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► **To cite this version:**

Sylvain Guillou, Jérôme Thiebot, Julien Chauchat, Romuald Verjus, Anthony Besq, et al.. The Filling Dynamics of an Estuary: From the Process to the Modelling. Sediment Transport in Aquatic Environments, intech, pp.125-146, 2011, 978-953-307-586-0. 10.5772/19933 . hal-00696510

**HAL Id: hal-00696510**

**<https://hal.science/hal-00696510v1>**

Submitted on 11 May 2012

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# The Filling Dynamics of an Estuary: From the Process to the Modelling

Sylvain Guillou<sup>1</sup>, Jérôme Thiebot<sup>2</sup>, Julien Chauchat<sup>3</sup>, Romuald Verjus<sup>4</sup>,  
Anthony Besq<sup>4</sup>, Duc Hau Nguyen<sup>4</sup> and Keang Sé Pouv<sup>4</sup>

<sup>1</sup>University of Caen Lower-Normandy

<sup>2</sup>BRGM

<sup>3</sup>University of Grenoble

<sup>4</sup>University of Caen Lower-Normandy  
France

## 1. Introduction

Estuaries are submitted to a natural filling caused by the settling of cohesive sediments ( $\phi < 63 \mu\text{m}$ ). Those sediments, coming mostly from the sea, are transported in estuaries by the tidal currents during ebb and flood flows. During slack water, fluid velocities vanish and particles are no more suspended by turbulent dispersion. Sediment particles settle towards the bottom, and then deposit on the bed and consolidate. Those particles could be resuspended by erosion when velocities are maximal. Flocculation processes (aggregation), erosion, deposition and compaction are major phenomena that must be considered for an accurate prediction of long term behaviour of estuarine sedimentary dynamics.



Fig. 1. A mudflat located in the up-stream part of the Rance estuary (France).

Several modelling approaches exist. A common way is based on the calculation of diluted particles-water mixture motion with a transport equation for the sediment concentration considered as passive (classical approach). A supplementary model is then used to model the phenomena in the sedimentary bed (i.e. the layer of mud with a high sediment concentration). A second way consists in modelling the behaviour of the particles and the water as two interacting effective fluids from the consolidated rigid bed to the water free surface: this is the two-phase approach.

The aim of this chapter is to summarise the main hydrosedimentary processes that take place in estuaries and to present different approaches for their modelling. Section 2 is dedicated to the description of the physical processes involved in estuaries. Section 3 presents the classical (single phase) approach focussing on different ways to model the bed evolutions; the rheological and erosion properties of mud are discussed. Section 4 deals with the two-phase model for sediment transport. Several applications are presented. Finally, the shortcomings of each modelling strategy are discussed and the perspectives are given in section 5.

## 2. Physical processes in estuaries

Cohesive sediments are constituted of granular organic and mineral solids. They are qualified of sticky or muddy materials (Winterwerp & Van Kesteren, 2004, and references herein). Cohesive sediments, or mud, are a mixture of clay, silt, fine sand, water, organic materials and colloidal particles. Sometimes the colloidal fraction is treated as a part of the clay. The percentages of all these fractions determine the specific properties of the mud. Due mainly to the size and shape of the particles as well as the electrical charge distribution, the clay minerals are largely responsible for cohesion. The clay particles are bound together because of the neutralisation of negative electrical charges on the particles by the sodium ions in seawater: Van Der Vaals attractive forces become predominant then. In presence of water, cohesive sediment particles aggregate and form flocs. A floc is constituted by thousands of clay particles and has high water content.

Particles agglomerate to produce flocs of greater size inducing a modification of the settling velocity. Flocculation process results from the mutual collisions and adherences during Brownian, settling or turbulent motions. The latter has the highest effect (Verney et al., 2006). High shears produced by turbulence lead to break up of flocs. The settling velocity of cohesive sediment is difficult to estimate because it is highly dependent on the time evolution of the flocs' sizes, their spatial and size distributions.

Fluvial water and seawater have different characteristics especially in terms of density. The mixing of those two types of waters creates a particular circulation that favours the appearance of very turbid zones called « turbidity maximums ». When the cohesive sediment contained in these zones settles rapidly on the bottom, mainly during slack periods, a layer called « fluid mud » is formed. It is a highly concentrated suspension that sometimes moves on the cohesive bed according to its slope or by entrainment by the current.

When the flocs reach the bottom, they are crushed by the flocs that deposit above them and the water contained in is putting out. A structuration appears which changes the properties of the material. This is the consolidation. The mud becomes a material close to a saturated

soil. It is more resistant to erosion. The main hydrosedimentary processes of cohesive sediments are schematized in Figure 2. According to their concentration, the different types of water-sediment mixtures can be classified into 3 types: mobile suspension, fluid mud and cohesive bed (Ross & Metha, as cited in Winterwerp & Van Kesteren, 2004). Mud suspensions with a concentration smaller than 10 g/l are called “mobile suspension”. This type of mixture corresponds to the blue zone in Figure 2. The upper limit (10 g/l) corresponds to a value that can be reached in turbidity maximum. Mobile suspensions can be considered as Newtonian fluids. Values of the order of the gel-point concentration ( $C_{gel} \approx 100$  g/l) are generally encountered near the bottom. Typically, mass concentration between 20 and 200g/l are reported for the fluid mud. Fluid mud appears frequently in navigation channel or in harbour basins. It can be horizontally mobile or stationary. Fluid mud is a plastic and shear thinning material (Coussot, 1997).

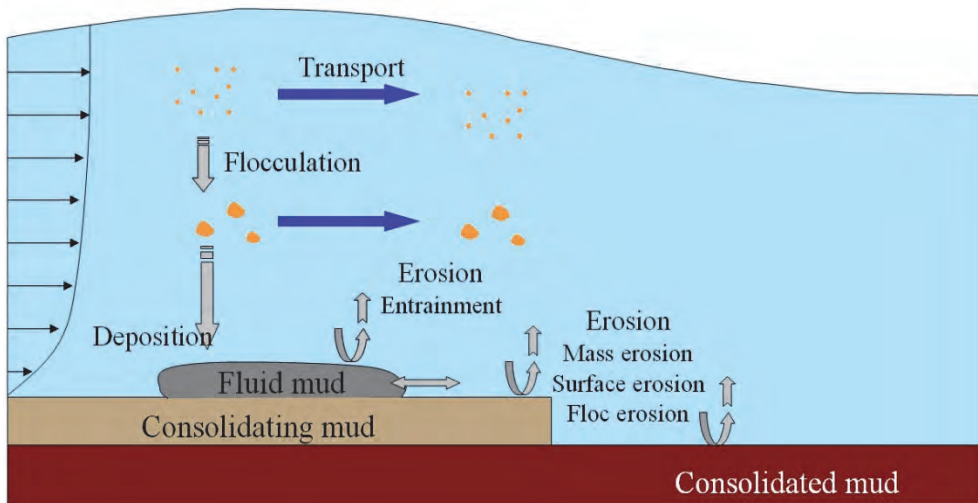


Fig. 2. Macroscopic description of the cohesive sediment transport processes.

For concentration higher than the gelling concentration, the water-sediment mixture has a solid (weak) structure. Its compaction under gravity effect is driven by consolidation process. Such a mixture is referred as consolidating bed or cohesive bed. Pore water is expelled from the cohesive bed while the solid structure strengthens: this is the self-weight consolidation leading to a slow compaction/densification of the bed. The consolidated bed is a pseudo plastic or a viscoelastic material.

Depending on the nature of the bed, four major types of erosion are distinguished. Erosion of a soft mud layer by turbulent water flow is called “entrainment”. It concerns the fluid mud. “Floc erosion” appears when flocs or part of flocs from the bed are individually disrupted or broken-up. It appears when there are peaks of shear stresses (Burst or sweep). “Surface erosion” is a consequence of a drained failure process (swelling of the surface) and of the liquefaction of the top of the bed. With the “mass erosion”, lumps of material are removed.

### 3. Modelling estuarine morphodynamics: The classical approach

The classic approach used to model estuarine morphodynamics consists in dividing the calculation domain into subdomains corresponding to different types of water-sediment mixtures. Each subdomain is associated to particular processes. The level of complexity and the realism of the model depend on the number of subdomains and on the way the interactions between them are accounted for. Following Ross and Metha, a description in three subdomains can be used. The first one is the mobile suspension ( $C < 10$  g/l) in which the cohesive particles are transported by the water flow. The second one is the fluid mud. It can be considered as a buffer that plays an important role in the exchanges between the suspension and the cohesive bed (erosion, deposition). The horizontal displacement of the fluid mud can be neglected in certain estuarine configurations. The lower subdomain is the cohesive bed. The dynamics of the bed is governed by the consolidation process. It is important to model consolidation because it modifies the resistance to erosion (the higher is the bed concentration, the higher is its erosion threshold) and because it determines the thickness of the bed (compaction). Under the cohesive bed, the non-erodible bed (or rigid bed) defines the lower limit of the calculation domain. The cohesive bed and the fluid mud layer constitute the stock of sedimentary materials which could be resuspended by erosion.

