades a non wetting one, is of equal practical  
96 importance. The discussion will be limited to media that are isotropic and homogeneous at large  
97 scales, and to cases where chemical reactions and adsorption or desorption between the fluids  
98 and the porous medium can be neglected. The structures of clusters of the moving fluid and the  
99 dynamics of drainage in porous media depend on several parameters like the density difference,

100 the surface tension, the wetting properties, the viscosities and the flow rates of the fluids  
101 involved. The various forces at play dominate on different length scales and their interplay give  
102 rise to separate scaling regimes. Up-scaling, which consists in relating the pore scale description  
103 to properties defined at the Darcy scale or even at the macroscopic scale, is a central topic within  
104 hydrology and petroleum engineering.

105         Only by understanding the scaling of the structures and dynamics within each regime,  
106 and the crossover lengths involved, it is possible to perform up-scaling. The structures involved  
107 are typically fractal within some scaling range; their fractal dimension depends on length scales,  
108 and often result from fluctuations occurring at smaller scales. At the end of this short review we  
109 provide an example of upscaling of recent experimental data; the experiments in question were  
110 aimed at studying the crossover between capillary and viscous fingering in a quasi two–  
111 dimensional (monolayer) porous medium.

112         This review summarizes the results of a series of works, published mostly in Physics  
113 journals, that are of interest to model flow in the vadose zone, or in general in hydrology. We  
114 will also illustrate on simple examples what type of microstructure and what properties of  
115 fingering of the flow control upscaling and the dynamic dependence of macroscopic capillary  
116 pressure on microscopic flow.

117

## 118         **2 Capillary and gravitational effects**

119

120         When drainage is performed in the limit of infinitely slow displacement velocities, the  
121 pressure drop accross the porous medium is controlled by the capillary pressure drop accross the  
122 interface between the two fluids. The criterion for advancement of the interface into a given pore

123 is that the capillary pressure drop is larger than the capillary pressure threshold needed to invade  
124 the pore neck that separates that pore from the already-invaded adjacent pore. The value of the  
125 capillary pressure threshold fluctuates from pore neck to pore neck, with a distribution function  
126 determined by the geometry of the porous medium. In the case of zero gravity or for a horizontal  
127 2D porous medium, the next pore throat /neck to be invaded will be, among the pores that touch  
128 the interface, the one whose pore throat has the smallest capillary pressure threshold. This idea is  
129 the basis of the invasion percolation algorithm (de Gennes and Guyon, 1978; Chandler et al.,  
130 1982; Wilkinson and Willemsen, 1983) where random numbers representing the capillary  
131 pressure threshold values are distributed on a lattice and where the front is moved at each time  
132 step at the location along the interface corresponding to the smallest threshold value. The fact  
133 that the fluid front always moves at the most easily invaded pore neck and nowhere else is  
134 actually not always true in real flows, even if it is a good approximation. What drives the  
135 advancement of the front is the capillary pressure build-up in the fluid. The capillary pressure  
136 will not relax immediately after invasion of a new pore but is controlled by a back contraction of  
137 the fluid interface. This is the reason for the so called Haines jumps which may lead to invasion  
138 of several pores in one jump (Haines, 1930; Måløy et al., 1992; Furuberg et al., 1996).

139 When the displaced fluid is incompressible (or lowly compressible), trapping takes place.  
140 Trapping is very important in two dimensions (2D) (Wilkinson and Willemsen, 1983) but much  
141 less significant in three dimensions (3D). Experiments addressing capillary fingering in 2D  
142 model systems were first performed by Lenormand et al. (1988; 1989) who found a mass fractal  
143 dimension of the invaded structures equal to  $D_c = 1.83$ , which is consistent with the results of  
144 numerical simulations based on invasion percolation (in the version of the model allowing  
145 trapping of the invaded fluid) (Wilkinson and Willemsen, 1983). In 3D, several experiments

146 have been performed (Chuoque et al., 1959; Paterson et al., 1984a; 1984b; Chen et al., 1992;  
147 Frette et al.1994; Hou et al., 2009; Mandava et al., 1990; Nsir et al., 2011; Yan et al., 2012). The  
148 fractal dimension found at small scale (between 2.0 and 2.6) is compatible with the dimension  
149  $D = 2.5$  found in three dimensional invasion percolation models (Wilkinson and Willemsen,  
150 1983).

151 Even though the capillary fingering structure is fractal, in practice it is well described by a fractal  
152 dimension only within a window of length scales ranging from the pore size up to a crossover  
153 length on a larger scale. In the case where the density difference between the two fluids is  
154 different from zero, but where viscous forces are small compared to the others, this crossover  
155 length corresponds to a scale at which the capillary threshold fluctuations become of the same  
156 order of magnitude as the difference in hydrostatic pressure drop between the two fluids. This  
157 means that the crossover will always occur when the fluid structures become large enough.  
158 When a lighter fluid is displacing a heavier one from above at a slow flow rate resulting in low  
159 viscous forces, a stable displacement is observed. In this case, the displacing fluid does not finger  
160 its way through the displaced fluid; the crossover length sets the width of the rough interface  
161 between the two fluids (Birovljev et al., 1991; Méheust et al., 2002; Løvoll et al., 2005), parallel  
162 to the average flow direction .

