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# The Deformometric Kit: a method and an apparatus for monitoring the deformation of wooden panels

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## **SUMMARY**

This paper describes the “Deformometric Kit” (DK), which is both a methodological approach and an equipment conceived, designed and made at DEISTAF (University of Florence). The DK’s main purpose is to carry out measurements and monitoring of the deformation dynamics of wooden objects. The monitoring can take place in virtually any environment, for any desired duration (minutes, hours, weeks, years, and so on). The measurement can be carried out mainly, but not exclusively, in connection with fluctuations of ambient temperature and relative humidity. The DK provides a reliable and accurate record (a first step towards understanding) of the behaviour of original panel paintings placed in their usual exhibition location, enabling curatorial staff and researchers to (a) obtain information about the behaviour of individual supports, in order to evaluate the impact of climate on their conservation state, help to make decisions for future restoration interventions; and (b) provide reference data for calibrating and validating numeric models. It can also provide data on the deformation of a panel while it is handled and transported, enabling an evaluation of stresses to which it is subjected during this operation. The device consists of two displacement transducers, which are fixed (in a low-impact, reversible way) to the back face of the panel, perpendicular to the grain, at different distances from the panel’s surface. The two transducers not only measure the in-plane shrinkage/swelling of the panel, but since they are located at different distances, their measurements can also be combined with simple geometrical calculations to indicate the amount of distortion (cupping) which the panel undergoes. This paper explains the geometrical principles on which the DK is based, as well as its construction. Some examples of the data which have been obtained during actual monitoring by the DK are also included.

**KEYWORDS:** Panel Paintings / Cupping of Wooden Supports / In-situ Monitoring / Environment-induced distortion / Mechanics of Panel Paintings / Wood Technology / Transient Deformation / Data-loggers / Displacement Transducers /

## **RESEARCH AIMS**

This paper describes a method used to monitor the deformation dynamics of panel paintings, based on the use of two transducers measuring the relative movement of two columns fixed on the back of the panel. It can be used in many contexts of conservation studies, like evaluating the effects of environmental variations or of handling and transportation, for supporting decisions about future restoration interventions, for providing reference data for calibrating and validating computer models.

## **1. Introduction**

1 Painted panels are basically wooden structures, of various sizes, that are often large and complex  
2 such as large altarpieces. The panels are formed by a wooden support (one or more planks), to  
3 which a ground layer (gesso, glue, and sometimes canvas) has been applied, which in turn holds  
4 the paint layers and protective varnishes. Conserving panels means maintaining the integrity of  
5 both the wooden support and paint layers, which have quite different mechanical and physical  
6 properties, and hence potential compatibility problems. There have been many different factors of  
7 panel degradation over the centuries, such as fire, accidents, vandalism, intentional modifications  
8 to adapt them to different display locations or contexts, inappropriate conservation interventions  
9 carried out even by well-intentioned restorers and biological degradation e.g. by wood-boring  
10 insects, the effects of light and air pollution on paint layers. In addition to these factors significant  
11 degradation can be produced by environmental factors – temperature and relative humidity (RH)  
12 and their fluctuations – which produce shrinkage/swelling and distortions of the wooden supports  
13 and hence may damage the supports themselves (disconnections, permanent distortions, internal  
14 stresses, cracks, etc.) and/or the paint layers which usually cannot fully follow the movements of  
15 wood. Knowledge of factors that control the climate-induced response of wooden supports enables  
16 optimizing decisions about the micro environment, showcases, protections, frames, restoration  
17 interventions, and so on. [1]

20 Wood science has been making increasing contributions to the conservation of panel paintings as  
21 the necessity for in-depth knowledge of wood as a material has become apparent. Mathematical  
22 modelling and experiments on mock-models made of “modern” wood have also been very  
23 important and useful tools. However, individual pieces of wood differ from each other; original  
24 supports are made of aged wood the story of which is seldom fully known; ground-layers and paint-  
25 layers influence the chemical and physical processes involved; therefore a reliable and reasonably  
26 accurate knowledge of the behaviour of original panel paintings can only be obtained by measuring  
27 and monitoring the original panel paintings themselves. The possibility of undertaking such  
28 measurements is constrained by a number of important considerations including: the need to not  
29 interfere with the public observing the artwork, to not damage the artwork as well as  
30 restricted/limited access to the artwork to install equipment and download data, and, of course,  
31 acceptable costs (in terms of both money and time) of the system and its upkeep. Few published  
32 papers deal with real-time deformometric monitoring of wooden artworks in a changing climate;  
33 among these, [2] and [3] may be cited. Also, selected devices developed towards this purpose  
34 have been briefly described in [4], including the Deformometric Kit dealt with in this paper, and in  
35 [5].

## 40 **2. The “Deformometric Kit”**

42 The “Deformometric Kit” (DK) is both a methodological approach and an equipment conceived,  
43 designed and made at DEISTAF (formerly DISTAF, a Department of the University of Florence). Its  
44 main purpose is to carry out measurements and to monitor the deformation dynamics of wooden  
45 objects in virtually any environment, for any desired duration (minutes, hours, weeks, years ...),  
46 mainly but not exclusively in relation to fluctuations of ambient temperature and humidity. The DK's  
47 principle was first tested on a Giotto's panel painting [6]. It was then used again and partly  
48 analyzed during further research at DISTAF [7] [8]. The conception of its present configuration has  
49 been developed and implemented mainly by the first and last authors of this paper. The DK has  
50 been used in several projects, including monitoring original panel paintings during their normal  
51 exhibition in museums [9] [10]; monitoring structural replicas in climatic chambers [11] and/or in  
52 museum rooms [9]; monitoring boards from Viking ships [12]; evaluating strain induced in painted  
53 panels during handling and transportation [unpublished report, see Fig. 6 below]; research on the  
54 ageing of wood [13]. This paper describes in greater detail the geometrical principles on which the  
55 DK is based, its construction, and its functional parameters.

### 59 **2.1 The working principle**

A "base line" perpendicular to the wood grain is chosen on the back of the panel; at its ends two metal "columns" are secured to the wood in such a way that they stand perpendicular to the wood surface (see Fig. 1). The distance between the two columns is measured by means of two displacement transducers, each hinged to the columns at fixed distances from the wood surface. As the panel contracts or expands, the two columns near or move away from each other, and as the panel cups the two columns rotate so that the angle between their two axes changes. As shown in Fig. 1, simple geometrical relationships convey/relate the following parameters: manufacturing and mounting dimensions of the equipment; measurements provided by the transducers (changing over time); expansion/contraction of the wood (occurring along the panel's thickness); cupping of the panel. From these geometrical relationships, several significant parameters can be computed, including cupping angle, curvature radius, expansion/contraction of wood at various depths across the board. Such calculations can be plotted over time, or against other variables, so as to provide a detailed picture of the panel's behaviour. The following parameters appear to be the most significant:

- cupping angle  $\varphi$
- length  $c$  of the "base line" on the back of the painting (i.e. where the DK is installed)
- length  $g$  of a virtual "base line" on the front of the painting (i.e. where the paint layers were applied)
- variations (expansion or contraction) of the above baselines, here called "elongation", which can be expressed as absolute values (mm) or better as percentage variation  $\varepsilon$

The following equations, used to compute the above parameters, have been derived by applying basic geometrical relationships to the schematic diagram shown in Fig. 1, under the hypotheses discussed in § 2.1.1.

