Lateral Dynamics Reconstruction for Sharp'71 Motorcycle Model with P2I Observer

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***Abstract*— The main objective of this paper is the reconstruction of lateral dynamics and both roll angle and steering torque of single track vehicles (motorcycle, scooter, etc.). For that purpose, the well-known motorcycle model developed by Sharp in 1971 is used. This model characterizes the lateral dynamics of a motorcycle [16]. The roll angle is not observable in the obtained structure, for overcoming this problem, the model is transformed in order to take into account the roll angle as an unknown input as well as the steering torque. A Proportional two integrals (P2I) observer is then proposed for estimating simultaneously all the variable states, the lateral forces and both roll angle and steering torque. This study is a part of the ongoing work of the research team on the design of preventive safety systems for motorcycles users. Simulation results and discussions are given in order to illustrate the effectiveness of the proposed observer.**

I. INTRODUCTION

Recently, the park of single track vehicles is constantly increasing, upsetting driving practices and road traffic. Unfortunately, this expansion resulted in a growth of traffic fatalities. The statistics endorse this statement and riders are considered as the most vulnerable road users. In 2010, the French Agency of Road Safety made a finding of around 1000 deaths (25% of traffic fatalities), while the traffic volumes of motorcycles does not exceed 1%, [14]. Many research projects are initiated to fulfill this issue in order to propose an enhancement in term of security through preventive and / or active safety systems, [1], [3].

The achievement of safety systems depends on the proper knowledge of: 1) the dynamics of single track vehicle, and 2) the evolution of its states (observation/estimation) , and to lesser extent, 3) the road geometry. Regarding the first point, several studies were carried out in order to understand the motorcycle dynamics [17], [6], stability analysis (eigenmodes) of PTW [2], optimal and safe trajectories [4] and the proposition of risk functions [19], [8] to control loss or equilibrium margins. These works are little sustainable if they are not supported by an efficient sensor system helping in the estimation of some dynamic states.

The use of sensors is not always possible for two main reasons: 1) instrumentation can be very expensive and leading inevitably to expensive new bikes, and 2) according to used technologies, the measurement noise can seriously compromise the future safety systems. Accordingly, we suggest the use of robust observation techniques to overcome the aforementioned shortcomings. Nevertheless, including all methodologies (Luenberger observer, Takagi-Seguno based observer, Extended Kalman filter), very few studies have been conducted on the estimation of motorcycle dynamic states [9], [20]. The present paper proposes a robust proportional two integral (P2I) approach, [11], [10], [12], [15], helping in states observation of linear motorcycle model and the reconstruction of rider's action (steering torque). Disturbance and the rider action are assumed to be almost affine. This assumption is quite realistic according to common variation of roads profile. An H_∞ performance index is included during the design in order to attenuate the effect of the second derivative of the Unknown Input (UI) on the estimation error.

This paper is organized as follows: section III is dedicated to the motorcycle model description. Section IV presents the robust estimation of the motorcycle states and UI reconstruction. Finally, simulation results and conclusions complete the paper.

II. OVERVIEW ON SAFETY QUANTIFICATION

One of the long-term objective of our studies is concern with the problem of quantifying the risk of loss-of-control over a motorcycle when cornering. To make a Safe Cornering, riders should observe some wariness : 1) maintain an appropriate speed when entering the corner, 2)choose a good position on the road, 3) use Counter steering phenomenon to lean the bike and 4) in curve, maintain a constant speed and smoothly accelerate when exit.

These principles must be inflected with other factors that may make the situation critical. Indeed, a road with limited adhesion, 2) poor weather conditions are dangerous.

Regarding early warning systems, loss of control, for standard cars, is expressed by the maximum speed at which a vehicle can be kept on the road while moving at a constant speed on a circular section giving by:

$$v_{max} = \sqrt{\frac{g \cdot \mu_{lat}}{\rho}} \tag{1}$$

where, g is the acceleration due to gravity and μ_{lat} is the maximum available side friction, v_max represents the authorized longitudinal velocity in cornering situation and ρ is the curvature of the road.

