Bidding among Friends and Enemies with Symmetric Information

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November 4, 2009

Abstract

We consider an auction setting, in a symmetric information framework, in which bidders, even if they fail to obtain the good, care about the price paid by the winner. We prove that the outcome of the first-price auction is not affected by identity independent price externalities while the outcome of the second-price auction is. In contrast, identity dependent price externalities affect the outcome of both auction formats. In any case, the second-price auction exacerbates the effects of price externalities.

JEL Classification: D44, D62, G32.
Keywords: auctions, symmetric information, externalities, toeholds, budget-constraints.

1 *Introduction*

In December 2004, the *Ligue Nationale de Football (LNF)* auctioned the retransmission rights of the French Soccer Championship for the next three years. The two major bidders were Canal+, the leader of the French pay-TV market and TPS, its challenger. At that time, TPS shareholders were thinking about selling TPS to Canal+. In order to raise the price that Canal+ would accept to pay for TPS, its managers wanted to show that the firm's independence was costly for Canal+. A way to do so was to be aggressive

*I would like to thank Marco Battaglini, Anne Duchêne, Françoise Forges, Tanjim Hossain, Eric Maskin, Moritz Meyer-ter-Vehn, Wolfgang Pesendorfer, Jérôme Pouyet, Ran Spiegler, Felix Vardy and the participants of the IAS, ETAPE, CORE and THEMA seminars and the EDP, SMYE, APET, EEA, ESEM and AMD conference for helpful comments and supports. Special thanks to Philippe Jehiel. All errors are mine.

during the auction to force Canal+ to pay a high price for the retransmission rights. Hence, conditional on losing this auction, TPS preferred Canal+ to pay a high price. Eventually Canal+ won the auction for a price of 600 millions per year (60% more than the precedent contract) and two years later, Canal+ and TPS announced their merging decision. The specific motivations of TPS during this auction is likely to have influenced his bidding behavior and indirectly Canal+'s. Was the high price reached partially due to TPS special motivations ? Was the choice of the first-price auction by the LNF optimal considering the specificity of the situation?

The standard auction theory analysis framework does not allow to answer these questions. As a matter of fact, there is a specific element in this setting. A bidder, here TPS, cared about the price paid by the winner even if he lost the auction. However, this interest in the price paid by another bidder may be a key element in many auction settings. Let us support this assertion with the following examples.

Cofiroute is a toll motorway firm. It has two major shareholders Vinci (82.3%) and Colas (16.7%), leading actors of the public works sector. Vinci and Colas participate in the tenders organized for the maintenance of its network or the creation of new roads. Both firms are interested in obtaining these markets for a high price. However, in their capacity of shareholders, they also prefer the price paid by Cofiroute for these works to be low. This motivation is even more important for Vinci with more than 80% of the capital.

In the summer of 1999, young Nicolas Anelka was a soccer rising star. He scored 17 goals during his season in Arsenal. SS Lazio, the second of the Italian championship made an offer to buy Anelka's contract. Juventus Torino, another Italian club, made counteroffers so that SS Lazio reacted with a 30 millions offer. Weeks later, a manager of Juventus Torino revealed that the club was not really interested in Anelka. Juventus' managers made a counteroffer in order to raise the price paid by SS Lazio. Considering that soccer clubs are budget-constrained, Juventus' strategy was indisputably rational. In any budget-constrained environment, firms competing in auction, conditional on losing the auction, prefer the winner to pay a high price.

In fact, empirical auctions specialists already noticed long ago that bidders may care about the price even they lose the auction. Cassady [1967, p170 and p54] made the following remarks:

> "It is well known that buyers at auction attempt to make competitors pay higher prices for good purchased for resale by bidding up a desired item."

> "Some dealers-buyers want to force competitors to buy at a high

> price at auction, and thus bid up an item or lot considerably above the underbidder's highest demand price. This tactic is found in antique auctions, but it is particularly relevant in sale for commodities such as fish, for a dealer who has just purchased goods at a particular price cannot afford to allow competitors to acquire supplies at a lower price and undersell him in the secondary market"

In all these cases, bidders, even if they fail to win the auction, care about the price paid by the winner. We call this concern of losing bidders about the price *a price externality* (PE). These examples highlight few different features of PE which will play a key role in the analysis.

First of all, there is a fundamental difference between the Anelka case and the Cofiroute case. In the Anelka case, Juventus' managers cared about the price paid, conditional on losing the auction only if SS Lazio won the auction. The price externality depends on the identity of the winner. In the Cofiroute case, both Vinci and Colas pay indirectly a fraction of the final price of the tender independently from the identity of the winner. The price reached by the tender affects through this channel these bidder's utility whoever the winner is, including themselves. Therefore, we define two categories of price externalities. When the identity of the buyer matters, it is an identity dependent price externalities (IDPE). When the identity of the winner does not matter, it is an identity independent price externalities (IIPE). Furthermore, the examples also show that PE may be decreasing or increasing functions of the price.

This paper examines how both types of price externalities affect the first-price and the second-price auction,[1] focusing on the two-buyers case[2]. We also assume that bidders have symmetric information about their preferences which allows to consider situations which could not be analyzed in an asymmetric information framework such as the Cofiroute case where shareholdings of Colas and Vinci are not identical.

We show that IIPE do not have any effect on the equilibrium of the first-price auction, while they generically have an effect in a second-price auction. As a matter of fact, IIPE, by definition, do not depend on the identity of the winner so that they do not affect the price for which bidders are indifferent between winning and losing. In a first-price auction, it turns out that equilibrium bids depend only on these indifference prices. Thus,

[1]Here, the descending and the ascending auction are equivalent to respectively the first-price auction and the second-price auction.

[2]With more bidders, we would have to distinguish between the effects of price externalities and the effects of allocative externalities.

IIPE do not affect the equilibrium of the first-price auction. On the other hand, in a second-price auction, a losing bidder may fix the price through his bid. If he strictly prefers the price to be the highest (resp: the lowest) possible, he will raise (resp: lower) his bid. As a result, in a second-price auction, IIPE may affect the equilibrium and the two auction formats are not equivalent.

With IDPE, things are slightly different. They affect the price for which bidders are indifferent between losing and winning the auction. As mentioned before, this indifference price is the only element that matters in a first-price auction. Therefore, IDPE affect the equilibrium of the first-price auction. However, even when there are only IDPE, the two auction formats are not equivalent either. The second-price auction is more *sensitive* to IDPE than the first-price auction. Again, this is due to the very structure of the second-price auction in which the loser, through his bid, determines the price paid by the winner. Hence, the amplification of the effects of price externalities with a second-price auction remains true with any type of price externalities.

Although there is no systematic study of auctions with price externalities, many specific cases of auctions with price externalities have been examined. In a symmetric information framework, Pitchik and Schotter [1988] study sequential auctions with budget-constrained bidders. They observe that the standard auction formats are not revenue equivalent. Benoit and Krishna [2001] also analyze sequential auctions with budget-constrained bidders but with a different perspective. Their paper emphasizes matters such as the best sequencing to sell goods while we are more focused on the situation in which the first seller has a unique good to sell. We take the environment as given and recommend an adequate format to sell his good.