#### 3.1 Modelling the mobile suspension dynamics

In the mobile suspension, the actions of the sediments on the fluid are neglected. Fluid motion is therefore calculated using Navier-Stokes Equations or Shallow Water Equations. Then the suspended sediment transport is modelled by the equation (1) where  $C_s$  represents the sediment concentration,  $\vec{u}$  is the water velocity,  $\vec{w}_s$ , is the vertical settling velocity,  $\vec{\nabla}$  is the gradient operator,  $D_{cs}$  is the diffusivity coefficient (horizontal and vertical diffusivities can be used),  $S$  is the source term,  $t$  is the time. The boundary conditions at the surface and the bottom are given by equations (2). The flux at the surface is nil whereas the flux between mobile suspension and fluid mud is equal to the sedimentation flux given by the difference between the deposition and the erosion fluxes  $F_d$  and  $F_e$ . Classically, those fluxes are modelled using the Krone's (1962) and the Partheniades' (1965) formulas respectively (4) and (5) where  $\tau_b$  is the bed shear stress,  $\tau_{cd}$  and  $\tau_{ce}$  are the critical shear stress for deposition and erosion respectively. The latter depends on the properties of the material to erode (density, structuration, rheology ...). The flocculation and the hindered settling processes are basically accounted for by linking the settling velocity to the concentration. The formulation of Thorn (1981) is an example of such a formulation (3). When the concentration is low, the fall velocity increases with the concentration as a consequence of flocculation whereas for higher concentrations, the fall velocity decreases as a consequence of hindered settling. Linking the settling velocity to the concentration only is a rough approximation because flocculation is known to depend on other parameters especially the salinity or the turbulence effects (Verney et al., 2006). Further investigations are required concerning the parameterisation of the settling velocity of cohesive sediments.

$$\frac{\partial C_s}{\partial t} + (\vec{u} - \vec{w}_s) \cdot \vec{\nabla} C_s = \vec{\nabla} \cdot (D_{cs} \vec{\nabla} C_s) + S \quad (1)$$

$$\left( K \frac{\partial C_s}{\partial z} + W_s C_s \right) \Big|_{\text{surface}} = 0 \quad \left( -K \frac{\partial C_s}{\partial z} - W_s C_s \right) \Big|_{\text{bottom}} = F_e - F_d \quad (2)$$

$$\begin{cases} W_s = 0,513C_s^{1,3} & \text{for } C_s \leq 3g.l^{-1} \\ W_s = 2,6.10^{-3}(1 - 0,008C_s)^{4,65} & \text{for } 3g.l^{-1} < C_s < 100g.l^{-1} \end{cases} \quad (3)$$

$$F_d = W_s.C_s \left( 1 - \frac{\tau_b}{\tau_{cd}} \right) \quad \text{for } \tau_b \leq \tau_{cd} \quad (4)$$

$$F_e = M \left( \frac{\tau_b - \tau_{ce}}{\tau_{ce}} \right) \quad \text{for } \tau_b \geq \tau_{ce} \quad (5)$$

### 3.2 Modelling the fluid mud dynamics

In some estuaries (e.g. the Loire estuary in France), the fluid mud is known to move horizontally. In that case the motion of the fluid mud on the cohesive bed should be considered in the modelling. The fluid mud can be seen as a dense mixture layer with a moving free surface and specific rheological properties. A model based on the Saint-Venant Equations (depth averaged equations) was proposed by Le Normant (2000). It uses the system of equations (6), in which  $h_{fm}$  is the thickness of the fluid mud layer,  $\rho_w$  and  $\rho_{fm}$ , the water density and fluid mud density,  $z_{cb}$  and  $z_s$ , the positions of the cohesive bed and of the free surface flow,  $\nu_{fm}$ , the viscosity coefficient of the fluid mud,  $\vec{f}_c$ , the Coriolis force,  $\vec{\tau}_{cb}$  and  $\vec{\tau}_s$ , the shear stresses at the interface cohesive bed - fluid mud and at the interface mobile suspension - fluid mud.  $dM_{fm}/dt$  represents the rate of mass exchange with the mobile suspension and with the cohesive mud.

$$\begin{cases} \frac{\partial h_{fm}}{\partial t} + \vec{\nabla}_h \cdot (h_{fm} \vec{U}_{fm}) - \frac{1}{C_{fm}} \frac{dM_{fm}}{dt} = 0 \\ \frac{\partial \vec{U}_{fm}}{\partial t} + (\vec{U}_{fm} \cdot \vec{\nabla}_h) \vec{U}_{fm} = - \frac{\rho_{fm} - \rho_w}{\rho_{fm}} g \vec{\nabla}_h (z_{cb} + h_{fm}) - \frac{\rho_w}{\rho_{fm}} g \vec{\nabla}_h (z_s) + \nu_{fm} \Delta_h \vec{U}_{fm} + \frac{\vec{\tau}_{cb} - \vec{\tau}_{sk}}{\rho_{fm} h_{fm}} + \vec{f}_c \end{cases} \quad (6)$$

The concentration of the layer  $C_{fm}$  is assumed to be constant vertically. In estuaries where the fluid mud does not move horizontally, there is no need to use such a model. Then only vertical exchanges with the mobile suspension and the cohesive bed must be taken into account.

### 3.3 Modelling the cohesive bed

Various strategies can be used to model the bed evolution. The simplest strategy consists in neglecting the consolidation. In this case, the cohesive bed is a sedimentary stock with a homogeneous concentration along the vertical. Basically, the thickness of the bed evolves according to erosion and deposition only. Using such a simple model, the compaction of the bed by consolidation is not accounted for. Moreover, the erosion threshold is described very roughly because it is assumed to be constant. A more precise approach is used by Teisson (1991), Teisson et al. (1993) and Lumborg & Windelin (2003). It consists in discretising the bed using a stack of layers having a fixed concentration and a variable thickness. The bed is divided into several layers characterized by their concentration  $C_c$  their residence time  $T_s$  (time after which the sediment passes to the layer of higher concentration) and their critical shear stress for erosion. These parameters are fixed. When deposition occurs, the mud

remains in the upper layer  $m$  until the time  $T_s(m)$  is reached. Then the mud is transferred to the layer located beyond  $(m-1)$  which has a higher concentration (more consolidated).

The thickness of the layer  $m$  increases as:  $E(m-1) = E(m-1) + E(m) C_c(m) / C_c(m-1)$  and the thickness of the layer  $m-1$  is then modified accordingly  $E(m) = 0$ . The residence time and the layers' concentrations are obtained from the consolidation curve (mean concentration versus the time; this curve is obtained from settling curve experiments). The critical shear stress for erosion associated with each layer follows a law deduced from experiment. One layer can be eroded only if all the upper layers are empty. The concentration of the deposition layer is set to  $100 \text{ g/l}$  corresponding to the fluid mud layer. This model was applied by Denot & Lang (2000) for the Rance estuary and by Le Normant (2000) for the Loire estuary.

Gibson's (1967) theory (7) constitutes the reference for soft soil consolidation modelling (Toorman, 1996) where  $k$  is the permeability,  $\sigma'$  is the effective stress,  $C_c$  is the sediment mass concentration,  $t$  is the time,  $\rho_w$  and  $\rho_s$  are the water and solid particle densities,  $g$  is the gravity acceleration, Eulerian coordinate  $z$  is taken as positive in the upward direction. Gibson's theory consists in describing the upward water flow using Darcy - Gersevanov's law and in characterizing the strength of the mud skeleton using Terzaghi's principle. Hypothesis is made that both the permeability  $k$  and the effective stress  $\sigma'$  depend on the sediment concentration only. Gibson's models require the constitutive relationships  $k(C_c)$  and  $\sigma'(C_c)$  to be determined. Equations (8) are commonly used. The  $A_i$  and  $B_i$  coefficients are dependent on the mud characteristics and should be determined from experiments following the technique proposed by Been & Sills (1981). The optimisation method of Thiébot et al. (2010) can be also used. Gibson's theory has been used in many consolidation models (Townsend & Mc Vay, 1990; Toorman, 1999; Bürger, 2000; Lee et al., 2000; Le Normant, 2000; Bürger & Hvistendahl, 2001; Bartholomeeusen, 2003; De Boer et al., 2007; Jeeravipoolvarn et al., 2009; Thiébot et al., 2010).

$$\frac{\partial C_c}{\partial t} - \frac{\rho_s - \rho_w}{\rho_s \rho_w} \frac{\partial}{\partial z} (k C_c^2) - \frac{1}{g \rho_w} \frac{\partial}{\partial z} \left( k C_c \frac{\partial \sigma'}{\partial z} \right) = 0 \quad (7)$$

$$\begin{cases} k(C_c) = k_0 \exp \left( \frac{\rho_s / C_c - A_1}{A_2} \right) & \sigma'(C_c) = B_1 \left( B_2 - \rho_s / C_c \right)^{B_3} \\ k_0 = 1 \text{ m/s} \end{cases} \quad (8)$$