Cupping angle	$\varphi = 2 \cdot \arcsin [(b-a) / 2 \cdot m]$	[radians]	(1)
Radius of curvature	$r = (e \cdot z) / (e - b)$	[mm]	(2)
Length of the front baseline	$g = \varphi \cdot (r + f)$	[mm]	(3)
Elongation of front face baseline	$\varepsilon = [(g - g_0) / g_0] \cdot 100$	[%]	(4)

The above equations have been entered into a spreadsheet named MK (from the Italian name "Maschera Kit"), which is used to process the downloaded data, and plot the resulting graphs, such as those in Figures 4, 5, 6.

### 2.1.1 Notes concerning the assumed geometry of the DK

The DK is usually mounted to measure the most significant deformations of a panel, which are typically "cupping" and expansion/contraction perpendicular to the wood grain. However, deformations in other directions can also be measured by means of the same principle and equipment. The following hypotheses are assumed to be true, at least for the area where the measuring system is mounted (see also Fig. 1): (a) the measured deformations and the geometrical axes of the DK elements all lie in the same plane perpendicular to the wood grain; (b) the "columns" are tightly connected to the boards and remain perpendicular to their surface in the mounting point at all times; (c) the shape of the cupped board may be considered approximately as an arc of a circle; (d) the boards forming the panel are of constant thickness, which does not vary over time; (e) in the cross-section of the boards, straight lines remain straight even during deformation, and straight lines originally perpendicular to the wood surface remain perpendicular even during deformation.

Experience and a further analysis show that in most cases hypotheses (a) (b) and (c) can be considered true, and associated errors may be neglected with good approximation. Hypotheses (d) and (e) are not essential for the basic geometry of the DK, and are required only for extrapolating measurements to compute deformations of the painted face. As for (d), in real situations the boards can feature different degrees of thickness in different points, and the wood does shrink and swell along the panel's thickness as well. As for (e), further analysis shows that due to wood

1 anisotropy some distortion takes place in the cross-section, unless the board was perfectly quarter  
2 sawn (i.e. perfectly radial). In both cases (d) and (e) the associated errors appear to be quite small  
3 and, if needed, could be corrected by means of basic but complicated calculations, requiring  
4 knowledge of the direction of growth rings. This paper does not consider these errors as the  
5 authors intend analyzing and quantifying the errors deriving from deviations from hypotheses (d)  
6 and (e) in a subsequent paper..

## 7 2.2 The equipment

### 8 2.2.1 Components

9 The DK can be designed and manufactured to fulfil various needs, in various situations and to  
10 pursue various measurement objectives. The most typical configuration is described here, and  
11 consists of the following components (see Fig. 2 and 3):

- 12 (a) Two identical displacement transducers, of the potentiometric type (Note: the potentiometric  
13 type was chosen mostly because of its low cost and low energy consumption, enabling a self-  
14 standing system, with very long battery life, to be assembled).
- 15 (b) Prolonged, as needed, by aluminium threaded rods, equipped with blocking nuts, allowing for  
16 length adjustment. Each transducer “line” (i.e. transducer + connecting pieces + rods)  
17 terminates at each end with a steel ball joint, enabling free rotation at connections with  
18 “columns”.
- 19 (c) Two identical “columns”, also made of aluminium, with a larger base to be fixed (by means of  
20 screws or other devices) on the back face of the panel. Distances between ball joints and  
21 wood surface are maintained constant by means of spacers.
- 22 (d) A self-powered data-logger, which powers the transducers, reads them and records data with  
23 desired frequency (typically every 15/30 minutes).

24 The equipment is designed so that any small offset of the columns’ axes from the plane (caused,  
25 for instance, by panel twisting) can occur without damaging the equipment nor significantly  
26 affecting the measurements.

### 27 2.2.2 Mounting the DK

28 The bases of the columns need to be firmly fixed against and perpendicular to the wood’s surface.  
29 However such fixation must not cause any significant damage to the panel and, most of all, it must  
30 be acceptable to the panel’s curator. In several cases three stainless steel panel screws, 3 mm  
31 diameter, 20-25 mm length, provided satisfactory fastening on poplar panels, where the wood  
32 density is about 0.4 g/cm<sup>3</sup> and no pre-boring is needed; when the screws are removed, almost no  
33 trace remains in the panel. In other cases (e.g. panels made of oak, wood density around 0.8  
34 g/cm<sup>3</sup>), small blocks of plywood were glued onto the back of the panel (more precisely, Japanese  
35 paper was glued to the panel by means of alimentary jelly, and plywood blocks were glued onto the  
36 paper by means of a quick set vinyl glue), then the DK’s bases were screwed on these blocks;  
37 when monitoring is complete, blocks can be easily chopped up by the restorer, and the jelly layer  
38 moistened and removed (Jean-Albert Glatigny, personal communication, 2011).

### 39 2.2.3 Setting the data logger

40 Transducer outputs are fed into a 2- or 4-channel data-logger, which cyclically reads all the  
41 channels at preset intervals (typically every 15/30 minutes for long-duration monitoring; or up to  
42 every 1 second / 1 minute for short-duration monitoring, such as during handling); two or four  
43 readings are performed at intervals of a few milliseconds, in practice at the same moment.  
44 Typically at least one 2-external-channels data-logger is used, which records two transducers plus  
45 air temperature and relative humidity at the same preset intervals. The system (transducers + data-  
46 logger) is powered by an internal timer at preset intervals only, when measurements are to be  
47 recorded; this allows for quite a long battery life, typically over 6 months – incidentally, one year is  
48 the shortest period recommended by present EN guidelines [14] for monitoring and characterizing  
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1 the micro-climate surrounding a work of art made of hygroscopic material such as wood. The data-  
2 logger's memory fills up over a varying range of time, depending on the preset sampling frequency.  
3 For example, a 64 kB memory in a 4-channel data-logger with a sampling frequency of 15 minutes,  
4 fills up and hence data need to be downloaded at approximately 96-day intervals. The recorded  
5 data can typically be downloaded by means of a USB cable connection; some loggers that transmit  
6 data by radio are being considered.

### 7 2.3 Accuracy of the DK measurements

8  
9 Given the complexity of the measurements taken with the DK, their accuracy is hard to assess,  
10 both theoretically and practically, and in any case depends on the DK's geometry. Some work is  
11 presently ongoing at LMGCC (Montpellier, France) where different measurement systems are being  
12 compared (Agnès Rouard, personal communication, 2011) under the supervision of one of the  
13 authors. However, a general approach showing the influence of the DK's geometry is presented in  
14 the following paragraphs.

#### 15 Systematic errors

16 Monitoring mainly implies comparing the values successively attained by the measured  
17 parameters; therefore, the possible systematic errors (e.g. inaccuracies in construction and  
18 mounting, or in determining calibration factors) tend to affect successive measurements in the  
19 same way, so that the trend and shape of resulting diagrams are not significantly affected. To  
20 summarize, accuracy is often less important than repeatability, since in most cases differences and  
21 variations are the most significant results.

#### 22 Accidental errors

23 The accuracy of the results (i.e. of the parameters resulting from the calculations performed by the  
24 MK, after it has processed the raw data provided by the transducers) is strongly affected by both  
25 the transducer's errors and the DK's geometry, as shown by the calculation in Annex, which in  
26 summary provides the results briefly discussed in the following § 2.4.