In an asymmetric information framework, there is a much developed literature on shareholdings and crossholdings. Bulow et al [1999] consider a setting in which bidders own a fraction of the good for sale. They assume that the value of the good is common and derive that small asymmetries among bidders -in terms of fraction of the good they own- may have dramatic effects. Their point is more related to the impact of asymmetries in a common value environment than specifically to price externalities. Hansen and Lott [1996] analyze the incentives of portfolio managers who owns shares in several competing or complementary firms. Burkart [1995], Singh [1998], Engelbrecht-Wiggans [1994], Maasland and Onderstal [2007] and Ettinger [2003] and Ettinger [2009] study the impact of some types of toeholds in a private value framework.

Our framework allows to consider any type of toehold distribution in a symmetric information which is not the case in this literature. Besides, we show that, in a perfect information framework, vertical toeholds (when bidders own a fraction of the seller) affect equilibrium bidding in a second-price auction and do not in a first-price auction. We also show that both types of auction formats are affected by the presence of horizontal toeholds (when a bidder owns a fraction of another bidder's capital). Another result which is specific to our framework, the allocation may be inefficient even in a second-price auction or an ascending auction[3].

Recently, a literature emerged on charity auctions with numerous theoretical, empirical and experimental contributions (see for instance Morgan and Sefton [2000], Goeree et al [2005], Engers and McManus [2007], Landry et al [2006], Lange et al [2007]. This literature focuses on lotteries (compared to auction procedures) and non standard auction formats (mainly all-pay). These papers show that, in an asymmetric information framework, these allocation mechanisms tend to perform better than standard auction formats. However, it is still interesting to study and compare standard auction formats since, in practice, auctioneers and bidders are used to these formats and it is difficult to impose new procedures for a specific sale. We show that, contrary to what have been obtained in the existing literature, with perfect information, the first-price auction is not affected by the specific motives present in a charity sale while the second-price auction is. Besides, the second-price auction gives the highest possible revenue if we restrain our attention to allocation mechanism in which only the winning bidder contributes.

In the literature on auction with externalities initiated by Jehiel et al [1996] and Jehiel and Moldovanu [1996], the key element is not the price but rather the identity of the winner. A losing bidder's utility may be affected by the identity of the winner. A key example of such a situation is the licensing of a patent for a cost reducing technology among Cournot competitors whose costs may be differently affected by the new technology (see, for instance, on this issue Jehiel and Moldovanu [2000], Das Varma [2003], Goeree [2003] or Fan et al [2009]). The literature on auctions with externalities has proven to be extremely fruitful. However, our framework differs because of its specific motivations. The main element in our context is the money spent (and not as a signaling device as in Goeree [2003] for instance). Besides, we exhibit differences between equilibrium outcomes (allocation and price) of the first and the second-price auction that would not arise with allocative externalities without a signaling motivation (see Jehiel and Moldovanu [2000] in which the outcomes of the two auction formats are similar provided that the good is sold with probability 1).

[3] Ettinger [2003] also observes inefficiencies in an asymmetric second-price auction with three bidders, not in a symmetric ascending auction with two bidders

The remainder of the paper is organized as follows. Section 2 introduces a general model of auctions with price externalities. Section 3 contains the study identity independent price externalities and section 4 the study of identity dependent price externalities. In section 5, we present some revenue considerations. At last, section 6 concludes. We choose to present separately IIPE and IDPE to ease results' presentation. All the proofs are in the appendix.

2 The Model

One good is sold through an auction process to two bidders, 1 and 2. Bidders' preferences depend on the identity of the winner and the price paid by the winner whoever the winner is. For $i = 1, 2$, bidder i's preference is represented by a utility function U_i. $U_i(k, p)$ stands for the utility of bidder i if the good is bought for a price p by bidder k, with $k = 1, 2$.

Without loss of generality, we can normalize utility functions so that if $i \neq j$, $U_i(j, 0) = 0$. If a bidder buys the good for the price zero, the other bidder derives a utility zero. Besides, we assume that utility functions are common knowledge among bidders (which makes the framework equivalent to a symmetric information setting) and that utilities are non transferable.

For convenience, we introduce v_i, $f_i(p)$ and $g_i(p)$ defined by: $v_i = U_i(i, 0)$, $g_i(p) = U_i(i, p) - (v_i - p)$ and $f_i(p) = U_i(j, p) - g_i(p)$. Utility functions can then be written: $U_i(i, p) = v_i - p + g_i(p)$ and $U_i(j, p) = f_i(p) + g_i(p)$.

Notice that, by definition, for $i = 1, 2$, $g_i(0) = f_i(0) = 0$. We will call v_i the "zero-value" of bidder i since it is equal to the utility derived by bidder i when he obtains the good for a price equal to zero. Now, the functions $g_i(p)$ and $f_i(p)$ are to be interpreted as follows. $g_i(p)$ is the identity independent price externality (IIPE) incurred by bidder i if the good is sold for the price p, whoever the buyer is. $f_i(p)$ is the identity dependent price externality (IDPE) incurred by bidder i if the good is sold for the price p specifically to bidder j. As a matter of fact, whoever the winner is, if the price paid is p, $g_i(p)$ appears in the utility function of bidder i. Therefore, it is the utility derived by bidder i from the good being sold at a price p, the IIPE. If bidder j buys the good for the price p, the utility of bidder i is $g_i(p) + f_i(p)$. Since $g_i(p)$ is the utility derived by bidder i from the good being sold at a price p, whoever the winner is, $f_i(p)$, the remaining element, represents the utility that bidder i derives specifically from bidder j paying p, the IDPE.

Apart from the f and g, everything corresponds to the standard case, v_i playing the role of bidder i's valuation. Whatever the shapes of $U_i(i, p)$ and $U_i(j, p)$ are, we do not lose any generality by representing the different types of price externalities in an additively separable fashion. We also remark that in the case without price externalities i.e. $f_1 = f_2 = g_1 = g_2 = 0$, $U_i(i,p) = v_i - p$ and $U_i(j,p) = 0$ as in the standard case.

We consider two auction formats, the first-price auction and the second-price auction. In both auction formats, each bidder submits simultaneously a bid $b \geq 0$ and the one who submits the highest bid obtains the good. In the first-price auction, the winner pays the amount of his bid, in the second-price auction, the second highest bid which reduces here to the bid of his opponent.