163 Gravitational effects can easily be accounted for in the invasion percolation model by mapping  
164 the system onto a problem where the capillary threshold values are modified linearly by the  
165 hydrostatic pressure difference between the two fluids (Wilkinson, 1984; Birovljev et  
166 al., 1991; Auradou et al., 1999). By using this theory of percolation in a gradient, Wilkinson  
167 (1984) predicted theoretically the scaling of the front width  $\xi$  observed in a gravitational field as

168 
$$\xi/a \propto Bo^{\frac{-\nu}{\nu+1}}, \quad (1)$$



169 where the Bond number  $Bo = \Delta\rho g a^2 / \gamma$  is the ratio of the difference of the hydrostatic pressure  
170 drops in the two fluids on the length scale of a single pore to the capillary pressure drop, and  $\nu$   
171 is the critical exponent associated to correlation length divergence in percolation theory ( $\nu = 4/3$   
172 in 2D, Wilkinson (1984) ). Here,  $\Delta\rho$  is the density difference between the fluids,  $g$  the  
173 gravitational acceleration,  $a$  the characteristic pore size and  $\gamma$  the surface tension between the  
174 two fluids. Gravitationally-stabilized fluid fronts occurring during very slow two-dimensional  
175 drainage have been studied both experimentally and by computer simulations (Birovljev et al.,  
176 1991). The results were found consistent with the theoretical prediction of Wilkinson (1984).  
177 When a lighter fluid is injected into a heavier fluid from below (Frette et al., 1992; Birovljev et  
178 al., 1995; Wagner et al., 1997), gravitational fingering of the displacing fluid through the  
179 displaced fluid occurs; a scaling behavior consistent with Eq. (1) has also been found in this  
180 unstable case: the characteristic length scale  $\xi$  then corresponds to the width of the unstable  
181 gravitational fingers, perpendicularly to the average flow direction. The same simple mapping to  
182 invasion percolation as described above can also be performed in the case of slow displacement  
183 in a rough fracture joint filled with particles (Auradou et al., 1999).

184 When comparing systems with different capillary pressure threshold distributions, Eq.(1)  
185 needs to be modified. From the phenomenology of the invasion process that we have explained  
186 above, it is quite intuitive that a gravity - stabilized front in a porous medium presenting a narrow  
187 capillary noise (i.e. a narrow distribution of the capillary thresholds) will give a flatter front than  
188 a porous medium with a wide capillary noise. Instead of equation Eq. (1), it has therefore been  
189 suggested to take into account a dimensionless fluctuation number  $F = \Delta\rho g a / W_t$  , in which  
190 capillary fluctuations are accounted for in terms of the width  $W_t$  of the capillary pressure  
191 distribution (Auradou et al., 1999; Méheust et al., 2002). Experiments to check the dependence

192 of the displacement process on the capillary noise ( $W_i$ ) are difficult, because controlling the  
193 distribution of threshold capillary pressures in the medium in a systematic way is not  
194 straightforward . These experiments therefore remain to be done.

195

### 196 **3 Capillary and viscous effects**

197

198 The crossover between capillary fingering and regimes for which viscous effects are  
199 dominant was first studied in the pioneering work of Lenormand (1988). He classified the  
200 different flow structures in a phase diagram depending on the viscosity contrast  $M = \mu_i/\mu_d$   
201 between the fluids and the capillary number  $Ca = \Delta P_{\text{visc}}/\Delta P_{\text{cap}}$ , which is the ratio of the  
202 characteristic viscous pressure drop at the pore scale to the capillary pressure drop. Here  $\mu_i$  and  
203  $\mu_d$  are the viscosities of the injected and displaced fluid, respectively. From the Darcy equation,  
204 the capillary number can be evaluated as

$$205 \quad Ca = \frac{\Delta P_{\text{visc}}}{\Delta P_{\text{cap}}} = \frac{a \nabla P_{\text{visc}}}{\gamma/a} = \frac{\mu a^2 v}{\kappa \gamma}. \quad (2)$$

206 where  $\gamma$  is the surface tension,  $a$  the characteristic pore size,  $\Delta P_{\text{visc}}$  is the viscous pressure drop  
207 at pore scale  $a$ , evaluated from the viscous pressure gradient  $\nabla P_{\text{visc}}$ ,  $\kappa$  the intrinsic permeability  
208 of the medium,  $v$  the seepage velocity associated to the imposed flow rate of the displaced fluid,  
209  $\mu$  the viscosity of the most viscous fluid, and  $\gamma/a$  the typical capillary pressure drop across the  
210 interface. Lenormand identified three flow regimes: (i) stable displacement, for which the  
211 interface roughness is not larger than one linear pore size, (ii) capillary fingering, which we have  
212 discussed in section 2, and (iii) viscous fingering, which occurs when large scale fingers of the

213 displacing fluid develop inside a more viscous defending fluid, resulting in a much faster  
 214 breakthrough of the displacing fluid. It is important to keep in mind that the observed structures  
 215 will depend on the length scale considered. For large systems it is therefore not meaningful to  
 216 talk about a sharp transition in a phase diagram between capillary and viscous fingering, because  
 217 one will always have both structures present, i.e. capillary fingering on small length scales, and  
 218 either viscous fingering or stable displacement on large length scales. When the two fluids  
 219 involved have different viscosities, the viscous pressure drop between two points along the fluid  
 220 interface will typically be different in the two fluids. This viscosity contrast will produce a  
 221 change in the capillary pressure along the fluid interface, therefore playing a role similar to that  
 222 of density contrasts in the presence of a gravitational field (see section 2). At sufficiently large  
 223 length scales, the difference in viscous pressure drop between the two sides of the interface will  
 224 become larger than the typical fluctuations in capillary pressure threshold. This means that at  
 225 sufficiently large length scales, and thus for a sufficient large system, viscous pressure drops,  
 226 rather than capillary forces associated to random capillary thresholds, determine the most likely  
 227 invaded pores; consequently, viscous fingering will always dominate at sufficiently large scales  
 228 when a viscous fluid is injected into another more viscous fluid. At these large scales, and in the  
 229 absence of a stabilizing gravitational effects, two-dimensional flows exhibit tree-like branched  
 230 displacement structures with a mass fractal dimension  $D_v = 1.6$  (Måløy et al., 1985). The fractal  
 231 dimension of the front, or growing hull, was found experimentally to be around 1, close to the  
 232 growing interface dimension in DLA models (Feder; 1988).

