Is the proton radius a player in the redefinition of the International System of Units?

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By Charge Radius Experiment with Muonic Atom collaboration :

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Résumé

It is recognized now that the international system of units will be redefined in terms of fundamental constants [1] [2] [3] [4] even if the occurring date is still under debate. Actually, the best estimate of fundamental constants values is given by a least square adjustment, carried out under the auspice of the Committee on Data for Science and Technology (CODATA) task group on fundamental constants. This adjustment provides a significant measure of the correctness and over-all consistency of the basic theories and experimental methods of physics using the values of the constants obtained from widely differing experiments. The physical theories that underlies this adjustment are assumed to be valid such as quantum electrodynamics (QED). Testing QED, one of the most precise theory is the aim of many accurate experiments. The calculations and the corresponding experiments can be carried out either on a boundless system (for example the electron magnetic moment anomaly) or on a bound system such as atomic hydrogen. The value of fundamental constants can be deduced from the comparison theory-experiment. For example using QED calculations [6], the value of the fine structure constant given by the CODATA mainly infers from the measurement of electron magnetic moment anomaly done in the group of G. Gabrielse [5]. The value of Rydberg constant is known from the two photon spectroscopy of hydrogen combined with accurate theoretical quantities. The Rydberg constant extracted from the theory-experiment comparison of atomic hydrogen is known with a relative uncertainty of 6.6×10^{-12} [7]. It is the most accurate fundamental constant up to date. A careful analysis shows that the knowledge of the electrical size of the proton is nowadays a limitation to this comparison. The aim of the muonic hydrogen spectroscopy was to get an accurate value of the proton charge radius. However, the value deduced from this experiment is in contradiction with the other less accurate determinations [8]. The problem is known as the proton radius puzzle. In this paper, we will look at the possible consequences of the proton puzzle to the redefinition of SI based units. After a short introduction of the proton properties, we will describe the muonic hydrogen experiment. An intense theoretical activity derives from our observation. A brief summary of possible theoretical explanations at the present date of the writing of the paper will be

given. Contribution of the proton radius puzzle to the redefinition of SI based units will be then examined.

Keywords : proton radius ; muonic hydrogen ; Rydberg constant ; Planck constant, Avogadro constant

1 The proton

Even though the proton is one of the most abundant constituent of the visible Universe, some of its properties are not well known. Study of its properties is an important issue of our deep understanding of the matter. Proton is made of three valence quarks (up, up, down) kept together by strong interactions. Ab-initio calculations can be down on this structure using quantum chromodynamics (QCD) theory.

An important step has been the calculation of the mass of the proton using QCD with a relative uncertainty better than 4% [9] [10]. Experimentally the absolute mass of the proton is known with a relative uncertainty of 5×10^{-8} [7].

The ab-initio calculation of the spin of the proton (i.e. 1/2) has been also undertaken. The present state of the problematic is known as the "spin crisis" [11]. It refers to the experimental finding that only a small contribution of the spin of the proton seems to be carried by the quarks [12] [13] [14]. Indeed the study of the spin structure of the nucleon is an important task of the particle physics either theoretically [15] or experimentally [16].

Some attempts have been also done to estimate the proton charge radius with QCD. Some values of r_p have been published [17] [18]. New and more reliable values of proton radius should be given in the future as the calculation capabilities are increasing with time.

2 Experimental determinations of the proton charge radius

2.1 Scattering experiments

In fact the best knowledge of the proton charge radius comes from experiments. First determinations were given by electron-proton elastic scattering experiments. The principle of those experiments is to measure the differential cross section of a scattered electron beam sent onto a thin hydrogen (H_2) target. The relevant parameter is the space-like momentum transfer -Q^2. The proton mean square charge radius is given by the slope of the Sachs form factor (G_E) of the proton at Q^2=0.

$$\langle r_p^2 \rangle = -6 \frac{\partial G_E(Q^2)}{\partial Q^2} |_{Q^2=0}. \tag{1}$$