Whatever the auction format is, if both bidders submit the same bid, b, the price paid is b and bidder i obtains the good if $v_i - f_i(b) > v_j - f_j(b)$. In this kind of situation, the standard hypothesis is that the limit of the discrete case is to allocate the good to bidder i if $v_i > v_j$. Here, what is important for a bidder is not his v_i but his utility difference for his obtaining the good or not $U_i(i,p) - U_i(j,p) = v_i - p - f_i(p)$. Comparing the values of this formula between the two bidders is equivalent to a comparison between $v_1 - f_1(p)$ and $v_2 - f_2(p)$. Hence, the tie-breaking rule. If $v_1 - f_1(b) = v_2 - f_2(b)$, the seller flips a fair coin to choose the winner.

We make the following assumptions. For $i = 1, 2$: A1. $U_i(i,0) > 0$, A2. $U_i(i,p)$ and $U_i(j,p)$ are differentiable in p, A3. for $p \geq 0$, $\partial_p U_i(i,p) < 0$, A4. for $p \geq 0$, $\partial_p U_i(i,p) < \partial_p U_i(j,p)$ and A5. $\nexists p$ such that $U_i(i,p) = U_i(j,p)$ and $U_j(j,p) = U_j(i,p)$.

Assumption A1 is equivalent to a strict preference for buying the good rather than leaving it to the other bidder at a price zero. Assumptions A3 and A4 suggest some limits to the extent to which bidders care about other agents' revenue. A3: Bidders have a strict preference for paying the lowest possible price, a limit to his interest in the seller's revenue. A4: For both bidders, the marginal disutility of paying ε more is always strictly higher than the marginal disutility of the other bidder's paying ε more. This is a limit now to the interest in the other bidder's revenue. A2 and A5 are technical assumptions. If A5 is satisfied, there is no unique price for which both bidders are indifferent between losing and winning the auction. In a standard framework, this would be equivalent to assuming that bidders' valuations differ. **Notation:** $v = (v_1, v_2)$, $f = (f_1, f_2)$ and $g = (g_1, g_2)$.

We only consider equilibria with pure and undominated strategies. If in any equilibrium, bidder i chooses a dominated strategy, we consider S_i^*, the set of pure strategies of bidder i that are components of Nash equilibria. Then all elements of S_i^* that are not weakly dominated in the game where players are restricted to strategies in S_i^* are considered solutions. A strategy is a bid $b \geq 0$ and an equilibrium is a couple (b_1, b_2).

Eventually, in order to ease the reading of the paper, we propose the following notation. $< i, p >$ with $i \in (1,2)$ and $p \in R^+$, is an outcome of the auction. In any specified environment (v, f, g), an outcome $< i, p >$ is an equilibrium outcome if and only if there exists an equilibrium of the auction such that the good is allocated with probability 1 to bidder i for the price p. By extension, the price p is an equilibrium price in the environment (v, f, g) if and only if, there exist an i such that $< i, p >$ is an equilibrium outcome and the allocation i is an equilibrium allocation in the environment (v, f, g) if and only if there exists a p such that $< i, p >$ is an equilibrium outcome.

3 Identity independent price externalities

We consider a setting in which there are no IDPE, $\forall x \in R^+$, $f_1(x) = f_2(x) = 0$ and observe that IIPE do not affect the outcome of the first-price auction while they generally affect the outcome of the second-price auction.

In order to simplify the presentation of our results, we assume throughout this section, without lost of generality, that $v_1 < v_2$.[4] We can also rule out the possibility of an equilibrium with both bidders obtaining the good with a probability $\frac{1}{2}$. As a matter of fact, suppose that (b, b) is an equilibrium such that both bidders obtain the good with probability $\frac{1}{2}$. If $b > v_1$, since g_1 is strictly continuous (Assumption A2), there always exists an $\varepsilon > 0$ small enough such that bidder 1 can profitably deviate submitting $b - \varepsilon$. Now if $b < v_2$, since g_2 is strictly continuous, there always exists an $\varepsilon > 0$ small enough such that bidder 2 can profitably deviate submitting $b + \varepsilon$.

3.1 The first-price auction

Proposition 1 *There is a unique equilibrium of the first-price auction: both bidders submit v_1 and bidder 2 buys the good for a price v_1.*

The equilibrium is the same as in a standard framework without price externalities. Both bidders submit a bid equal to the second highest zero-value. The bidder with the highest zero-value obtains the good for a price equal to the second highest zero-value. IIPE do not affect bidders' equilibrium strategies. To illustrate and understand this result, let us consider an example.

Example 1 . *A good is auctioned by a charitable organization to either bidder 1 or 2. The value of the good is 5 for bidder 1 and 10 for bidder 2. Both bidders derive a specific utility $\frac{t}{3}$ when the organization receives t because it finances a public good. This can be represented by the following framework: $v = (5, 10)$, $f_1(p) = f_2(p) = 0$, $g_1(p) = \frac{p}{3}$ and $g_2(p) = \frac{p}{3}$. Both*

[4] $v_1 = v_2$ is impossible since it induces $U_1(1, v_1) = U_1(2, v_1) = g_1(v_1)$ and $U_2(2, v_1) = U_2(1, v_1) = g_2(v_1)$ which is impossible because of assumption A5.

bidders, conditional on losing, prefer the price to be high since $0 < g_1', g_2'$. *In a first-price auction, at the equilibrium, bidder 2 wins the good and pays a price 5. This is equivalent to what would have happened without price externalities.*

Both bidders would like the charity organization to receive the highest possible amount of money. However each bidder always prefers a dollar in his pocket than a dollar given to the charity organization. As a result, bidders are indifferent between winning and losing the auction when the price is equal to their zero-values. It is a dominated strategy for bidder 1 to submit more than v_1. Bidder 2 knows it. As a result, he can win the auction and buy the good for a price equal to v_1, the second highest zero-value, as in the standard case.

This result is specific to the symmetric information framework. In an asymmetric framework, bidders' caring about the amount of money raised by the charity organization has a positive effect on the equilibrium price of the first-price auction (see Goeree et al [2005]). Therefore, only the conjunction of altruistic motives and asymmetric information affects equilibrium bidding.

We would observe exactly the same phenomenon if bidders preferred the price to be low. In that case, the losing bidder would like the other bidder to win for a low price, below v_1. However, if bidder 2 were to submit a bid b strictly smaller than v_1, bidder 1 would always be better off overbidding him and obtaining the good for a price slightly higher than b.

3.2 The second-price auction

In a second-price auction, with two bidders, the losing bid determines the price. As a result, contrary to what we observed in the first-price auction, IIPE may affect the losing bid and the price. Before presenting a general analysis of this issue, a simple example will illustrate that point and give some intuitions about the differences between the two auction formats.

Example 2 $v = (10, 15)$ *and for* $p \geq 0$, $f_1(p) = f_2(p) = g_2(p) = 0$. *Whatever* g_1 *is, the equilibrium in a first-price auction is* $(10, 10)$ *with bidder 2 obtaining the good. Now, for* g_1 *defined as follows:* $g_1(p) = -\frac{p^2}{2k} + p$ *with* $0 < k < 15$, *there is a unique equilibrium of the second-price auction:* $(k, 15)$.