The MDM (Mud Deposit Model) has been designed to simulate the evolution of homogeneous mud deposits in hydro-sedimentary simulations (Thiébot & Guillou, 2006). The MDM uses the same physics as Gibson's theory but the equations have been rewritten to account for the multilayer discretisation. In the MDM the mud deposit is represented by a stack of layers of specified concentrations  $C_{c_i}$  (low concentrated layers on top of the deposit) and variable thicknesses  $Ep_i(t)$ . At each time step,  $Ep_i(t)$  varies according to the solid mass fluxes at the boundaries of the layer  $i$ :  $F_i(t)$  and  $F_{i+1}(t)$ .  $F_i$  is negative as it is downward-oriented. The evolution of the thicknesses is given by equation (9). The calculation of the mass flux between two layers is made with relation (10). Interested readers should refer to Thiébot et al. (2010).

$$Ep_i(t + \Delta t) = Ep_i(t) + \frac{(F_i(t) - F_{i+1}(t)) \Delta t}{C_{c_i}} \quad (9)$$

$$F_i(t) = -\frac{(V_{s,i-1}(t) - V_{s,i}(t)) C_c C_{i-1}}{C_{i-1} - C_c} \tag{10}$$

$$V_{s,i}(t) = -k(C_c) C_c \left( \frac{1}{\rho_s} - \frac{1}{\rho_w} \right) \tag{11}$$

$$V_{s,i}(t) = -k(C_c) C_c \left( \frac{1}{\rho_s} - \frac{1}{\rho_w} \right) + \frac{k(C_c)}{g\rho_w} \frac{\sigma'(C_c) - \sigma'(C_{i-1})}{(E p_i(t) + E p_{i-1}(t)) / 2} \tag{12}$$

The MDM was originally developed to simulate the vertical dynamics of the cohesive bed under consolidation effect. However, it can also be used to simulate the sedimentation of particles in the fluid mud. Been & Sills (1981) pointed out that Gibson’s equation becomes a sedimentation equation when the effective stress is neglected. This is used in the MDM. A sediment concentration value  $C_i$  is used to distinguish sedimentation from consolidation. In the layers where  $C_c$  is smaller than  $C_i$ , the effective stress is neglected and the MDM resolves a sedimentation equation. Otherwise, the MDM resolve a consolidation equation. From a physical point of view,  $C_i$  is the representative value of the transition between a fluid-supported suspension and a (soft) soil which has a solid structure (i.e. the concentration is greater than the gelling concentration). So, in the MDM, the flux is linked to the solid particle velocities in layers  $i$  and  $i-1$  which are estimated with relation (11) during the sedimentation process (when  $C_c < C_i$ ) and with the relation (12) during the consolidation process (when  $C_c > C_i$ ).

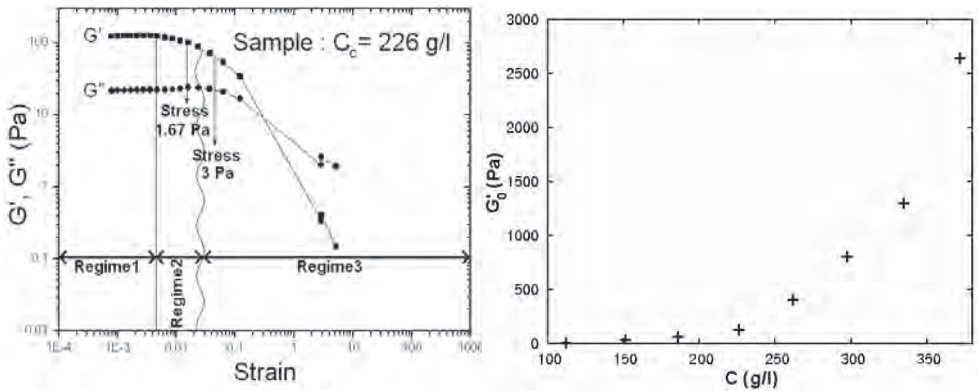


Fig. 3. Dynamic response of a mud sample of the Rance estuary: Evolution of the storage module  $G'$  and of the loss module  $G''$  relating to the imposed deformation for one concentration (left); Evolution of the storage modulus plateau  $G'_0$  at low deformation for different concentrations (right).

For the mud of Rance, the existence of  $C_i$  has been justified using rheometric tests performed with samples of mud collected from the Rance estuary (Thiébot et al., 2006). Dynamic oscillation tests have been performed. This type of experiments aims at characterising the viscoelastic behaviour of materials. When the material is submitted to oscillatory solicitations, a part of the energy is transmitted to the structure of the material (it



is characterised with the elastic modulus  $G'$ ), the other part is dissipated (it is characterised with the loss module  $G''$ ). Basically, a dynamic oscillation test involving cohesive sediments consists in studying the response of the materials for increasing strain where the sediment changes from having predominantly solid-like properties immediately after a rest period, to having predominantly liquid-like properties once set. The test represented in Figure 3 is an example. For lower strains, the response of the material is mostly elastic which gives rise to a plateau on the storage modulus (regime 1 in Fig. 3a). For intermediate strains, the storage module decreases revealing a weakening of the structure (regime 2 in Fig. 3a). One deals with rearrangements but the strength of 3D network is not broken. Finally, for great strain, the structure of the material is destroyed: it is a liquefaction process (regime 3 in Fig. 3a).

For the present purpose, we intend to find a transition behaviour for a given mass concentration value. The aim is to justify the use of a concentration which characterises the appearance of effective stress. Dynamical oscillation tests have been performed with samples of increasing concentrations. For each sample, the storage module during the regime 1 (i.e. when the solid structure is intact) has been estimated: it is  $G'_0$ . The results are presented in Figure 3b.  $G'_0$  becomes significant and increases steeply when the concentration exceeds approximately  $200 \text{ g/l}$ . The change in the trend of  $G'_0(C)$  indicates an evolution of the material structure when a given concentration value is exceeded. From a physical point of view, the  $200 \text{ g/l}$  value is interpreted as the appearance of a "sufficiently stiff" solid structure. This justifies the use of a concentration transition  $C_t$  in our sedimentation-consolidation model.

The MDM has been validated using the experimental data of Bartholomeeusen et al. (2002). An example of comparison between numerical and experimental results is represented in Figure 4.

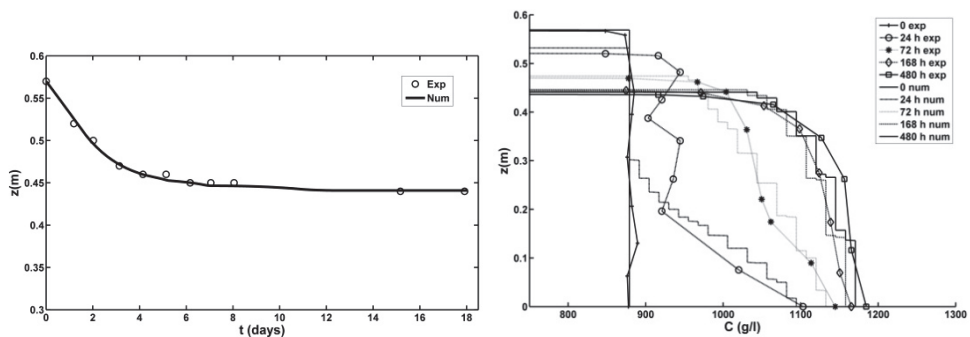


Fig. 4. Experimental ("Exp") and simulated ("Num") settling curves (left) and mass concentration profiles (right) along the time. The initial height of the settling test is  $H_0 = 0.57 \text{ m}$ ; the initial concentration is  $C_0 = 879 \text{ kg/m}^3$ . The relations (8) were used with following parameters:  $A_1 = 7.28$ ;  $A_2 = 0.298$ ;  $B_1 = 2590 \text{ kg/m/s}^2$ ;  $B_2 = 3.31$  and  $B_3 = 9.25$ .

### 3.4 Modelling the filling the Rance estuary: An example

In this section, a way of modelling the filling of estuary is presented. The Rance estuary model is used to illustrate. The Rance estuary (20 km long) is located in the north of Brittany (France). A tidal power station was built in the sixties. It modifies the hydrodynamic regime

and the sedimentary processes (Bonnot-Courtois et al., 2002). As the silting up is intense in the most upstream part of the Rance estuary, dredging works are regularly carried out. Originally, the hydro-sedimentary model of the Rance aims at optimizing these operations (Denot & Lang, 2000). Simulations are performed with the modelling system Telemac (Hervouet, 2007). The mesh contains 6250 nodes. The mesh contains 6250 nodes. Telemac2d and Subief2d are used. The hydrodynamics is calculated from the flows given at the Châtelier lock (upstream boundary of the calculation domain) and at the tidal power plant (downstream boundary of the calculation domain).

$$\begin{cases} \tau_{\alpha}(C_c) = 2,238.10^{-6} \cdot C_c^{1,953} & \text{if } C_c < 367 \text{ g/l} \\ \tau_{\alpha}(C_c) = 4,110.10^{-10} \cdot C_c^{3,41} & \text{otherwise} \end{cases} \quad (13)$$

In the Rance model, the erosion flux is calculated with the widely used Partheniades (1962) formula (5). The use of this formula implies to define a critical shear stress for erosion. Equation (13) is used. It has been determined from experiments on the mud of Rance following Migniot (1968). The Partheniades formula (5) also involves a parameter  $M$  which has to be tuned according the mud characteristics. Several values have been tested until the concentration of suspended sediment calculated by the model fits well to the suspended sediment concentration measured in-situ. Using  $M = 1.875 \cdot 10^{-3} \text{ kg m}^{-2} \text{ s}^{-1}$ , the mean concentration reaches 200 mg/l in the turbidity maximum which is consistent with measurements (Thiébot, 2008). In our model, the settling velocity of suspended sediment  $W_s$  is supposed to be constant and equals 0.1 mm/s. The sediment concentrations at the boundaries of the domain are set to 0 because turbidity measurements indicate that the sediment inputs are quasi-negligible except for particular meteorological conditions.