Other risk function are proposed by the National Highway Traffic Safety Administration (NHTSA) which recommend a maximum safe speed governed by the following equation:

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$$v_{max} = \sqrt{\frac{g}{\rho}\left(\frac{\phi_r+\mu_{lat}}{1-\phi_r\mu_{lat}}\right)} \quad (2)$$

where ϕ_r is the road super-elevation angle. Please refer to [19] for more precise models adapted for motorcycles.

In general, computing the lateral friction μ_{lat} involves all the dynamic states of the bike and a good interpretation of the tire-road contact. This makes the success of such warning system strongly dependent on the availability of dynamic states of the motorcycle (efficient sensors or robust observers).

III. Dynamic description and model of motorcycle

To study the dynamics of Two Wheels vehicle, often it is assumed that overall vehicle and driver consists of interconnected rigid bodies with the possibility of rotation of each body around predefined axes[7]. According to the assumed number of rigid bodies, a number of corresponding degrees of freedom which characterizes the system motion is considered. Then, motorcycle's motion can be characterized by two main modes: in-plane mode representing movements in its plane of symmetry (longitudinal and pitch displacements) and the out-of-plane mode represented by the lateral dynamics when cornering [17].

In this work we exploited the reference model Sharp'71 [16], which is a linear one when assuming both a constant forward speed and Pacejka's linear form of the lateral forces. The study concerns the normal driving situation with where the driver do not make high maneuvers (small roll and steering angle) and not sudden movement (small variations of the steering torque) which justifies all the linearizations leading to a linear model.

The considered model of the motorcycle contains two bodies: the rear body G_r which includes the chassis, engine and the rear wheel, the front body G_f which represents the steering assembly and the front wheel. This configuration allows 4 degrees of freedom: the lateral displacement y, the yaw rotation ψ, the roll inclination ϕ and the handlebar steering angle δ with respect to the rider torque input τ applied on the motorcycle's handlebar (Figure 1).

According to all these assumptions, The resulting set of four second-order differential equations of motion is compactly written as:

$$\mathbf{M}\begin{bmatrix}\dot{v}_y\\ \ddot{\psi}\\ \dot{F}_{yf}\\ \dot{F}_{yr}\\ \ddot{\delta}\\ \ddot{\phi}\end{bmatrix}+\mathbf{H}\begin{bmatrix}v_y\\ \dot{\psi}\\ F_{yf}\\ F_{yr}\\ \dot{\delta}\\ \dot{\phi}\end{bmatrix}+\mathbf{P}\begin{bmatrix}\delta\\ \phi\end{bmatrix}+\mathbf{G}\begin{bmatrix}\tau\end{bmatrix}=\mathbf{0} \quad (3)$$

A full description of all terms of matrices is given in the Appendix.

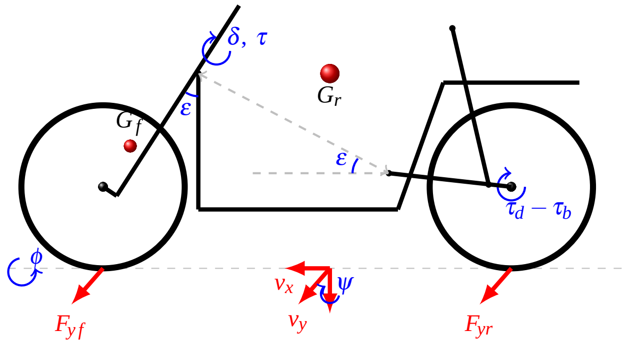

Fig. 1. Geometrical representation of the Sharp's motorcycle model

Now we recast the equation of motion in state space formulation as follows:

$$\dot{\tilde{x}}(t) \quad = \quad \mathscr{A}\tilde{x}(t)+\mathscr{B}\tau \quad (4)$$

The components of the state variable vector $\tilde{x}(t)$ are motion coordinates lateral forces and velocities, such that $\tilde{x}(t)=[\delta \quad \phi \quad v_y \quad \dot{\psi} \quad F_{yf} \quad F_{yr} \quad \dot{\delta} \quad \dot{\phi}]^T$.
The Linear Time Invariant matrices $\mathscr{A}$ and $\mathscr{B}$ are given by:

$$\mathscr{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{P} & -\mathbf{M}^{-1}\mathbf{H}\end{bmatrix},\ \mathscr{B}=\begin{bmatrix}\mathbf{0}\\ -\mathbf{M}^{-1}\mathbf{G}\end{bmatrix}$$

