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Provability and the Continuum Hypothesis. A Letter.

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Provability and the Continuum Hypothesis. A Letter. Institute of Actuaries Scotland. J.I. Pillay.¹

Abstract

A recent development in the theory of \mathcal{M} -sets has paved a new means of an approach to CH. Here we further clarify the theory and form part two of our previous paper[P].

Notation

Object o_i, B_i

\mathcal{M} -set A set of Mechanisms.

Object sets $\mathcal{O}_i, \mathcal{B}_i$

Information sets \mathcal{I}

The set \mathcal{B}_i will be used to represent binary numbers.

A set of positions associated with a one symbol we will denote by pos

Elements of pos we will denote by $p(1)_j$.

Sets of transformations T_i .

A knowable set H is denoted by $K(H)$

A Predictably knowable set H is denoted by $PK(H)$

A Writable set H is denoted by $W(H)$

$Pow(n) n|x^n$ for a free variable x .

RX Reasons for statement X .

$Arith(S)$ a statement made in arithmetic.

1.1 M-sets.

Definition 1.1.1.1. Object-Representation \mathcal{R}_o .

An entity that makes use of symbols and spatial positions as parameters for use in distinguishing one object from another within an object set \mathcal{O} . In addition, the mapping between $\mathcal{O} \rightarrow \mathcal{I}$ the object set to the set of information that we wish to map each object to, is implicitly established with the entity.

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Definition 1.1.1.2. \mathcal{I} -Representation \mathcal{R} .

A representation that conjuncts symbols and placements for these referred to as spatial positions for use in structuring the mapping $T(\mathcal{R}_i) \mapsto j_i \{j \in \mathcal{I}\}$.

Definition 1.1.2. ϕ -operation.

Given an ordered set $\mathcal{S} := \{o_i\}$ of objects o_i , the ϕ -operation is the most fundamental of operations, that transforms one object of $\{\mathcal{S}\}$ to the next.

Definition 1.1.3. Mechanism \mathcal{M} .

A transformation resulting in a single change in symbol or spatial ordering of a representation, is known as the Mechanism responsible .

1.1.2 Multiplicative M-sets.

With the aim of forming a more complete understanding of arithmetic operations, we consider the \mathcal{M} -sets associated with natural numbers raised to a specific power.

An extremely beautiful observation we discovered, that when used in combination with the multinomial theorem aides us greatly in our undertaking of this task.

Observation 1.1.2.1.

B_i is simply a summation of the form : $\Sigma 10^{j_i} | j \in N$. B_i^n can be expressed via the use of the multinomial expression as : $\Sigma C_i^n (10)^{k_i}$.

$$(1010)^n = (\quad \cdot \quad | C_i^1 \times (1000)^{k_1} \times (10)^{k_2} \quad | \quad \cdot \quad | C_i^2 \times (1000)^{k_3} \times (10)^{k_4} \quad | \quad \cdot \quad \cdot \quad)$$

Figure 1.

1.2 Heritability, predictability and comparability.

Definition 1.2.1. Formal Writable Statements.

If a mapping $R \rightarrow \mathcal{I}$ is formed, then \mathcal{I} is writable. Written $W(\mathcal{I})$

Such statements are representable or \mathcal{R} -writable with respect to elements of \mathcal{I} .

Definition 1.2.2. Formal Comparability.

If a language \mathcal{L} can be established that enables the transcribing of the differences or equality between $\{I_j \in \mathcal{I} | \forall j\}$, the sets being compared, then the

set \mathcal{I} is said to be comparable, written $comp(\mathcal{I})$.

Definition 1.2.3.1 Formal Predictability.

Given a functional \mathcal{F} , the mapping $m_i \mapsto T(O_i)|\{o_i \forall i \in K\}$, is predictable writable if $\forall m_j \in T_F \mapsto F(S)$, is knowable $\forall o_i$. Such functionals are predictable, or predictable writable, written $PW(\mathcal{R})$.

Observation 1.2.3.2 Predictable Information.

