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Non-uniform Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) state

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ABSTRACT

We provide a general review of the properties of the non uniform superconducting Fulde Ferrell Larkin Ovchinnikov (FFLO) phase. Special emphasis is made on the orbital and crystal structure effects which may result in the quantum transitions between the higher Landau level states and should be responsible for the strong modification of the anisotropy of the critical field. The FFLO type instability may be also expected in ultracold Fermi gases. In these systems it is caused not by the Zeeman interaction but by the tuning of the population imbalance between two lowest hyperfine states of the atoms. We also briefly discuss their properties.

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1. Introduction

The non uniform superconducting state has been predicted independently by Fulde Ferrell [1] and Larkin Ovchinnikov [2] in 1964. This phase is called now the Fulde Ferrell Larkin Ovchinnikov (FFLO) phase and in their original works Fulde Ferrell and Larkin Ovchinnikov considered superconductors in the magnetic field acting on the electron spins only (the case of the paramagnetic effect). Usually it is an orbital effect which is most important and this made difficult the experimental observation of the FFLO phase. Moreover the superconductor must be in the clean limit because electron scattering is detrimental for the FFLO phase [3]. The orbital effect may be weakened in heavy fermion superconductors or in quasi 2D superconductors when magnetic field is applied parallel to the superconducting planes. That is why the evidences of the FFLO phase has been revealed in the heavy fermion superconductor CeCoIn₅ (see Ref. [4] and references cited therein), and in organic quasi 2D superconductors λ (BETS)₂ FeCl₄

[5] and κ (BEDT TTFS)₂ Cu(NCS)₂ [6] as well as in some other organic superconductors.

The physical reason of the FFLO state formation is that due to the Zeeman splitting the Fermi momenta of the electrons with opposite spin orientation are different. The singlet Copper pair consists of two electrons ($\mathbf{k}\uparrow$, $\mathbf{k}\downarrow$) and the total momentum of the pair is zero. In the presence of Zeeman splitting the electrons on the Fermi surface forming the Cooper pair should have different momenta ($\mathbf{k}\uparrow$, $\mathbf{k}+\mathbf{q}\downarrow$), where $q \sim 2\mu_B H / (\hbar v_F)$ (μ_B is the Bohr magneton), and the resulting momentum of the Cooper pair q is non zero. This results in the formation of the superconducting condensate with a wave function modulated in space. While it is rather difficult to observe the FFLO phase in superconductors, the same physics is at the origin of the oscillatory like proximity effect between a superconductor and a ferromagnet and leads to many interesting phenomena observed in experiment [7]. We may illustrate the physics of the FFLO state also on the basis of Ginzburg Landau (GL) approach. All the coefficients of the GL functional could be calculated from the microscopical theory and it occurs that starting from some limiting value of the ratio $\mu_B H / T > 1.89$ the gradient term in the GL functional changes its sign. The FFLO state can be described in the framework of the modified Ginzburg Landau (MGL) functional [8] with negative gradient terms and additional higher derivative terms if the FFLO

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modulation vector is small $q \ll \xi_0^{-1}$

$$\mathcal{F} = \alpha |\Psi|^2 \left[g \sum_{i=1}^3 |\Pi_i \Psi|^2 + \gamma \left| \sum_{i=1}^3 \Pi_i^2 \Psi \right|^2 \right], \quad (1)$$

where $\alpha(H, T) = \alpha_0(T - T_{cu}(H))$ and $T_{cu}(H)$ is transition temperature of the uniform state, $\Pi_i = i\hbar \partial / \partial x_i$ ($2e/c$) A_i are momentum operators and A_i are the components of the vector potential. Looking for the solution of the form $\Psi \sim \exp(i\mathbf{q}\mathbf{r})$, we immediately see that the condition of the superconducting transition is $\alpha_0(T - T_{cu}(H)) gq^2 + \gamma q^4 = 0$, and the highest transition temperature $T_{ci} = T_{cu}(H) + (g^2/4\alpha_0)\gamma$ corresponds to the transition to the modulated state with the wave vector $q_0 = \sqrt{g/2\gamma}$. The transition into FFLO state occurs at $T < T^* = 0.56T_c$ and $\mu_B H > H^* = 1.06T_c$.

