Discussion of the paper: ”Rock nail reinforcement of a free surface”

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The paper by Papamichos and Vardoulakis represents a contribution to the mechanical modelling of a rock material reinforced by the introduction of metallic bolts or nails, with a special emphasis on the way the interactions between the nails and the rock may be captured by means of a specific constitutive law. Leaving aside Section 2, which is devoted to the analysis of a single nail embedded in a rock specimen subject to plane-strain compression, our comments and criticisms will be focused on Section 3 (and secondarily Section 4), where the compression of a rectangular block of rock reinforced by an array of regularly distributed nails, is considered.

Referring to the original *shear-lag* concept of Cox dealing with fibre composites materials, the approach developed by the Authors is based on two equilibrium equations as well as two constitutive equations relating to the rock material and the rock-nail interaction, respectively, which are briefly recalled here for the sake of clarity. Referring to figure 5 of their article, where the x-axis is taken parallel to the nail orientation, the first equilibrium equation writes (the equation numbers are those of the original paper):

$$f_x = m\frac{\mathrm{d}N}{\mathrm{d}x} \qquad (14)$$

where N is the axial force in the nail, m the number of nails per unit transverse area and f_x a body force volume density representing the action of the rock on the nail. Correspondingly, the equation governing the equilibrium of the rock is:

$$\frac{\mathrm{d}\sigma_x}{\mathrm{d}x} + f_x = 0 \qquad (15)$$

where σ_x is the normal stress component in the rock material along the x-axis. The linear elastic behaviour of the rock is expressed by Eq. (18) in the form of the classical Hooke's law formulated in the context of plane-strain conditions, while the rock-nail interaction law is written as:

$$\frac{\mathrm{d}N}{\mathrm{d}x} = M(u_n - u_x) \tag{1}$$

where M is constant parameter, u_n is the nail displacement and u_x is the rock displacement.

This brief, but necessary presentation calls for several remarks.

1) The approach advocated by the Authors and defined by the above set of governing equations, appears to be nothing but a particular case of the *multiphase approach* which has been proposed more than ten years ago for describing the macroscopic behaviour of soils or rocks strengthened by linear inclusions (see among other references: [1], [2] or [3]). According to this model, the reinforced soil or rock is described *at the macroscopic scale* as the superposition of two mutually interacting continua (called "phases", hence the denomination of the model); the *matrix phase* represents the soil or rock material, while the *reinforcement phase* is the three-dimensional continuum equivalent of the array of uniformly distributed parallel inclusions.

2) The above equations could therefore be revisited in the light of this multiphase model, in the sense that all the force/stress or displacement/strain variables introduced above, are to be regarded as *macroscopic quantities*. Thus, mN appearing in the equilibrium equation (14), is a density of axial force per unit transverse area, with the dimension of a stress, while in Eq. (15) σ_x should be regarded as a macroscopic stress variable, which can be computed as the average value of the same stress component over the representative volume of nailed rock (called "volume of the nail influence" in the paper). The same comment applies to the rock displacement variable u_x and to the associated strain variables as well.

3) As shown in [4] and [5], this multiphase model can be regarded as an extension of the homogenization approach, where the reinforced ground is considered as a *single homogeneous anisotropic medium*. In the problem considered by the Authors, this situation is recovered by assigning the same kinematics to both phases (u_n=u_x) or alternatively by making parameter M in Eq. (1) tend to infinity (*perfect bonding* assumption).

4) One of the key features of the multiphase model is the interaction constitutive law, which is wrongly formulated as Eq. (1) by the Authors. Indeed, the later equation is a combination of the interaction law, which relates the interaction body force density to the difference between the phase displacements, and of the equilibrium equation of the reinforcement phase (14). Furthermore, the definition proposed by the Authors of the displacement u_x involved in Eq. (1) is rather confusing. First defined as "the displacement of the rock at the same point *if the nail was absent*" (quotation from paragraph just above Eq. (1)), a quite different, but finally correct definition is adopted in the analytical developments of Section 3, notably Eq. (22): u_x is the *actual* displacement experienced by the matrix phase, representing the rock, *in the presence of the nail reinforcement*.

5) The Authors assume that, due to the fact that the nail is much stiffer than the surrounding rock, the former can be considered as rigid, that is inextensible, in the analysis. This assumption is questionable, since it is proved for instance in [4] or [5], that the relevant stiffness parameter to be compared with the rock elastic moduli, is not the elastic modulus E_n of the nail constituent material, but this *elastic modulus multiplied by the reinforcement volume fraction*, which can also be calculated as the product of the individual nail stiffness E_nA_n by the nail density m. Since this volume fraction is small, the later parameter, called *reinforcement axial stiffness density* is of the same order as the rock elastic moduli, and the reinforcement can hardly be considered as rigid under such conditions.

6) The analytical developments performed by the Authors in Section 3 should therefore constitute a particular case of the complete solution of an almost identical problem given in [4], where the extensibility of the reinforcements is taken into account. Unfortunately, it appears that, due to an initial sign error (A^2u_n should be changed in $-A^2u_n$ in Eq. (24) of the paper), the subsequent analysis, and notably the calculated value of u_n given by Eq. (36), is not valid.

7) One last final remark refers to Section 4 where the results of the previous analysis are used for evaluating the strength enhancement due to the presence of the nails. The Authors rightly point out that in both lateral reinforced zones of the rock specimen, the tensile forces generated in the nails provide an horizontal compressive confining stress to the rock, thus increasing its compressive strength derived from a Mohr-Coulomb criterion, evaluated from the average "free surface strength" (Eq. (40)). But the Authors fail to notice that no strength enhancement of the rock located in the central unreinforced zone of the specimen is to be

expected, since owing to simple equilibrium considerations the horizontal stress component remains equal to zero in this zone. The conclusion would of course be different for an axisymmetric (and not a plane strain) compressed specimen.

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