Identification of a probabilistic model of the geometrical and mechanical properties for a nonhomegeneous cortical bone using in vivo measurements in ultrasonic range

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IDENTIFICATION OF A PROBABILISTIC MODEL OF THE GEOMETRICAL AND MECHANICAL PROPERTIES FOR A NONHOMEGENEOUS CORTICAL BONE USING IN VIVO MEASUREMENTS IN ULTRASONIC RANGE

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The purpose of this paper is the construction of an uncertain probabilistic model for the mechanical properties of the cortical bone. The main objective is firstly to propose a probabilistic simplified model adapted to the ultrasonic axial transmission technique and secondly to present an experimental identification using this technique. The simplified model is constructed as a fluid-solid semi-infinite multilayered system in which the solid layer (the cortical bone) is a homogeneous anisotropic elastic material and the two others semi-infinite layers are fluids. This model is a rough approximation of the real biomechanical model and it can be improved by taking into account the uncertainties related to the elasticity tensor with a probabilistic model. In this paper, the parameters of the probabilistic model are the mean elasticity tensor and a dispersion coefficient. A complete application is presented for the human cortical bone for which an experimental database is available.

1. Introduction

Biomechanical systems such as the cortical bone, are very complex systems which are difficult to model in regard to their constitutive material at the microscopic scale. Such a biomechanical system can be modeled using a mechanical model which can be more or less sophisticated using or not a multiscale approach. Nevertheless, assumptions yielding modeling simplifications and approximations are introduced and therefore the developed model is always a rough approximation of the real biomechanical system. In this paper, such sources of uncertainties are taken into account in order to extend the domain of validity of a simplified model sind, therefore, a probabilistic model is constructed to take into account uncertainties in the model of elasticity tensor of the cortical bone.

This construction is carried out using Information Theory with the available information derived from the mechanical and probabilistic properties for the random elasticity tensor. The parameters of the probabilistic model are the mean elasticity tensor and a dispersion parameter characterizing the level of uncertainties. The identification of these parameters is performed by using *in vivo* measurements obtained with the axial transmission technique: an acoustic pulse is applied on the skin layer in the ultrasonic range and the velocity of the first arriving signal is measured. Thus, the purpose of this paper is the experimental identification and validation of the probabilistic model developed to take into account uncertainties in the elasticity tensor of the cortical bone using the ultrasonic axial transmission technique. In this paper, the simplified mechanical model is constructed as a fluid-solid semi-infinite multilayered system in which the solid layer (representing a cortical bone) is a homogeneous anisotropic elastic material and the two others semi-infinite layers are fluids (representing the skin and the marrow bone). This model is obviously quite simple in regard to the real biomechanical system for which the cortical layer is, for instance, heterogeneous. Thus, uncertainties related to the cortical bone have to be taken into account to improve the validity domain of such a simplified model that allows the velocity of the first arriving signal to be accurately predicted. The solver used for this problem is presented in [1]. The experimental database is obtained by *in vivo* ultrasonic axial transmission on cortical bones of a given set of patients. A computational optimization problem is then introduced, consisting in minimizing a cost function with respect to the parameters of the probabilistic model. This cost function is defined by taking into account the type of experimental observations. The stochastic solver used to solve the optimization problem is based on the Monte Carlo method for which the simplex algorithm is used at each iteration. The complete stochastic model is presented with its experimental validation.
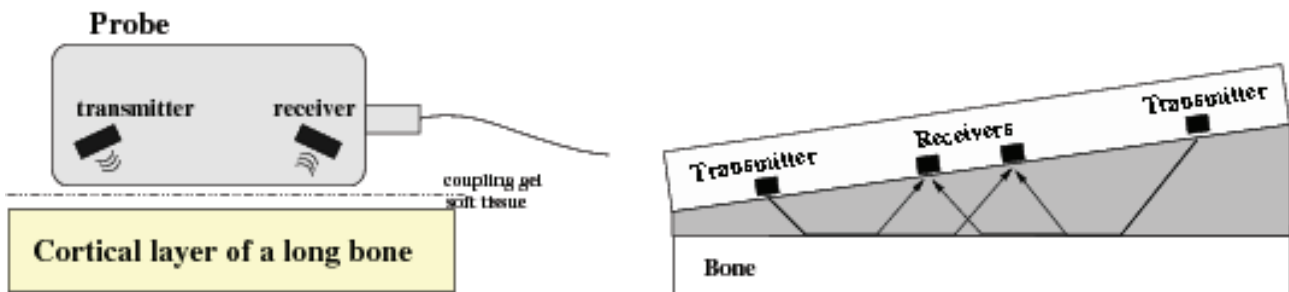
2. Experimental database