The suspended sediment concentrations in the Rance estuary (typically smaller than 200 mg/l) are small in comparison to many estuaries. As a consequence, the fluid mud layer is generally thin and appears only during slack periods. During the ebb and flood flows, the fluid mud rapidly disappears by erosion. In such configuration, the horizontal movements of the fluid mud can be neglected and therefore the fluid mud can be considered as a part of the bed. In the Rance model, the bed evolution is simulated with the MDM: ten layers are used. The upper layer corresponds to the fluid mud; its concentration is set to 100 g/l which is consistent with the earlier works of Ross & Metha. The concentrations in the other layers vary uniformly from 200 g/l to 1000 g/l. The constitutive relationships  $k(C_c)$  and  $\sigma'(C_c)$ , given by (14), have been determined from experimental results obtained with mud of Rance following the method presented in Thiébot et al. (2010). The effective stress  $\sigma'$  is set to zero in the fluid mud layer because this material does not have any solid structure (according to the rheological tests, the solid structure appears for a concentration value of approximately 200 g/l). The maximum concentration of the multilayer bed is 1000 g/l. This value is the concentration beyond which the mud does not evolve much neither by consolidation nor by erosion.

$$k(C_c) = \left( \frac{1}{181,5} \left( \frac{\rho_s}{C_c} - 1 \right) \right)^{4,62} \quad \begin{cases} \sigma'(C_c) = 0 & \text{if } C_c < 200 \text{ g/l} \\ \sigma'(C_c) = 5,81.10^{-3} \cdot C_c^{1,81} & \text{otherwise} \end{cases} \quad (14)$$

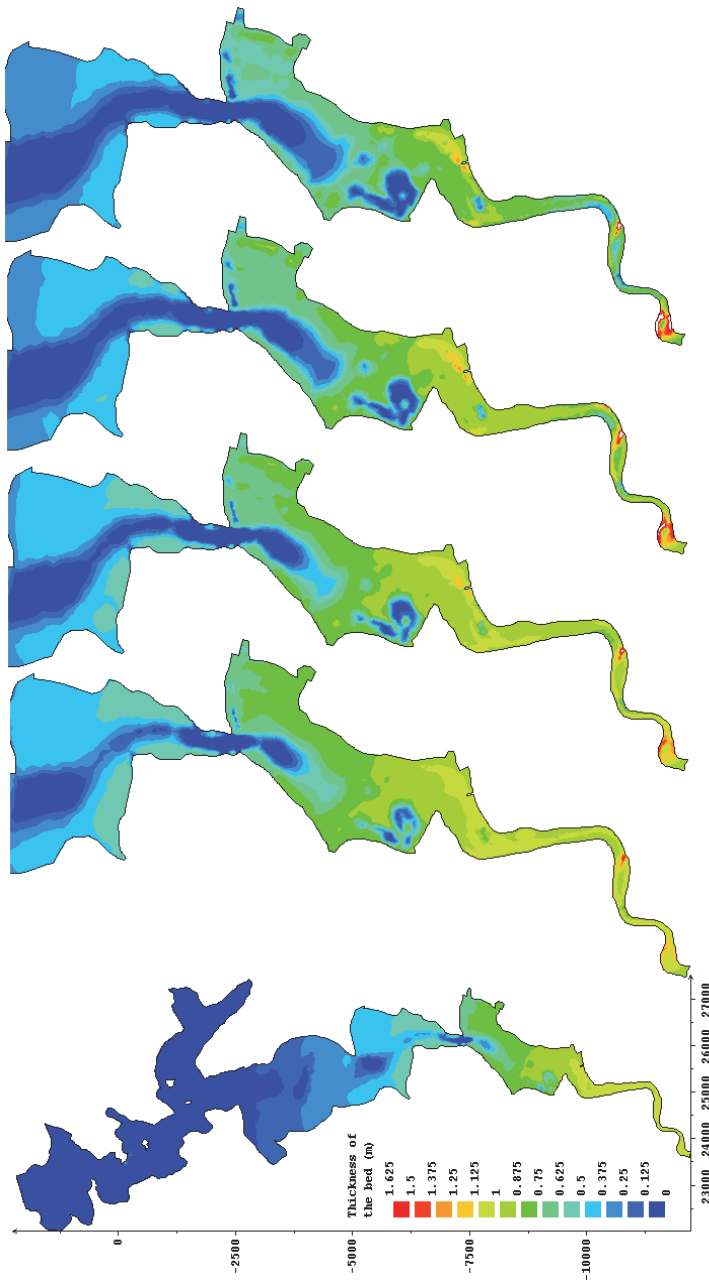


Fig. 5. Thicknesses of the solid bed in the Rance estuary: left) initial deposit in the complete estuary; right) evolution of the bed thickness in the up-stream part after 1, 2, 5 and 10 lunar cycles.

For the morphodynamics modelling of the estuary, a preliminary simulation is used in order to introduce the sediment in the system and to obtain a configuration representative of the actual state of the estuary. This state is characterised by a greater stock of cohesive sediment in the upstream part of the estuary and by a turbidity maximum with sediment concentration of the order of  $200 \text{ mg/l}$ . At the beginning of the preliminary simulation, the sediment is introduced in the  $500 \text{ g/l}$  -layer. A variable thickness has been used to account for the fact that the sediment stock is greater in the upstream part of the estuary. At the beginning of the preliminary simulation, the bed thickness varies from  $0$  to  $1 \text{ m}$  from the downstream to the upstream boundaries of the estuary. The preliminary simulation lasts a lunar cycle ( $\approx 29.5$  days). At the end of this simulation, we consider that the influence of the initial procedure vanishes and start to simulate a 10 lunar cycles period.

Figure 5 shows the evolution of the bed thickness during the simulation. The model shows that the sediment is progressively re-distributed within the estuary. In the middle of the estuary, the channel progressively widens while in the most upstream part, the channel and the mudflat accrete. This evolution is consistent with the in-situ observations. Additional investigations are required in order to validate the model results quantitatively.

#### 4. Two-phase model for sediment transport

In the two-phase approach, the flow is composed of two phases: the fluid phase and the solid phase (sediment). Continuity and momentum equations are solved for each phase with the introduction of interaction terms between the two phases (fluid-solid particles, particle-particle interactions, particle-wall collision). Wallis (1969) was the first, who applied two-phase equations to the sedimentation problem in the late 60s. More recently, Toorman (1996) has presented a unifying theory of sedimentation-consolidation derived from the two-phase equations that allows to recover Kynch's sedimentation theory at low sediment concentration and Gibson's consolidation theory at high sediment concentration.

The two-phase approach was then applied to modelling sediment transports in the 90s (Teisson et al., 1992, Vilaret & Davis, 1995, Greimann et al., 1999, Barbry et al., 2000). The key idea is that the computing domain extends from the "true" non-erodible bottom to the free water surface which is a great advantage compared with the classical three-layer approach. More recently, the interest in this approach has increased and led to numerous publications mainly for non-cohesive sediment transport (*i.e.* sand) (Greimann & Holly, 2001, Dong & Zhang, 2002, Hsu et al., 2003, Hsu & Liu, 2004, Jiang et al., 2004, Amoudry et al., 2005, 2008, Longo, 2005, Chauchat & Guillou, 2008, Nguyen et al., 2009). These studies show encouraging results concerning the suspended-load transport mainly by integrating the influence of the sediment particles on the fluid turbulence and the collisions between particles (two-way and four-way coupling). Hsu et al. (2007) and Torres -Freyermuth & Hsu (2010) have applied a simplified two-phase flow model to boundary layer and gravity-driven fine sediment transport under tidal and wave forces. At the same time, first applications of 2-D X/Z two-phase model for fine-sediment transport in estuaries (Nguyen et al., 2009) have been published.