The equilibrium of the first-price auction is not affected by IIPE. The equilibrium price of the second-price auction varies between 0 and 15 depending on the shape of IIPE. In a sense, the losing bidder chooses the price he prefers his opponent to pay in the interval between 0 and the other bidder's bid. Thus, the preferences of the losing bidder regarding the price paid matter.

Result 1 *IIPE may affect the outcome of the second-price auction.*

We can even be more explicit.

Proposition 2 *Any equilibrium price of the second-price auction lies in the interval* $[0, v_2]$. *Conversely, for any* $(v_1, v_2) \in R_+^2$ *and for any* $t \in [0, v_2]$ *, there always exist a couple* (g_1, g_2) *such that* t *is an equilibrium price.*

Corollary 1 *In presence of identity independent price externalities, the first-price auction and the second-price auction are not revenue equivalent.*

In a second-price auction, depending on the shape of price externalities, the equilibrium price may have any value in the interval $[0, v_2]$. Compared to the first-price auction, the losing bidder has an extra means of action. He can choose the price he prefers his opponent to pay between zero and the bid of his opponent. Hence the sensitivity of this auction format to price externalities. In the charity sale example, with a second-price auction, at the equilibrium, bidder 2 obtains the object and pays a price equal to v_2[5]. v_1 and v_2 are no longer a sufficient statistic to determine the equilibrium. For more precise results, new constraints on the structure of price externalities are necessary. We propose the following assumption that we will consider as verified for the rest of this section: **B1.** g_1 **and** g_2 **are strictly monotonic**. Independently from the amount of money that he spends, a bidder prefers the price to be the lowest possible or the highest possible.

Proposition 3 *If* g_1 *and* g_2 *are both strictly increasing, there is a unique equilibrium:* (v_2, v_2). *Bidder 2 obtains the good and pays* v_2.

If g_1 *and* g_2 *are both strictly decreasing, there is a unique equilibrium price: 0.* $< 2, 0 >$ *is always an equilibrium outcome and* $< 1, 0 >$ *is an equilibrium outcome if an only if* $v_2 - v_1 + g_2(v_1) \leq 0$.

If g_1 *is strictly decreasing and* g_2 *is strictly increasing,* $< 2, 0 >$ *is the only equilibrium outcome.*

If g_1 *is strictly increasing and* g_2 *is strictly decreasing,* $< 1, 0 >$ *is always an equilibrium outcome and* $< 2, t >$ *is an equilibrium outcome if and only if* $v_2 - t + g_2(t) \geq 0$ *and* $t \geq v_1$. *There are no other equilibrium outcomes.*

If both bidders prefer the price to be high, there is a unique equilibrium. Bidder 2 wins the good and pays v_2. Bidder 1 makes bidder 2 pay the highest possible price for which he prefers buying the good rather than leaving it to his opponent. On the other hand, if both bidders prefer the price to be low, the equilibrium price is always equal to zero. Zero is an equilibrium price in many cases. A sufficient condition is that at least one bidder prefers the

[5]Bidder 2 submits v_2 and bidder 1 also because he knows that he can raise the price paid by bidder 2 that way.

price to be low. Conditional on losing the auction, the best thing to do for a bidder who prefers the price to be low is to bid zero. Therefore, if the losing bidder prefers the price to be low, he bids zero. Since the other bidder knows it, he can take advantage of it and win the auction for a low price.

While IIPE do not affect the first-price auction, they play a key role in a second-price auction. Depending on the shapes of the IIPE, the equilibrium price varies between zero and the highest valuation. This difference between the two auction formats is due to the structure of the second-price auction. As mentioned before, in a first-price auction, only a winning bid affects the outcome of the auction. In a second-price auction, bidders have an extra leverage. When they lose the auction, they fix the price with their bids. In a standard framework, this has strictly no incidence, since bidders do not care about the price paid by their opponents. Here, losing bidders do care about the price paid and uses this extra leverage with its suitability.

4 Identity dependent price externalities

We consider a setting in which there are no IIPE, $\forall x \in R^+$, $g_1(x) = g_2(x) = 0$. IDPE affect the outcome of both auction formats. However, the outcomes of the two auction formats still differ.

First, let us observe that, in this context, v_i is not the price for which bidder i is indifferent between obtaining or not the good for sale. It represents the difference in utility for bidder i between obtaining the good at a price zero and leaving it to the other bidder for a price zero. That is why we introduce e_i, bidder i's indifference price defined as follows: $U_i(i, e_i) = U_i(j, e_i) \Leftrightarrow e_i = v_i - f_i(e_i)$. Bidder i is indifferent between the two events: "Bidder i buys the good for a price e_i" and "Bidder j buys the good for a price e_i". Note that without price externalities, $e_i = v_i$. The existence and uniqueness of a strictly positive e_i follows from assumptions A1, A2 and A4. Furthermore, the genericity assumption A5 implies that $e_1 \neq e_2$.This allows us to rule out the possibility of an equilibrium with both bidders obtaining the good with a probability $\frac{1}{2}$. Suppose that (b, b) is an equilibrium with both bidders obtaining the good with probability $\frac{1}{2}$. $e_1 \neq e_2$, then $\exists i$ such that $e_i \neq b$. In a first-price auction, if $b < e_i$, $v_i - b + g_i(b) > f_i(b) + g_i(b)$ and, by continuity (Ass. A4), $\exists \varepsilon > 0$ such that bidder i is better off bidding $b + \varepsilon$. Reciprocally, if $b > e_i$, bidder i is better off bidding e_i.In a second-price auction, if $b < e_i$, $v_i - b + g_i(b) > f_i(b) + g_i(b)$ and bidder i is better off bidding e_i. If $b > e_i$, as $v_i - b + g_i(b) < f_i(b) + g_i(b)$, by continuity (Ass. A4), $\exists \varepsilon > 0$ such that bidder i is better off bidding $b - \varepsilon$. Then, without loss of generality, we will assume in this section that $e_1 < e_2$. Notice that it follows from A4 that for $p < e_i$, bidder i prefers buying the good and for $p > e_i$, he prefers leaving it to bidder j.

4.1 The first-price auction

PROPOSITION 4 *If $e_1 < e_2$, there is a unique equilibrium of the first-price auction: both bidders submit e_1 and bidder 2 buys the good for a price e_1.*

Contrary to our observations with IIPE, IDPE have an impact on the outcome of the auction. The equilibrium depends on the indifference prices, functions of the price externalities. The bidder with the highest indifference price wins the auction and pays the indifference price of his opponent. This equilibrium derives from two constraints. First, it is a dominated strategy for bidders to submit more than their indifference prices. Second, the winning bid cannot be lower than the indifference price of the loser. Otherwise, the loser could profitably overbid it. Indifference prices play the part that valuations play in a standard setting. In fact, in any context, what really matters is the price for which bidders are indifferent between winning and losing the auction. In the absence of IDPE, this indifference price is equal to the utility of a bidder if he obtains the good for free. That is why, these two notions are usually considered as equivalent. Here, there is a difference between these two notions. IDPE affect the equilibrium of the first-price auction, we can also examine how a change in these IDPE modifies the equilibrium.