Measuring the roll angle rate $\dot{\phi}(t)$, one can note that it is not possible to obtain directly $\phi(t)$ by integration. Indeed, computing $\phi(t)$ from its derivative requires the knowledge of the initial conditions which are not necessarily known. Then the second component $\phi(t)$ of the state vector is not observable. To overcome this limitation, the system's equations are rewritten by separating the observable and non observable state variables. In the obtained new model, the dimension of the state vector is reduced and it does not contain the roll angle $\phi(t)$. The roll angle together with the steering torque are considered as unknown inputs. One obtains the following model:

$$\dot{x}(t) \quad = \quad Ax(t)+B_1\ \phi+B_2\ \tau \quad (5)$$

$$y(t) \quad = \quad Cx(t) \quad (6)$$

where the new state vector $x(t)$ is the state vector given by $[\delta \quad v_y \quad \dot{\psi} \quad F_{yf} \quad F_{yr} \quad \dot{\delta} \quad \dot{\phi}]^T$.

$$A=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{P(:,1)} & -\mathbf{M}^{-1}\mathbf{H}\end{bmatrix},\quad B_1=\mathbf{P(:,1)}$$

$$B_2=\begin{bmatrix}\mathbf{0}\\ -\mathbf{M}^{-1}\mathbf{G}\end{bmatrix}$$

$$C=\begin{bmatrix}1&0&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{bmatrix}$$

As a conclusion, the motorcycle model, is expressed as a state space linear model with two unknown inputs: the roll angle and the steering torque. In the next section, a Proportional two Integrals observer (P2I) is designed for estimating simultaneously the state and unknown inputs.

IV. Proportional Two Integrals Observer Design

Observers design for linear models is a problem which has been dealt with intensively since Luenberger's work [13]. And Proportional-Integral (PI) observer is one of the extensions based on adding an integral action of the estimation error which makes it effective in estimating constant and slowly time varying unknown inputs. The PI observer can be extended for the estimation of non constant UI having a polynomial form by the use of Proportional Multiple Integral loops (PMI). Here it is assumed that the two inputs (roll angle and steering torque) are unknown. The proposed observer considers two integral actions which leads to a proportional two-Integrals (P2I) observer which is able to estimate the two inputs and their first derivatives. Consequently, The form of the unknown inputs for which the asymptotic convergence can be obtained are in first order polynomial form. This correspond to the condition that the second derivatives of the unknown inputs are zero. In order to extend the class of unknown inputs which can be estimated by the P2I observer, it is possible to relax the last condition by assuming that the second derivatives of the UI are not zero but bounded. The observer is then designed in such a way to estimate the unknown inputs, their first derivatives and to minimize the effect of the bounded second derivatives on the state and unknown input estimation errors.

For that purpose the previous system (5) is augmented by considering additional state variables $\dot{\xi}_1 = \ddot{\phi}$, $\dot{\xi}_2 = \xi_1 = \dot{\phi}$, $\dot{\xi}_3 = \ddot{\tau}$, $\dot{\xi}_4 = \xi_3 = \dot{\tau}$ which leads to the augmented system described by state vector, $\bar{x} = \begin{bmatrix} x^T & \xi_1 & \xi_2 & \xi_3 & \xi_4 \end{bmatrix}^T$. The new state space system is given by:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}\bar{u}(t) && (7) \\ y(t) &= \bar{C}\bar{x}(t) && (8) \end{aligned}$$

with

$$\bar{A} = \begin{bmatrix} A & \mathbf{0} & B_1 & 0 & B_2 \\ \mathbf{0} & 0 & 0 & 0 & 0 \\ \mathbf{0} & 1 & 0 & 0 & 0 \\ \mathbf{0} & 0 & 0 & 0 & 0 \\ \mathbf{0} & 0 & 0 & 1 & 0 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\bar{u}(t) = \begin{bmatrix} \chi(t) \\ \alpha(t) \end{bmatrix} \text{and } \bar{C} = \begin{bmatrix} C & \mathbf{0} \end{bmatrix}$$

where $\chi(t) = \ddot{\phi}(t)$ and $\alpha(t) = \ddot{\tau}(t)$.