Given that the mapping $T\mathcal{R} \mapsto \mathcal{I}$ is onto. In addition if the series of mechanisms by which the transformation operates is also 'finite-knowable', then \mathcal{I} is predictable-knowable.

$PW(\mathcal{R})$ requires that the set \mathcal{I} associated is knowable, by this we mean that elements of \mathcal{I} associated with the transformation $\mathcal{I} \mapsto (X)$ is ascertainable with no restriction or ambiguity and is available for the mapping $T(\mathcal{R}) : \rightarrow \mathcal{I}$. If this is the case, then we say that \mathcal{I} is knowable, written : $K(\mathcal{I})$.

In addition to the above definition, if information is predictably knowable, we write $PK(\mathcal{I})$.

Definition 1.2.5.

An arithmetic theory is a set $(\mathcal{R}, \mathcal{L})$ of representations and a language \mathcal{L} where $\mathcal{L} \mapsto T$.

Definition 1.2.6.

The \mathcal{L} on a theory \mathcal{T} is a set of transformations $T(\mathcal{R}) : \rightarrow \mathcal{I}$ along with the representations $\mathcal{R}_i \mapsto T$, that are used to describe the transformations T .

2.1 Arithmetic Language.

Observation 2.2.1.1.

Let A_i be a set of C_i elements. We have from our previous observations that, $PK(\forall p_j(1) \in A_i)$ which enables us to construct a set $C_Y = \Sigma|p_j(1)|$ for all combinations of C_i occupying positions such that $\cap\{p_j(1) \in C_i | Pos_s\} \neq 0$ for some set of positions occupied by C_i . This set can be used for structuring mappings of the form $\cap|p_j(1)| \in A_i \mapsto u \in U_Y$. In cases where subsequent to the initial such mappings, there are again overlapping positions so much so that $\cap|p_j(1)| \in A'_i$ for the now transformed A_i denoted by A'_i , exceeds in magnitude, values available for the mapping $\cap|p_j(1)| \in$

$A'_i \mapsto u \in U_Y$, then we transform A'_i at each such junction where overlapping positions present by combining only magnitudes available for mapping $\cap|p_j(1)| \in A'_i \mapsto u \in U_Y$ and subsequently follow the process on $\cap|p_j(1)| \in A''_i \mapsto u \in U_Y$. As these can be built up alongside S so as to be able to call upon these on demand for deductive purposes, we can conclude that if $PK(|\cap p_j \in A_i|)$, then $PK(|\cap p_j \in A_i|) \mapsto U_Y \Rightarrow PK(R_Y(U_Y))$ where $R_Y(U_Y)$ is the set of recursive or follow up uses required of U_Y . Importantly also $PK(Pos(A_i)) \Rightarrow PK(Pos(u) \cap Pos(A_i))|u \in U_Y$. This implies a chain on knowledge from only the initial knowledge $Pos(A_i)$. To elaborate on the previous mapping $K(\cap Pos(A_i)) \Rightarrow K(\bigcap_{\forall p_j \in Pos(A_i)} p_j \mapsto U_Y, (Consequent(Pos))) \Rightarrow K(\bigcap_{\forall p_j \in Pos} p_j \mapsto U_Y, (Consequent(Pos')))$, so on. As such, the knowledge of the mapping to U_Y implies the knowledge (with no calculation) to its consequent mapping to U_Y . This process is denoted by R_U . The above arguments amount to one thing, which is that, predictably knowing the distribution of (1) will result in predictably knowing where the transformation will result. As such, all associated information is available for mapping with a language, also, the information being predictable ensures the finiteness of the associated language.

Observation 2.1.1.2

For growing n , C_i vary in accordance with the combinatorial expression:

$$J_k := \binom{m}{m - x_i}$$

In general, the $|C_i|$ terms follow the schematic $\{p(p-1)(p-2)..\}$ in magnitudes. The associated magnitudes are thus knowable for growing $Pow(n)$. Given any $p \in H := \{h, h-1, h-2..\}$ in L -form, that is, of the form $\Sigma 2^r$, for arbitrary r , the remainder of such forms can easily be 'known' from the recursive pattern :

$$\begin{array}{|c} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{array}$$

Here each subsequent row is one minus its previous. The proof of the pattern being recursive is trivially obtained via mathematical induction and will be omitted here.