The proton radius is defined as

$$r_p = \sqrt{\langle r_p^2 \rangle}. \tag{2}$$

The main contribution to r_p comes from low momentum measurements. Even if conceptually the experiment is simple, practically it is not. The space-like momentum transfer cannot be made arbitrary small. The electron must cross the target in which a single elastic scatter has to occur. Measurements nearby Q^2=0 must not be affected by the direct electron beam. Therefore the proton radius can be only obtained from an extrapolation of the differential cross section at Q^2=0. This extrapolation is strongly model-dependent. A first determination of r_p have been obtained in 1955 [19]. In the 80s, an experiment has been specially built in Mainz to get an accurate value of r_p from low momentum transfers. The analysis of the data done in Mainz gives r_p=0.862(12) fm [21] in strong disagreement with an accurate previous determination [20]. Up to 2000's, no new scattering experiment have been carried out at low momentum transfers. All the different values of r_p which have been published come from a re-analysis of the low momentum scattering data associated or not to high momentum scattering data (for example [22] [23]). The re-analysis considered by the CODATA task group has been obtained by I. Sick [24]. This work takes into account the world data on elastic electron proton scattering, the Coulomb distortion, and uses a parametrization that allows to deal properly with the higher moments. In the 2000's a sophisticated experiment has been carefully designed at Mainz to measure accurately r_p [25]. Taking advantage of three high-resolution spectrometers, it was possible to measure the elastic electron-proton scattering cross section with a statistical precision of better than 0.2%. The value of r_p published in 2010, deduced from an analysis done in Mainz with their data is 0.879(8) fm [26]. Another value of r_p from scattering experiment has been published very recently [27]. This value of the proton radius is deduced from the global analysis done in [28] in which the new measurements done at "high" Q^2 [27] are included.

2.2 Hydrogen spectroscopy

At short distance, the Coulombian electrostatic potential is "shielded by the finite size of the proton". Consequently the energy levels of atomic hydrogen are slightly shifted. Therefore a value of r_p can be obtained from high resolution spectroscopy of hydrogen. A simplified but powerful analysis of the main hydrogen data has been presented in [29] using only the most accurate experimental data : the frequency of the 1S-2S transition measured in Garching [30], the 2S-8S/8D transitions measured in Paris [31] and the $1/n^3$ dependence of QED corrections [32] [33] (where n is the principal quantum number of hydrogen theory). A precise value of the 1S Lamb shift of hydrogen can be extracted from a proper linear combination of those three quantities. Assuming the exactness of QED calculations, an accurate value of r_p can be deduced. A complete analysis of all the spectroscopic measurements is also regularly done by the CODATA task group on fundamental constants (see for example [7]). The last resulting spectroscopic value of r_p is 0.8760(78) fm.

2.3 Muonic hydrogen spectroscopy

The aim of the muonic hydrogen spectroscopy of the 2S-2P splitting was to determine more accurately the proton radius. Muonic hydrogen (μ-p) is an exotic atom in which the muon (μ^-) replaces the electron "orbiting around" the proton in normal atomic hydrogen. As electron, muon is a lepton but 207 times heavier and its lifetime is only 2.2 μs. Because of the mass dependence, the Bohr radius of μ-p is around 200 times smaller than the one of electronic hydrogen, the sensitivity to finite size of the proton is then largely enhanced. The contribution due to the finite size of the proton is around 2 % to the 2S-2P splitting of muonic hydrogen whereas it is only 1.4×10^{-4} for the $2S_{1/2}$-$2P_{1/2}$ of electronic hydrogen. Another important issue in the comparison between muonic and electronic hydrogen, is the vacuum polarization which is the dominant contribution to the 2S-2P splitting of μ-p. As the Bohr radius is 200 times smaller, the overlap of S state wave-functions with the distribution of virtual electron-positron pairs is more important, consequently the contributions of the vacuum polarization corrections are larger. The wavelength of the 2S-2P transition in μ-p is 6 μm and the oscillator strength 10^{-7} of the one of electronic hydrogen. Moreover the population of muonic hydrogen in 2S state is small. Because of those considerations this experiment has been really challenging.