The P2I observer has the following form:

$$\left\{ \begin{array}{rcl} \dot{\hat{x}}(t) &=& A\hat{x}(t) + B_1\hat{\xi}_2 + B_2\hat{\xi}_4 + K_p(y(t) - \hat{y}(t)) \\ \dot{\hat{\xi}}_1(t) &=& K_{i1}(y(t) - \hat{y}(t)) \\ \dot{\hat{\xi}}_2(t) &=& \hat{\xi}_1 + K_{i2}(y(t) - \hat{y}(t)) \\ \dot{\hat{\xi}}_3(t) &=& K_{i3}(y(t) - \hat{y}(t)) \\ \dot{\hat{\xi}}_4(t) &=& \hat{\xi}_3 + K_{i4}(y(t) - \hat{y}(t)) \end{array} \right. \quad (9)$$

It is clear that $\hat{\xi}_1 = \dot{\phi}$ is estimated in the initial state vector $\hat{x}(t)$, so we can use it to compute directly $\hat{\dot{\phi}}$ without need to minimize $\ddot{\phi}$.

Then the new obtained P2I observer can be expressed as follows:

$$\left\{ \begin{array}{rcl} \dot{\hat{x}}(t) &=& A\hat{x}(t) + B_1\hat{\xi}_2 + B_2\hat{\xi}_4 + K_p(y(t) - \hat{y}(t)) \\ \dot{\hat{\xi}}_2(t) &=& \hat{\dot{\phi}} + K_{i1}(y(t) - \hat{y}(t)) \\ \dot{\hat{\xi}}_3(t) &=& K_{i2}(y(t) - \hat{y}(t)) \\ \dot{\hat{\xi}}_4(t) &=& \hat{\xi}_3 + K_{i3}(y(t) - \hat{y}(t)) \end{array} \right. \quad (10)$$

where K_p, K_{i1}, K_{i2} and K_{i3} are the observer gain matrices. By defining the new augmented state vector $\bar{x} = \begin{bmatrix} x^T & \xi_2 & \xi_3 & \xi_4 \end{bmatrix}^T$, the state space observer becomes:

$$\dot{\hat{\bar{x}}}(t) = \bar{A}\hat{\bar{x}}(t) + L(y(t) - \bar{C}\hat{\bar{x}}(t)) \quad (11)$$

where

$$\bar{A} = \begin{bmatrix} A & B_1 & 0 & B_2 \\ \Psi & 0 & 0 & 0 \\ \mathbf{0} & 0 & 0 & 0 \\ \mathbf{0} & 0 & 1 & 0 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} \mathbf{0} \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{and } \bar{C} = \begin{bmatrix} C & \mathbf{0} \end{bmatrix}$$

and

$$\Psi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The gain of the observer is given by the matrix K to be designed which includes all the gains K_p, K_{i1}, K_{i2} and K_{i3}. Let us define the state estimation error $e(t) = \bar{x} - \hat{\bar{x}}$. Its time derivative is given by:

$$\begin{aligned} \dot{e}(t) &= \dot{\bar{x}}(t) - \dot{\hat{\bar{x}}}(t) && (12) \\ &= (\bar{A} - K\bar{C})e(t) + \bar{B}\alpha(t) && (13) \end{aligned}$$

Consider the Lyapunov function:

$$V(t) = e^T(t)Pe(t) \quad (14)$$

where P is a symmetric and positive definite matrix. The time derivative of the Lyapunov function is given by:

$$\begin{aligned} \dot{V} &= e^T(t)[P(\bar{A} - K\bar{C}) + (\bar{A} - K\bar{C})^T P]e(t) + \alpha^T \bar{B}^T Pe(t) \\ &+ e(t)^T P\bar{B}\alpha(t) && (15) \end{aligned}$$

The observer gain K is computed in order to stabilize the system generating the state estimation error and also to attenuate the effect of the second derivative $\alpha(t)$ of the Unknown Input on the estimation errors, namely $\|T_{e\alpha}\|_\infty < \gamma$, $\gamma > 0$, where $T_{e\alpha}$ represents the transfer from $\alpha(t)$ to the error $e(t)$.