2. FFLO state in the pure Pauli limit

The exact solution for the FFLO state is known only for the 1D or quasi 1D superconductors [9–11]. The superconducting order parameter is described by the Jacobi elliptic functions $\Delta(z) \sim sn(z, k)$. This type of the soliton lattice solution has been proposed previously for the doped polyacetylene [12]. Near the transition from the normal to the superconducting state the $sn(z, k)$ function is transformed into the usual $\sin(z)$ function, while at the transition from FFLO (soliton lattice) state to the uniform state the $sn(z, k)$ function corresponds to the large domains of the uniform phase separated by the domain walls. At low temperature the critical field of the FFLO transition diverges and the paramagnetic limit is lifted.

In 2D superconductors near the tricritical point (H^*, T^*) the FFLO phase is the modulated structure with a single wave vector [13, 15, 14]. With a decrease of the temperature this 1D pattern transforms into a square modulation of the order parameter, and then the 3 vectors, the 4 vectors, etc. order parameter spatial structures appear in sequence [15]. In the 2D case at $T=0$ the paramagnetic limit for the FFLO phase is $\mu_B H_p^{2D} = 2\Delta_0$ [16], which exceeds the standard Chandrasekhar Clogston limit $\mu_B H_p(0) = \sqrt{2}\Delta_0$, where Δ_0 is BCS gap at $T=0$ [17].

In 3D superconductors near the tricritical point the FFLO phase also appears as a 1D modulated structure, while at lower temperature the modulated phase should be square and then cubic [14, 18]. In 3D case at $T=0$ the paramagnetic limit for the FFLO phase is $\mu_B H_p^{3D} = 1.51\Delta_0$ and only slightly exceeds the Chandrasekhar Clogston limit see Fig. 1.

All calculations of the FFLO structure in 2D and 3D have been performed within the framework of the isotropic model. However the isotropic model may be hardly applied for real superconductors because the higher derivative terms in general functional will create the anisotropy even for cubic crystal symmetry and will leave the degeneracy over the directions of the FFLO modulation

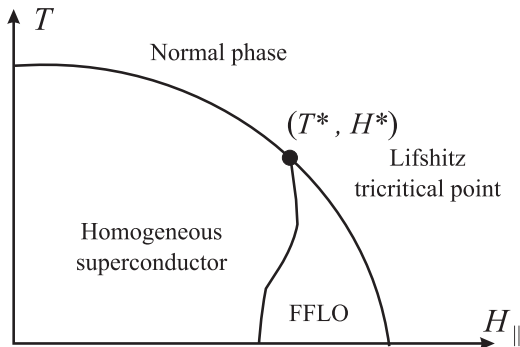


Fig. 1. Schematic phase diagram of the FFLO state in the plane (H, T) for a 2D superconducting film. Magnetic field is parallel to the film surface.

vector [19, 20].

$$\mathcal{F}^{cubic} = \alpha |\Psi|^2 \left[g \sum_{i=1}^3 |\Pi_i \Psi|^2 + \gamma \left| \sum_{i=1}^3 \Pi_i^2 \Psi \right|^2 + \varepsilon \sum_{i=1}^3 |\Pi_i^2 \Psi|^2 \right]. \quad (2)$$

Note that in the case of the d wave superconducting pairing the additional anisotropy should appear due to the pairing potential. It is worth mentioning that the fluctuational regime describing superconducting correlations near the tricritical point (H^*, T^*) by Eq. (2) is very different from the usual BCS one case [21].