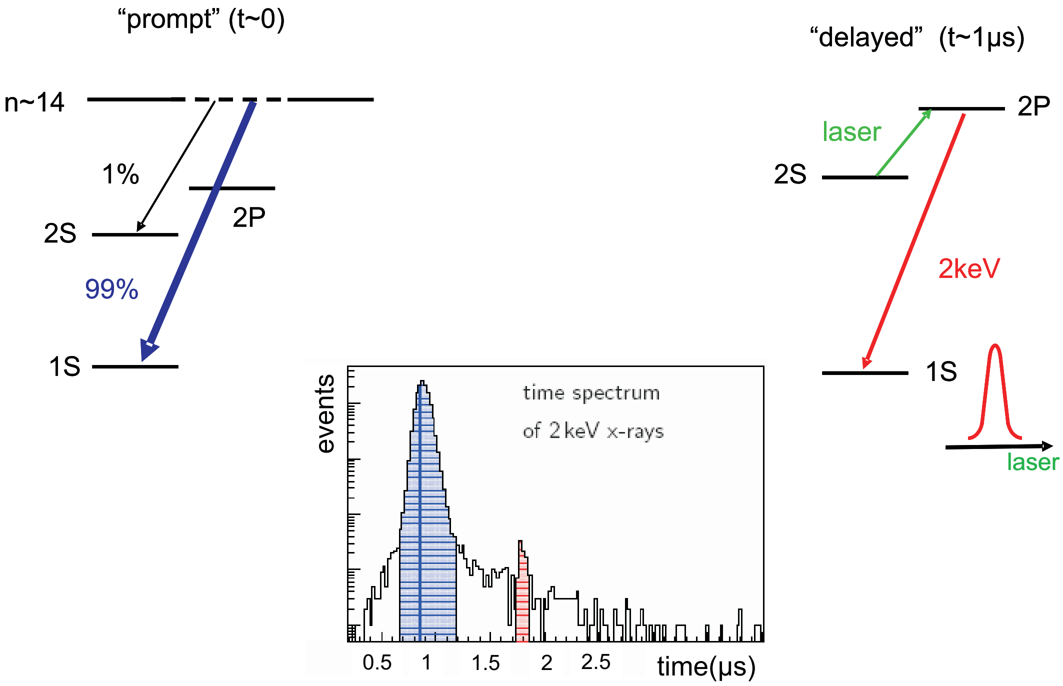

FIG. 1 – *Principle of the muonic hydrogen experiment. Muons are stopped in 1 hPa H_2 gas. 99% goes under radiative cascade to 1S state, producing a large prompt peak of 2 keV radiation. The 2S-2P transition is detected with the smaller peak due to the laser induced X rays, at the time of the laser arrival ($\sim$0.7 μs) to the muon stop volume.*

3 Muonic hydrogen experiment

3.1 Principle of the experiment

The principle of the experiment has been described in many papers [34]. Let us recall the mains step. Muons (μ^-) beam is send into a H_2 gas target. Muonic hydrogen atom is formed at principal quantum number around n=14 : the muon goes from the continuum to a bound state by ejection of an Auger electron. Subsequently the H_2 molecule breaks up and the muonic cascade of the neutral μ-p atom begins. Radiative and collisionally induced deexcitation processes bring it down to 2S or 1S state within a few nanoseconds. This cascade has been carefully studied [35] specially the collisional quenching of the 2S state [36] [37] [38]. 99% of atoms decay to 1S state, producing a large 2 keV fluorescence prompt peak. About 1% of muonic hydrogen are formed in a long-lived 2S state [39]. A laser pulse triggered by muons (see after) is send into H_2 target. Because of the short delay between laser trigger and output of the laser, a time-delayed 2 keV fluorescence peak is observed if the 2S-2P transition has been excited by the 6 μm light.