### 27 2.4 Influence of design parameters on accuracy and resolution of the DK measurements

28 From the calculations in Annex one can derive the following guidelines for choosing design  
29 parameters that optimize the DK's performance (accuracy and resolution):

- 30 - high precision transducers
- 31 - high precision and resolution data-logger
- 32 - large distance  $m$  between the transducers
- 33 - small distance  $z$  between the lower transducer and the back face of the panel
- 34 - large initial (i.e. mounting) distance  $e_0$  between the two columns

35 Furthermore, the performance of the DK will be influenced, *inter alia*, by the following factors:

- 36 - thickness of the panel (the thicker the panel, the less accurate the DK in relation to the  
37 measurements extrapolated on the front face)
- 38 - calibration accuracy of the individual transducers.

39  
40 It should therefore be emphasized that the above parameters can be different for each individual  
41 DK, so that trying to characterize all DKs by means of one general figure, expressing their  
42 accuracy or resolution, would be meaningless. Table 1 in Annex shows the geometrical data and  
43 the resulting accuracies (estimated maximum errors) and resolutions (see § 2.5 below) which can  
44 be obtained from five actual case-studies. In summary, the following accuracies (maximum  
45 estimated errors) were found; figures in parentheses refer to one case, where high precision  
46 calibration (HPC) was made:

- 47 - accuracy of the individual transducer lines (transducer + data-logger + calibration): ranging  
48 between approx.  $\pm 60$  and  $\pm 110$  microns (HPC:  $\pm 9$ )
- 49 - accuracy of variation of cupping angle: ranging between approx.  $\pm 60$  and  $\pm 110$  thousands  
50 of a degree (HPC:  $\pm 15$ )

- accuracy of elongation of the back face: ranging between approx.  $\pm 60$  and  $\pm 70$  microns (HPC:  $\pm 14$ )
- accuracy of elongation of the front face: ranging between approx.  $\pm 90$  and  $\pm 140$  microns (HPC:  $\pm 19$ )
- accuracy of percent elongation of the front face: ranging between approx.  $\pm 0.017\%$  and  $\pm 0.022\%$  (HPC:  $\pm 0.005\%$ ).

## 2.5 Resolution of the DK measurements

Since the resolution of the transducers is virtually infinite, the resolution of the displacement readings depends on the resolution of the data-logger. With the equipment which has mostly been used (see § 2.2.1), such resolution is 7.3 microns (see Annex § C). The resolution of the whole DK depends both on the resolution of transducers and data-loggers, and on the DK's geometry and dimensions. Resolution values for the same actual study cases as in § 2.4 are also reported in Annex, Table 1.

## 3. Application examples

In order to show configurations and resulting data, some examples are briefly outlined in Figures 4, 5 and 6, which all refer to data obtained from DKs mounted on the back face of original panels exhibited in a hall (Sala dell'Ospizio) of the San Marco museum, Florence, Italy. Fig. 4 shows data measured (temperature and relative humidity of the surrounding air) and calculated from DK's measurements (cupping angle  $\varphi$  and elongation of front face  $\epsilon$ ) plotted against/over time. The time period shown starts in June 2008 and ends in November 2009 (approximately 17 months). Data were recorded every 15 minutes. To make them more distinct, the plots show data smoothed by a 6-hour moving average. The yearly cycle of the surrounding climate and of the panel's deformations is clearly visible, together with a surprisingly prompt and accurate deformational response of the panel (inverse relationship between RH variations and the panel's cupping angle). Analysis (not discussed here) confirms that such deformations originate from the formation of moisture gradients along the panel's thickness, particularly when such gradients are asymmetric due to the presence of paint layers only on the painted face of the panel. These are transient deformations that fade over time; however, in a real museum environment the climatic fluctuations never cease, and the diagrams clearly show the continuously changing deformation of the panels, which constantly adapt themselves to the changing environment, and are never in a state of equilibrium in terms of uniform moisture and temperature distribution across their structure. In order to show with greater detail the daily variations and the prompt deformational response, a 10-day period from the data of Fig. 4 is shown in Fig. 5.

Fig. 6 shows quite a different application of the DK, that is monitoring the handling operations of an original panel painting (Case 2 in Table 1 in Annex), and its effects on the panel strains. To enable a detailed analysis, data were recorded every minute, and no smoothing was performed on the resulting plot. The order of magnitude of the strains produced by handling can be well identified. Also the different deformations pertaining to the two resting positions (leaning and vertical) are highlighted. The jolts visible on the two plots highlight the resolution limit of the equipment, respectively  $\sim 0.01$  degrees for  $\varphi$  and  $\sim 0.011\%$  for  $\epsilon$ , (see Table 1 in Annex).

## 4. Final comments

### 4.1 Usefulness of the DK

In sum, the Deformometric Kit is a stand-alone device, equipped with electronic data logger and probes measuring temperature and relative humidity, specially conceived to perform long-term and/or short-term monitoring of the deformational response of painted panels. Monitoring can be carried out in nearly any environment, and for any desired duration (minutes, hours, weeks, years...). The only significant requirement is that some space, of the order of 8-12 cm, be available

1 behind the panel (e.g. distance from the wall). It is not invasive, either from a technical point of  
2 view (no damage to the panel), or from an aesthetical point of view (no interference with visitor  
3 experience). It provides reliable and significant data showing the history of the panel's deformation  
4 under the influence of factors such as environmental fluctuations, handling or transportation. It can  
5 therefore be considered a new, powerful method for studying the deformational response of panel  
6 paintings and other cultural wooden objects. The reliable and reasonably accurate knowledge of  
7 the behaviour of individual, original panel paintings placed in their usual display locations, enables  
8 evaluating the impact of climate on their conservation, helps to make decisions for future  
9 restoration interventions, and provides reference data for calibrating and validating numerical  
10 models.

#### 11 4.2 Comments about use and potential developments of the DK

12 To study the reactions of a panel to the fluctuations of its surrounding environment, it is necessary  
13 to cover at least a one-year period of monitoring, according to present technical guidelines [14]. It  
14 should be emphasized that the measurements obtained by means of the DK only represent the  
15 behaviour of the board(s) at the location in which the equipment has been installed; this location,  
16 therefore, must be chosen accurately, and in accordance with monitoring objectives. Experience  
17 shows that each and all details in manufacturing and installing the DK are essential for obtaining  
18 reliable results. Firmness and stability of both mechanical and electrical connections require  
19 special care, in order to prevent loss of performance of the electrical contacts and the undesirable  
20 onset of relative movements among the mechanical parts, leading to deteriorated and hence  
21 unusable data. Moreover, at the present time, analysis of the data requires demanding  
22 commitment from a well-trained researcher. Possible future developments of hardware and  
23 software will enable routine examination of the main data by personnel that are not so highly  
24 trained; however, in-depth exploitation of the data should remain the realm of well-trained  
25 researchers. Further research needs to be carried out on the accuracy of the measurements, as  
26 influenced by several factors, including the DK's geometry and the anisotropy of wood  
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3 securing system for DK on Flemish oak panels
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9  
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11 developed within COST Action IE0601 "Wood Science for Conservation of Cultural Heritage",  
12 which also financially supported some STSMs (Short Term Scientific Missions) intended to  
13 disseminate the DK's applications.

## 14 **SUPPLIERS**

15  
16 Data-loggers: Hobo U12-013 (temperature, RH, 2 external channels), U12-006 (4 external  
17 channels), Onset Computer Corporation (USA). Linear displacement sensors  
18 SLS095/0030/1.2K/R/50, Penny & Giles Controls Ltd. (UK). Digital caliper used for calibration of  
19 transducer lines: Mitutoyo Digimatic, 550 series, 600 mm. Manufacturing drawings of components  
20 not commercially available, as well as the MK spreadsheet, can be obtained from the authors at  
21 DEISTAF, the University of Florence.  
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## FIGURE CAPTIONS

**Fig. 1** – Schematic diagram of the main geometrical parameters of the DK. The convention concerning positive (front face convex) and negative (front face concave) values of the cupping angle  $\varphi$  is also shown.