COROLLARY 2 *If $\overline{f}_1$ and $\underline{f}_1$ are such that for $p > 0$, $\overline{f}_1(p) > \underline{f}_1(p)$, then, for any v, f_2[6], the equilibrium price of $(v, (\overline{f}_1, f_2))$ is lower than the equilibrium price of $(v, (\underline{f}_1, f_2))$.*

If for any $p > 0$, $f_1(p)$ increases, it means that the utility bidder 1 derives if bidder 2 buys the good for a price p is higher. Therefore, he is less eager to win the auction since his utility is higher if he loses the auction. This lowers his indifference price and his bid. Bidder 2 takes it into account and submits a lower winning bid. This result is reminiscent of what was observed with fixed allocative externalities (see JEHIEL AND MOLDOVANU [1996]). In that case also, with two bidders, the larger the externality that the loser derives conditional on losing, the lower the final price. We illustrate this result with an example.

EXAMPLE 3 . *Two risk-neutral bidders, bidder 1 and 2, are competing in two sequential auctions, first for good A and then for good B. The valuations for both goods are, respectively: $V_A^1 = 70$, $V_B^1 = 100$, $V_A^2 = 80$, $V_B^2 = 100$. Bidder 2 has a strict budget constraint of 100 and bidder 1 has no budget constraint.*

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[6]Not to lose any generality, we keep $\overline{e}_1 < e_2$ but do not impose $\underline{e}_1 < e_2$.

Good A is sold at date $t = 1$. At date $t = 2$, with a probability β, good B is sold. With a probability $1 - \beta$, it is not sold. Let us denote by q the money spent by bidder 2 in the first auction, then, after the first auction, before knowing if the second auction will take place or not, the utility that bidder 1 expects to derive from the second auction is βq. By backward induction, we can apply our model to the auction for good A. At date $t = 1$, preferences can be represented as follows: $v = (70, 80)$, $f_2(p) = g_1(p) = g_2(p) = 0$ and $f_1(p) = \beta \min(p, 100)$. Then, $e_1 = \frac{70}{1+\beta}$ and $e_2 = 80$.

At the equilibrium of the first-price auction (see Proposition 4), the price paid is $\frac{70}{1+\beta}$ which is a decreasing function of β. If β increases, it is more important for bidder 1 that bidder 2 buys good A for a high price as it becomes more and more likely that the second auction will take place. However, the equilibrium price goes in the opposite direction. The larger β is, the smaller is the price paid by bidder 2 for good A. For higher values of β, it is more important for bidder 1 that bidder 2 spends a higher fraction of his budget on the first auction. For any additional dollar spent by bidder 2 in the first auction, the expected gain of bidder 1 in the second auction increases by β dollar. However, this gain exists also if the price is low. This effect dominates and the larger β is, the less credible is bidder 1 if he threatens bidder 2 with submitting a high bid. This would be a dominated strategy because bidder 1 does not want to win the auction unless the price is extremely low.

4.2 The second-price auction

Some elements of the analysis of IIPE remain true with IDPE. In a second-price auction, losing bids directly affect the price. Hence, the equilibrium varies strongly according to losing bidder's preferences regarding the price paid by his opponent. Even though IDPE affect the first-price auction, the equilibrium outcomes of the two auction formats still differ in presence of IDPE.

PROPOSITION 5 *Any equilibrium price of the second-price auction lies in the interval $[0, e_2]$. Conversely, for any $(t, x_1, x_2) \in R_+^3$ such that $x_1 < x_2$ and $t \leq x_2$, there always exist a couple (v, f) such that $(e_1, e_2) = (x_1, x_2)$ and t is an equilibrium price.*

COROLLARY 3 *In presence of identity dependent price externalities, the first-price auction and the second-price auction are not equivalent.*

As in the IIPE case, with this degree of generality, we cannot obtain more than a higher bound on the equilibrium prices. The equilibrium price may have any value in the interval $[0, e_2]$. Eventually, with IDPE, whatever the auction format is, what really matters are the indifference prices. However, indifference prices are not a sufficient statistic to determine the equilibrium. For more precise results, we must put more constraints on the structure of IDPE. We propose the following assumption that we will consider as verified for the remaining part of this section: **B2. f_1 and f_2 are strictly monotonic**. Each bidder is either *benevolent* or *malevolent* towards the other bidder. In this restricted framework, we are able to give a more precise description of the shape of the equilibria.

PROPOSITION 6 *If f_1 and f_2 are both strictly increasing, there is a unique equilibrium: (e_2, e_2). Bidder 2 obtains the good for a price e_2.*

If f_1 and f_2 are both strictly decreasing, there is a unique equilibrium price: 0. $< 2, 0 >$ is always en equilibrium outcome and $< 1, 0 >$ is an equilibrium outcome if an only if $v_2 - e_1 \leq 0$.

If f_1 is strictly decreasing and f_2 is strictly increasing, $< 2, 0 >$ is the only equilibrium outcome.

If f_1 is strictly increasing and f_2 is strictly decreasing, $< 1, 0 >$ is always an equilibrium outcome and $< 2, t >$ is an equilibrium outcome if and only if $t \in [e_1, v_2]$. There are no other equilibrium outcomes.

Equilibria have the same qualitative characteristics as with IIPE. Equilibrium prices are as extreme except that e_2 replaces v_2 (e_2 is higher than v_2 if f_2 is decreasing and lower than v_2 if f_2 is increasing). If both bidders are mutually benevolent, the equilibrium price is always zero and if they are mutually malevolent, the bidder with the highest indifference price wins the good and pays his indifference price. If bidder i is benevolent towards bidder j and bidder j is malevolent towards bidder i, for any values of e_i and e_j, there always exists an equilibrium in which bidder j obtains the good for a price zero. Bidder j can always turn the benevolence of bidder i to his advantage. With both type of PE, the second-price auction exacerbates the effect of PE.

Now, since IDPE affect both auction formats, it is possible to compare their impact on the equilibria of the two auction formats. What is the most striking is the difference between the extreme equilibrium prices in the second-price auction and the intermediate values of the equilibrium price in the first-price auction. Besides, in the second-price auction, unlike in the first-price auction case, if the loser prefers that the winner pays a high (resp: low) price, the winner does not pay a lower (resp: higher) price, quite the reverse. If the loser prefers the price to be high (resp: low), the price is actually at its maximum (resp: minimum). This is the complete reversed as compared to the observations with the first-price auction in corollary 2 and example 3.