3.2 Apparatus

The challenges of the muonic hydrogen experiment were the production of long-lived muonic hydrogen in 2S state, the realization of the laser at 6 μm randomly trigerrable with a short delay and the analysis of the small signal awaited (few counts/hours).

To reduce collisional quenching of the 2S state, muons are stopped in H_2 gas at pressure of 1 hPa. For that a special low energy muon beam ($\sim$keV) has been designed and built at Paul Scherrer Institut to efficiently stop muons at this ultra-low gas pressure within the small target volume required for efficient laser excitation. At 1hPa, the lifetime of the 2S metastable state is around 1 μs [36] [37]. Many details of this muon line can be found in [40] [35] [41] [42] [43] [44]. We just emphasis that a special care has been put on the quality of the signal triggering the laser. The detection of keV-muons without stopping them is a non trivial task. It is done with thin carbon foils stack which simultaneously acts as muon detector and improves the beam quality by frictional cooling [45].

The design of the laser is dictated by the need of a tunable light source at 6 μm triggerable within 1 μs after a random trigger by incoming muons. The laser chain used in 2009 run derives from many development of the initial laser chain used in 2003 [41]. The fast and powerful triggerable pulsed laser at 515 nm is used to pump an oscillator-amplifier titanium sapphire (TiSa) laser. Three consecutive vibrational Raman scattering in H_2 in a multiple-pass cell are used to convert the 708 nm light into 6 μm pulse. The 6 μm light is then send, 20 m away, in a specially design multi-pass non resonant cavity surrounding the H_2 target in which μ-p atoms are formed. The later cavity is used to illuminate all the stopping volume of the muon in the target ($5\times15\times190$ mm^3). The frequency of the 6 μm light is driven by the one of the TiSa oscillator which is seeded by a cw-TiSa laser. The frequency of the cw-laser is permanently controlled with two wavemeters, a very stable Fabry-Perot cavity and atomic/molecular lines (I_2, Cs, Rb). The cw laser is permanently locked on a Fabry-Perot fringe. The step of the scanning of the laser frequency is a multiple of the free spectral range of this Fabry-Perot interferometer [8]. Whereas the design of TiSa ensemble remains the same between the first and the last runs [46], the design of the green pump laser has to be changed drastically. A designed thin disk laser has to be built specially for our experiment [47].

The analysis of small signal requires a very good knowledge of the background. Since the early stage of the experiment, the 2 keV detectors (Large Area Avalanche Photodiodes, LAAPDs) have been well studied in order to maximize their time and energy resolutions [42] [43] [48]. During the data analysis, background events are efficiently rejected. The final rate of about 1 background event per hour originates mainly from electrons from muon decay which are wrongly identified as 2 keV Lyman-α x-rays.

3.3 Results

At least, many muonic hydrogen lines have been observed during the fourth beam time period. The first line which has been observed and analysed is the most intense one : $2S_{1/2}$(F=1)-$2P_{3/2}$(F=2) of muonic hydrogen (see figure 2). The signal is clearly above the noise floor. 550 events are measured in the resonance where 155 background events are expected. A Lorentzian profile is adjusted on the data to give the frequency of the line versus Fabry-Perot fringe. The absolute frequency of the line is performed from absolute calibration of the light at 6 μm using the H_2O well known spectroscopy and the free spectral range of the Fabry-Perot interferometer (see figure 3). A careful and detailed analysis of the uncertainty budget of the centroid position of this line has been done in [8] [44].

The quantity determined by our experiment is the absolute frequency of $2S_{1/2}$(F=1)-$2P_{3/2}$(F=2) resonance of muonic hydrogen : 49 881.88(76) GHz. The main contribution to the uncertainty is statistical; it is given by the fit of the Lorentzian shape on the experimental data. The relative uncertainty of the centroid position (1.6×10^{-5}) corresponds to 4% of the line width ($\sim$18 GHz) which exhibit the fact that our experiment is far away from a high resolution spectroscopy one, but which also means a very weak dependance of the centroid of the resonance to the line shape model.