Let us define the L_2-gain of the system as the quantity:

$$\sup \frac{\|e(t)\|_2}{\|\alpha(t)\|_2}, \quad \|\alpha(t)\|_2 \neq 0 \quad (16)$$

where the L_2-norm of $e(t)$ and $\alpha(t)$ are defined by:

$$\|e(t)\|_2 = \left(\int_0^\infty e(t)^T e(t)dt\right)^{1/2} \quad (17)$$

$$\|\alpha(t)\|_2 = \left((\int_0^\infty \alpha(t)^T \alpha(t)dt)\right)^{1/2} \quad (18)$$

Then, if the inequality

$$\dot{V}(t) + e^T e - \gamma^2 \alpha^T \alpha \ < 0 \quad (19)$$

holds, the state estimation error dynamics is stable and the transfer from $\alpha(t)$ to $e(t)$ is bounded by γ. By replacing (15) in (19) one obtains:

$$\begin{aligned} & e^T(t)[P(\bar{A} - K\bar{C}) + (\bar{A} - K\bar{C})^T P + I]e(t) \\ + \quad & \alpha^T(t)\bar{B}^T Pe(t) + e^T(t)P\bar{B}\alpha(t) - \gamma^2 \alpha^T(t)\alpha(t) < 0 \end{aligned} \quad (20)$$

Since the final objective is to derive LMI conditions, let us consider the change of variable $Z = PK$. The inequality (20) is equivalent to:

$$\begin{aligned} & e^T(t)[\bar{A}^T P + P\bar{A} - \bar{C}^T Z^T - Z\bar{C} + I]e(t) \\ + \quad & \alpha^T(t)\bar{B}^T Pe(t) + e^T(t)P\bar{B}\alpha(t) - \gamma^2 \alpha^T(t)\alpha(t) < 0 \end{aligned} \quad (21)$$

In matrix formulation, (21) is equivalent to:

$$\begin{pmatrix} e(t) \\ \alpha(t) \end{pmatrix}^T \begin{pmatrix} \bar{A}^T P + P\bar{A} - \bar{C}^T Z^T - Z\bar{C} + I & P\bar{B} \\ \bar{B}^T P & -\gamma^2 I \end{pmatrix} \begin{pmatrix} e(t) \\ \alpha(t) \end{pmatrix} \quad (22)$$

The quadratic form (22) is negative definite if and only if the linear matrix inequality:

$$\begin{pmatrix} \bar{A}^T P + P\bar{A} - \bar{C}^T Z^T - Z\bar{C} + I & P\bar{B} \\ \bar{B}^T P & -\gamma^2 I \end{pmatrix} < 0 \quad (23)$$

holds. Finally, given a scalar γ, if there exists a symmetric and positive definite matrix P and a matrix Z such that the LMI (23) is satisfied, then the system generating the state estimation error is stable and the transfer from $\alpha(t)$ to $e(t)$ is bounded by the L_2 gain γ. Furthermore, in order to enhance the performances of the observer, it is possible to minimize the transfer gain γ subject to LMI constraints. The following optimization problem is then stated

$$\min \bar{\gamma}$$

s.t.

$$\begin{pmatrix} \bar{A}^T P + P\bar{A} - \bar{C}^T Z^T - Z\bar{C} + I & P\bar{B} \\ \bar{B}^T P & -\bar{\gamma} I \end{pmatrix} < 0 \quad (24)$$

by choosing γ as a variable and using the change of variables $\bar{\gamma} = \gamma^2$. After solving this optimization problem, the gains of the observer are obtained by

$$K = P^{-1} Z \quad (25)$$

and the attenuation gain is given by $\gamma = \sqrt{\bar{\gamma}}$.