#### 4.1 Mathematical modelling

In an Eulerian-Eulerian or two-fluid formulation, a set of equations (continuity and momentum equations) are written for each phase. If  $k$  is the index of the phase ( $k$  is "f" for the fluid phase and "s" for the solid phase), they are:

$$\left\{ \begin{array}{l} \frac{\partial(\alpha_k)}{\partial t} + \vec{\nabla} \cdot (\alpha_k \vec{u}_k) = 0 \\ \frac{\partial(\alpha_k \vec{u}_k)}{\partial t} + \vec{\nabla} \cdot (\alpha_k \rho_k \vec{u}_k \otimes \vec{u}_k) = \frac{1}{\rho_k} \vec{\nabla} \cdot (\alpha_k (-p_k \bar{I} + \bar{\tau}_k + \bar{\tau}_k^{\text{Re}})) + \alpha_k \vec{g} + \frac{1}{\rho_k} \bar{M}_k \end{array} \right. \quad (15)$$

where  $\alpha_k$  represents the volume fraction of the  $k$ -phase, with  $\alpha_s + \alpha_f = 1$ ,  $\rho_k$  is the averaged density, and  $\vec{u}_k$  is the averaged velocity vector of  $k$ -phase.  $\vec{g}$  is the gravity acceleration and  $\bar{M}_k$  is the momentum exchanged between these two phases.  $p_k$  is the pressure,  $\bar{\tau}_k$  is the shear stress tensor and  $\bar{\tau}_k^{\text{Re}}$  is the turbulent Reynolds stress tensor for  $k$ -phase. The viscous shear stresses depend on the strains of the fluid and of the solid ( $\bar{D}_f$  and  $\bar{D}_s$ ) by relation (18).  $\mu_{ff}$ ,  $\mu_{fs}$ ,  $\mu_{sf}$  and  $\mu_{ss}$  designate effective viscosity coefficients (17), which are proportional to the fluid's viscosity  $\mu_f$ .  $\beta$  is the amplification factor of viscous stresses and depends (18) on  $\xi$  - the distance between solid particles. Where  $\alpha_{s,\max}$  is equivalent to the maximum value of the solid-particle concentration. The interactions between the phases are given by the transfer laws (20).  $p_{ki}$  and  $\bar{\tau}_{ki}$  are the pressure and the shear-stress tensor of  $k$ -phase at the interface (21).  $\bar{M}'_s = -\bar{M}'_f$  represents all the forces exerted by the fluid on the solid particles. For application to sediment particles in water the Drag force is dominant and given by (22), in which  $\vec{u}_r$  is the relative velocity of fluid-particles, and  $\tau_{fs}$  is the particle relaxation time, which represents the time needed by a particle initially at rest submitted to a constant fluid velocity to reach its steady state. The relative velocity of fluid-particles is defined as  $\vec{u}_r = \vec{u}_s - \vec{u}_f - \vec{u}_d$  where  $\vec{u}_d = \langle \vec{u}_f \rangle_s$ , the drift velocity, represents the correlation between the fluctuating velocity of the fluid phase and the instantaneous spatial distribution of the solid phase (Deutch & Simonin, 1991). A complete description of the model was made in Nguyen et al. (2009). A study of several turbulence models was made by Chauchat & Guillou (2008).

$$\alpha_f \bar{\tau}_f = \mu_{fs} \bar{D}_s + \mu_{ff} \bar{D}_f \quad \alpha_s \bar{\tau}_s = \mu_{ss} \bar{D}_s + \mu_{sf} \bar{D}_f \quad (16)$$

$$\mu_{ff} = \alpha_f \mu_f \quad \mu_{fs} = \alpha_s \mu_f \quad \mu_{ss} = \alpha_s \beta \mu_{fs} \quad \mu_{sf} = \alpha_s \beta \mu_{ff} \quad (17)$$

$$\beta = \frac{5}{2} + \frac{9}{4} \left( \frac{1}{1 + \xi/d} \right) \left[ \frac{1}{2\xi/d} - \frac{1}{1 + 2\xi/d} - \frac{1}{(1 + 2\xi/d)^2} \right] \frac{1}{\alpha_s} \quad (18)$$

$$\xi/d = \frac{1 - (\alpha_s / \alpha_{s,\max})^{1/3}}{(\alpha_s / \alpha_{s,\max})^{1/3}} \quad (19)$$

$$\bar{M}_f = p_f \bar{\nabla} \alpha_f - \bar{\tau}_f \bar{\nabla} \alpha_f + \bar{M}'_f \quad \bar{M}_s = p_{si} \bar{\nabla} \alpha_s - \bar{\tau}_{si} \bar{\nabla} \alpha_s + \bar{M}'_s \quad (20)$$

$$\bar{\tau}_{si} = \bar{\tau}_f = \beta \bar{\tau}_f \quad (21)$$

$$\bar{M}'_s \approx \bar{F}_D = \frac{\alpha_s \rho_s}{\tau_{fs}} \bar{u}_r \quad \bar{u}_r = \bar{u}_s - \bar{u}_f - \bar{u}_d \quad \tau_{fs} = \frac{4d\rho_s}{3\rho_f C_D \|\bar{u}_r\|} \quad (22)$$

**4.2 Applications to non-cohesive sediment transport**

Dredging operations of navigation channels and harbours are regularly planned to maintain the nautical depth and ensure therefore the navigation safety. The dumping operation of dredged sediment could affect the environment by increasing the turbidity of water or burying the biological habitats. After the release, the sediments settle under a cloud of very high concentration (more than 350 g/l at the beginning). This settling step is followed after impact on the bed by the formation of turbidity current. In Guillou et al. (2011), we used our two-phase model to study this phenomenon by performing simulations of the experiment of Villaret et al. (1998). In this paper only sand release with no horizontal current is considered (Fig. 6). The initial concentration was 350 g/l with an injection velocity of 0.6m/s. The different steps of the process are qualitatively simulated by the model such as the formation and the displacement of the turbidity current.

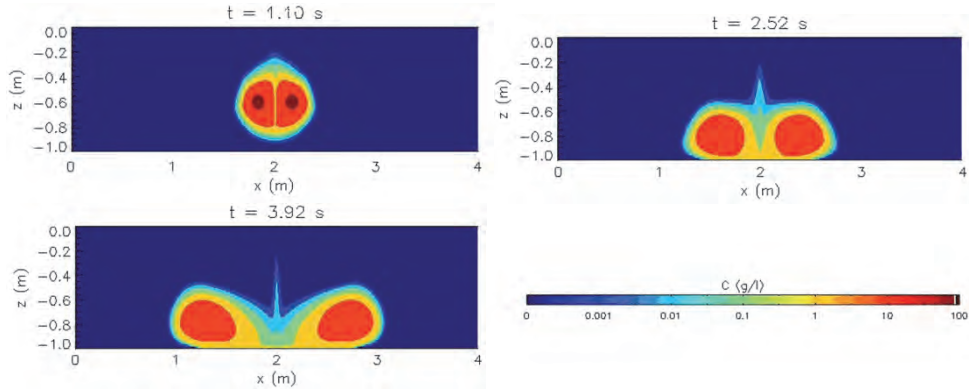


Fig. 6. Isoconcentration of sand for several instants (test e6).

One application was tried in the Seine estuary (Chauchat et al., 2009) to simulate motion of the turbidity maximum dynamics. The computational domain extends from the extremity of the semi-submersible dykes to the dam of Poses (160 km). A mixing length model is used to model the turbulence. The particle’s diameter is equal to 16 μm with a density of 1700 kg/m<sup>3</sup>. The simulations have been performed over a semi-lunar cycle with a river discharge of 300 m<sup>3</sup>/s. A one meter thickness layer with a concentration of 25 g/l between 20 and 60 km from the river mouth is imposed initially. The hydrodynamics is well reproduced. A Turbidity Maximum is simulated in the estuary. Its location is in coherence with observations. One main interest of this simulation is the appearance of a very high concentration layer close to the bottom with concentrations in relation with the one of a

fluid mud and with a horizontal motion (Fig. 7). One major default of this model is its computational cost.

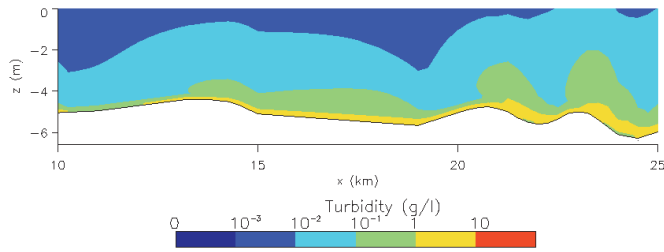


Fig. 7. Zoom near the bottom during spring tide (High water).

The concentration in the TM is not as high as in the reality. This may be in relation with the turbulent model used. Thus Chauchat & Guillou (2008) have shown that the way the turbulence is modelled had a very important impact on the capacity of the flow to keep in suspension particles. A better modelling of the turbulence in estuarine application could certainly overcome this problem. Other phenomena, not considered here, such as flocculation or consolidation processes should provide more realistic results.

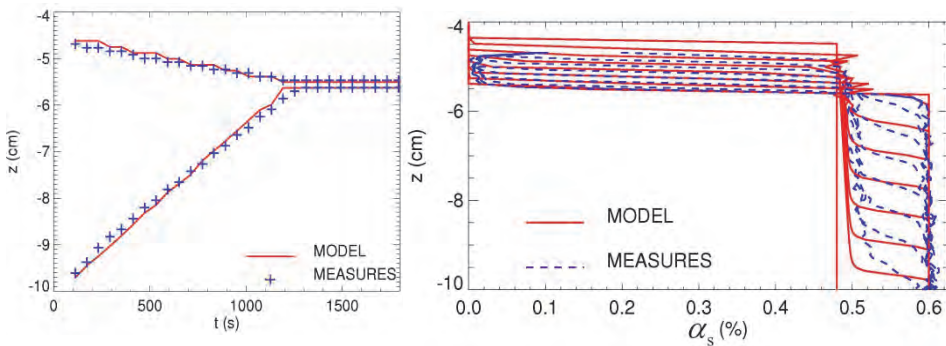


Fig. 8. Sedimentation of spherical particles: Left) Time evolution of the interface clear water and mixture and of the lutocline; right) instantaneous profiles of the solid volume fraction at different times. Comparison with the experimental data of Pham Van Bang et al. (2006).