Key to symbols:

- a** distance between the axes of the two columns, at distance  $m+z$  from the back face of panel, as measured by the upper transducer (variable in time)
- b** distance between the axes of the two columns, at distance  $z$  from the back face of the panel, as measured by the lower transducer (variable in time)
- c** length of the baseline (assumed to be an arc of a circle), lying on the back face of the panel, joining the axes of the two columns (variable in time)
- e** distance between the axes of the two columns, where they intersect the back face of panel (variable in time)
- f** thickness of the wooden panel (assumed constant)
- g** length of the virtual baseline (assumed to be an arc of a circle), lying on the front (painted) face of the panel, joining the axes of the two columns (variable in time)
- g<sub>0</sub>** reference value of **g** (typically the value of **g** at the start of the monitoring)
- m** distance between the centres of the ball joints of the two transducers on the same column (constant, determined by construction)
- r** radius of curvature of the baselines (variable in time)
- z** distance between the centre of the ball joint of the lower transducer, and the back face of the panel, along the axis of the column (constant, determined by construction)
- $\varphi$**  “cupping angle” between the axes of the two columns (assumed to be coplanar) (variable in time) (radians)

**Fig. 2** – A simplified construction drawing of a DK, in actual scale, mounted on a flat panel (deformation and cupping might intervene later).

Key to symbols:

- h** height of columns (in this drawing: 70 mm); in order to house the device, the distance behind the panel should be larger than **h**
- z** distance between the centre of the ball joint of the lower transducer, and the back face of the panel, along the axis of the column (in this drawing: 14.5 mm)
- m** distance between the centres of the ball joints of the two transducers on the same column (in this drawing: 33 mm)
- d** transducer’s shaft displacement (with this type of transducer, the measuring range is approximately between  $d=3$  and  $d=33$  mm)
- e<sub>0</sub>** distance between the axes of the two columns, where they intersect the back face of panel, at time of mounting (in this drawing: 224 mm)
- 1** rod end spherical bearing (ball joint  $\varnothing$  6 mm)
- 2** fitting, connecting transducer’s back end to the back spherical bearing
- 3** lower transducer (also named “b”)
- 4** upper transducer (also named “a”)
- 5** extension rod, connecting the transducer’s shaft to the rod end spherical bearing
- 6** rod end spherical bearing (ball joint  $\varnothing$  6 mm)
- 7** base, connecting the column to the wooden panel ( $\varnothing$  30 mm, with 6 holes  $\varnothing$  3 mm for at least 3 panel screws)
- 8** spacer
- 9** column

**Fig. 3** – A DK mounted on the back face of the original panel “Trittico di San Pietro Martire” (~1425), exhibited in a hall (Sala dell’Ospizio) of San Marco museum, Florence, Italy. The panel is made of poplar (*Populus alba* L.) wood boards, 28-30 mm thick

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**Fig. 4** – Data measured (temperature and relative humidity of the surrounding air) and calculated (cupping angle  $\phi$  and elongation of front face  $\epsilon$ ) from a DK placed on the “Trittico di San Pietro Martire”, plotted against time. The time period shown starts in June 2008 and ends in November 2009 (approximately 17 months).

**Fig. 5** – A 10-day period in the data of **Fig. 4**, showing daily variations more clearly.

**Fig. 6** – Monitoring the handling of an original panel painting (“Pala del Bosco ai Frati” ~1450-52 , by Beato Angelico, San Marco museum, Florence, Italy), from a leaning position on a trolley, to its vertical exhibition position, on the wall.

Figure 1  
[Click here to download high resolution image](#)

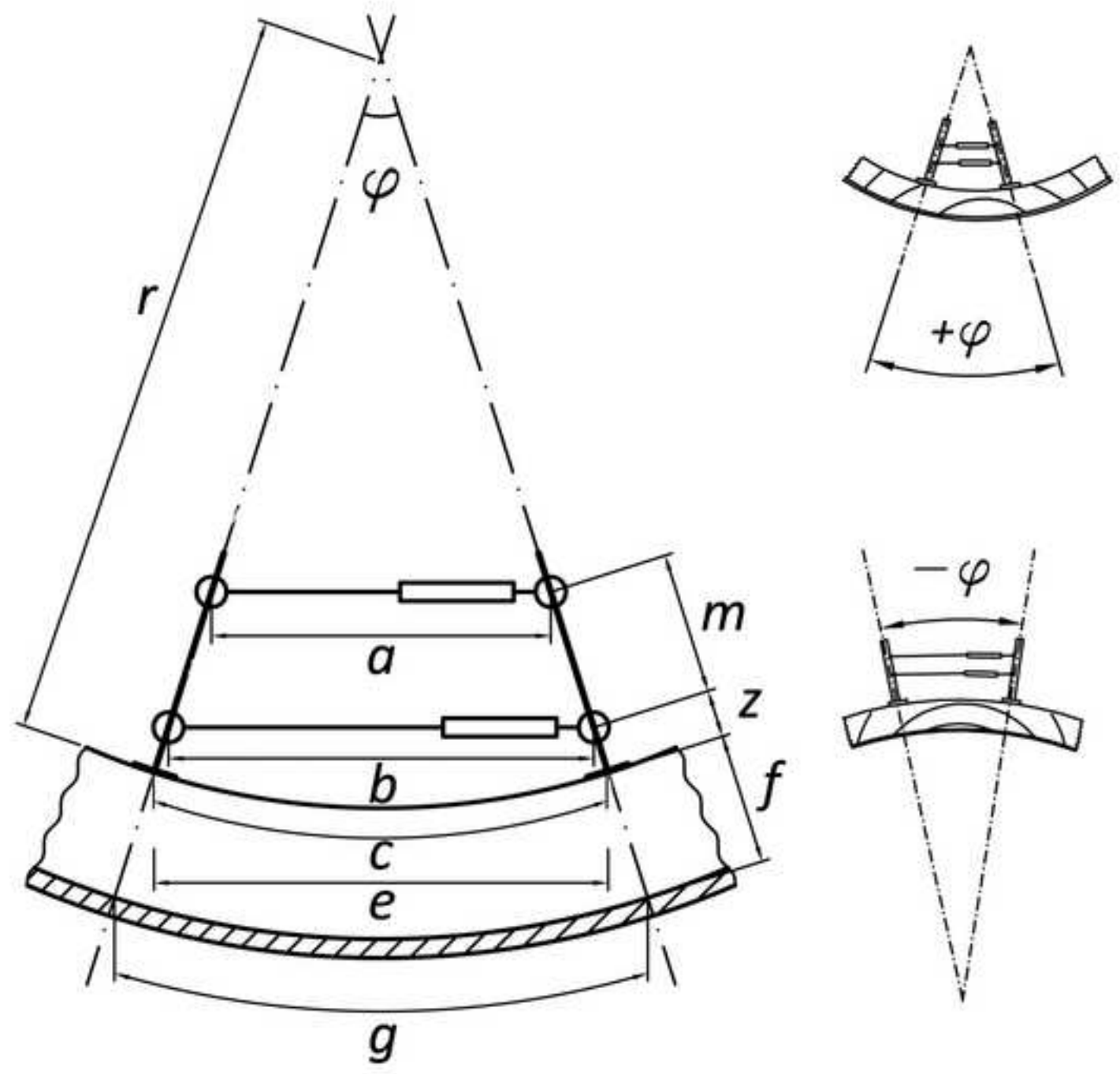
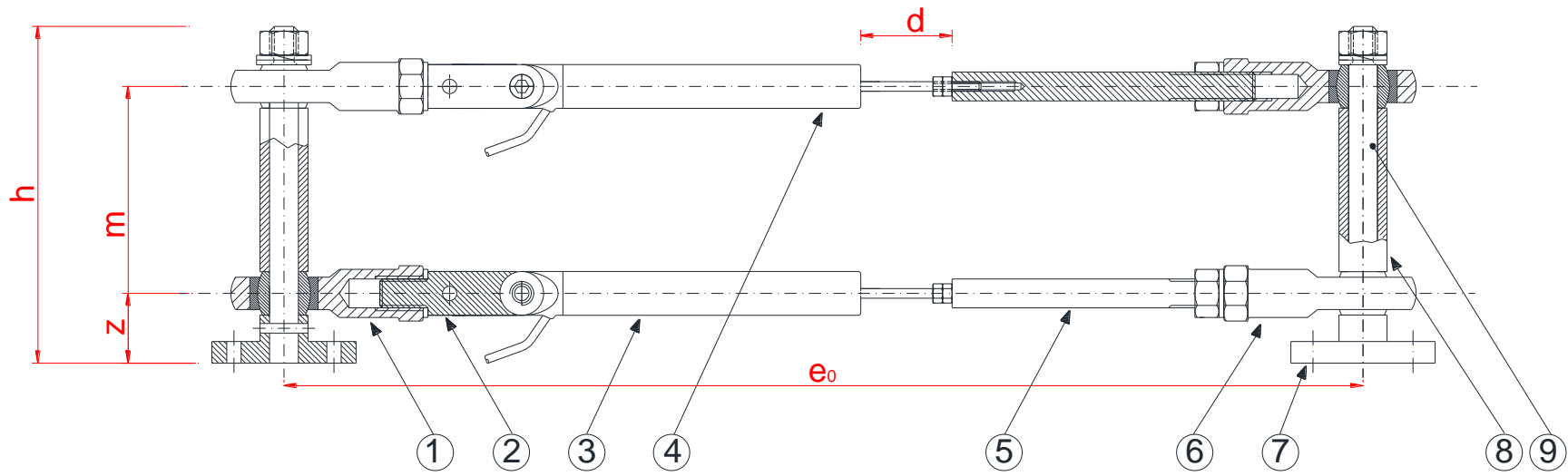