Figure 1. Experimental configuration

The ultrasonic axial transmission technique is used to construct an experimental database (see for instance [3-5]). The experimental configuration is described by Fig. 1. A device has been designed and is made up of several receivers and transmitters. A coupling gel is applied at the interface between the device and the skin of the patient. Each transmitter generates an acoustical impulse in the ultrasonic range that propagates in the coupling gel, the skin, the muscle, the cortical bone and the marrow. The axial transmission technique consists in recording these signals at several receivers located in the device. The first arriving contribution of the signal (FAS) is considered. Following the signal processing method used with the experimental device, the velocity of FAS is determined from the time of flight of the first extremum of the contribution. Figure 2 shows a part of a simulated signal and the FAS. Each velocity measurement is considered as a realization of a random variable $V^{\exp}$ corresponding to the random variable V^{mod} obtained with the stochastic simplified model. The mean value of $V^{\exp}$ is $v^{\exp} = E\{V^{\exp}\}$ and its coefficient of variation $\Delta^{\exp}$ is defined by $(\Delta^{\exp})^2 = E\{(V^{\exp})^2\}/(v^{\exp})^2 - 1$ in which $E\{\cdot\}$ is the mathematical expectation.
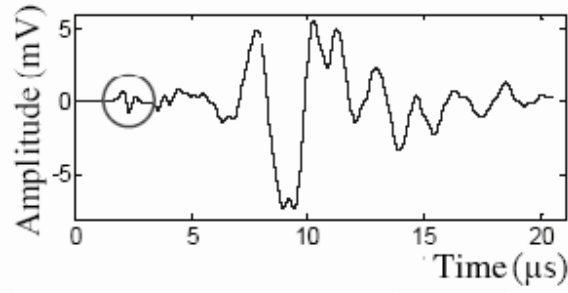
Figure 2. First arriving signal

3. Simplified model

A simplified model of the biomechanical system made up of the coupling gel, the skin, the cortical bone and the marrow has been developped in [1, 2]. This simplifed model is composed of an elastic solid semi-infinite layer between two acoustic fluid semi-infinite layers (see Fig. 3). The
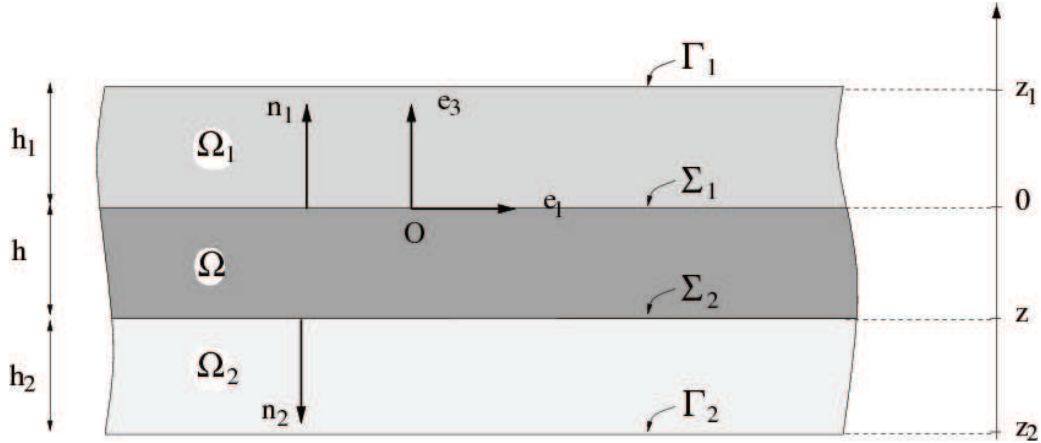

Figure 3. Geometry of the multilayer system

thicknesses of the layers are denoted by h_1, h and h_2. Let $\boldsymbol{u}$ be the displacement field of the solid elastic and let p_1 and p_2 be the pressure fields in the two acoustic fluid layer for which the masses density are denoted by ρ_1, ρ_2 and the celerities are denoted by c_1 and c_2. Let ρ be the mass density of the solid. Let $[C]$ be the effective elasticity matrix of the solid layer for which the components are the coordinates of the elasticy tensor. A line source modeling the acoustical impulse is applied in the top acoustic fluid layer at a distance x_3^S from the interface between the top acoustic fluid layer and the elastic solid layer. The central frequency of the impulse is $f_c = 1\,\text{MHz}$. In Fig.4, the time history and the modulus of its Fourier transform are shown. At time $t = 0$, the system is assumed to be at rest. For a given effective elasticity matrix $[C]$, the displacement field in the solid layer and the pressure fields in the two fluids are calculated using the fast and efficient hybrid solver presented in [1]. For a given mean elasticity matrix $[C]$, this solver allows the displacement field $\boldsymbol{u}$ in the elastic solid layer and the pressure fields p_1 and p_2 in the two acoustic layers respectively, to be calculated. Then, the velocity v^{velo} of the first arriving signal is deduced. Consequently, it is possible to construct a mapping g_{velo} such that