Naturally each $C_j \in A_i$ is associated with set of the form $H := \{h, h -$

$1, h - 2..$ }. If we assume the form of h of C_j which is the largest of all other h' of all other $c_j \in A_i$, then $K(L - form(C_j)\forall j)$ is established as the L-form of h is knowable along with the expansion of the product $\prod_{k \in H} L - form(\{k|H \mapsto C_j\})$, as the form of all elements of H are knowable

along with the nature of the products of the form $\Sigma 2^r$.

The order of the elements of the set H are thus 'knowable' and 'predictable writable'. \mathcal{I} here is associated with the set $(L - form(p_i))$ which the recursive pattern shows is $PK(\mathcal{I})$ thus $\mathcal{I} := (ord(A_i), L - form(C_i))$ and $PK(\mathcal{I})$, where A_i here denotes the positional distribution of all C_i .

We note further that there will be in amount, the number of terms as there are x_i in the combinatorial expression. These will have a maximum number less than or equal to n , the exponential power to which B_i is raised.

Thus 'knowing' the associated L -forms of the set $\{p, (p - 1), (p - 2), ..\}$ allows us to 'know' the progression of C_i as n grows, from Observation 2.2.1. the step down process along with the R_U mappings being knowable for such systems as well. In summation, expressions involving growing n on $Pow(n)$ associated with terms of a statement, again have predictable information associated. In order to better see how one can build a rep around R_U , is by taking note of the type of groupings possible of $\cap pos$. For instance, take note of the number of ones along the columns of the above matrix, we can easily know what progression in number will appear along the columns (as these are inductive in nature). A blueprint now exists of the number of ones that will appear along such columns starting from any one B_i , this enables us to 'know' what types of numbers are possible of B_i^n , in terms of ΣC_i thus enabling us to know this finally in terms of the representation : $\Sigma 2^p$ -Rep. Note importantly that B_i^n will have utmost i in C_i overlapping positions. These along with overlapping positions of ΣB_i^n can be represented. Since the matrix is always of the same structure regardless of size and is additionally finite in nature, we can always 'know' the progression (what is possible of) of R_U thus making it representable. This exact same technique is possible of $\cap pos(C_i)$ as these are also predictably knowable. Predictably knowability of one thing implies PK of the same thing in a different representation.

Observation 2.1.1.3 (Relative Positions)

Given an initial set of positional values, the information of positions associated with $C_i \mapsto B_i^n$ is 'knowable' via the multinomial distribution. Furthermore, the positions at which C_i appears are mapped to powers of $\{u^{t+p} | u \in B_i, t \in Po(B_i), t \in Pow\}$, where the set Pow is that of the powers

of terms appearing in the multinomial distribution. Since the distributional expansion is writable, we have: $PK(\mathcal{I} := \{Pow(\mathcal{B}_i)\})$.

$$\{X^n | B_i \in B, n \in N\} \mapsto \frac{C_i^1 - C_i^2 - - C_i^3 - C_i^4 C_i^5}{P_o^1 - P_o^2 - - P_o^3 - P_o^4 P_o^5}$$

Figure 4.

Observation 2.1.1.4

An entire side of a statement is a summation of the form : $\Sigma C_i(P_o)$. Their relative positions is what is altered by a change in variable values. A convenient aspect of this representation that enables comparability is that both $\{S_L, S_R\}$ of the statement $Arith(S)$ can be written in terms of $L - form(base(2^r))$. In addition, $Pos(C_i)$ associated with variables of the form xy is simply their dot product : $(Pos(C_i))_x \odot (Pos(C_i))_y$. This follows simply from the rules associated with the expansion of products of the form $(\Sigma A)(\Sigma B)$. Their relative position constraints are in fact also their dot product respectively, thus $PK(\mathcal{I} \mapsto \{Pos(xy)\})$. This gives indication that the an entire side of a statement is predictably writable with respect to some r in 2^r as a base, which is what we aimed to establish.