233 Méheust et al. (2002) have introduced a generalized fluctuation number

$$234 \quad F = \frac{\Delta\rho g a - \frac{a\mu v}{\kappa}}{W_t} \quad (3)$$

235 which is the ratio of the typical total pressure drop in the fluids over one pore, including both  
236 viscous and gravitational pressure drops, to the width of the capillary pressure threshold  
237 distribution  $W_t$ . The experiments by Méheust et al. (2002), identical to those by Birovljev et al.  
238 (1991) but performed at larger flow rates and therefore under significant viscous effects, showed  
239 that the characteristic width of the rough interface parallel to the macroscopic flow could be  
240 characterized with the fluctuation number according to an equation analog to Eq.(1):

$$241 \quad \xi/a \propto F^{\frac{-\nu}{\nu+1}} \quad (4)$$

242 where the exponent  $\nu/(1+\nu) = 4/7$  is consistent with percolation theory ( $\nu = 4/3$ ) in 2D. Since  
243 the viscous pressure field is not homogeneous like the gravitational field, this result is not  
244 obvious. Note that in this case  $\xi$  can be interpreted as the length scale at which the sum of the  
245 viscous and gravitational pressure drop becomes of the same order of magnitude as the spatial  
246 fluctuations of the capillary pressure threshold. In terms of fluid-fluid interface,  $\xi$  corresponds  
247 to the crossover scale between capillary fingering structures at small scale and the stabilized  
248 structure, which is linear (dimension 1) at large scales. When the displacement is large enough  
249 for viscous forces to play a role, the fractal dimension typical of viscous fingering structures is  
250 also seen at intermediate scales (Méheust et al., 2002). Even in the case where the two fluids  
251 involved have the same viscosity, the width of the front was found to be consistent with Eq.(4)  
252 (Frette et al.,1997). As observed in the experiments of Frette et al. (1997), the effect of trapping  
253 turns out, at least in two dimensions, to be of central importance. The trapped islands result in a  
254 decrease in the relative permeability of the invaded fluid, which is equivalent to having a fluid  
255 with a higher viscosity (as was shown by Frette et al. (1997) by the comparison to simulations  
256 allowing trapping or not, with growth along the whole external perimeter of the invader or  
257 restricted to the hull). This is the effect responsible for the well-known decreasing dependence of

258 a soil's matric potential on its water content. This result is consistent with the scaling relation  
259 Eq.(4), which is expected from theoretical arguments for percolation in a stabilizing gradient (Xu  
260 et al., 1998; Wilkinson, 1984; Lenormand, 1989). Note that other scaling relations have been  
261 derived theoretically and observed experimentally by other authors, as reported by Wu et al.  
262 (1998). As mentioned previously, when a viscous fluid is injected into a 2D medium filled with a  
263 more viscous fluid, viscous fingers occur. The scaling of the finger width was studied  
264 experimentally by Løvoll et al. (2004) and Toussaint et al. (2005). The measurements were  
265 found to be consistent with a scaling law in the form

$$266 \quad \xi/a \propto Ca^{-1} \quad (5)$$

267 This result is different from the scaling laws that can be explained from the theory of percolation  
268 in a gradient, as observed from stabilizing viscous or gravitational forces. In the experiments of  
269 Løvoll et al. (2004) and Toussaint et al. (2005), contrarily to what happens in a pressure field  
270 arising from gravitational effects (as in Méheust et al. (2002)), the viscous pressure field is  
271 highly inhomogeneous, constant in front of the fingers, and screened by the fingers behind the  
272 invasion front (i.e. the pressure gradient concentrates around the finger tips and decays behind  
273 the finger tips, in the stagnant zones). This may explain why the behaviour expected from  
274 percolation in a gradient is not observed, but a rather simpler one instead. Note however that  
275 scaling laws based on the theory of percolation in a gradient are still expected for some types of  
276 unstable flows (Xu et al. 1998).

277 The scaling law, Eq. (5), can be explained from a simple mean field argument: consider  
278 an approximation for the pressure field for which the pressure gradient  $\nabla P$  is homogeneous  
279 around the mobile sites at the boundary between the two fluids. Let us consider two of these  
280 sites, separated by a distance  $l$  along the direction of  $\nabla P$ ; the difference between the drops in