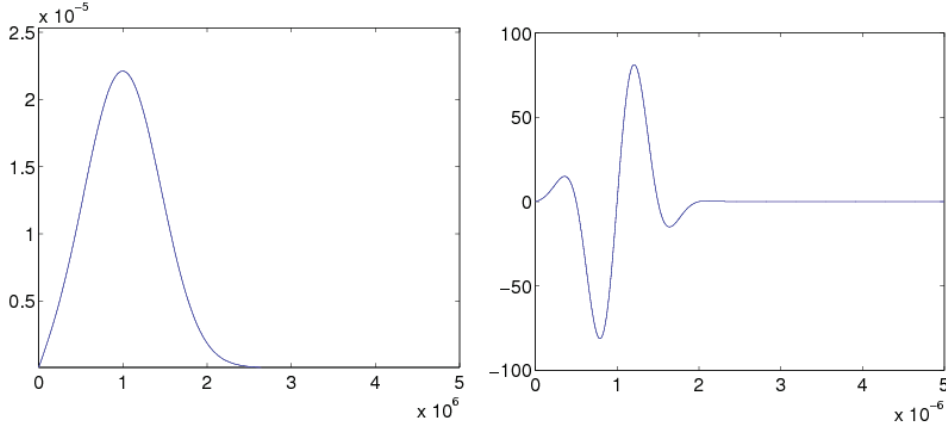$$v^{\text{mod}} = g_{\text{velo}}([C]) \quad . \tag{1}$$

Figure 4. Time history (right) and modulus of the Fourier transform of the pulse (left): horizontal axis in seconds (right) and in MHz (left)

4.   Stochastic simplified model

It is assumed that uncertainties are only related to the components of the effective elasticity tensor. The introduced probabilistic model is presented in [2]. Then, the construction of the probabilistic model consists in substituting $[C]$ by a random matrix $[\mathbf{C}]$ for which the probability density function is constructed using the information theory (see [6, 7]) with the available information defined as follows: (1) the random matrix $[\mathbf{C}]$ is a second-order random variable with values in the set $\mathbb{M}^+(\mathbb{R})$ of all the (6×6) real symmetric positive-definite matrices; (2) the mean value of random matrix $[\mathbf{C}]$ is the mean elasticity matrix $[C]$; (3) the norm of the inverse matrix of $[\mathbf{C}]$ is a second-order random variable. It has been shown in [8, 9] that the random matrix $[\mathbf{C}]$ can then be written as

$$[\mathbf{C}] = [L]^T[\mathbf{G}][L] \quad , \tag{2}$$

in which the (6×6) upper triangular matrix $[L]$ corresponds to the Cholesky factorization $[C] = [L]^T[L]$ and where the probability density function $p_{[\mathbf{G}]}$ of random matrix $[\mathbf{G}]$ is written as

$$p_{[\mathbf{G}]}([G]) = \mathbb{1}_{\mathbb{M}^+(\mathbb{R})}([G]) \, c \, (\det[G])^b \, \exp\{-a\mathrm{tr}[G]\} \,, \tag{3}$$

where $a = 7/(2\delta^2)$, $b = a(1 - \delta^2)$, $\mathbb{1}_{\mathbb{M}^+(\mathbb{R})}([G])$ is equal to 1 if $[G]$ belongs to $\mathbb{M}^+(\mathbb{R})$ and is equal to zero if $[G]$ does not belong to $\mathbb{M}^+(\mathbb{R})$, $\mathrm{tr}[G]$ is the trace of matrix $[G]$ and where positive constant c is such that