3. FFLO and orbital effects

In the framework of the isotropic model with s wave pairing the critical field for the FFLO phase in presence of the orbital effect has been calculated by Gruenberg and Gunther [22]. They demonstrated that the FFLO state may only exist if the ratio of pure orbital effect $H_{c2}^{orb}(0)$ and pure paramagnetic limit $H_p(0)$ is larger than 1.28, i.e. the Maki parameter $\alpha_M = \sqrt{2}H_{c2}^{orb}(0)/H_p(0)$ is larger than 1.8. In Ref. [22] the exact solution for the order parameter represents the FFLO phase with the modulation vector directed along the magnetic field and the zero Landau level function for the coordinates in perpendicular plane. Further analysis [23] revealed that the higher Landau level solutions (LLS) became relevant for large values of Maki parameter $\alpha_M > 9$ and the $H_{c2}(T)$ curve may represent the regions described by different LLS. This results obtained for isotropic model are readily generalized for the case when the electron spectrum anisotropy is described by elliptic Fermi surface [24]. In such case the Maki parameter becomes angular dependent and the transitions between different LLS phases may occur with a change of the magnetic field orientation. Therefore we may expect the transition between the usual FFLO state with zero Landau levels [22] to the state with higher Landau levels [23, 28] in quasi 2D superconductors when the magnetic field is tilted from the perpendicular orientation to the parallel one. Note that as has been shown in Ref. [24] the scaling transformation of the coordinates completely maps the FFLO problem with the elliptic Fermi surface into that with the spherical one and therefore the direction of the FFLO modulation vector is arbitrary in the absence of orbital effect.

Higher LLS naturally appear in 2D superconductors [16, 23, 25, 26] and result in the oscillatory like angular dependence of the critical field in the FFLO state [25]. The vortex lattices corresponding to the higher LLS are pretty special they may carry several flux quanta per cell and include vortices with different vorticity [29, 27] see Fig. 2.

For the adequate description of the FFLO state in real compounds the form of the Fermi surface as well as the type of the superconducting pairing play a very important role because it determines the direction of the FFLO modulation vector. This circumstance has been demonstrated in Ref. [20] within the framework of the general phenomenological approach based on the modified MGL functional [8]. It occurs that the higher LLS may be realized for arbitrary values of Maki parameter in contrast with the isotropic model. This is because of the special mechanism of the higher Landau level phase formation in 3D systems. Moreover depending on various types of deviation of the Fermi surface from isotropic form three possible solutions for the FFLO state can be realized: (a) maximum modulation occurs along the magnetic field with zero Landau level state, (b) both modulation and higher Landau level state, and (c) highest possible Landau level and no modulation along the field (or modulation with very small wave vector).

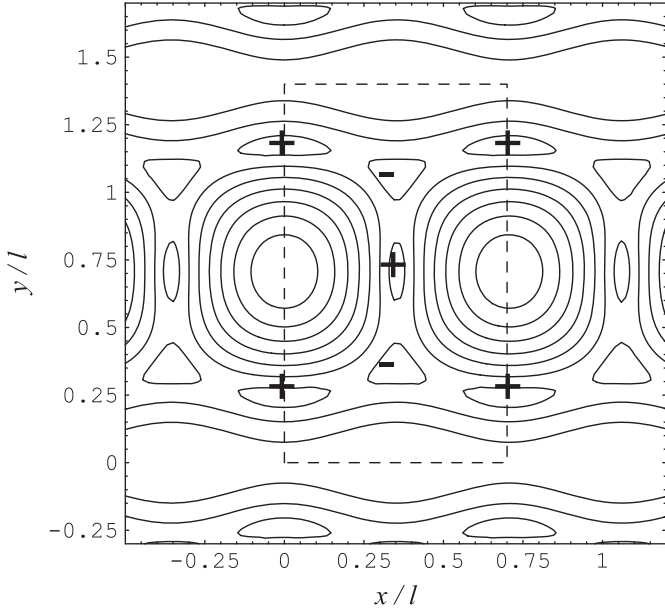


Fig. 2. Square asymmetric vortex lattice described by the LLS with $n = 2$ formed in the presence of orbital and spin effects. The profiles of the amplitude of the superconducting order parameter are shown. The dashed line represents the unit cell with five zeros [27].