281 viscous pressure across the interface at the two sites is  $l\nabla P \approx lCa\gamma/a^2$ . This relation holds not  
 282 only when the viscosity of one fluid can be neglected with respect to that of the other one (for  
 283 example, for air and water), in which case a non-zero viscous pressure difference between the  
 284 two sites occurs in the defending liquid only, and the definition of the capillary number is given  
 285 by Eq. (2). It also holds in the general case of two viscous fluids, in which case the capillary  
 286 number can be defined from Eq. (2) by replacing the viscous pressure difference by the  
 287 difference in the viscous pressure drop across the interface at the two sites, or equivalently by the  
 288 differential viscous pressure drop  $|\Delta P_v^{(i)} - \Delta P_v^{(d)}|$ , where the  $|\Delta P_v|$  are viscous pressures difference  
 289 between the two sites in each of the phases, the superscript <sup>(i)</sup> and <sup>(d)</sup> denoting respectively the  
 290 invading- and defending- phase. If the differential viscous pressure difference between these two  
 291 sites exceeds the characteristic random fluctuations of the capillary pressure threshold from one  
 292 pore throat to another along the interface, then the viscous pressure field is dominant in  
 293 determining at which of the two points considered new pores are going to be invaded. On the  
 294 contrary, if the random fluctuations of capillary threshold exceed the differential viscous  
 295 pressure drop between the two points, then this random pressure difference component  
 296 dominates. Assuming that its magnitude  $W_l$  is of the same order as the average capillary pressure  
 297 value,  $\gamma/a$  we conclude that capillary effects are expected to dominate for scales  $l$  such that  
 298  $lCa\gamma/a^2 < \gamma/a$ , whereas viscous effects will dominate for larger scales, such that  $lCa\gamma/a^2 > \gamma/a$  :  
 299 this explains the observed cross over scale  $\xi = a/Ca$  between the structures characteristic of  
 300 capillary fingering and those characteristic of viscous fingering. Using a pore-scale simulation  
 301 where they can tune capillary noise, Holtzman and Juanes (2010) determined a phase diagram of  
 302 the displacement regime (capillary or viscous fingering) as a function of capillary noise and  
 303 capillary number, and observed the same crossover, which they explained in a similar manner,

304 only expressing the prefactor for the scaling law of the crossover scale in terms of typical  
305 capillary pressure threshold fluctuations rather than in terms of the mean capillary pressure  
306 threshold (that is, they did not assume that  $W_t \approx \gamma/a$ ).

307

## 308 **4 An example of upscaling from capillary to viscous fingering**

309

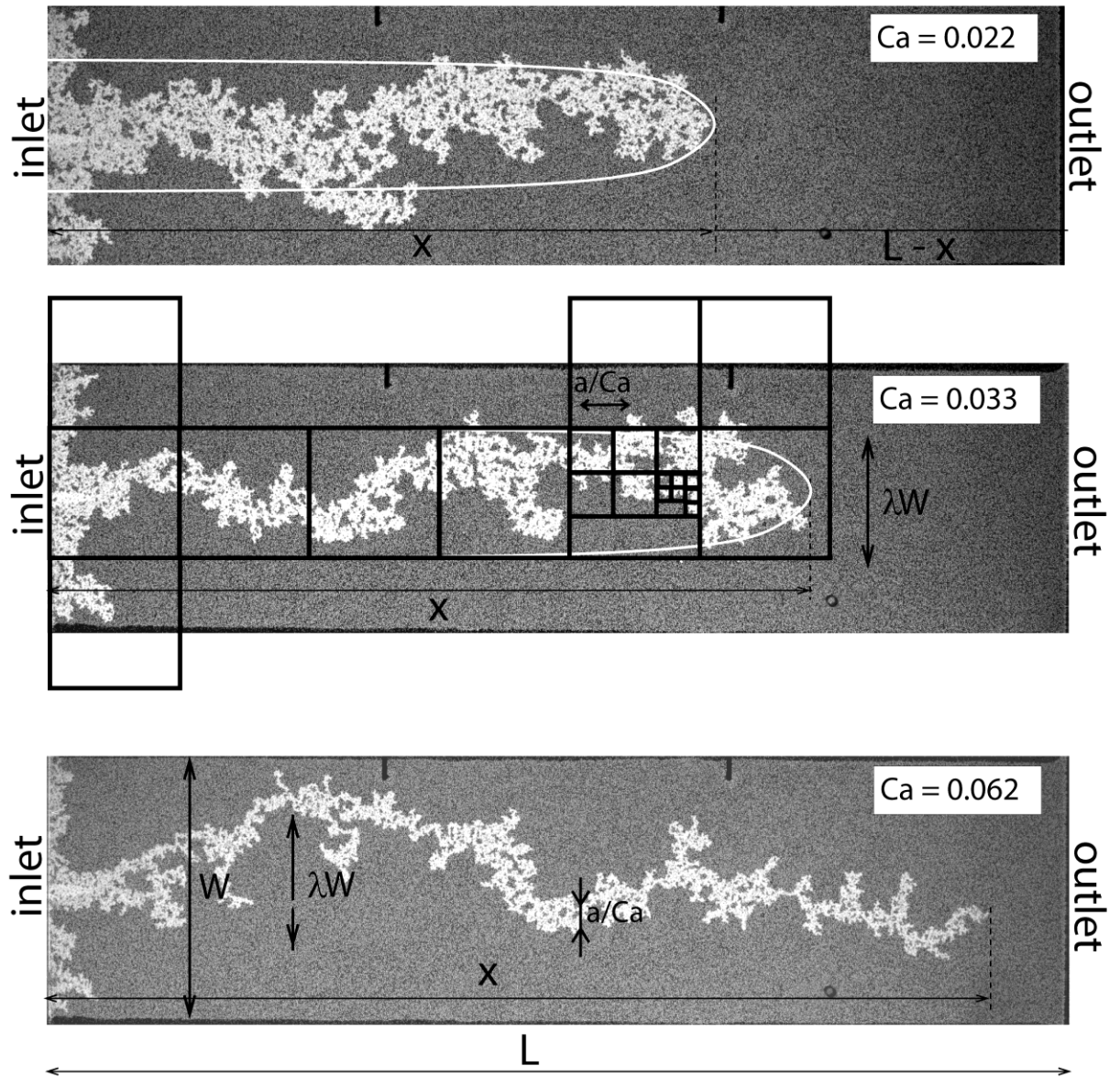
310 In many situations of non-miscible biphasic flow in porous media, the flow gets  
311 organized in fingering structures (preferential paths); in these unstable configurations, the fluids  
312 arrange in fractal geometries with nontrivial fractal dimensions depending on the observation  
313 scale, and the scale range over which each dimension is observed depends on the imposed  
314 boundary conditions (such as the macroscopic fluxes). This microscopic structuration has far  
315 reaching consequences for the upscaled relationships between, for example, saturation and the  
316 pressure difference between the two phases.