A way to consider the latter is to study sedimentation in settling column. Nguyen et al. (2009) have published some results about the sedimentation of suspended polystyrene particles (diameters of  $290 \pm 30 \mu\text{m}$ , density of  $1.05 \text{ kg.m}^{-3}$ ), falling through a tank of silicon oil (viscosity of  $20 \text{ mPa.s}^{-1}$ , density of  $0.95 \text{ kg.m}^{-3}$ ). Its initial solid volume fraction was 0.48. The process in that case is well reproduced (Fig. 8).

### 4.3 Study of cohesive sediment transport

In the case of cohesive particles, the previous formulation of the two-phase flow model fails to simulate the hindered settling process as well as the consolidation process. Only the hindered settling regime is usually considered and parameterized using a hindrance

function (Richardson & Zaki, 1954). Improvements in modelling sedimentation and consolidation processes are needed for progressing two-phase modelling of sediment transport in estuaries. In particular, closure laws for the two-phase equations are required and need to be checked by comparison with experiments. The consistency of the excess pore pressure calculated from the two-phase equations should be checked as well.

In a recent work, Chauchat et al. (2011) proposed closure law for the two-phase flow model that allows to capture the essential features of sedimentation-consolidation processes.

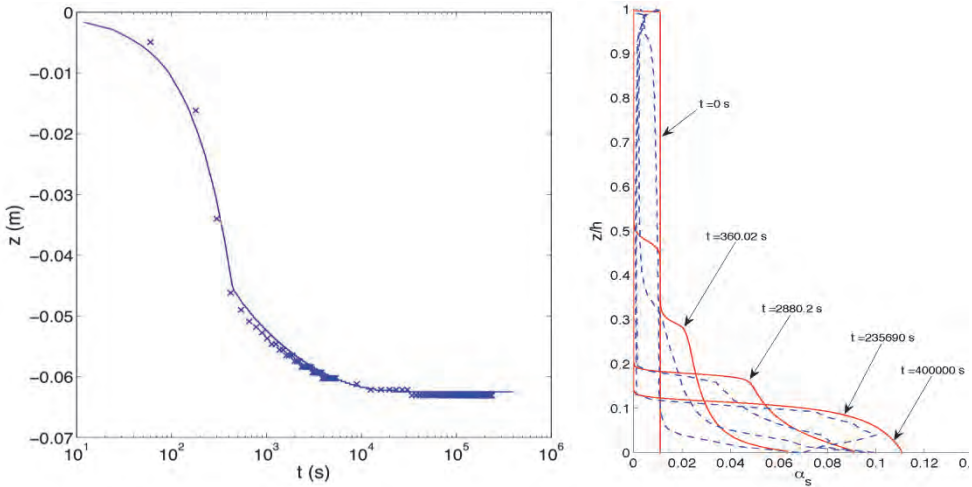


Fig. 9. Comparison of two-phase model results (—) with experiments (xxx) (Pham Van Bang, 2007) for initial concentrations  $\alpha_s^0=1.2\%$ : a) Settling curves: time evolution of the mud-clear water interface position and b) solid volume fraction profiles.

In a dense mixture the pressure of the solid phase  $p_s$  is the sum of the fluid pressure, the so-called pore pressure  $p_f$ , and of a particle pressure or effective stress  $\sigma'$  induced by inter-particle contacts in the solid network (23) (Concha et al., 1996, Burger, 2000). The particle pressure exists when the solid volume fraction exceeds a percolation value: it is the maximum packing  $\alpha_{s,max}$  for non-cohesive particles, and is the gelling point for cohesive ones. The particle pressure should vanish rapidly when  $\alpha_s$  decreases below this percolation threshold. For a high value of the volume fraction of solid-particles, the fluid flow can be viewed as the flow in a porous media constituted by the particles network. In this case, the total pressure of the mixture  $p$  is partially supported by the solid skeleton and partially by the fluid filling the pores, and leads to the relation (24) corresponding to the principle of Terzahi.

$$p_s = p_f + \sigma' / \alpha_s \tag{23}$$

$$p = \alpha_f p_f + \alpha_s p_s = p_f + \sigma' \tag{24}$$

$$\bar{M}_s' \approx \frac{\rho_f g}{K} (\bar{u}_s - \bar{u}_f) \tag{25}$$



$$\sigma' = \begin{cases} 0 & \text{if } \alpha_s < \alpha_s^{gd} \\ \sigma_0 \left[ \left( 1 - \frac{\alpha_s - \alpha_s^{gd}}{\alpha_{s,max}} \right)^{\frac{2}{3-n}} - 1 \right] & \text{if } \alpha_s \geq \alpha_s^{gd} \end{cases} \quad (26)$$

Following Toorman (1996) the Darcy-Gersevanov's semi-empirical expression for the drag force (25) is used in the two-phase model, where  $K$  (in  $m/s$ ) represents the permeability. Therefore the closure issue consists in finding closure laws for the permeability and the effective stress. The effective stress represents both permanent contacts between particles in concentrated suspension and inter-particle collisions during sedimentation. The relation (26) is proposed. The permeability is fixed regarding the settling velocity which can be estimated following the method proposed by Camenen & Pham Van Bang (2011). The results presented in Figure 9 indicate that the two-phase model is able to reproduce almost quantitatively the experimental. This work is still in progress and will be implemented in a 2D vertical two-phase model to study mud-flow interactions in estuaries.

## 5. Shortcomings of the modelling and perspectives

With the classic approach the different kind of estuarial water - sediment mixtures are represented by three subdomains associated with a given range of sediment concentration (mobile suspension, fluid mud and cohesive bed). When the horizontal displacements of the fluid mud can be neglected, the fluid mud can be treated in the same manner as the cohesive bed. The simulation of the consolidation processes in the cohesive bed is of importance. A convenient way of modelling the vertical evolution of the fluid mud and the cohesive bed consists in considering the bed as a stack of layers having fixed concentrations and having thicknesses that vary according to erosion, deposition, sedimentation and consolidation. The MDM is a model of that type. It has recently been introduced in the *Sisyphé* model (Villaret et al., 2010).

The major shortcoming of the classic approach is that a lot of parameters have to be determined for the formulation of the settling velocity, the permeability, the effective stress, the erosion flux ... Those parameters are specific for a given type of mud that is why, their determination requires us to collect experimental data from either from laboratory experiments (rheology, settling experiments) or field measurements (turbidity measurements). Furthermore, the results of the model are often much sensitive to the values of the parameters. Modelling the estuarine morphodynamics in an operational manner is still a challenge. Additional investigations are therefore required especially regarding the link between the erosive properties of the mud and the structuration of the material, *i.e.*, and the rheological behaviour of the mud (Thiébot et al., 2006). Current experimental works are focussed on that link (Pouv et al., 2010).

Two-phase models provide *a priori* a more general framework that allows the representation of the physical processes involved from the suspension to the consolidating bed such as interactions between fluid-solid particles, fluid-bottom as well as particle-particle interactions. No erosion/deposition fluxes are needed to be empirically prescribed.

First development about two-phase sediment transport model was one dimensional vertical. Recent improvements tend to develop 2D and 3D models. These models are based on the theory of granular flows. Then, the great majority of them suppose the sediment as non-cohesive. Due to this physics, the motions of very high concentrated suspension can be

simulated realistically. It is interesting to note that fluid mud could be simulated by this modelling. New achievement must concern the simulation of cohesive sediments. Our recent works (Chauchat et al., 2011) show that it is necessary to introduce appropriate closure laws to simulate correctly the consolidation process in a settling column test (Darcy drag expression and effective stress). In particular, the explicit calculation of the fluid pressure from the suspension to the consolidated bed represents a major advantage of the two-phase approach compared with the classical Kynch or Gibson approach.

The flocculating and deflocculating processes must be also introduced. Attentions must be paid to fractal description of mud (Merckelbach & Kranenburg, 2004). Turbulent modelling in two-phase models is really complex even for mono-disperse non-cohesive particles (Chauchat & Guillou, 2008). It is a challenge for the future in the case of cohesive particles (Hsu et al., 2007; Torres-Freyermuth & Hsu, 2010). Finally, these two-phase models are very time consuming even in a 2D case (Nguyen et al., 2009). At this time it is not possible to use them as operational tools for studying sediment transport in an open estuary. Nevertheless, it is a powerful tool to study the interaction of particles and turbulence (fluid-particles turbulent interactions, flocculation, structuration of sediment bed, mud-flow interactions).