Figure 2



**Figure 3**  
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Figure 4  
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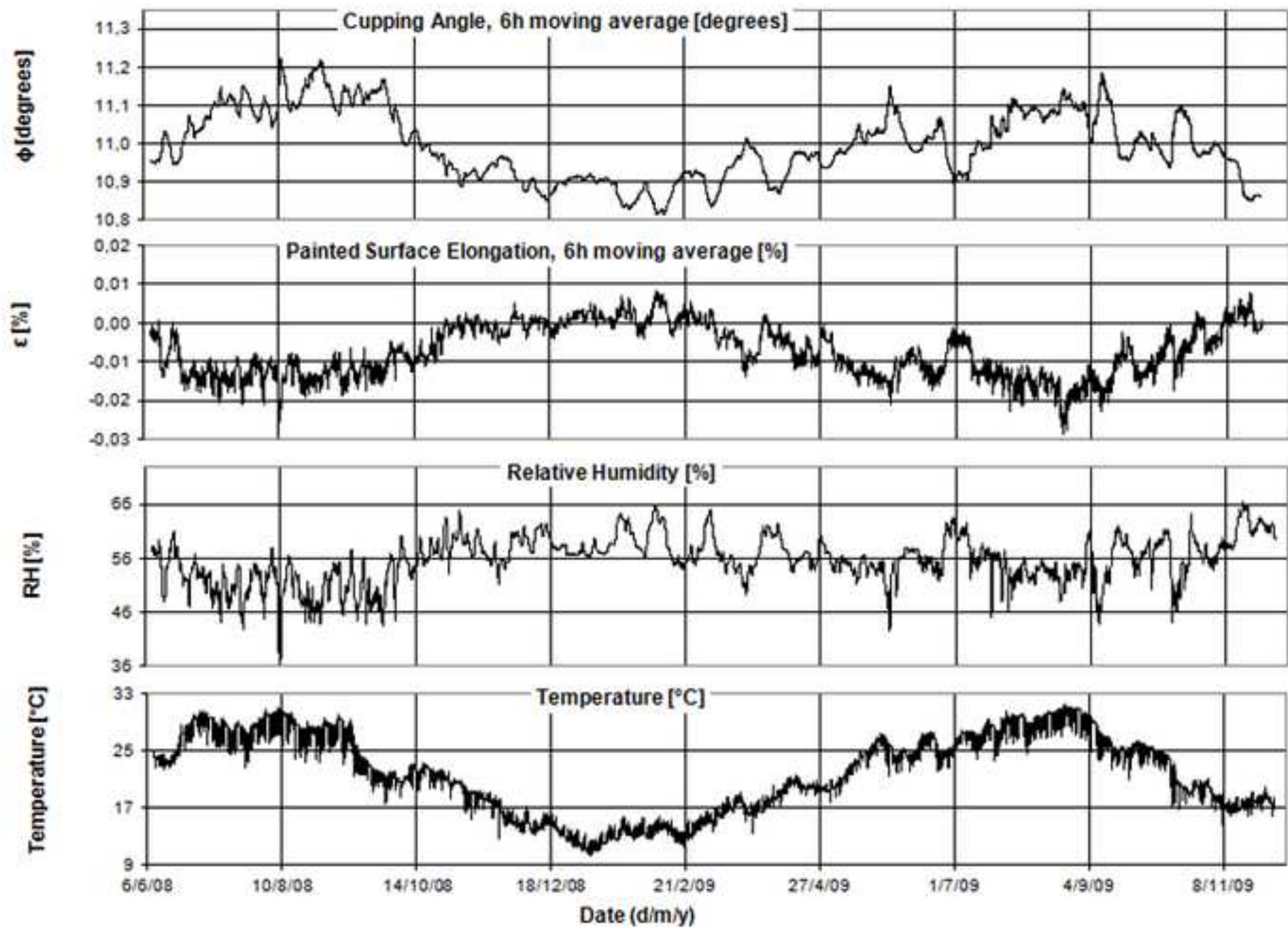




Figure 5  
[Click here to download high resolution image](#)