From the observations made, one thing seems clear, any arithmetic equation can be broken into a series of transformations on O_i that are knowable and predictable. Specifically we have shown that the mechanisms by which the transformations operate for any statement are finite in measure and additionally, these are predictable on O_i , meaning that an onto mapping of these to O_i exists. This enables one the opportunity to ascertain whether, for any O'_i a corresponding exists for this result, and additionally one could also ascertain whether . This is because we can know the mechanisms by which T 's form O'_i , and knowing $PK(Form(output))$ means that one can know all types of outputs possible, which enables one to form a generic type of predictable condition/s c_k that need to be met in order for $pos(LHS) = pos(RHS)$. This we call the generic conditioning property. This again follows from the logic that knowing form output is possible in a finite descriptive manner as the transformations associated thereof are finite in nature. Using this, and the fact again that transformations of the other side of the equation is also finite. one can use the finite nature of the structure associated to see whether o_i can be constructed that results in the finite generic needs of C_k .

Sufficiency Theorem 2.1.1.1

All statements that can arithmetically be made in terms of \mathcal{F}^{-1} can be rewritten in terms of \mathcal{F} .

Proof

This follows from the nature of the formation of the Fundamental Theorem of Algebra.

Theorem 2.1.1.2

Statements \mathcal{S} made with arithmetic entities are predictably knowable and thus have associated theories capable of proving them.

Proof

From observations (2.1.1.1) to (2.1.1.4) and the generic conditioning property, we have that $\forall m_i \in T(S), K(m_i \mapsto T)$. Thus $K(\mathcal{I}_T(m_i \mapsto T))$ as we may simply associate the \mathcal{M} -sets associated with the transformation to elements of \mathcal{I}_T . Furthermore, the sum total of all our observations establish $PK(\mathcal{I}_P \mapsto \{T(S)|o_i \in \mathcal{O}\})$. The nature and finiteness of the information set \mathcal{I}_P , as we can note from its definition contains only the knowable information surrounding $(Pos(in), Pos(out))$ via the knowledge of $K(m_j \mapsto T(S))$, and additionally the fact that these are finite and knowable from the onset of S , exposes a set of conditions c_j ; which are knowable in terms of its associated \mathcal{M} -set required of the statement. Representations \mathcal{R}_I can be associated with all $\mathcal{I}_j \in Arith(S)$, as we showed that $PK(\mathcal{I}_j)$ on the associated conditions, which makes

$comp(\mathcal{I}(PK(Pos(S_L))), \mathcal{I}(PK(Pos(S_R), Eq)))$ possible.

Finally since the existence of $\mathcal{R}_{\mathcal{I}}$ was established associated languages are eminent, which establishes $\mathcal{T}(\cup \mathcal{R}_{\mathcal{I}}, \mathcal{L})$.

Theorem 2.2.3.

If theories $\cup \mathcal{T} := (\mathcal{R}, \mathcal{L}, \mathcal{M})$ are established, where $\cup \mathcal{L}$ are capable of transcribing $s \in Arith(S)$, and $s \mapsto \{True\}$ then the expression $X(\cup \mathcal{L}) \mapsto \{True\}$ where $X(\cup \mathcal{L}) := \{X_i, X_{i-1}, ..\}$, if and only if $s \mapsto \{True\}$.

Proof (Assuming equality as the connective)

$s \mapsto \{True\} \Rightarrow \mathcal{M}_s \mapsto s|B_i$ for free variables B_i . Now if $s_L = s_R$ then $Pos_L = Pos_R|B_i$ for some $B_i \subseteq \mathcal{B}$. Furthermore if $Pos_L = Pos_R|B_i$ this implies that $\mathcal{M}_s \mapsto s_L(B_i) = \mathcal{M}_s \mapsto s_R(B_i)$. It is easy to see from the previously established that individual mechanisms are unambiguous, specifically they mean one single thing, additionally $\cap Pos(\mathcal{B}) = 0$ thus $\cap \{\mathcal{M} \mapsto$