317 An example of such a situation has been mentioned briefly earlier in this review: if  
318 gravity is negligible, only capillarity and viscosity play a role on the flow; when a fluid of lower  
319 viscosity displaces a more viscous one, the fluid-fluid interface is unstable due to viscous  
320 pressure gradients increasing at the most advanced parts of the invader, so that fingers naturally  
321 arise. The development of such an interface instability occurring during drainage was studied  
322 optically in Hele Shaw cells, at several controlled injection rates (Løvoll et al.2004; Toussaint et  
323 al.2005). Air was injected into an artificial porous medium composed of a monolayer of immobile  
324 glass beads sandwiched between two glass plates, and initially filled with a wetting dyed  
325 glycerol-water solution. The Hele-Shaw cell dimensions were denoted  $L \times W \times H$ ,  $H = 1$  mm  
326 being the cell thickness as well as the typical glass bead diameter; flow was imposed along the

327 length  $L$ , with impermeable lateral boundaries defining a channel of width  $W$  (see Fig. 1). An  
328 occupancy (also termed occupation probability) was defined in the reference frame moving at the  
329 average finger speed between the system boundary; in that referential, the invasion structure is  
330 seen as a finger fluctuating during the experiment behind its stationary tip: for each point in this  
331 reference frame, the proportion of the time where this point is occupied by the invading fluid is a  
332 measure of the occupancy. It was shown in these experiments that the pathway of the air, defined  
333 as the locations where the occupancy probability exceeds half its maximum value, was a finger  
334 of width  $\lambda W$  positioned in the centre of the channel, with  $\lambda = 0.4$ . This was attributed to a  
335 similarity between the process of selection of the pore throats to be invaded and a Dielectric  
336 Breakdown Model of exponent 2 (Niemeyer et al., 1984), that is, a growth process in which the  
337 growth velocity is proportional to the gradient of the driving effect to the power of 2. The  
338 presence of a disorder in capillary threshold turns out to be important to enforce a boundary  
339 condition analogous to a growth probability along the invader proportional to  $[(P - P_{air})/a]^2$ ,  
340 with a field  $P$  satisfying Laplace equation due to mass conservation: This result was justified  
341 theoretically by computing the average invasion speed taking into account the width of the  
342 capillary threshold distribution (Toussaint et al., 2005). This type of invasion structure is  
343 illustrated in Fig. 1.

344





345

346 **Figure 1:** Invasion structure of a fluid with a low viscosity (white) into a much more  
 347 viscous one (dark grey) during drainage in an artificial 2D porous medium of width  $W$  and  
 348 extent  $L$ , at three different extraction speeds. The position of the invasion tip is denoted  $x$ .  
 349 Characteristic crossover scales between fractal regimes,  $\lambda W$  and  $a/Ca$ , separate a straight  
 350 finger structure, a viscous fingering geometry, and a capillary fingering geometry, down to the  
 351 pore scale  $a$ . The black square of various dimensions in the central figure illustrate the types of  
 352 boxes used in the box counting measure of the fractal dimension: for boxes of a certain side size  
 353  $l$ , one counts the number of boxes  $N(l)$  needed to cover the structure. This is done for various  
 354 sizes, from system size down to pixel size. The scaling of this number as function of the size,  
 355  $N(l) \sim l^{-D}$  defines the mass fractal dimension  $D$ . The sizes  $W$  and  $a/Ca$  turn out to be the limits of  
 356 scale-ranges with well defined fractal dimensions:  $D = 1.00$  above  $W$ ,  $D_v = 1.60$  between  $W$   
 357 and  $a/Ca$ , and  $D_c = 1.83$  below. Modified from Løvvoll et al. (2011).

358 From approximations on the shape of the pressure around this finger, mostly controlled  
 359 by the viscous pressure drop, one can derive an upscaled pressure-saturation relation (Løvøll et  
 360 al.2010).

361 Indeed, the pressure presents to first order a linear viscous pressure drop from the tip of  
 362 the invasion cluster, at position  $x$ , to the outlet of the system, at position  $L$ . Over the rest of the  
 363 system, the pressure gradient is screened by the finger, rendering the pressure in the wetting fluid  
 364 essentially constant at a value close to the sum of the air pressure and the entrance pressure  $\gamma/a$ .  
 365 Hence, the pressure difference between the two phases, with a pressure in the wetting fluid  $P_w$   
 366 measured at the outlet, the one in the non wetting phase equal to the atmospheric pressure  $P_{n.w.}$ ,  
 367 and a correction to this entrance pressure, writes as

$$368 \quad \Delta P^* = P_{n.w.} - P_w - \gamma/a = (L-x)\nabla P \quad (6)$$

$$369 \quad = (L-x)\Delta P_{\text{visc}}/a \quad (7)$$

$$370 \quad = (L-x)\Delta P_{\text{cap}} Ca/a \quad (8)$$

$$371 \quad = (L-x) \frac{\gamma Ca}{a^2} \quad (9)$$

372 where  $\Delta P_{\text{visc}}$  and  $\Delta P_{\text{cap}}$  are considered at the pore scale. Thus, there is a linear relationship  
 373 between the viscous pressure drop across the cell and the distance between the finger tip and the  
 374 outlet.