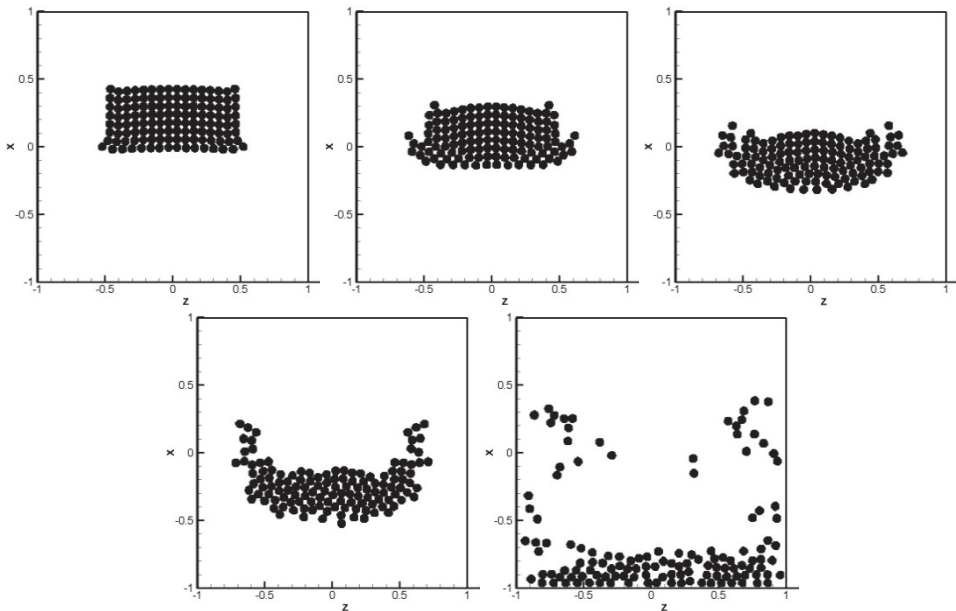


Fig. 10. Snap shots at different time steps of the sedimentation of 128 solid particles in water with Direct Numerical Simulation. The density of particles is 1.5.

Both single and two-phase models need some closure relationships for each process. Those relationships are based either on theoretical basis (*e.g.* kinetic theory of granular flows) or empirical considerations (*e.g.* permeability, effective stress). Thanks to the constant improvements in computer sciences, it is now possible to simulate directly the motion of the particles in a fluid. “Direct” numerical simulations for particulate flows have received a great attention for fifteen years in various domains like chemical engineering, petroleum

industry. In these methods the flow field around each particle is resolved and hydrodynamical forces between particles and fluid are results of simulation (Glowinsky et al., 1999, Peskin, 2002, Yu & Shao, 2007). Of course, it will not be possible to apply this method to an estuary, but it can be useful to better understand the physics at the scale of the particles (collision, flocculation ...) and then to improve Eulerian models: classical models or two-phase flow models. Figure 10 provides an example of sedimentation of a cloud of particles with our model (Verjus et al., 2011).

## 6. Acknowledgment

The authors thank EDF GEH Ouest, IIBSN, AgroParisTech - ENGREF and COEUR who funded J. Thiébot's Ph.D at Laboratory LUSAC, the French research ministry for their funding for the Ph.D. of R. Verjus and K.S. Pouv, The Vietnamese research ministry for their funding for the Ph.D of D.H. Nguyen, and the CETMEF for its financial support of the Ph.D. of J. Chauchat. We thank the Syndicat Mixte du Cotentin for their financial support.

## 7. References

- Amoudry, L.; Hsu, T.J. & Liu, L.F. (2005). Schmidt number and near-bed boundary condition effects on a two-phase dilute sediment transport model. *Journal of Geophysical Research*, Vol. 110, C09003, 12 pp. DOI:10.1029/2004JC002798
- Amoudry, L.; Hsu, T.J. & Liu, P. L.F. (2008). Two-phase model for sand transport in sheet flow regime. *Journal of Geophysical Research*, Vol. 113, C03011, 15 pp. DOI: 10.1029/2007JC004179
- Barbry, N.; Guillou, S. & Nguyen, K. D. (2000). Une approche diphasique pour le calcul du transport sédimentaire en milieux estuariens. *Comptes Rendus Mécanique, Série II.b*, Vol. 328, pp. 793-799.
- Bartholomeeusen, G. (2003). *Compound shock waves and behaviour in sediment beds*. PhD thesis, St. Catherine College, Oxford, 229 p.
- Bartholomeeusen, G. ; Sills, G. C. ; Znidarcic, D. ; Van Kesteren, W. ; Merckelbach, L. M. ; Pykes, R. ; Carrier, W. D. ; Lin, H. ; Penumadu, D. ; Winterwerp, H. ; Masala, S.; Chan, D. (2002). Sidere: numerical prediction of large strain consolidation. *Géotechnique*, Vol. 52, No. 9, pp. 639-648.
- Been, K. & Sills, G. C. (1981). Self-weight consolidation of soft soils: an experimental and theoretical study. *Géotechnique*, Vol. 31, No. 4, pp. 519-535.
- Bonnot-Courtois, C. ; Caline, B. ; L'Homer, A. & Le Vot, M. (2002). *La Baie du Mont-Saint-Michel et l'Estuaire de la Rance – Environnements sédimentaires, aménagements et évolution récente*. Bull. Centre Rech. Elf Explor. Prod., Mém. 26, 256 p.
- Bürger, R. & Hvistendahl, K. (2001). On some upwind difference schemes for the phenomenological sedimentation-consolidation model. *Journal of Engineering Mathematics*, Vol. 41, No. 2-3, pp. 145-166.
- Bürger, R. (2000). Phenomenological foundation and mathematical theory of sedimentation - consolidation processes. *Chemical Engineering Journal*, Vol. 80, pp. 177-188.
- Camenen, B. & Pham van Bang, D. (201). Modelling the settling of suspended sediments for concentrations close to the gelling concentration. *Continental Shelf Research*, doi:10.1016/j.csr.2010.07.003

- Chauchat, J. & Guillou, S. (2008). On turbulence closures for two-phase sediment-laden flows models. *Journal of Geophysical Research*, Vol. 113, C11017, doi:10.1029/2007JC004708.
- Chauchat, J., Guillou, S., Barbry, N., & Nguyen K. D. (2009). Simulation of the turbidity maximum in the Seine estuary with a two-phase flow model. *Comptes Rendus Geosciences, Série II.a.*, Vol. 341, pp. 505-512. doi: 10.1016/j.crte.2009.04.002.
- Chauchat, J.; Guillou, S.; Pham Van Bang, D. & Nguyen, K.D. (2011), A one-dimensional two-phase flow approach for sedimentation-consolidation modelling, *Journal Hydraulic Engineering, ASCE*, (submitted)
- Concha, F.; Bustos, M.C. & Barrientos, A. (1996). Phenomenological theory of sedimentation. In: *Sedimentation of Small Particles in a Viscous Fluid*, Tory, E. (Ed.), pp. 51-96, Computational Mechanics Publications.
- Coussot, Ph (1997). *Mudflow rheology and dynamics*, IAHR monograph series, Balkema, Rotterdam.
- De Boer, G. J.; Merckelbach, L. M.; Winterwerp, J. C. (2007). A parameterised consolidation model for cohesive sediments. *Proceedings in Marine Science*, Vol. 8, pp. 243-262.
- Denot, T. & Lang, P. (2000). Apports de la modélisation numérique à l'étude de la dynamique hydro-sédimentaire de l'estuaire de la Rance, *Journées Nationales Génie Civil Génie Côtier*, pp. 147-154. DOI:10.5150/jngcgc.2000.013-D
- Deutsch, E. & Simonin, O. (1991). Large eddy simulation applied to the motion of particles in stationary homogeneous fluid turbulence. *Turbulence Modification in Multiphase Flows ASME-FED*, Vol. 110, pp. 35-42.
- Dong, P. & Zhang, K. (2002). Intense near-bed sediment motions in waves and currents. *Coastal Engineering*, Vol. 45, No. 2, pp. 75 - 87.
- Gibson, R. E. (1967). The theory of one-dimensional consolidation of saturated clays. *Géotechnique*, Vol. 17, pp. 261-273.
- Glowinski, R.; Pan, T.-W.; Hesla, T.I. & Joseph, D.D. (1999). A Distributed Lagrange multiplier/fictitious domain for particulate flows. *International Journal of Multiphase Flow*, Vol. 25, pp. 755-794
- Greimann, B. P. & Holly, F.M. (2001). Two-phase flow analysis of concentration profiles. *ASCE Journal of hydraulic Engineering*, Vol. 127, No. 9, 753-762.
- Greimann, B.P., Muste, M., Jr., F. M.H. (1999). Two-phase formulation of suspended sediment transport. *J. Hydraul. Res.*, 37, 479 - 500.
- Guillou, S.; Chauchat, J.; Pham Van Bang, D.; Nguyen, D.H. & Nguyen, K.D. (2011). Simulation of the dredged sediment's release with a two-phase flow model, *Bulletin of the Permanent International Association of Navigation Congresses*, Vol. 142, 25-33, ISSN 0374-1001.
- Hervouet, J.M. (2007). *Hydrodynamics of free surface flows: modelling with the finite element method*. Ed. John Wiley & Sons, New-York.
- Hsu, T.; Jenkins, J.T. & Liu, L.F. (2003). On two-phase sediment transport: Dilute flow. *Journal of Geophysical Research*, Vol. 108, 3057, 14 pp., DOI:10.1029/2001JC001276
- Hsu, T.-J. & Liu, P. L.F. (2004). Toward modelling turbulent suspension of sand in the nearshore. *Journal of Geophysical Research*, Vol. 109, C06018, 14pp., DOI: 10.1029/2003JC002240