V. Simulation Analysis

The main objective of this work is to reconstruct the lateral dynamics, especially, the roll angle, the lateral forces and the steering torque. To limit the effect of the oscillatory phenomenon in the transit phase the poles of the matrix $A - LC$ are assigned in a LMI region S defined by $S = \{z \in C \mid \mathrm{Re}(z) < -a, \ |z| < R\}$, which is an intersection between the left plan defined by the $\mathrm{Re}(z) < -a$ and the disc with center $(0,0)$ and radius R. Thus, we solve simultaneously the proposed optimization problem and the LMI constraints, corresponding to the LMI region, given as follows (for more details see [5]).

$$\bar{A}^T P + P\bar{A} - \bar{Z}C - C^T \bar{Z}^T + 2aP < 0 \quad (26)$$

$$\begin{bmatrix} -RP & P\bar{A} - \bar{Z}C \\ \bar{A}^T P - C^T \bar{Z}^T & -RP \end{bmatrix} < 0 \quad (27)$$

In all the simulations we fixed $a = 10$ and $R = 50$.

Simulation results prove the efficiency of the motorcycle model used here as illustrated in figure 2 where we show that the equilibrium condition [18] is verified. The torque has been chosen in such a way to excite the lateral vehicle dynamics which corresponds to a cornering maneuver as shown in the same figure.

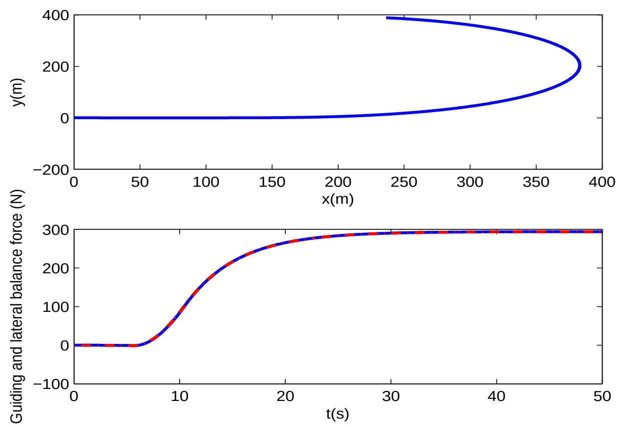

Fig. 2. Vehicle trajectory (top) and Comparison between guiding force and lateral balance force (bottom)

A. Results without measurement noise

The two figures (3 and 4) show the evolution of the state variables and the estimate of each state. We see that the observer converges quickly and the state estimation errors converge accurately to real states. Figure 5 represents the roll angle estimation of the vehicle and the reconstruction of the steering torque, satisfactory results are then obtained.

B. Results with measurement noise

Now, consider the same observer in the presence of measurement noise of order 10% of maximal value of each output. The obtained results are depicted in figures 6, 7 and 8. One can conclude that even if the the measurements are affected by noises, an acceptable state and unknown input estimations are obtained.

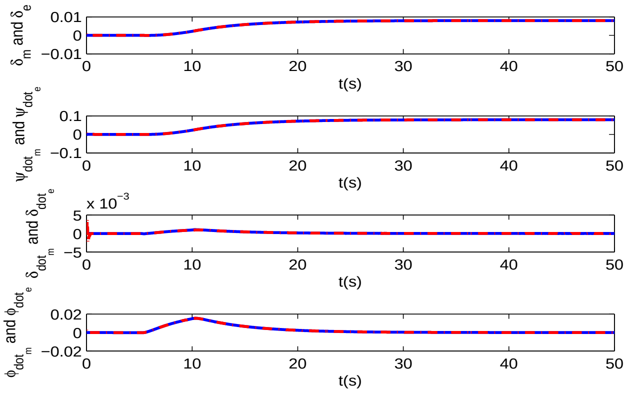

Fig. 3. mesurments (blue line) and her estimations (dashed red line)

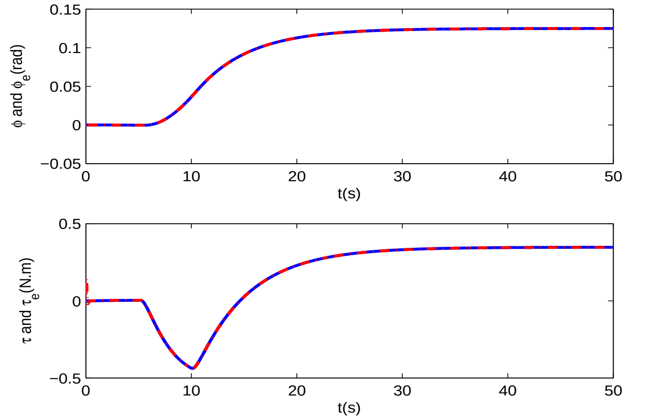

Fig. 5. Roll angle and its estimation (top)and Steering torque and its estimation (bottom)