We can interpret this difference between the two auction formats in terms of credibility. In both auction formats, one of the two bidders would like to be able to commit but he cannot. If f_1 is increasing, in the first-price auction, bidder 1, the losing bidder, would like to commit to a bid of e_2. That way, he would force bidder 2 to bid and pay e_2. In the same case, with a second-price auction, bidder 2 would like to commit to a bid of e_1. That way, he would force bidder 1 to bid e_1 which would allow bidder 2 to obtain the good for the price e_1. None of these commitments are credible. They require bidders playing dominated strategies. Thus, the ruling out of dominated strategies constrains the losing bidder more in the first-price auction and the winning bidder more in the second-price auction. The burden of the credibility is on a different bidder in each auction format.

5 *Implications and Extension*

5.1 *Formats comparison and recommendation to the seller*

We did not observe any general revenue ranking of the two auctions formats. However, the second-price auction is more sensitive to the presence of price externalities than the first-price auction. This claim is obvious in cases of IIPE, since they only affect the equilibrium of the second-price auction. However, the phenomenon is more general and the second-price auction also magnifies the effects of IDPE. The following proposition (a corollary of preceding propositions) states it.

PROPOSITION 7 *If at least one bidder has strictly decreasing price externalities (IDPE or IIPE), 0 is an equilibrium price in a second-price auction while it is never an equilibrium price in a first-price auction. If both bidders have (strictly) decreasing price externalities of any type, the equilibrium price is (strictly) higher in a second-price auction than in a first-price auction.*

The equilibrium price is often more extreme with a second-price auction than with a first-price auction because. This extreme sensibility of the second-price auction has clear-cut effects on recommendations that can be made to the seller. As a matter of fact, across the paper, the underlying assumption that the seller does not know the exact value of (v, f, g) was made. Otherwise, he would not use an auction since he would be better off making take-it-or-leave-it offers. Nevertheless, in general, the seller has, at least, a qualitative perception of the kind of price externalities bidders are facing. Let us consider that the seller perceives two polar cases. In the first case, price externalities are increasing in the price. In the second case, price externalities are decreasing in the price. In the first case, in order to take advantage of price externalities, the seller should choose the second-price auction. With a first-price auction, he would obtain the lowest indifference price while with a second-price auction, he would obtain the highest indifference price. In the second case, he should choose the first-price auction in order to secure a revenue equal to the second lowest indifference price rather than zero. To illustrate these results, let us apply them to situations mentioned in the introduction.

In both the LNF case and the Anelka case, one of the bidders had strictly increasing IDPE. In these cases, either this aggressive bidder has the highest indifference price and the revenue is the same with both auction formats (equal to the second highest indifference price) or this aggressive bidder has the second highest indifference price and the second-price auction gives a higher revenue than the first-price auction. In the Cofiroute case, both Vinci and Colas have increasing IIPE. Therefore, a second-price auction would reduce the price paid by Cofiroute. However, since Vinci has the effective control of Cofiroute, it is not certain that Cofiroute will end up choosing a second-price auction.

5.2 Extension to the n bidders case

The choice of the two bidders case is an obvious limitation of our model. Hence, an interesting extension of the model would consist in studying a setting with $n > 2$ bidders.

A first step would be to consider situations in which the utility of a losing bidder depends on the price paid by the winner but not on which other bidder obtains the good. We would define $U_i(i, p)$ as the utility of bidder i if he obtains the good for a price equal to p and $U_i(-i, p)$, the utility of bidder i if any other bidder obtains the good for a price equal to p. The setting is equivalent to the two-bidders case (except that $U_i(-i, p)$ replaces $U_i(-j, p)$) so that we can define v_i, f_i and g_i the same way except that $f_i(p) = U_i(-i, p) - g_i(p))$. An environment is fully defined by a triplet (v, f, g) with $v = \{v_1, v_2, ..., v_n\}$, $f = \{f_1, f_2, ..., f_n\}$ and $g = \{g_1, g_2, ..., g_n\}$. We should also assume that assumptions A1-A5 are satisfied for any $\{i, j\} \in \{1, 2, ..., n\}^2$ with $i \neq j$. e_i defined by $v_i - e_i = f_i(e_i)$ is still the price for which bidder i is indifferent between losing and winning the auction. Bidders should also be rearranged so that $i < j$ if and only if $e_i < e_j$. We derive results close to what was obtained with two bidders.[7] There is a unique

[7]We introduce key results of the n-bidder analysis but do not present results for all the possible configurations of f and g. The proofs are close to the ones presented in the paper and obtainable from the author.

equilibrium outcome of the first-price auction: $< n, e_{n-1} >$. In a second-price auction: (1) For any $(x_1, x_2, ..., x_n, t)$ such that $x_1 < x_2 < ... < x_n$ and $t \leq x_n$, there always exist a (v, f, g) such that $(e_1, e_2, .., e_n) = (x_1, x_2, ..., x_n)$ and t is an equilibrium price. Conversely, there cannot exist an equilibrium price strictly higher than e_n. (2) If $f_n + g_n$ is not decreasing and if there exists an $i \in \{1, 2, ..., n-1\}$ such that $f_i + g_i$ is strictly increasing, there is a unique equilibrium outcome, $< n, e_n >$. (3) If $\forall i \in \{1, ..., n\}$, $f_i + g_i$ is strictly decreasing, the only equilibrium price is zero. (4) If $\exists (i, j) \in \{1, 2, ..., n\}^2$ with $i < j$ such that $f_i + g_i$ and $f_j + g_j$ are strictly increasing, then any equilibrium price p must satisfy $p \geq e_j$.

In the first-price auction, at the equilibrium, the good is always allocated to the bidder with the highest indifference price for a price equal to the second highest indifference price. Thus, as in the two-bidders case, in a first-price auction, indifference prices have the same role as valuations in a context without price externalities. IIPE have still no impact on the equilibrium. This result differs from what happens with allocative externalities that depend on the identity of the winner but not on the price. In that case, with more than two bidders, there may be more than one equilibrium outcome with the first-price auction (see Jehiel and Moldovanu [1996]). Here, there is a unique equilibrium outcome.

In the second-price auction, the situation is slightly more complex. The equilibrium depends on both types of price externalities. However, these results also are not qualitatively different from the results in the two-bidders case. Depending on the shape of price externalities, the equilibrium may take any value between zero and the highest indifference price. Thus, our results seem to be robust to an increase in the number of bidders.

6 Conclusion

Price externalities affect equilibrium strategies through two channels. First, they change the price for which bidders are indifferent between winning and losing the auction. It is no longer equivalent to the utility level of a bidder if he obtains the good for a price of zero. This effect only arises with IDPE and has an impact on the equilibrium of both auction formats. Second, they change the preferences of losing bidders. By definition, this is true with both types of price externalities. However, this only affects the second-price auction. Besides, the impact of the preferences of a bidder when he loses the auction is qualitatively the same whether they are due to IDPE or IIPE.

Regarding the equilibrium itself, the essence of our results can be summarized in the four following points: (1) The two auction formats are not revenue equivalent. The difference between equilibrium prices can be large.