Nevertheless, the present result is good enough to extract the proton radius with the smallest uncertainty up to date. The value of the proton radius is obtained from the comparison of our determination of the frequency

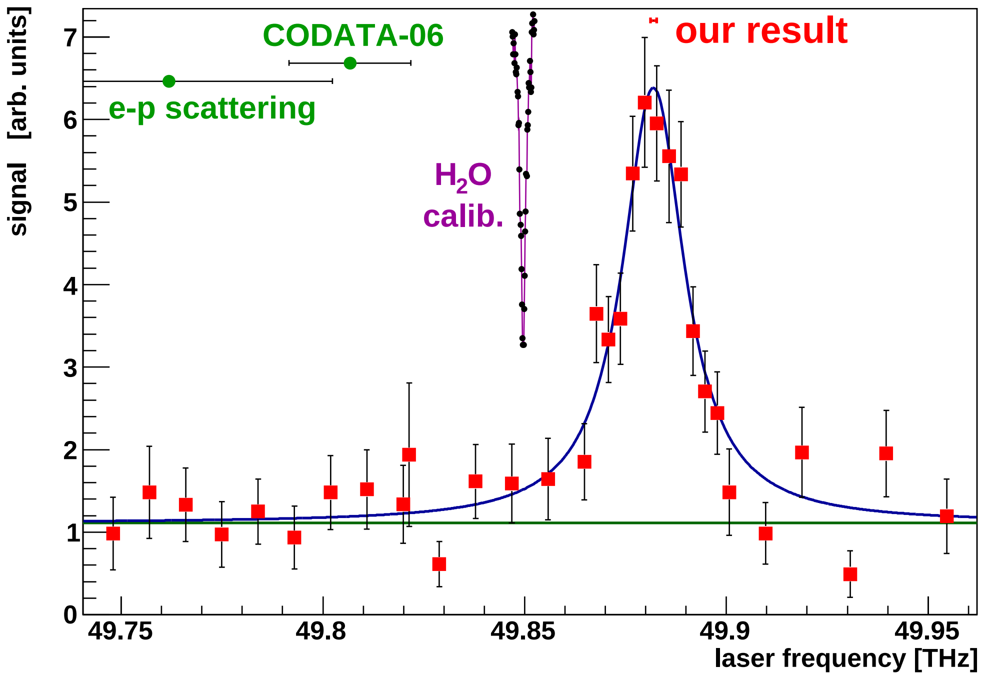

FIG. 2 – *$2S_{1/2}(F{=}1)$-$2P_{3/2}(F{=}2)$ resonance. The discrepancy with the others determinations is clearly visible. The frequency gap is around 75 GHz with the position of the line expected with electronic hydrogen. The inset calibration H_2O line has been recorded at 30 hPa.*