$$c = \frac{(2\pi)^{-15/2}a^{6a}}{\prod_{j=1}^6 \Gamma(\alpha_j)} \,, \tag{4}$$

in which $\alpha_j = 7/(2\delta^2) + (1 - j)/2$ and where Γ is the Gamma function. The parameter δ allows the dispersion of the random matrix $[\mathbf{C}]$ to be controlled. Thus, the parameters of the probabilistic model of uncertainties for the elasticity matrix are the components of $[C]$ and the coefficient δ. The velocity of the FAS constructed using this stochastic simplified model is a random variable denoted by V^{mod} that corresponds to the random experimental velocity V^{exp} introduced in Section 2 and we have (see Eq. (1))

$$V^{\mathrm{mod}} = g_{\mathrm{velo}}([\mathbf{C}]) \quad . \tag{5}$$

5. Optimization problem for the identification

The mean elasticity matrix of the simplified mechanical model is assumed to be related to a transverse anisotroopic elastic medium. Such a medium is defined by (1) the longitudinal and transversal Young moduli e_L and e_T, (2) the longitudinal and transversal shear moduli g_L and g_T and (3) the longitudinal and transversal Poisson coefficients ν_L and ν_T that are such that $g_T = e_T/2(1 + \nu_T)$. We then have $[C] = [C(\mathbf{a})]$. Let $\mathbf{a}$ be the vector such that $\mathbf{a} = (\rho, e_L, \nu_L, g_L, e_T, \nu_T)$. The identification problem consists in finding vector $\mathbf{a}$ and coefficient δ such that the stochastic model can represent the experimental database in a statistical sense. The optimal values $(\mathbf{a}^{\text{opt}}, \delta^{\text{opt}})$ for $(\mathbf{a}, \delta)$ is given by solving the following optimization problem

$$(\mathbf{a}^{\text{opt}}, \delta^{\text{opt}}) = \arg \min_{(\mathbf{a}, \delta)} F^{\text{cost}}(\mathbf{a}, \delta) \quad , \tag{6}$$

in which $F^{\text{cost}}(\mathbf{a}, \delta)$ is the cost function which has to be defined. The cost function F^{cost} adapted to the optimization problem is written as

$$F^{\text{cost}}(\mathbf{a}, \delta) = \frac{(v^{\text{exp}} - v^{\text{mod}}(\mathbf{a}, \delta))^2}{(v^{\text{exp}})^2} + \frac{(\Delta^{\text{exp}} - \Delta^{\text{mod}}(\mathbf{a}, \delta))^2}{(\Delta^{\text{exp}})^2} \quad , \tag{7}$$

in which $v^{\text{mod}}(\mathbf{a}, \delta) = E\{V^{\text{mod}}(\mathbf{a}, \delta)\}$ and

$$\Delta^{\text{mod}} = \sqrt{\frac{E\{(V^{\text{mod}}(\mathbf{a}, \delta)^2\}}{(v^{\text{mod}}(\mathbf{a}, \delta))^2} - 1} \quad . \tag{8}$$

The optimization problem defined by Eq. (6) is solved by the simplex algorithm. For each iteration of the simplex algorithm, the cost function is calculated in solving the stochastic equations with the Monte Carlo numerical simulation method (see for instance [1, 2]).

6. Experimental validation of the stochastic simplified model

This section is devoted to the experimental validation of the stochastic simplified model. The stochastic simplified model must be able to simulate the experimental database in a statistical sense. The experimental validation is performed with the *in vivo* experimental database presented in Section 2 and is made up of $N = 2747$ mesurements $V^{\text{exp}}(\hat{\theta}_1), \ldots, V^{\text{exp}}(\hat{\theta}_N)$ plotted in Fig. 5. The identification of the vector $\mathbf{a} = (\rho, e_L, \nu_L, g_L, e_T, \nu_T)$ and the coefficient δ is carried out using the method presented in Section 5 with $h_1 = 10^{-2}$m, $h = 4 \times 10^{-3}$m, $h_2 = 10^{-2}$m, $\rho_1 = \rho_2 = 1000$ kg.m^{-3} and $c_1 = c_2 = 1500$ m.s^{-1} The solution $\mathbf{a}^{\text{opt}} = \left(\rho^{\text{opt}}, e_L^{\text{opt}}, \nu_L^{\text{opt}}, g_L^{\text{opt}}, e_T^{\text{opt}}, \nu_T^{\text{opt}}\right)$ and δ^{opt} are such that $\rho^{\text{opt}} = 1598.8$ kg.m^{-3}, $e_L^{\text{opt}} = 17.717$ GPa, $\nu_L^{\text{opt}} = 0.3816$, $g_L^{\text{opt}} = 4.7950$ GPa, $e_T^{\text{opt}} = 9.8254$ GPa, $\nu_T^{\text{opt}} = 0.4495$ and $\delta^{\text{opt}} = 0.1029$. For $\mathbf{a} = \mathbf{a}^{\text{opt}}$ and $\delta = \delta^{\text{opt}}$, the realizations $V^{\text{mod}}(\theta_1), \ldots, V^{\text{mod}}(\theta_N)$ of random velocity V^{mod} are constructed with the stochastic simplified model and then, the probability density function $v \mapsto p_{V^{\text{mod}}}(v)$ of V^{mod} is estimated. Figure 6 compares the graphs of the probability density function $v \mapsto p_{V^{\text{exp}}}(v)$ of the random variable V^{exp} estimated with the $N = 2747$ experimental realizations $V^{\text{exp}}(\hat{\theta}_1), \ldots, V^{\text{exp}}(\hat{\theta}_N)$ and the graph of $v \mapsto p_{V^{\text{mod}}}(v)$ in logarithm scale. This figure shows that the stochastic simplified model is able to predict in a statistical sense the velocity of the first arriving signal in a good accordance with the experimental tests.