375 In addition, the relationship between saturation and capillary number can be inferred  
 376 from the fractal structure of the non-wetting invading fluid. At scales above the width  $\lambda W$ , the  
 377 finger is a linear structure of dimension 1. Between the scale  $\lambda W$  and  $a/Ca$ , the structure has a  
 378 viscous fingering geometry of fractal dimension  $D_v = 1.60$  (Måløy et al.,1985; Løvøll et  
 379 al.,2004). Between the crossover scale  $a/Ca$  and the pore scale  $a$ , the structure has a capillary

380 fingering geometry of fractal dimension  $D_c = 1.83$ . Hence, the total number of pores invaded by  
 381 the non viscous fluid (n.v.f.) can be evaluated as a function of these fractal dimensions, the ratio  
 382 of the finger length to its width,  $x/(\lambda W)$ , and the ratios of the latter length to the two others  
 383 lengths, the crossover length and the pore size. This leads to:

$$384 \quad N_{n.w.} = \frac{x}{\lambda W} \left( \frac{\lambda W}{a \text{Ca}^{-1}} \right)^{D_v} \left( \frac{a \text{Ca}^{-1}}{a} \right)^{D_c} \quad (10)$$

385  
 386 Together with the relationship between the total number of pores and the characteristic  
 387 model dimensions,

$$388 \quad N_{tot} = \frac{LW}{a^2} \quad (11)$$

389 and the relationship between the wetting phase and non-wetting phase saturation  $S_{n.w.}$ ,

$$390 \quad S_{n.w.} = 1 - S_w = \frac{N_{n.w.}}{N_{tot}}, \quad (12)$$

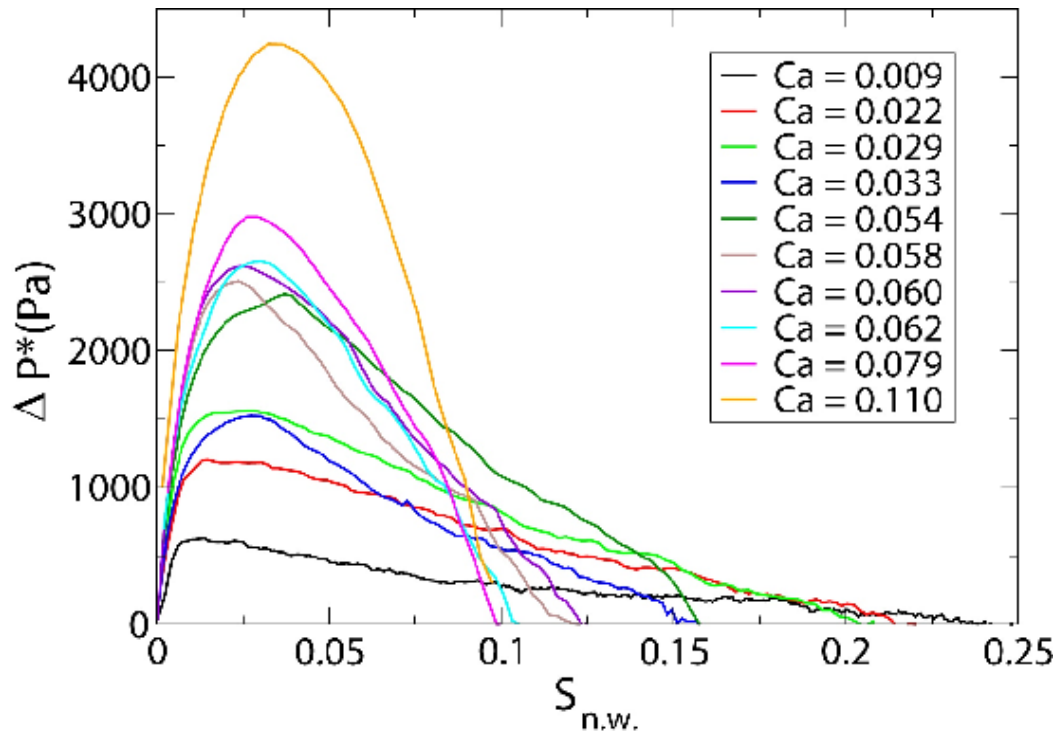
391 Eq. (10) leads to

$$392 \quad 1 - S_w = S_{n.w.} = \lambda^{D_v - 1} \left( \frac{a}{W} \right)^{2 - D_v} \text{Ca}^{D_v - D_c} \left( 1 - \frac{a^2 \Delta P^*}{\gamma L \text{Ca}} \right) \quad (13)$$

394  
 395 This relationship allows to collapse all the pressure difference curves measured as a  
 396 function of saturation in the set of experiments performed by Løvoll et al. (2010) onto a unique  
 397 master curve for capillary numbers ranging from around 0.008 to 0.12. Note that the rescaled  
 398 pressure  $P^* = a^2 \Delta P^* / (\gamma L \text{Ca}) = (\Delta P^* / M) / (\Delta P_{visc} / a)$  is simply the ratio of the gradient in viscous  
 399 pressure defined at the scale of the model to the gradient in viscous pressure defined at the pore

400 scale. Apart from the normalization by  $\Delta P_{visc} / a$ , it is nothing else than what is usually defined  
 401 at the scale  $L$  of the experimental model as the capillary pressure. In other words, Eq. (13)  
 402 defines the dependence on Darcy/seepage velocity of what is commonly denoted as dynamic  
 403 capillary pressure, measured at scale  $L$ . This example shows how both viscous and capillary  
 404 effects play a role in constraining the geometry of the invasion structures, resulting in a dynamic  
 405 capillary pressure, as it is traditionally called (Hassanizadeh et al., 2002), that is simply due to  
 406 the upscaling of the invasion structure, with only capillary and viscous effects seen at the REV  
 407 scale, and without any dynamic capillary/wetting effects occurring at the pore scale.