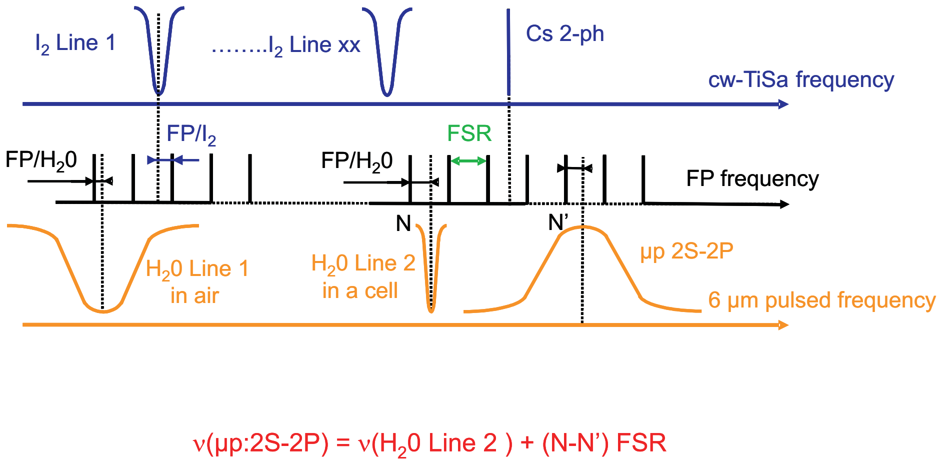

FIG. 3 – *Principle of the determination of the absolute frequency of the muonic hydrogen line. The absolute calibration has been made with 5 H_2O lines, 17 I_2 lines, 5 Cs two-photon atomic line and 3 Rb lines. Absolute calibration between 5.49 µm and 6.04 µm is performed with H_2O lines with the pulsed laser source while the free spectral range of the Fabry-Perot interferometer is determined in cw regime in the 695 nm-780 nm range with well known atomic/molecular lines. To detect an eventual drift of the Fabry-Perot interferometer, this one has been regularly compared to iodine lines before, during and after the beam time. The integer numbers (N,N') can be determined without any doubt thanks to our two wave-meters.*

of $2S_{1/2}$(F=1)-$2P_{3/2}$(F=2) muonic line with the theoretical prediction. The theoretical prediction derives for a large part from the bound state QED one of electronic hydrogen scaled with the mass of the muon. Those predictions account for radiative, recoil, proton structure fine and hyperfine contributions. A detailed list of the contributions can be found in [49]. The predicted frequency of the $2S_{1/2}$(F=1)-$2P_{3/2}$(F=2)transition of muonic hydrogen is :

$$\nu(GHz) = 50772.43(1.18) - 1263.69 \times r_p^2 + 8.39 \times r_p^3. \tag{3}$$

The deduced value of r_p is : 0.84184(36)(56) fm where the first and second uncertainty originate respectively from experimental uncertainty and theoretical uncertainty. This result 0.84184(68) fm differs significantly ($\sim$5 σ) with others determinations (see figure 4).

4 The proton radius puzzle

The confusing situation about r_p is known as the proton radius puzzle. It has stimulated an intense activity in the community. It is impossible to make an exhaustive list of all the contributions already available on the ArXiv base, we apologize to authors whose papers are not cited. We can only point out some of the searches at the present date.

4.1 Definition of the proton radius

An obvious concern in the comparison is to make sure that the proton radius extracted from various experiments have the same meaning. A first positive answer has been given in [50] : "*a conceivable accidental incompatibility of the conventions used in references [...] for the proton radius therefore cannot be the reason for the observed discrepancy*". [...]refereing to all the proton radius determinations.

4.2 Charge distribution in the proton

The charge radius distribution is related to G_E by the Fourier transformation. One attempt has been done to reevaluate the third Zemach moment of the proton $\langle r_p^3 \rangle$ to solve the proton radius puzzle. However the reevaluation of $\langle r_p^3 \rangle$ with the charge distribution presented in [53] is in contradiction with electron-proton scattering data [54] [55].