$\mathcal{B}\} = 0$. We have previously established that $R_U(W(Base(2^k), Pos_L)) \mapsto R_U(W(Base(2^k), Pos_R))$ is onto. Additionally $\mathcal{R}X$ merely derives from $\mathcal{I} \mapsto \mathcal{R}$ and in no way enforces structure on the mapping $\mathcal{M} \mapsto s$. \mathcal{I} here, falls under the mapping $\mathcal{I}_{RIV} \mapsto \{PK(Pos_V), PK(Pos \mapsto (L - form))\}$ for $V := (L, R)$. Thus if $(L)_T \mapsto \mathcal{I}_{RI}$ then for \mathcal{T} such that $(\mathcal{I}_{RI}, \mathcal{L}) \in \mathcal{T} := (\mathcal{R}, \mathcal{L}, \mathcal{M}), \mathcal{R}X(\mathcal{L}_{RI}) \mapsto \{True\}$ if and only if $Pos_L = Pos_R|B_i$. Since this is also when $s \mapsto \{True\}$, $s \mapsto \{True\} \Rightarrow \mathcal{R}X(\mathcal{L}_{RI}) \mapsto \{True\}$. Furthermore, since $\bigcup_{\forall i} X_i \in \mathcal{R}X(\mathcal{L}_{RI}), s \mapsto \{True\} \Rightarrow X \mapsto \{True\}$. Here since $\forall x \in \bigcup_{\forall i} X_i, x \mapsto \mathcal{L}(\mathcal{I})$ where $\mathcal{L}(\mathcal{I})$ is the linguistic representation of $\mathcal{R} \mapsto \mathcal{I}$. This mapping $\mathcal{L}(\mathcal{I}) \mapsto \mathcal{I}$ is one to one since \mathcal{L} expresses $\mathcal{R} \mapsto \mathcal{I}$. Thus the series $\bigcup_{\forall i} X_i$ makes true or false inferences from $\mathcal{L}(\mathcal{I})$, where, to restate, $\mathcal{L}(\mathcal{I})$ is associated with $\{PK(Pos_V), PK(Pos \mapsto (L - form))\}$. Information associated with the structure of $\bigcup_{\forall i} m_i \mapsto \{L - Form\}$ is derived from $PK(Pos \mapsto (L - form))$, thus a series of $L - statements$ forms a mapping of the form $\mathcal{L} \mapsto \{\mathcal{I} \mapsto PK(Pos \mapsto (L - form))\}$. $PK(Pos \mapsto (L - form))$ is derived from $\mathcal{I} \mapsto \{T(\mathcal{M})\}$. Thus since there exists an association between $\mathcal{L} \mapsto (I)$ and since \mathcal{I} merely associates mechanism to information, if the mechanism can only under certain \mathcal{M} -sets map $s \mapsto \{True\}$, then \mathcal{L} can only transcribe the circumstances under which this is so.

We conclude our ideas on \mathcal{M} -sets by concluding the proof of Theorem 2.1 of [p]. To recap, we managed to show that between any pair of C_{inf} -type elements exists another innumerous set of C_{inf} -type elements.

A simple attempt in enumerating these shows us that a consequence of the proof of Theorem 2.1[p], is that between any such enumeration elements not in this set, and thus we can extend Cantor's diagonalization argument directly to the aforementioned proof.

References

[G] Marvin.J.Greenberg, (1997). Euclidean and Non-Euclidean Geometries: Development and History. 3 US:W.H.Freeman.

[H] S.Hawkings. God Created the Integers.

[P] J.Pillay, (2012). Inner Moden and the Continuum Hypothesis. HAL.

[S] Stephen.G.Simpson, (2008). Mathematical Logic. : Lect. Notes.

[Sm] Raymond.M.Smullyan, (1992). Gödel's Incompleteness Theorems. US:Oxford University Press.

[T] A.Tarski, (1971). Undecidable Theories. 3 Netherlands:North-Holland Publishing Company.

Dedicated to my Sister, Uncle, Parents and Family in India