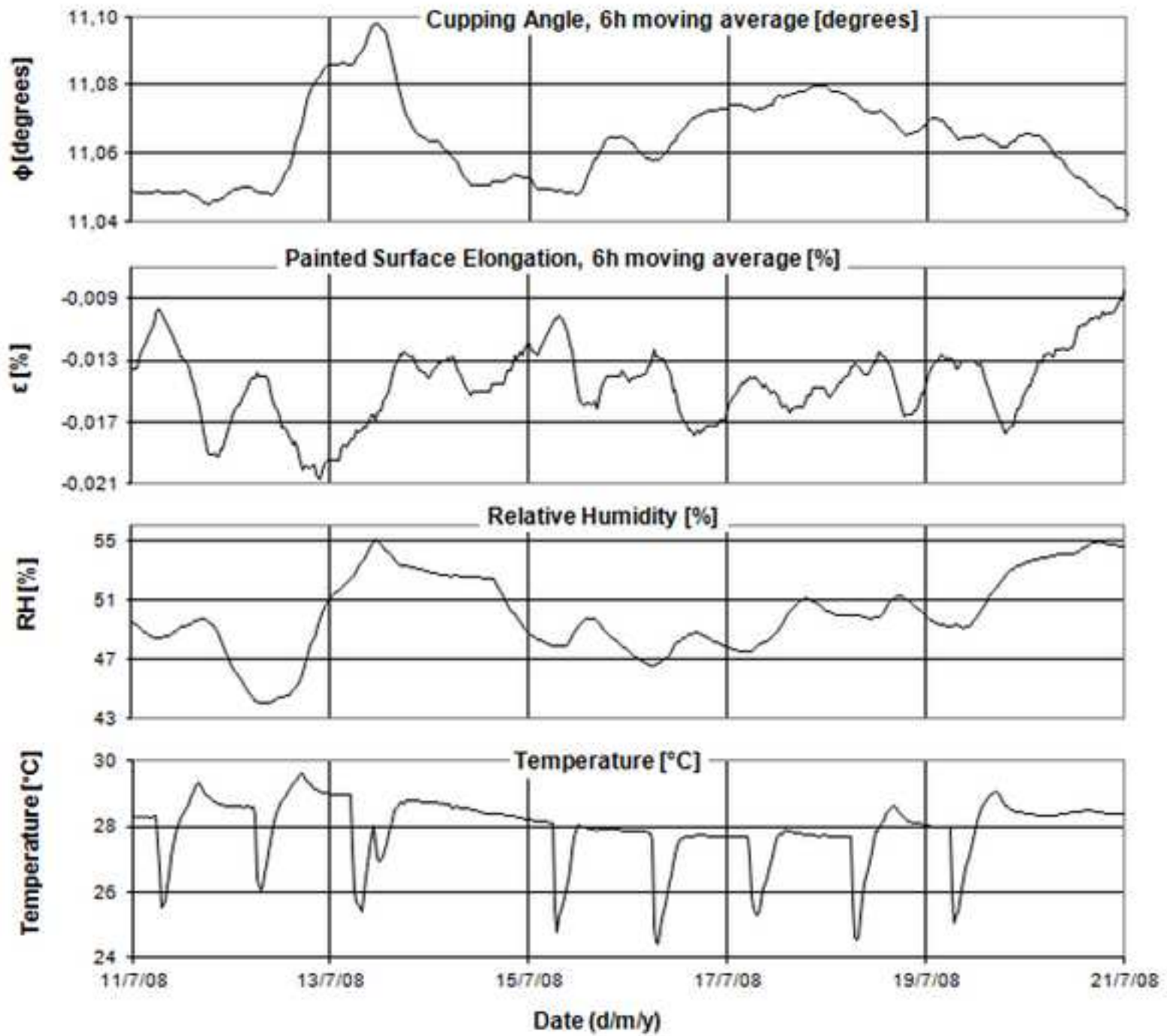
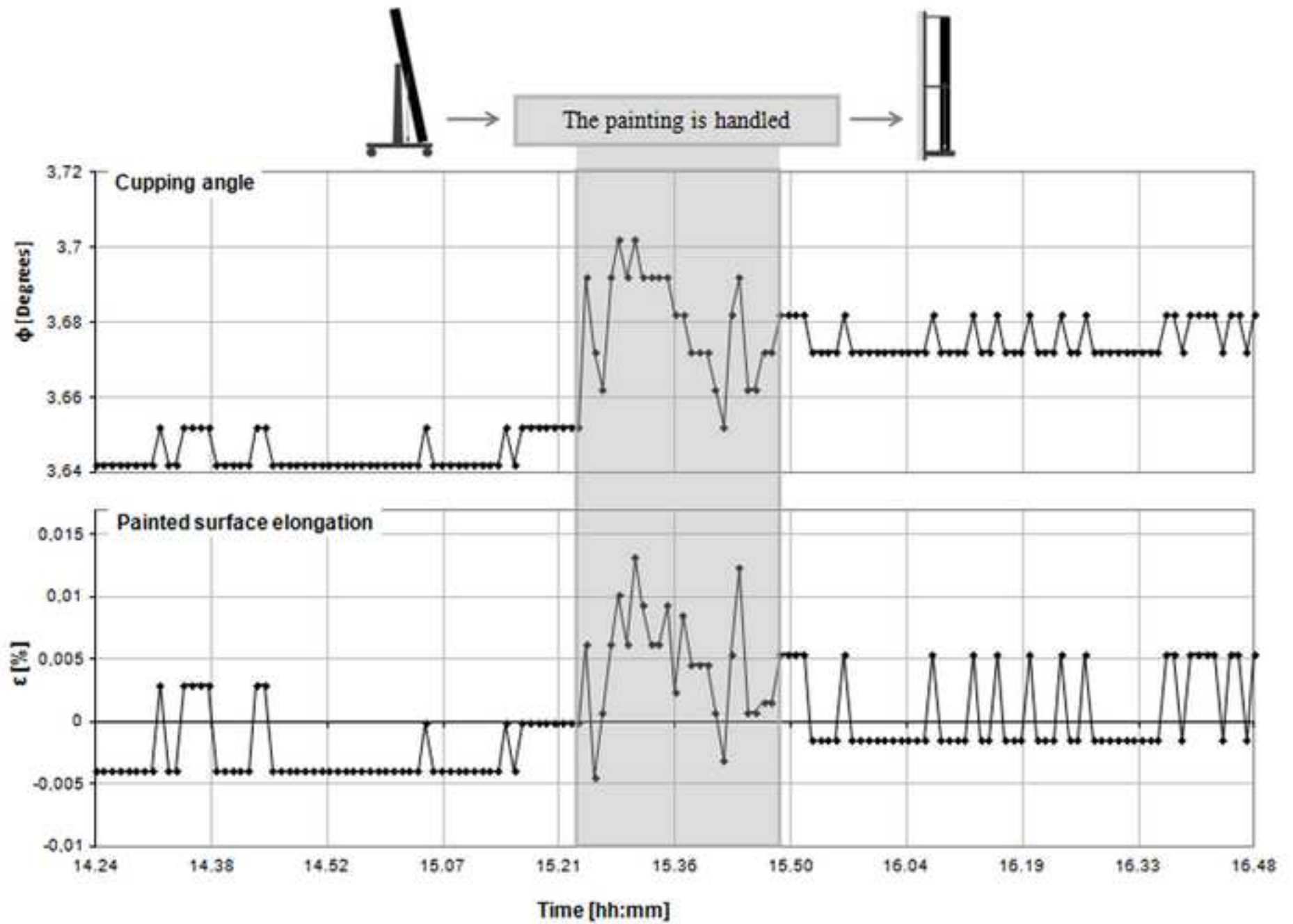


Figure 6  
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## Uzielli et Al The Deformometric Kit Annex

((Note: this Annex will be treated as “supplementary material”, i.e. it will not be printed, but will be made available online as a PDF file))

### **ANNEX – Estimation of how the DK’s geometry affects the propagation of errors, and hence the criteria for designing individual DK applications.**

In the DK, we need to consider the resolution and accuracy both of the individual transducers, and the whole DK (including the MK calculations) considered as one measuring system.