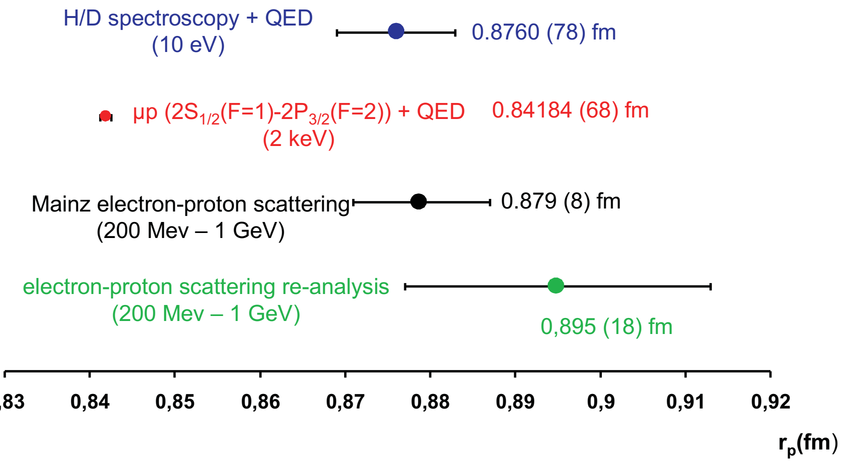

FIG. 4 – *Comparison of the various determinations of the proton radius. Defining the proton radius as the slope of the Sachs form factor, experimental results are sort from low momentum transfer (electronic hydrogen) to high momentum transfer (scattering experiments) followings [73]. This presentation emphasis the surprising result of muonic hydrogen which can be view as a "dip in the form factor slope" at the corresponding* Q^2 *momentum of muonic hydrogen experiment.*

4.3 QED corrections

The contradiction between the proton radius extracted from electronic hydrogen and from muonic hydrogen is intriguing for QED specialists. Because of the mass scaling law, QED of muonic hydrogen has always been considered to be simpler than the one of the hydrogen atom. Nevertheless, the proton radius puzzle has stimulated either careful checks of present QED corrections or evaluation of new corrections for hydrogen atoms [56] [57] [58]. Up to now, the proton radius puzzle can not be solve with new or wrong QED corrections.

4.4 Rydberg constant

The Rydberg constant is the scaling factor of atomic level. Assuming the correctness of QED calculations in electronic hydrogen atom, the problem can be solved by shifting by 5 standard deviations the value of the Rydberg constant, known with an uncertainty of 22 kHz, using the most accurate frequency measured in hydrogen [30]. However, the CODATA Rydberg constant is not only determined with 1S-2S and 2S-8S frequencies transitions but with others frequencies measurements certainly less accurate but all highly consistent together and with theory [7]. Moreover the recent measurement of the 1S-3S transition in hydrogen [59] is in very good agreement with the theoretical estimate [60]. At the present time there is no indications of a disagreement between theory and experiments in electronic hydrogen. But it is clear that the accuracy of the hydrogen experiments other than the 1S-2S one have to be improved to clarify the situation. It can be seen in figure 5 in which all the value of r_p deduced from 1S-2S transition frequency, the $1/n^3$ law and only one of the 2S-n(S,P,D) transitions frequency which has been measured. Individually, there is not a large discrepancy with the muonic determination of r_p. On the other hand, most of the values are pointing in the same direction...

Many ongoing experiments are carried out to test bound state QED. Results of the NPL experiment on the 2S-6S/D transitions [68], currently analyzed may bring an important and independent contribution to the hydrogen spectroscopy and so to the proton radius puzzle. Combined with the ongoing experiment at Garching, which aims at measuring the 1S-2S transition in He^+ [69], the experiment planed at Paul Scherrer Institut in order to measure the muonic helium ion Lamb shift (μHe^+) may illuminate the pro-