(2) The equilibrium of the first-price auction does not depend on IIPE while they do affect the equilibrium of the second-price auction. (3) The burden of credibility is on a different bidder for each auction format. On the loser in the first-price auction, on the winner in the second-price auction. Consequence: in a first-price auction, a losing bidder, if he prefers the price to be the lowest possible, cannot credibly commit to bid less than his indifference price, e_1. Therefore, the price is e_1. In a second-price auction, he will bid 0 which will be the final price.[8] (4) The second-price auction magnifies the effect of price externalities while the first-price auction tempers them. The second-price auction which was designed partly in view of his robustness properties (it relies on dominant strategies) is more sensitive than the first-price auction to the introduction of price externalities.

A Appendix

A.1 Proof of proposition 1

We may first note that, because of Assumption A3, there cannot exist an equilibrium with the two bidders submitting different bids since the bidder submitting the highest bid could profitably deviate with a lower bid. Therefore, at the equilibrium, the two bidders submit the same bid, b.

Suppose that $b < v_1$. With our tie-breaking rule, bidder 2 obtains the good. By continuity of g_1, there always exists an $\varepsilon > 0$ small enough so that: $g_1(b) < g_1(b+\varepsilon)+v_1-b-\varepsilon$. Therefore, bidder 1 can strictly improve his utility submitting $b+\varepsilon$ rather than b and (b,b) cannot be an equilibrium.

Suppose that $b > v_2$, With our tie-breaking rule, bidder 2 obtains the good. Since $b > v_2$, $g_2(b) > g_2(b) + v_2 - b$. Bidder 2 can strictly increase his utility submitting strictly less than b rather than b so that (b,b) cannot be an equilibrium.

Now, for any $b \in [v_1, v_2]$, (b,b) is an equilibrium with bidder 2 obtaining the good (because of our tie-breaking rule). Bidder 2 cannot make a profitable deviation since he prefers winning the auction than losing it for any price in the interval $[v_1, v_2]$. Bidder 1 cannot make a profitable deviation since he prefers losing auction than winning it for any price in the interval $[v_1, v_2]$.

However, for any $b > v_1$, submitting v_1 is a dominated strategy for bidder 1 (by a strategy consisting in bidding v_1). If $g_1'(v_1) > 0$, bidding v_1 is not a dominated strategy and (v_1, v_1) is the only equilibrium with undominated strategy. If $g_1'(v_1) \leq 0$, bidding v_1 is a dominated strategy. However, if we consider $S_1^* = [v_1, v_2]$, the set of equilibrium strategies of bidder 1, $b = v_1$ is the only element of this set which is not dominated by any element of the set. Q.E.D.

[8] We introduced, in section 4, the symmetric case: the loser prefers the price paid by his opponent to be high, see example 3.

A.2 Proof of proposition 2

Suppose that there exists a $p > v_2$ such that p is an equilibrium price. This means that there exists an equilibrium in which bidder i obtains the good for a price p. Since $p > v_2 > v_1$, because of the continuity of g_1 and g_2, for $i = 1, 2$, there always exists an $\varepsilon > 0$ such that $g_i(p - \varepsilon) > g_i(p) + v_i - p$. Thus, bidder i can profitably deviate submitting $p - \varepsilon$ and $p > v_2$ cannot be an equilibrium price.

Now, for any $(v_1, v_2) \in R_+^2$ and for any $t \in [0, v_2]$, we can build a g such that t is an equilibrium price. As a matter of fact, if g_1 and g_2 are defined as follows: $\forall x \in R$, $g_2(x) = 0$, $\forall x \leq t$, $g_1(x) = \frac{x}{5}$ and $\forall x > t$, $g_1(x) = \frac{2x-t}{5}$, (t, v_2) is an equilibrium and t is an equilibrium price. Q.E.D.

A.3 Proof of proposition 3

g_1 and g_2 strictly increasing. First, let us remark that there cannot exist an equilibrium with the two bidders submitting different bids since the bidder submitting the lowest bid could profitably deviate with a bid in the opened interval between the value of the two bids. His utility would be higher since he prefers his opponent to pay a higher price. Thus, at the equilibrium, the two bidders submit the same bid, b. Now, for bidder 2, submitting a bid lower than v_2 is dominated by a strategy consisting in submitting v_2 since g_2 is increasing. At last, suppose that $b > v_2$. With our tie-breaking rule, bidder 2 obtains the good. By continuity of g_2, there always exist an $\varepsilon > 0$ such that: $g_2(b) + v_2 - b < g_2(b - \varepsilon)$. Then, bidder 2 could strictly improve his utility submitting $b - \varepsilon$ rather than b and (b, b) cannot be an equilibrium. Therefore, (v_2, v_2) is the only possible equilibrium and it is an equilibrium since no bidder can profitably deviate.

g_1 and g_2 strictly decreasing. The equilibrium price cannot be strictly positive otherwise the losing bidder could profitably deviate submitting 0, since he prefers the price to be low. $(0, v_2)$ is always an equilibrium, thus $< 2, 0 >$ is always an equilibrium outcome. Now, for $t > 0$, $(t, 0)$ is an equilibrium with bidder 1 obtaining the good if and only if $g_2(t) + v_2 - t \leq 0$ (otherwise bidder 2 could profitably deviate) and $t \leq v_1$ (otherwise, bidding t is a dominated strategy for bidder 1). Since $g_2^{'} < 1$ (Assumption A3), there exists an equilibrium with bidder 1 obtaining the good for a price if and only if $g_2(v_1) + v_2 - v_1 \leq 0$.

g_1 strictly decreasing and g_2 strictly increasing. Since g_2 is increasing, it is a dominated strategy for bidder 2 to submit a bid lower than v_2. Now, for any bid higher than v_2, bidder 1 has a unique best response: submitting zero. Therefore, there is a unique equilibrium outcome $< 2, 0 >$.

g_1 strictly increasing and g_2 strictly decreasing. $(v_2 + 1, 0)$ is an equilibrium. Bidder 1's strategy is not dominated, he prefers the price to be high. Thus, $< 1, 0 >$ is an equilibrium outcome. Besides, no equilibrium can exist with bidder 1 obtaining the good for a strictly positive price otherwise bidder 2 could profitably deviate submitting zero. Now, for any equilibrium in which bidder 2 obtains the good, bidder 1 submits the same bid as bidder 2 since he prefers the price to

be high. Thus, for $t > 0$, $< 2, t >$ is an equilibrium outcome if and only if $v_2 - t + g_2(t) \geq 0$ (otherwise bidder 2 could profitably deviate submitting 0) and $g_1(t) \geq v_1 - t + g_1(t) \Leftrightarrow t \geq v_1$ (otherwise bidder 1 could profitably deviate submitting v_1). Q.E.D.

A.4 Proof of proposition 4

First note that there cannot exist an equilibrium with the two bidders submitting different bids since the bidder submitting the highest bid could profitably deviate with a lower bid. At the equilibrium, the two bidders submit the same bid, b.