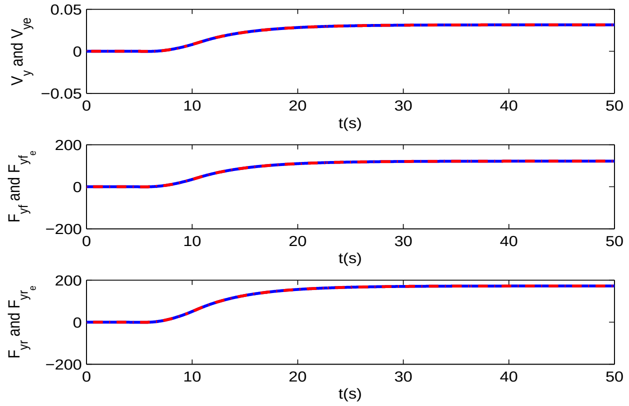

Fig. 4. no mesured states (blue line) and her estimations (dashed red line)

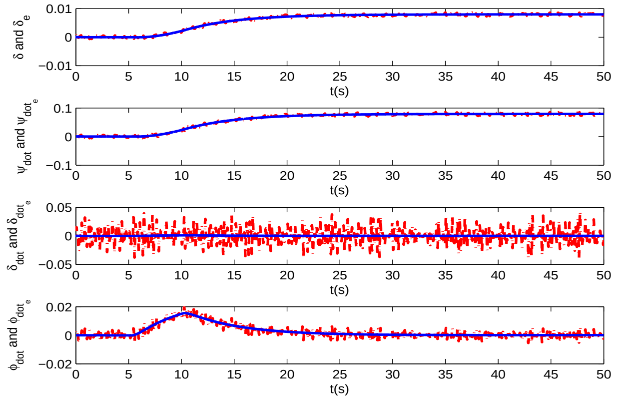

Fig. 6. mesurments (blue line) and her estimations (dashed red line)

VI. Conclusion and Future work

In this paper, a synthesis of P2I observer was presented to estimate the dynamic states of a two-wheeled vehicle, the roll angle that is often inaccessible to measure and reconstruct the steering torque which is the control input (rider's action on the handlebar). The stability of the observer is studied with Lyapunov theory and LMI conditions are established. Finally, the efficiency of the observer are illustrated by some simulation results. For Future works, it will be interesting to extend the approach for a nonlinear model of the motorcycle by using Takagi-Sugeno fuzzy structure and Linear Parameter Varying (LPV) model in order to take into account some nonlinear behaviors and the variation of the longitudinal velocity. In addition, it remains to achieve these results through a validation of the prototype test available in the laboratory, this aims to test some risk functions developed for strong risks related at preventive security systems development.

Appendix

$$\mathbf{M} = \begin{bmatrix} M_f+M_r & M_f k & 0 & 0 & M_f e & M_f j+M_r h \\ M_f k & M_f k^2+I_{rz}+I_{fx}\sin\varepsilon^2+I_{fz}\cos\varepsilon^2 & 0 & 0 & M_f ek+I_{fz}\cos\varepsilon & (M_f jk-C_{rxz}+(I_{fz}-I_{fx})\sin\varepsilon\cos\varepsilon \\ 0 & 0 & \sigma_f/V & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_r/V & 0 & 0 \\ M_f e & M_f ek+I_{fz}\cos\varepsilon & 0 & 0 & I_{fz}+M_f e^2 & M_f ej+I_{fz}\sin\varepsilon \\ M_f j+M_r h & M_f jk-C_{rxz}+(I_{fz}-I fx)\sin\varepsilon\cos\varepsilon & 0 & 0 & M_f ej+I_{fz}\sin\varepsilon & (M_f j^2+M_r h^2+I_{rx}+I_{fx}(\cos\varepsilon^2+I_{fz}\sin\varepsilon^2) \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} 0 & (M_f+M_r)V & -1 & -1 & 0 \\ 0 & M_f kV & -l & b & -(i_{fy}/R_f)\sin\varepsilon V \\ C_{f1}/V & C_{f1}l/V & 1 & 0 & -C_{f1}t/V \\ C_{r1}/V & -C_{r1}b/V & 0 & 1 & 0 \\ 0 & (M_f e+(i_{fy}/R_f)\sin\varepsilon V & t & 0 & K \\ 0 & (M_f j+M_r h+(i_{fy}/R_f)+(L/R_r))V & 0 & 0 & (i_{fy}/R_f)\cos\varepsilon V \end{bmatrix}$$