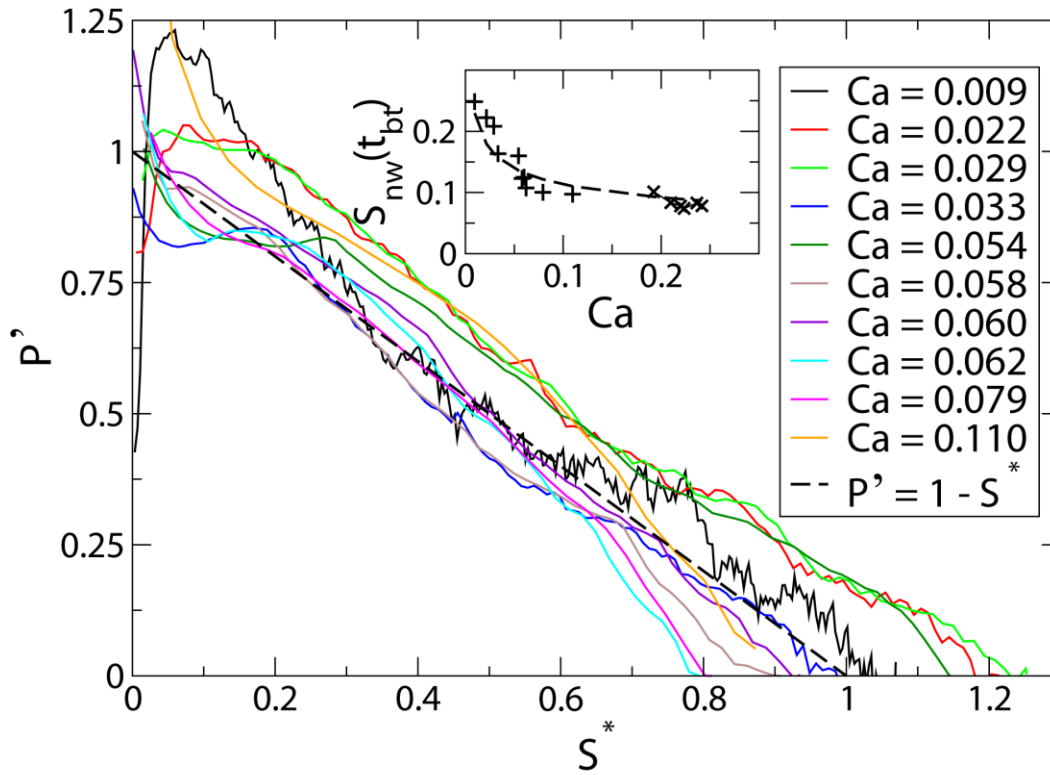
408 Figures 2 and 3 illustrate respectively the raw measurements at several injection speeds  
 409 and how Eq. (13) allows to collapse these curves of saturation versus pressure: with a reduced  
 410 saturation  $S^* = \lambda^{1-D_v} (a/W)^{-2+D_v} Ca^{D_c-D_v} S_{n,w.}$  and the rescaled pressure  
 411  $P' = a^2 \Delta P^* / (\gamma L Ca) = (\Delta P^* / M) / (\Delta P_{visc} / a)$ , Eq. (13) predicts that  $P' = 1 - S^*$ , which is the  
 412 theoretical straight line in Fig. 3. This is well followed by the experimental data collapse.



413

414 **Figure 2:** Dependence of the pressure difference between the two phases and the  
 415 saturation of the invading fluid after removal of the average capillary pressure drop,  
 416  $\Delta P^* = P_{n.w.} - P_w - \gamma/a$ , at different injection speeds. Adapted from Løvoll et al. (2011).

417



418

419 **Figure 3:** The collapse of the relationship between the reduced pressure difference  
 420 (between the two phases),  $P' = a^2 \Delta P^* / (\gamma L Ca)$ , and the reduced saturation of the invading fluid,  
 421  $S^* = \lambda^{1-D_v} (a/W)^{-2+D_v} Ca^{D_c - D_v} S_{n.w.}$ , at different injection speeds, shows the influence of the  
 422 structure on the upscaling. Note that plots corresponding to lower Ca values have been rescaled  
 423 more and therefore appear more noisy. Dashed curve: prediction. Inset: Residual saturation at  
 424 breakthrough. Adapted from Løvoll et al. (2011).

425

426 The viscous pressure drop across the cell drops linearly as the finger progresses into the  
 427 cell, from a maximum value at the beginning of the invasion of  $\gamma L Ca / a^2$ , down to 0 at  
 428 breakthrough of the invasion finger. In the previous equation, the wetting saturation is indeed  
 429 initially 1 as it should be at initial total saturation, but we also obtain the final and maximum

430 value of the residual wetting saturation as

431 
$$1 - S_{w.r.} = S_{n.w.r.} = \lambda^{D_v - 1} \left( \frac{a}{W} \right)^{2 - D_v} \text{Ca}^{D_v - D_c}. \quad (14)$$

432 This relation between the residual saturation and the capillary number is indeed consistent with  
433 the observed residual saturations, as shown in the inset of Fig.3.