#### **A) Accuracy of individual transducers**

The data sheet of the potentiometric transducers (see § “Suppliers” at the end of the referred paper), specifies that, regardless of the stroke length, their “resolution” is virtually infinite, and “hysteresis (repeatability)” is less than 0.01 mm. For 30 mm stroke length (the most used in the DKs described in this paper) “independent linearity” is given as  $\pm 0.25\%$  (i.e.  $\pm 0.0025 \cdot 30 = \pm 0.075$  mm =  $\pm 75$   $\mu$ m). Moreover, the actual accuracy of the transducers in DKs depends on the accuracy of the data-loggers used to read them. The data sheets of the data-loggers (see again § “Suppliers” at the end of this paper) specify that the accuracy of external input channels is  $\pm 2$  mV  $\pm 2,5\%$  of absolute reading. However, further inquiries clarified that some part of such inaccuracies (approximately  $\pm 2\%$ ) can be considered systematic and held constant for each individual instrument, while other parts depend on random errors and on the instrument’s resolution. Finally, the data sheet of the 600 mm digital calliper used for calibration of the transducer lines (see below) states that the resolution is  $\pm 0.01$  mm, and the error is  $\pm 0.05$  mm.

Combining the above accuracies, each in its “worst case”, would lead to an accuracy (or rather “inaccuracy”) of the whole system much larger than would be acceptable for the measurements to be performed; moreover, inaccuracy would be much larger than what was found when calibrating the individual transducer lines. In fact a calibration procedure is performed once for all before each DK is installed, by comparing the electrical readings (on the same data-logger and channel actually used for monitoring) with the actual displacement (measured with the 600 mm calliper, resolution  $\pm 0.01$  mm); the calibration factor is obtained for each individual combination of transducer-and-channel, by interpolating the measured points with a straight line, by the least squares method. Global accuracy for each calibration is then computed as the maximum difference between the actual measurement and the corresponding straight line value. By the above described calibration procedure (which, on the whole, accounts for inaccuracies of the transducers, the data-loggers, the calibration calliper, and the possible relative movements among the mechanical parts forming the transducer lines), the global accuracy for individual combinations of transducer-and-channel was found to be of the order of 0.04 mm = 40  $\mu$ m for the DKs used both in museums and in the laboratory in Florence, and of the order of 0.009 mm = 9  $\mu$ m for the DKs used in the laboratory in Montpellier; such values have therefore been used to calculate the accuracies reported in Table 1.

#### **B) Accuracy of the whole DK**

The accuracy of the whole DK (including the MK calculations), considered as one measuring system, is affected by (1) the accuracy of transducers, (2) the DK’s ideal geometry (as described in Fig. 1), (3) inaccuracies in construction, in mounting, and in functioning of the mechanical joints, and (4) by the electronic circuits used to read the transducers. The two latter factors depend on construction accuracy and choice of components, and their discussion is beyond the scope of the present paper. This section deals with the first two factors, which can be considered as prevailing:

- how the transducer’s accuracy and the DK’s design parameters affect the accuracy of the whole DK’s measurements, and
- hence, the effect of individual design parameters on the DK’s performance.

Such values are then used to calculate the accuracies reported in **Table 1**

A short explanation of the symbols used:

- “ $\Delta$ ” represents the error, i.e. the difference between the measurement result and the true value of the measured quantity;
- “d” represents the differential, i.e. a very small variation of the measured quantity (e.g. da, db, dg etc.);
- “ $\Delta d$ ” represents therefore the error on the variation of the concerned quantity, be it a reading or the result of a calculation (e.g.  $\Delta da$  is the possible error of the reading of transducer “a”;  $\Delta d\phi$  is the propagated error of the cupping angle  $\phi$ );
- since the two transducers are identical, we can assume that also their possible errors are also identical, hence  $\Delta b = \Delta a$  and  $\Delta db = \Delta da$ .

B1) Influence of the transducer’s possible errors ( $\Delta a$ ,  $\Delta b$ ) and of the construction parameter  $m$  on the error ( $\Delta d\phi$ ) for the variation of the cupping angle  $d\phi$

From equation (1) in § 2.1, the following equation can be obtained:

$$\sin(\phi/2) = (b - a) / (2 \cdot m)$$

Since  $m$  is a constant, deriving such an equation with respect to time yields:

$$(1/2) \cdot \cos(\phi/2) \cdot d\phi = (db - da) / (2 \cdot m)$$

and hence:

$$d\phi = (db - da) / (m \cdot \cos(\phi/2))$$

Since  $\phi$  is small, we can assume  $\cos\phi \approx 1$

The most unfavourable situation entails the summation of the modules of the errors:

$$\Delta d\phi = (\Delta db + \Delta da) / m = 2 \cdot \Delta da / m$$

The error ( $\Delta d\phi$ ) on the variation ( $d\phi$ ) of the cupping angle  $\phi$  is therefore:

- proportional to the error of the linear measurements provided by the transducers ( $\Delta da$  or  $\Delta db$ )
- inversely proportional to the design parameter  $m$  (distance between the two transducers)

B2) Influence of the transducer’s possible errors ( $\Delta da = \Delta db$ ) and of the construction parameters  $z$  and  $m$  on the error ( $\Delta dc$ ) of the back face baseline’s ( $c$ ) elongation

From equation [2] in § 2.1, the following equation can be obtained:

$$(e - b) / z = (b - a) / m$$

Since  $m$  and  $z$  are constants, deriving such an equation with respect to time yields:

$$(de - db) / z = (db - da) / m$$

$$de = [1 + (z / m)] \cdot db - (z / m) \cdot da$$

The most unfavourable situation entails the summation of the modules of errors:

$$\Delta de = [1 + (z / m)] \cdot \Delta db + (z / m) \cdot \Delta da$$

The two transducers are identical, hence we can assume  $\Delta db = \Delta da$

$$\Delta de = \{[1 + (z / m)] + (z / m)\} \cdot \Delta da = [1 + 2 \cdot (z / m)] \cdot \Delta da$$

In situations where the DK is used, the difference between chord **e** and arc of circle **c** is typically very small, hence  $\Delta de \approx \Delta dc$ , therefore

$$\Delta dc = [1 + 2 \cdot (z / m)] \cdot \Delta da$$

The error ( $\Delta dc$ ) on the variation of the elongation (**dc**) of the back face baseline **c** is therefore:

- proportional to the error in the linear measurements provided by the transducers ( $\Delta da$  or  $\Delta db$ )
- proportional to the design parameter **z** (distance between the lower transducer and the back face of the panel)
- inversely proportional to the design parameter **m** (distance between the two transducers)

B3) Influence of the transducer's measurement errors ( $\Delta da = \Delta db$ ) and of the construction parameters **z** and **m** on the error ( $\Delta dg$ ) of the front face baseline's (**g**) elongation.