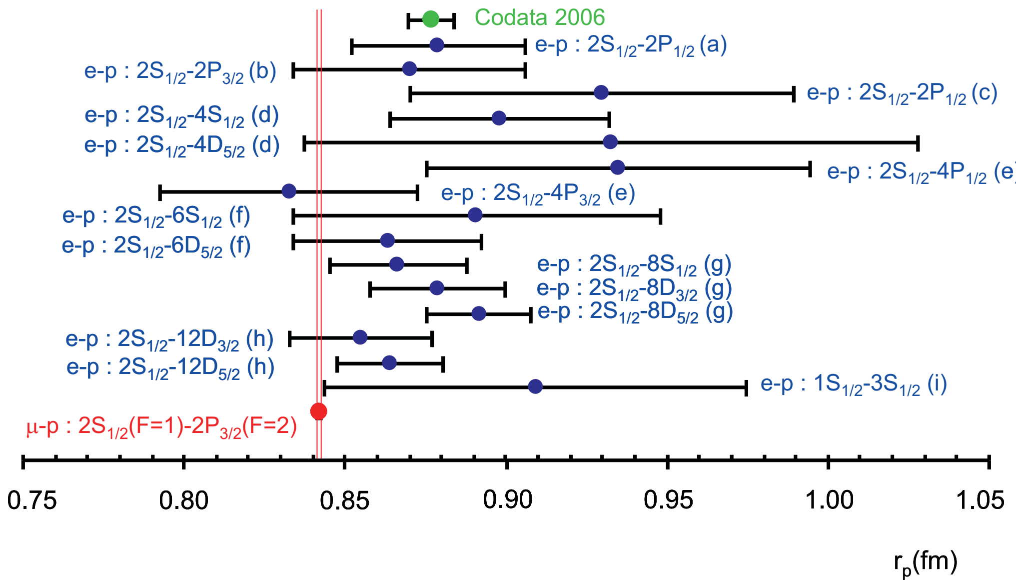

FIG. 5 – *Comparison of various determinations of the proton radius from hydrogen spectroscopy. Each value is obtained from the 1S-2S transition frequency, the $1/n^3$ law and one of the other hydrogen experimental data from 2S-n(S,P,D). (a) is from [61], (b) is from [62], (c) is from [63], (d) is from [64], (e) is from [65], (f) is from [66] combined with [59], (g) is from [31], (h) is from [67] and (i) from [59]. The double red line corresponds to the uncertainty of the proton radius determination obtained from muonic hydrogen spectroscopy.*

ton radius conundrum [70]. A determination of the Rydberg constant nearly independent on the nucleus structure has also started at NIST Gaithersburg with the study of circular states of $^{20}Ne^{9+}$ [71].

4.5 New physics

A possible way to solve the proton radius would be to introduce new "particles" which are coupled differently to the muon compared to the electron. However, this new physics search is already constrainted by many low energy data [72] [73] [74]. Indeed for example, recent laboratory-sized experiments are enough accurate to set a limit to the internal structure of the electron or to the existence of new dark matter particles assuming the exactness of QED calculations or testing for the first time the muon and hadrons contributions to the electron anomaly a_e [5] [75]. A deviation to the Coulomb's law in muonic hydrogen is also ruled out by measurements in electronic hydrogen [76].

5 The proton and the redefinition of SI units

As discussed above, the new determination of the proton radius may affect the value of the Rydberg constant R_∞. This constant is related to many fundamental constants. For example the estimate of the mass of the electron m_e in the CODATA adjustment is derived from the relation

$$m_e = \frac{2R_\infty h}{c\alpha^2} \tag{4}$$

where α is the fine structure constant, c the velocity of light and h the Planck constant. The Rydberg constant is also linking the two possible ways proposed for the redefinition of the kilogramme, the Avogadro constant N_A and the Planck constant h :

$$N_A \times h = \frac{c}{2}\frac{A_e \alpha^2 M_u}{R_\infty} \tag{5}$$

However, the current relative uncertainty on the experimental determinations of N_A [77] or h [78] is in the order of few part of 10^8. This is three orders of magnitude larger than the "possible" shift of the Rydberg constant may be disclosed by the new value of proton size radius from muonic hydrogen. The proton radius puzzle will not interfere in the redefinition of the Kilogramme.

For many years, hydrogen atom has been advocated as an "ideal" clock (see for example [79]) as in principle it can be calculated in contrary to the others clocks. Should the second being redefined with hydrogen atom in term of fundamental constants ? Today, calculations for hydrogen are about 10000 times less accurate than current experimental caesium clock precision, due to the growing complexity of the calculations of higher order corrections. The best experimental determination is also 10^3 times worse than the best optical clocks, which are front-runners to redefine the second in several years time (see P.Gill article in this issue). Improved theory with higher order corrections to two or three more orders is needed before the calculable clock approach for hydrogen becomes competitive.

Nevertheless, the finding solution of the proton radius puzzle is a step in the long quest to decode hydrogen spectrum : the "Rosetta stone of modern physics" [80].

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