7. Conclusion

A simplified elastoacoustic model has been developed to simulate the ultrasonic wave propagation in a complex biomechanical system made up of multilayered media. In order to improve
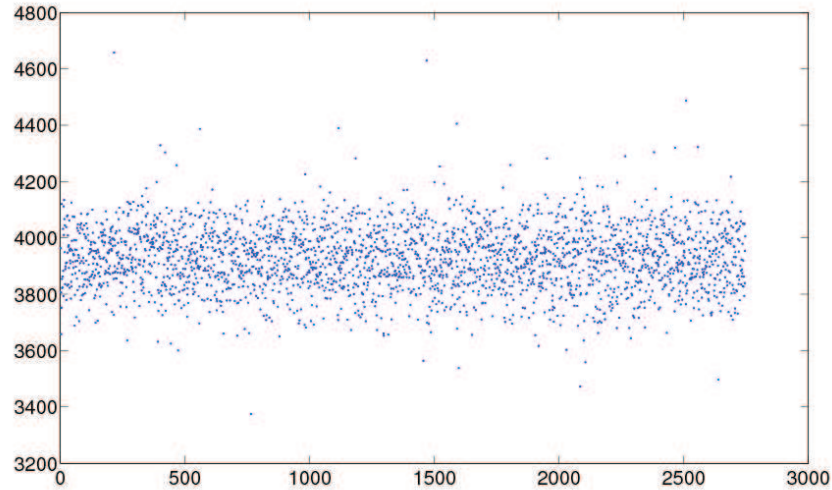
Figure 5. Graph of the realizations $V^{\exp}(\hat{\theta}_1), \ldots V^{\exp}(\hat{\theta}_N)$

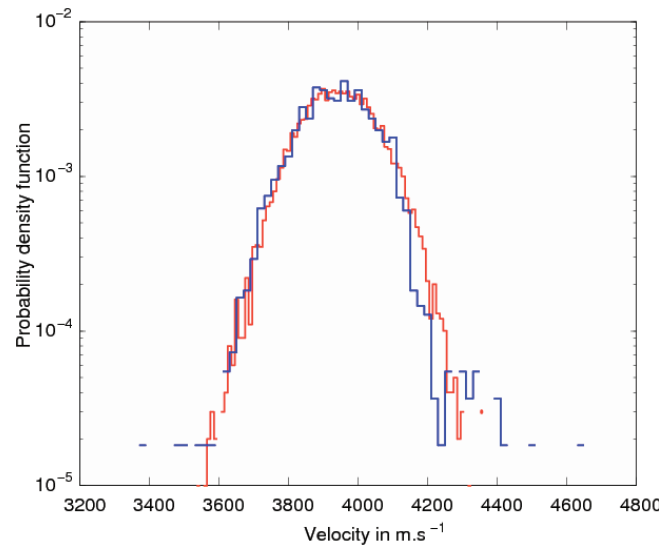**Figure 6.** Graphs of $v \mapsto \log(p_{V^{\exp}}(v))$ and $v \mapsto \log(p_{V^{\exp}}(v; \mathbf{a}^{\mathrm{opt}}, \delta^{\mathrm{opt}}))$.

the simplified model, the uncertainties related to the solid layer have been taken into account using a probabilistic approach. A method has been presented to identify the parameters of the stochastic simplified model. The capability of the proposed stochastic simplified model to predict the velocity of the first arriving signal in the statistical sense has been demonstrated using a large experimental *in vivo* database.

8. Acknowledgments

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