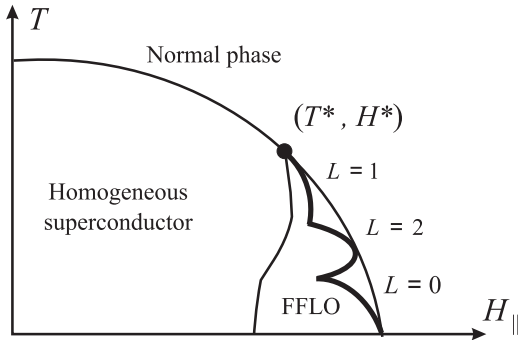


Fig. 3. Schematic phase (H, T) diagram for a superconducting disk of a finite radius in parallel magnetic field [33]. The transitions between the different orbital states (L) are related to the commensurability effects between the period of FFLO modulation and the disk radius R .

4. FFLO and cold gases

During the last decade a lot of attention has been attracted to the ultracold Fermi gases in magneto optical traps – a new type of superfluid systems, which are considered as promising playground for the study of this intriguing phenomenon [30]. The FFLO type instability in these systems is caused not by the Zeeman interaction but by the tuning of the population imbalance between two lowest hyperfine states of ^6Li atoms. Experimentally this population imbalance is governed by the radiofrequency signal inducing transitions between the hyperfine states. Thus, changing the population imbalance we should get the inhomogeneous FFLO state with a certain intrinsic length scale and this phenomenon is not masked by any kind of the orbital effect. The orbital effect in such neutral atomic condensates is associated not with magnetic field but with system rotation which is known to be an important part of the experimental procedure of detection of superfluidity in the ultracold gases. The interesting predictions regarding the properties of such cold gas FFLO state has been

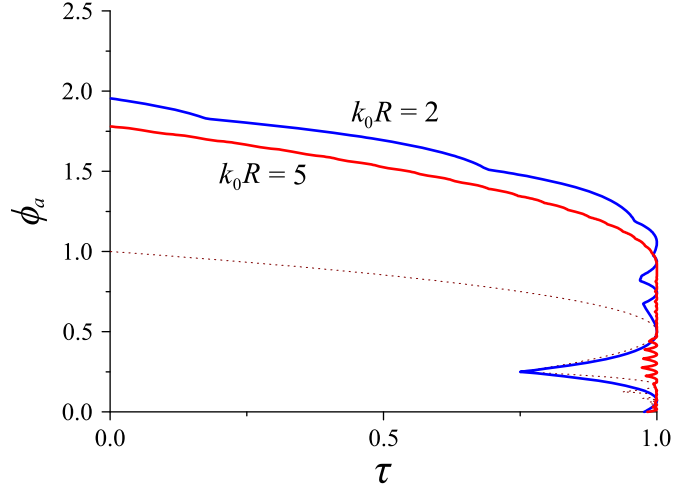


Fig. 4. Typical phase diagrams for 2D disk in the plane (H, T) for disk radii $k_0 R = 2; 5$, where k_0 is a wave vector of FFLO modulation [33]. The magnetic field is described by the flux through the disk $\phi_a = \pi R^2 H / \Phi_0$.

made for the 1D like traps – see Ref. [31] and references therein, and for 2D traps [32,33]. We may consider a superconducting disk as a rough model of a cold gas in 2D trap. In such a case we could expect a realization of FFLO state with different winding numbers (orbital states $L \neq 0$) – Figs. 3 and 4 [33]. However at the moment the experimental observations of the FFLO phase in imbalanced cold gases are still lacking.

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