Suppose that $b < e_1$. One of the bidder, bidder i obtains the good with probability at least 1/2. By continuity of f_j, there always exists an $\varepsilon > 0$ small enough so that: $v_j - b - \varepsilon > f_j(b + \varepsilon)$. Therefore, bidder j can strictly improve his utility submitting $b + \varepsilon$ rather than b and so that (b, b) cannot be an equilibrium.

Suppose that $b > e_2$, one of the bidder, bidder i, obtains the good at least with a probability 1/2. Since $b > e_2 > e_1$, we have $f_i(b) > f_i(b) + v_i - b$. Therefore, bidder i can strictly increase his utility submitting strictly less than b rather than b so that (b, b) cannot be an equilibrium.

Now, for any $b \in [e_1, e_2]$, (b, b) is an equilibrium with bidder 2 obtaining the good (because of our tie-breaking rule). However, for any $b > e_1$, submitting b is a dominated strategy for bidder 1 (dominated by a strategy consisting in bidding e_2). If $f_1' < 0$, bidding e_1 is not a dominated strategy and (e_1, e_1) is the only equilibrium with undominated strategy. If this property is not verified, bidding v_1 may also be a dominated strategy. However, if we consider $S_1^* = [e_1, e_2]$, the set of equilibrium strategies of bidder 1, $b = e_1$ is the only element of this set which is not dominated by any element of the set. So that (e_1, e_1) is the only equilibrium in which strategies are not dominated by any other equilibrium strategies. Q.E.D.

A.5 Proof of corollary 2

Let us define $\overline{e}_1$ (resp: $\underline{e}_1$) as the indifference price for $f_1 = \overline{f}_1$ (resp: $\underline{f}_1$). From proposition 4, we derive that $\overline{e}_1$ is the equilibrium price of $(v, (\overline{f}_1, f_2))$. Suppose that $\underline{e}_1 < e_2$, then the equilibrium price of $(v, (\underline{f}_1, f_2))$ is $\underline{e}_1$. For $p > 0$ $\overline{f}_1(p) > \underline{f}_1(p)$ and $v_1 - \underline{e}_1 = \underline{f}_1(\underline{e}_1)$, we derive that $v_1 - \underline{e}_1 < \overline{f}_1(\underline{e}_1)$ and $\overline{e}_1 < \underline{e}_1$. Now, suppose that $\underline{e}_1 > e_2$, equilibrium prices are e_2 and $\overline{e}_1$ and by definition $\overline{e}_1 < e_2$. Q.E.D.

A.6 Proof of proposition 5

Suppose that there exists a $p > e_2$ such that p is an equilibrium price. Thus, there exist an equilibrium in which bidder i obtains the good for a price p. $p > e_2 > e_1$, for $i = 1, 2$, bidder i strictly prefer losing the auction than winning it when the price is higher than his indifference price and f_i is continuous. Then, there always exists an $\varepsilon > 0$ such that $f_i(p - \varepsilon) > v_i - p$. Thus, bidder i can profitably deviate submitting $p - \varepsilon$ and $p > e_2$ cannot be an equilibrium price.

Now, let us prove that for any $(t, x_1, x_2) \in R_+^3$ such that $x_1 < x_2$ and $t \leq x_2$, there exists a v and a f such that $(e_1, e_2) = (x_1, x_2)$ and t is an equilibrium price. As a matter of fact, if we define f_1 and f_2 as follows: $\forall x \in R$, $f_2(x) = 0$, $\forall x \leq t$, $f_1(x) = \frac{x}{10}$, $\forall x > t$, $f_1(x) = \frac{x-2t}{10}$ and $v_1 = x_1 + f_1(x_1)$, $v_2 = x_2$, then $(e_1, e_2) = (x_1, x_2)$ and (t, v_2) is an equilibrium which means that t is an equilibrium price. Q.E.D.

A.7 Proof of proposition 6

f_1 and f_2 strictly increasing. First, note that there cannot exist an equilibrium with the two bidders submitting different bids since the bidder submitting the lowest bid could profitably deviate with a bid in the opened interval between the value of the two bids. His utility would be higher since he prefers his opponent to pay a higher price. Thus, at the equilibrium, the two bidders submit the same bid, b.

Now, for bidder 2, submitting a bid lower than e_2 is dominated by a strategy consisting in submitting e_2 since f_2 is increasing. At last, suppose that $b > e_2$. With our tie-breaking rule, bidder 2 obtains the good. For any price strictly higher than e_2, bidder 2 strictly prefers losing the auction than winning it and f_2 is continuous and there always exist an $\varepsilon > 0$ such that: $v_2 - b < f_2(b - \varepsilon)$. Then, bidder 2 could strictly improve his utility submitting $b - \varepsilon$ rather than b so that (b, b) cannot be an equilibrium. Therefore, (e_2, e_2) is the only possible equilibrium and it is an equilibrium since no bidder can profitably deviate.

f_1 and f_2 strictly decreasing. The equilibrium price cannot be strictly positive otherwise the losing bidder could profitably deviate submitting 0, since he prefers his opponent to pay the lowest possible price. $(0, e_2)$ is always an equilibrium, thus $< 2, 0 >$ is always an equilibrium outcome. Now, for $t > 0$, $(t, 0)$ is an equilibrium with bidder 1 obtaining the good if and only if $v_2 - t \leq 0$ (otherwise bidder 2 could profitably deviate submitting e_2) and $t \leq e_1$ (otherwise, bidding t is a dominated strategy for bidder 1). There exists an equilibrium with bidder 1 obtaining the good for a price zero if and only if $v_2 - e_1 \leq 0$.

f_1 strictly decreasing and f_2 strictly increasing. Since f_2 is increasing, it is a dominated for bidder 2 to submit a bid lower than e_2. Now, for any bid higher than e_2, bidder 1 has a unique best response: submitting zero. Therefore, there is a unique equilibrium outcome $< 2, 0 >$.

f_1 strictly increasing and f_2 strictly decreasing. $(e_2 + 1, 0)$ is an equilibrium. The strategy of bidder 1 is not dominated since he prefers bidder 2 to pay the highest possible price. Thus, $< 1, 0 >$ is an equilibrium outcome. Besides, no equilibrium can exist with bidder 1 obtaining the good for a strictly positive price otherwise bidder 2 could profitably deviate submitting zero. Now, for any equilibrium in which bidder 2 obtains the good, bidder 1 submits exactly the same bid as bidder 2 since bidder 1 prefers bidder 2 to pay the highest possible price. Thus,

for $t > 0$, $< 2, t >$ is an equilibrium outcome if and only if $v_2 - t \geq 0$ (otherwise bidder 2 could profitably deviate submitting 0) and $f_1(t) \geq v_1 - t$ equivalent to $t \geq e_1$ (otherwise bidder 1 could profitably deviate submitting a bid higher than t). Therefore, $< 2, t >$ is an equilibrium outcome if and only if $t \in [e_1, v_2]$. Q.E.D.

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