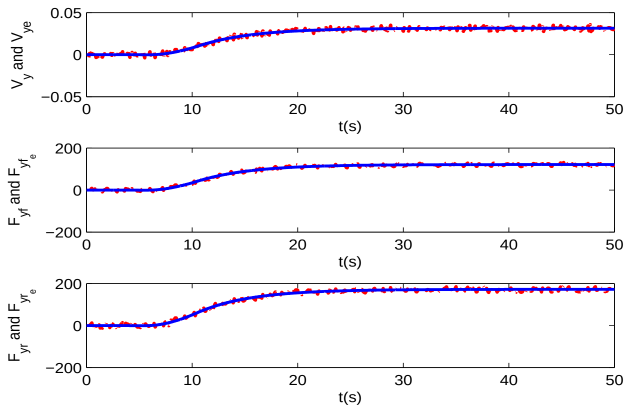

Fig. 7. no mesured states (blue line) and her estimations (dashed red line)

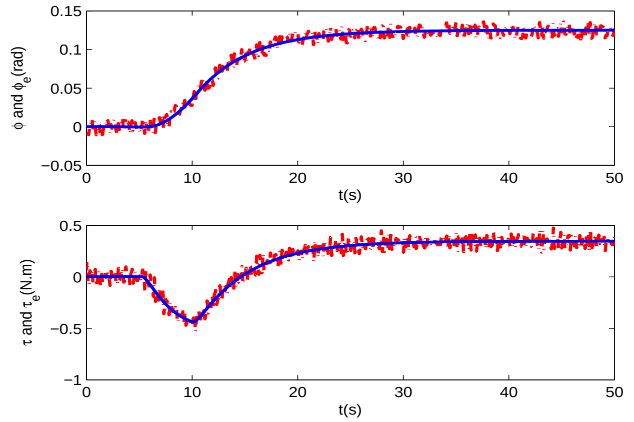

Fig. 8. Roll angle and its estimation (top)and Steering torque and its estimation (bottom)

$$\left. \begin{matrix} 0 \\ -((i_{fy}/R_f)+(L/R_r))V \\ 0 \\ 0 \\ -(i_{fy}/R_f)\cos\varepsilon V \\ 0 \end{matrix} \right]$$

$$\mathbf{P} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -(C_{f1}\cos\varepsilon + C_{f2}\sin\varepsilon & -C_{f2} \\ 0 & -C_{r2} \\ (tZ_f - M_f eg)\sin\varepsilon & (tZ_f - M_f eg) \\ tZ_f - M_f eg & -(M_f j + M_r h)g \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

where the parameters are defined as follows:

M_f, M_r Mass of front/rear frame ($M = M_f + M_r$)

j Distance between the center of gravity of the front frame and ground

k Distance between the centers of gravity of each frame

L_f, L_r Distance between the center of gravity

e Distance between the fork and the center of gravity

h Height of the center of gravity

i_{fy}, i_{ry} Polar moment of inertia of front/rear wheel

I_{rz} Camber inertia of rear wheel.

R_f, R_r radius of the front/rear wheel

g Acceleration due to gravity.

σ_f, σ_r Front and rear tire relaxation lengths respectively

C_{f1} C_{f2} Front and rear tire cornering stiffnesses respectively

C_{r1} C_{r2} Front and rear tire camber stiffnesses respectively

t, V Pneumatic trail and Forward speed

Z_f, ε Front wheel load and Steering head angle

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