434 Besides the relation between saturation and the macroscopic pressure difference between  
435 the phases, other macroscopic relations can be obtained via upscaling, as e.g., in some situations,  
436 the relative permeability. For example, in other experiments where both fluids were injected at  
437 the same time, with both drainage and imbibition happening in the flow simultaneously, the  
438 trapped structures of wetting fluids were observed to be fractal up to a certain cutoff depending  
439 on the imposed flux (Tallakstad et al., 2009a; 2009b). A scaling law was observed for the  
440 relative permeability of wetting viscous fluid in these experiments, with an observed dependence  
441 on the imposed flux that can be expressed as  $\kappa_{rel} \propto \text{Ca}^{-1/2}$ . The upscaling explaining the cutoff  
442 and the structures allowed to explain this measured scaling law.

443 For the pure imbibition case, the macroscopic capillary pressure also presents a dynamic  
444 dependence: it was found by Stokes et al. (1986) and Weitz et al. (1987) that the fingering also  
445 occurs when the lower viscous fluid displaces the more viscous one, and that the finger width  
446 scales as  $\text{Ca}^{-1/2}$ , a result that is still largely unexplained.

447

## 448 **5 Conclusion**

449

450 We have discussed the local flow structures that are observed experimentally during  
451 drainage in a disordered porous medium. When the viscosity contrast between the two fluids is

452 high (as for air displacing water), the flow structures are fractal, with a fractal dimension that  
453 depends on the observation scale. At small scales, capillary fingering exhibits a fractal dimension  
454 of 1.8 for two-dimensional media, and between 2 and 2.6 for three-dimensional media. At larger  
455 scales a branched structure characteristic of viscous fingering is seen, with a fractal dimension  
456 1.6 for two-dimensional systems. The crossover between the two behaviors occurs at a length  
457 scale for which the differential viscous pressure drop equals the typical capillary pressure  
458 threshold in the medium. This means that for horizontal flow, unstable viscous fingering is  
459 always seen at large enough scales, even if the medium exhibits no permeability heterogeneities  
460 at the Darcy scale. From the definition of the crossover length, it follows that it scales as the  
461 inverse of the capillary number, which explains why experiments performed at a given  
462 experimental scale and at very slow flow rates have evidenced capillary fingering, while those  
463 performed at very large flow rates have evidenced viscous fingering. As for the effect of gravity,  
464 it can be to either destabilize or stabilize the interface, depending on which fluid is the densest.  
465 In the latter case, it acts against capillary effects and, when the displacing fluid is the most  
466 viscous, against the destabilizing viscous forces, resulting in an amplitude of the interface  
467 roughness that scales as a power law of the generalized fluctuation number (or generalized Bond  
468 number  $Bo - Ca$ , as introduced by Méheust et al. (2002)). In horizontal two-dimensional flows,  
469 viscous fingering is observed to occur up to another characteristic length that is a fixed fraction  
470 of the width of the medium. Upscaling of the local flow structures is possible once one knows  
471 the fractal dimensions typical of the flow regimes, and the relevant length scale range for each of  
472 them. We have given an example of how the measured capillary pressure can be related  
473 theoretically to water saturation, a relation that is confirmed by measurements. In that example,  
474 the capillary pressure measured at the scale of the experimental setup exhibits dynamic features,



475 i. e., a dependence on the flow rate, that is fully explained by the geometry of the upscaling,  
476 without any dynamic effects in the physical capillary pressure as defined at the pore/interface  
477 scale.

478

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480

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483

484

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670  
671 Figure Captions:

672 **Figure 1:** Invasion structure of a fluid with a low viscosity (white) into a much more  
673 viscous one (dark grey) during drainage in an artificial 2D porous medium of width  $W$  and  
674 extent  $L$ , at three different extraction speeds. The position of the invasion tip is denoted  $x$ .  
675 Characteristic crossover scales between fractal regimes,  $\lambda W$  and  $a/Ca$ , separate a straight  
676 finger structure, a viscous fingering geometry, and a capillary fingering geometry, down to the  
677 pore scale  $a$ . The black square of various dimensions in the central figure illustrate the types of  
678 boxes used in the box counting measure of the fractal dimension: for boxes of a certain side size  
679  $l$ , one counts the number of boxes  $N(l)$  needed to cover the structure. This is done for various  
680 sizes, from system size down to pixel size. The scaling of this number as function of the size,  
681  $N(l) \sim l^{-D}$  defines the mass fractal dimension  $D$ . The sizes  $W$  and  $a/Ca$  turn out to be the limits of  
682 scale-ranges with well defined fractal dimensions:  $D = 1.00$  above  $W$ ,  $D_v = 1.60$  between  $W$   
683 and  $a/Ca$ , and  $D_c = 1.83$  below. Modified from Løvoll et al. (2011).  
684

685 **Figure 2:** Dependence of the pressure difference between the two phases and the  
686 saturation of the invading fluid after removal of the average capillary pressure drop,  
687  $\Delta P^* = P_{n.w.} - P_w - \gamma/a$ , at different injection speeds. Adapted from Løvoll et al. (2011).  
688

689 **Figure 3:** The collapse of the relationship between the reduced pressure difference

690 (between the two phases),  $P' = a^2 \Delta P^* / (\gamma LCa)$ , and the reduced saturation of the invading fluid,  
691  $S^* = \lambda^{1-D_v} (a/W)^{-2+D_v} Ca^{D_c - D_v} S_{n.w.}$ , at different injection speeds, shows the influence of the  
692 structure on the upscaling. Note that plots corresponding to lower Ca values have been rescaled  
693 more and therefore appear more noisy. Dashed curve: prediction. Inset: Residual saturation at  
694 breakthrough. Adapted from Løvoll et al. (2011).  
695