By performing similar calculations as in case (B2) above, we obtain:

$$\Delta dg = \{1 + [2 \cdot (f + z) / m]\} \cdot \Delta da$$

The error ( $\Delta dg$ ) on the variation of the elongation (**dg**) of the front face baseline **g** is therefore:

- proportional to the error in the linear measurements provided by the transducers ( $\Delta da$  or  $\Delta db$ )
- proportional to the design parameter **z** (distance between the lower transducer and the back face of the panel)
- proportional to the thickness **f** of the panel
- inversely proportional to the design parameter **m** (distance between the two transducers)

B4) Influence of the initial distance **e<sub>0</sub>** between the columns (mounting distance) on the percentage error ( $\Delta d\epsilon$ ) of the front face baseline's (**g**) elongation

Carrying out calculations similar to those performed above would lead to quite complicated equations from which the desired results could not be easily derived. A simpler, although rougher approach can be considered as follows. If we consider a uniformly cupped panel (i.e. with the same radius of curvature **r<sub>0</sub>**), increasing the initial distance **e<sub>0</sub>** between the columns entails a larger cupping angle  $\phi_0$ , such that  $e_0 = r \cdot \phi_0$

From Fig. 1, considering the similar triangles:  $r / e = (r-z) / b$ ; hence  $e = b \cdot r / (r-z)$

In order to simplify, if **r** remain constant, but the panel undergoes an elongation **de**:

$$de = db \cdot r / (r-z)$$

and when  $r \gg z$  for small values of  $\phi$  we can assume  $e \approx c \approx b \approx g$  and hence  $de \approx dc \approx db \approx dg$

However, if we express the elongation as a percentage of its original dimension, we obtain (from equation (4)):

$$d\epsilon = [dg / g_0] \cdot 100 = [db / g_0] \cdot 100 \approx [db / e_0] \cdot 100$$

and the influence of the error ( $\Delta db$ ) on the variation of the elongation (**dg**) of the front face baseline **g** is therefore:

$$\Delta d\epsilon = [\Delta db / g_0] \cdot 100 \approx [\Delta db / e_0] \cdot 100$$

Therefore we can see that  $\Delta d\epsilon$  is inversely proportional to **g<sub>0</sub>** i.e. to **e<sub>0</sub>**.

### C) Resolution of individual transducers

1 As reported in § A above, the data sheet specifies that the resolution of the transducers is virtually  
2 infinite. However, the actual resolution in reading the transducers is given by the resolution of the  
3 data-loggers. For instance, for the equipment which has mostly been used (see § 2.2.1):

- 4 - the data-logger provides a measuring range of 2.5 V and a resolution of 12 bits; since  $2^{12} =$   
5 4 096, the reading resolution (1 digit) corresponds therefore to  $2\,500/4\,096 = 0.61\text{ mV}$
- 6 - the transducers have a measuring range of 30 mm
- 7 - the reading resolution corresponds therefore to 1 digit =  $30 / 4\,096 = 0.0073\text{ mm} = 7.3\text{ }\mu\text{m}$ .

#### 8 9 10 **D) Resolution of the whole DK**

11 Since the final formulas derived above express the propagation of errors through the geometry of  
12 the DK, the DK's resolution for each of the computed parameters ( $\varphi$ ,  $c$ ,  $g$ ,  $\epsilon$ , ...) can also be  
13 obtained from such formulas, by replacing the estimated error  $\Delta a$  or  $\Delta b$  with the resolution of the  
14 data-logger readings of the transducers. However, such formulas are affected by some  
15 approximation, and some results might be inaccurate in some way. Another method, less general  
16 but more intuitive and accurate, is the following, applicable for any individual case: if the DK's  
17 geometric dimensions are known, the resolution can also be obtained through computing by  
18 means of the "Maschera Kit" the output variations produced by inputting a variation of plus or  
19 minus 1 digit (by definition, the resolution) of the measurements provided by the transducers  
20 (parameters "a" and/or "b") in their most extreme combination. The resolution figures reported in  
21 **Table 1** have been computed according to this latter method.

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25 **Insert Table 1**

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1 **TABLE CAPTIONS**

2  
3 **Table 1** - Geometrical data and the resulting measurement parameters for five actual DK case-  
4 studies (Case 1: Replica panel, named “A”, kept in DEISTAF’s climatic chambers. Cases 2-3-4:  
5 original panel paintings by Beato Angelico, exhibited in San Marco museum, Florence; 2-“Pala del  
6 Bosco ai Frati”, 3-“Trittico di San Pietro Martire”, 4-“Armadio degli Argenti”. Case 5: test panel in  
7 LMGC laboratory, Montpellier).  
8

9  
10 Key to the headings of the table, and related measurement units:

11  
12 **I - DK’s geometric parameters** (see Fig. 1):

13      $e_0$      Mounting distance between the columns  
14      $z$        Distance between the lower transducer and the back face  
15      $m$        Distance between the two transducers  
16      $f$         Thickness of the board  
17

18  
19  
20 **II - Accuracy of the individual transducer lines (transducer + data-logger + calibration)** (see  
21 § A) (microns):

22      $\Delta da = \Delta db$   
23

24 **III - Accuracy of the calculated parameters** (see § B):

25      $\Delta d\varphi$    Accuracy (maximum estimated error) of variation  $d\varphi$  of cupping angle  $\varphi$  (thousands  
26                   of a degree)  
27      $\Delta dc$      Accuracy (maximum estimated error) of elongation  $dc$  of the back face (microns)  
28      $\Delta dg$      Accuracy (maximum estimated error) of elongation  $dg$  of the front face (microns)  
29      $\Delta d\varepsilon$     Accuracy (maximum estimated error) of percent elongation  $\varepsilon$  of the front face (%)  
30  
31

32 **IV - Resolution of the individual transducer lines (transducer + data-logger)** (see § C)  
33 (microns)  
34

35 **V - Resolution of the calculated parameters** (see § D):

36      $\varphi$         Resolution of the measurement of variation of cupping angle  $\varphi$  (thousands of a  
37                   degree)  
38      $c$         Resolution of the measurement of length  $c$  of the back face (microns)  
39      $g$         Resolution of the measurement of length  $g$  of the front face (microns)  
40      $\varepsilon$        Resolution of the measurement of percent elongation  $\varepsilon$  of the front face (%)  
41  
42

43 As mentioned in § D, the resolutions reported in this Table have been obtained through computing  
44 by means of the “Maschera Kit” the output variations produced by inputting a variation of plus or  
45 minus 1 digit (by definition, the resolution) of the measurements provided by the transducers (“a”  
46 and/or “b”) in their most extreme combinations, described below:

- 47     - column (i): variation by +1 digit of the input from transducer (a) alone  
48     - column (ii): variation by +1 digit of the input from transducer (b) alone  
49     - column (iii): variation by -1 digit of the input from transducer (a) and by +1 digit of the input  
50       from transducer (b)  
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Table 1 (belongs to the Annex)

**Table 1**

	I				II	III				IV	V										
Case	$e_0$	z	m	f	$\frac{\Delta da}{\Delta db}$	$\Delta d\phi$	$\Delta dc$	$\Delta dg$	$\Delta d\epsilon$		$\phi$	c			g			$\epsilon$			
	(mm)	(mm)	(mm)	(mm)	( $\mu\text{m}$ )	(Degrees $\cdot 10^{-3}$ )	( $\mu\text{m}$ )	( $\mu\text{m}$ )	(%)	( $\mu\text{m}$ )	(Degrees $\cdot 10^{-3}$ )		(μm)			(μm)			(%)		
											(i)=(ii)	(iii)	i	ii	iii	i	ii	iii	i	ii	iii
1	184	19.5	70	38	±40	±65.5	±62	±106	±0.022	7.3	0.006	0.012	2	9	11	6	13	19	0.003	0.008	0.011
2	239	14.5	43	38	±40	±106.6	±67	±138	±0.017	7.3	0.01	0.02	2	10	12	9	16	25	0.004	0.007	0.011
3	238	14.5	58	28	±40	±79	±60	±99	±0.017	7.3	0.007	0.014	2	9	11	6	13	18	0.002	0.006	0.008
4	238	20.8	70.9	23	±40	±64	±63	±89	±0.017	7.3	0.006	0.012	2	9	12	4	12	16	0.003	0.009	0.012
5	184	19.5	70	20	±9	±14.7	±14	±19	±0.005	7.3	0.006	0.012	2	9	11	4	12	16	0.002	0.007	0.009