- Hsu, T.J.; Traykovski, P.A. & Kineke, G.C. (2007). On modeling boundary layer and gravity-driven fluid mud transport. *Journal of Geophysical Research*, Vol. 112, C04011. doi:10.1029/2006JC003719
- Jeeravipoolvarn, S.; Chalaturnyk, R.J. & Scott, J.D. (2009). Sedimentation-consolidation modeling with an interaction coefficient. *Computers and Geotechnics*, Vol. 36, No. 5, pp. 751-761.
- Jiang, J.; Law, A. W.-K. & Cheng, N.-S. (2004). Two-phase modeling of suspended sediment distribution in open channel flows. *J. Hydraul. Res.*, Vol. 42, No. 273 - 281.
- Krone, R. B. (1962). *Flume studies of the transport of sediment in estuarine shoaling processes*. Technical report, Hydraulic Engineering Laboratory, University of California: Berkeley, CA, 110p.
- Le Normant, C. (2000). Three-dimensional modeling of cohesive sediment transport in the Loire estuary. *Hydrological processes*, Vol. 14, pp. 2231-2243.
- Lee, S. R.; Kim, Y. S. & Kim, Y. S. (2000). Analysis of sedimentation - consolidation by finite element method. *Computers and Geotechnics*, Vol. 27, pp. 141-160.
- Li, M.Z. & Gust, G. (2000). Boundary layer dynamics and drag reduction in flows of high cohesive sediment suspensions. *Sedimentology*, Vol. 47, pp. 71-86.
- Longo, S. (2005). Two-phase flow modeling of sediment motion in sheet-flows above plane beds. *Journal of Hydraulic Engineering*, Vol. 131, No. 5, pp. 366-379.
- Lumborg, U. & Windelin, A. (2003). Hydrography and cohesive sediment modelling: application to the Rømø Dyb tidal area. *Journal of Marine Systems*, Vol. 38, No. 3-4, pp. 287-303.
- Merckelbach, L.M. & Kranenburg, C. (2004). Determining effective stress and permeability equations for soft mud from simple laboratory experiments. *Géotechnique*, Vol. 54, No. 9, pp. 581-591.
- Migniot, C. (1968). Etude des propriétés physiques de différents sédiments très fins et de leur comportement sous des actions hydrodynamiques. *La Houille Blanche*, Vol. 7, pp. 591-620.
- Nguyen, K. D.; Guillou, S.; Chauchat, J. & Barbry, N. (2009). A two-phase numerical model for suspended-sediment transport in estuaries. *Advances in Water Resources*, Vol. 32, pp. 1187-1196, DOI:10.1016/j.advwatres.2009.04.001
- Pane, V. & Schiffman, R.L. (1985). A note on sedimentation and consolidation, *Géotechnique*, Vol. 35, No. 1, pp. 69-72
- Partheniades, E. (1965). Erosion and deposition of cohesive soils. *Proceedings of the American Society of Civil Engineers, Journal of the Hydraulics Division*, Vol. 91, pp. 105-139.
- Peskin, C.S. (2002) - The immersed boundary method. *Acta Numerica*, pp. 479-517.
- Pham Van Bang, D. (2007). Rhéophysique des vases : rhéologie et sédimentation. Tech. rep., LCPC-CETMEF.
- Pham Van Bang, D.; Lefrançois, E.; Ovarlez, G. & Bertrand F. (2006). Mri experimental and 1d fe-ct numerical investigation of the sedimentation and consolidation, in *7th International Conference on Hydroinformatics*.
- Pouv, K. S.; Besq, A. & Guillou, S. (2010). Caractérisation rhéométrique des conditions de transition solide/liquide de sédiments cohésifs : vers un lien avec le comportement en érosion ?, *XIèmes journées nationales Génie côtier et Génie Civil*, Sable d'Olonnes, Juin 2010, p 529-538. DOI:10.5150/jngcgc.2010.062-P

- Richardson, J.F. & Zaki, W.N. (1954). Sedimentation and fluidization: Part I. *Trans. Instn. Chem. Engrs*, Vol. 32.
- Teisson, C. (1991). Cohesive suspended sediment transport: feasibility and limitations of numerical modelling. *Journal of Hydraulic Research*, Vol. 29, No. 6, pp. 755-769.
- Teisson, C.; Ockenden, M. C.; Le Hir, P.; Kranenburg, C. & Hamm, L. (1993). Cohesive sediment transport processes. *Coastal Engineering*, Vol. 21, pp. 129-162.
- Teisson, C.; Simonin, O.; Galland, J.C. & Laurence, D. (1992). Turbulence and mud sedimentation: A Reynolds stress model and a two-phase flow model. *Proceedings of 23rd International Conference on Coastal Engineering*, ASCE, pp. 2853-2866.
- Thiébot, J. & Guillou, S. (2006). Simulation of processes acting on water-sediment mixtures in estuaries. *Flow simulation in hydraulic engineering / Dresdner Wasserbaukolloquium*, edited by H.-B. Horlacher and K.-U.Graw, Dresden, mars 2006, pp. 141-148.
- Thiébot, J. (2008). *Modélisation numérique des processus gouvernant la formation et la dégradation des massifs vaseux - Application à l'estuaire de la Rance et aux berges de la Sèvre Niortaise*. PhD Thesis, AgroParisTech-ENGREF, in French, 139p.
- Thiébot, J., Besq, A., Qi, X., Guillou, S. & Brun-Cottan, J.-C. (2006). Sédimentation et consolidation des sédiments cohésifs estuariens: influence des propriétés rhéologiques. *41ème Colloque Annuel de Groupe Français de Rhéologie*, Cherbourg, octobre 2006, pp. 19-22.
- Thiébot, J.; Guillou, S. & Brun-Cottan, J.C. (2010). An optimisation method for determining permeability and effective stress relationships of consolidating cohesive sediment deposits, *Continental Shelf Research*. doi:10.1016/j.csr.2010.12.004
- Thorn, M.F.C. (1981). Physical processes of siltation in tidal channels, *Proceedings of Hydraulic Modelling applied to Maritime Engineering Problems*, ICE, London, 47-55.
- Toorman, E. A. (1999). Sedimentation and self-weight consolidation: constitutive equations and numerical modeling. *Géotechnique*, Vol. 49, No. 6, pp. 709-726.
- Toorman, E.A. (1996). Sedimentation and self-weight consolidation: general, unifying theory. *Géotechnique*, Vol. 46, No. 1, pp. 103-113.
- Torres-Freyermuth, A. & Hsu, T.-J. (2010). On the dynamics of wave-mud interaction: A numerical study. *Journal of Geophysical Research*, Vol. 115, C07014, 18 p., DOI: 10.1029/2009JC005552}
- Townsend, F. C.; McVay, M. C. (1990). SOA: large-strain consolidation predictions. *Journal of Geotechnical Engineering*, 116(2), 166-176.
- Verjus, R.; Guillou, S. & Ahamadi, M. (2011). Simulation of sedimentation of rigid particle suspensions: Towards a multi scale analysis. *International conference 'Two-phase Modelling for Sediment Dynamics'*, Paris, 26-28 April, 4 p.
- Verney, R.; Brun Cottan J. C.; Lafite R.; Deloffre J. (2006). Tidally-induced shear stress variability above intertidal mudflats. Case of the macrotidal Seine estuary. *Estuaries and Coasts*, Vol. 29, No. 4, pp. 653-664.
- Villaret, C. & A. G. Davies (1995). Modelling of sediment-turbulent flow interactions. *Applied Mechanics Review*, Vol. 48, pp. 601-609.
- Villaret, C.; Anh Van, L.; Huybrechts, N.; Pham Van Bang, D. & Boucher O. (2010). Consolidation effects on morphodynamics modelling: application to the Gironde estuary. *La Houille Blanche*, Vol. 6, pp. 15-24.

- Villaret, C.; Claude B. & Du Rivau J.D. (1998). Etude expérimentale de la dispersion des rejets par clapage. He-42/98/065/a, LNHE, EDF.
- Wallis, G., (1969). *One-dimensional Two-Phase Flow*. McGraw-Hill.
- Winterwerp, J. C. & Van Kesteren, W. G. (2004). *Introduction to the Physics of Cohesive Sediment in the Marine Environment*. Developments in Sedimentology Series, Elsevier, Amsterdam.
- Yu, Z. & Shao, X. (2007). A direct-forcing fictitious domain method for particulate flows, *Journal of computational physics*, Vol. 227, pp. 292-314.