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# INTERNAL GRAVITY WAVES AND BOUNDARY LAYERS FROM AN OSCILLATING CIRCULAR DISC

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The generation of internal gravity waves by oscillating bodies, a classical topic from the late 1960s and early 1970s, has been revived about a decade ago in connection with the generation of the internal tide by the oscillation of the barotropic tide over deep-ocean topography, an important topic for ocean mixing and the energy balance of the Earth–Moon system [1–3].

Most investigations of the problem are inviscid, based on the free-slip boundary condition at the body or topography and the inviscid internal wave equation within the fluid. Simple geometries were studied first, namely circular or elliptical cylinders [4–5] in two dimensions and a sphere [6–8] or a spheroid [9–10] in three dimensions, appropriate for solution in separable coordinates. For more involved geometries or real ocean topographies, a general-purpose method is required that may be implemented numerically. The boundary integral method was proposed in [11] in a steady formulation and applied to the cylinder [12], and in [13,14] in an unsteady formulation and applied to various bodies including a plate either horizontal [15], inclined [16] or vertical [17] in two dimensions and a horizontal circular disk [18] in three dimensions. However, it is only two decades later, after independent introduction in an oceanographic context for topographies of increasing complexity [19–22], that the method finally gained visibility.

Such inviscid approaches provide the radiated energy but not the wave profiles. For the latter, *a posteriori* addition of the viscosity is necessary, implemented in [23–27] and compared with experiment for the cylinder [28–30] and the sphere [31–33]. The addition is not fully consistent, though, in that the effect of viscosity is taken into account on the propagation of the waves (in the wave equation) but not on their generation (in the boundary condition). This approximation rests on the large value of the Stokes number  $S = \omega a^2 / \nu$ , with  $a$  the size of the body or topography,  $\omega$  the frequency of oscillation and  $N$  the buoyancy frequency. Physically, it implies that only the waves are retained while the other two components of the motion are neglected: the Stokes boundary layer at the body or topography, and an internal boundary layer within the fluid [34]. In order to obtain all three components, explicit consideration of the no-slip condition is required.

For specific geometries and on the approximation that the no-slip condition holds not only at the body or topography but also at its continuation through the fluid, the full viscous problem has been solved for an inclined plate [35,36] in two dimensions and a vertical cylinder [37,38], an inclined plate [39] and a horizontal circular disc performing torsional [40] or translational [41] oscillations in three dimensions. In each case the results were compared with experiment. The approach is summarized in [42,43]. For translational oscillations of the disc, the introduction of techniques inherited from rotating flows [44,45], Stokes flows [46,47] and acoustic waves [48,49] allows the approximation to be relaxed and the condition of continuity of the stresses to be imposed through the fluid in the plane of the disc [50].

The present communication considers the connections between these approaches for the only configuration to which all approaches have been applied: the vertical oscillations of a horizontal circular disc. With the disc as a limit of an oblate spheroid, the inviscid solution of [9] in separable coordinates is shown to coincide with an original solution by the boundary integral method, of double-layer type, and the alternative solution in [18] to be in error. Addition of viscosity according to [26] yields wave profiles compared in Fig. 1 to the experiments of [41]. Owing to the geometry, the Stokes boundary layer is absent and the internal boundary layer inherits its properties. The role of this layer is assessed using [50]. In the limit of large  $S$  the flow is seen to reduce to the superposition of waves forced by the free-slip condition at the disk, and a boundary layer of thickness  $1/S$  ensuring adaptation from the actual no-slip condition at the disk to the effective free-slip condition for the waves.

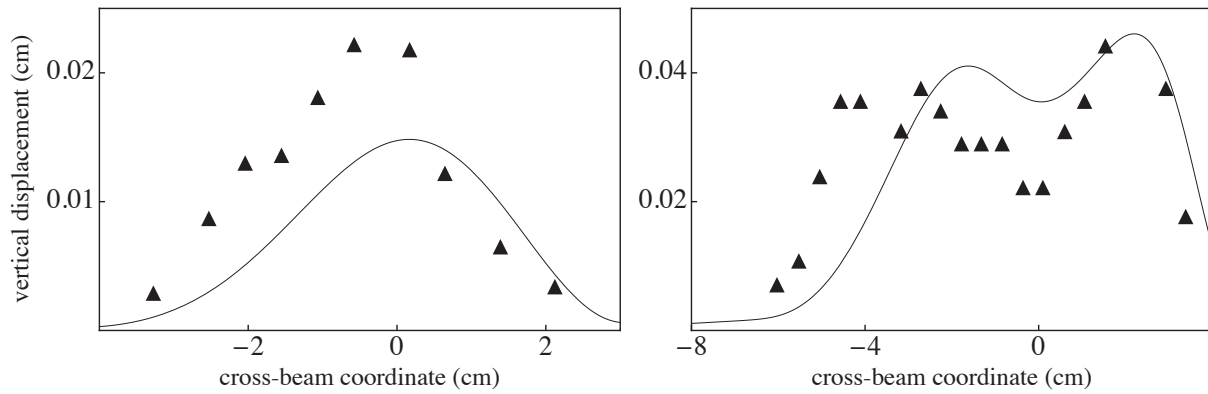


Fig. 1. Wave profiles along a vertical line at the distance  $r_0 = 8.5$  cm from the centre of a disc oscillating at the velocity  $u_0 = 1$  mm/s in a fluid of kinematic viscosity  $\nu = 1$  mm<sup>2</sup>/s, for a small disc of radius  $a = 1.75$  cm with  $\omega = 0.7$  s<sup>-1</sup> and  $N = 1.1$  s<sup>-1</sup> so that  $S = 170$  (left), and a large disc of radius  $a = 4.0$  cm with  $\omega = 0.628$  s<sup>-1</sup> and  $N = 0.9$  s<sup>-1</sup> so that  $S = 1000$  (right). The location of each measurement is specified in terms of its distance to the centreline of the wave beams, not in terms of its vertical coordinate. The triangles represent the data of [41], and the continuous lines the present theory.

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