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## **AN ENERGY-BASED AUTOMATIC SUB-STRUCTURING METHOD IN LOW- AND MEDIUM-FREQUENCY RANGES FOR UNCERTAIN COMPUTATIONAL VIBROACOUSTIC MODEL.**

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### **ABSTRACT**

*In this work, an energy-based automatic sub-structuring method is proposed. The established energy formulation is applicable in the low- and medium-frequency ranges when using a stochastic vibroacoustic model. The idea evolved from recent work which aimed to take into account system parameters and model uncertainties in vibroacoustic modeling by applying a probabilistic approach of uncertainties. Numerical and experimental results showed statistical properties of the frequency response functions in low- and medium-frequency ranges in a stochastic context. Thus, although inspired from the Statistical Energy Analysis (SEA) method, the used energy formulation is a quite different method and it has the advantage of being applicable in low-frequency ranges where the SEA and its derivations fail to apply. The stochastic model is constructed using the non-parametric probabilistic approach. A simplified model is constructed based on the proposed energy formulation. This model is robust regarding model and system parameters uncertainties as well as from the conception point of view thanks to a less dispersed response which is independent of the direction of excitation. Based on this model the automatic substructuring method is developed using the theory of information. The method was verified numerically on a vehicle structure.*

### **1 INTRODUCTION**

Vibroacoustic design of automotive vehicles is becoming more and more complex due to the increasing demand of acoustic comfort. Thus, car makers are forced to increase the quality of the design process while keeping the cost within affordable limits. However, in the current design process, the complexity of an automotive vehicle structure as well as the numerous noise sources leads to study a large number of Frequency Response Functions (FRF), making the analysis a very exhausting process. Moreover, the existence of modeling uncertainties (data and model errors) as well as product uncertainties increases the difficulty of analyses. In a context of development costs reduction, the need for more robust and simplified models becomes a necessity.

In the past years statistical methods such as the Statistical Energy Analysis (SEA) [1] and its derivations were adopted as a powerful tool for the analysis of vibroacoustic behavior of complex structures in the high-frequency range. These methods are very efficient, because they synthesize the dynamical behavior of the structure in a very few, robust quantities such as power exchange or energies see for example [2][3][4][5][6] [7][8][9][10]. Unfortunately, they do not apply in the low- and medium-frequency ranges, due to some specific assumptions or pre-requisites such as space and frequency averaging.

Recently developed probabilistic vibroacoustic models now allow investigating the ensemble statistical properties of the dynamic responses of an industrial product, including design diversity as well as manufacturing uncertainties.

In the present work, a sub-structuring method based on an energy approach is presented. The energy method is based on a probabilistic computational model. The use of the probabilistic model is essential as it allows a statistical analysis without having to average over space or frequency. This permits the application of the energy approach in the low- and medium- frequency ranges. In this method the vibroacoustic system usually represented by, input forces, output velocities related using Frequency Response Functions (FRF), is represented using input and response power spectral density functions related using a dimensionless FRF. These dimensionless FRF are independent of the excitation and observation directions. It is shown that the vibroacoustic system, observed using these dimensionless FRF, has a zone behavior. This means that neighbor points lying in one zone have similar behavior regarding an input excitation to the system. Thus, a simplified model is constructed leading to an automatic sub-structuring method of the vibroacoustic system of a complex structure.

In the first part of the paper, the probabilistic computational model is presented. The energy method is presented in the second part. In the third part, the simplified model is presented. In the fourth part, the numerical validation is given. finally a conclusion is given in the last part of the paper.

## 2 CONSTRUCTION OF THE PROBABILISTIC VIBROACOUSTIC COMPUTATIONAL MODEL

The vibroacoustic problem to be solved consists in a three dimensional damped elastic structure without rigid body displacements and is coupled with an internal damped acoustic cavity. The mean vibroacoustic system is not show in this paper the reader is referred to [11] for the general formulation and to [12] and [13] for the formulation devoted to automotive structures. For all angular frequency  $\omega$  belonging to the frequency band of analysis with  $B = [\omega_{min}, \omega_{max}]$  with  $\omega_{min} > 0$ , the reduced mean computational vibroacoustic model [11] is written as

$$\mathbf{u}^s(\omega) = \Psi \mathbf{q}^s(\omega) \quad , \quad \mathbf{p}^f(\omega) = \Phi \mathbf{q}^f(\omega) \quad , \quad (1)$$

in which the  $\mathbb{C}^n$ -vector  $\mathbf{q}^s(\omega)$  of the generalized structural coordinates associated with the  $n$  first structural elastic modes constituting the matrix  $\Psi$  and the  $\mathbb{C}^m$ -vector  $\mathbf{q}^f(\omega)$  of the generalized acoustical coordinates associated with the  $m$  first acoustic modes constituting the matrix  $\Phi$  which includes the constant pressure mode at zero eigenfrequency, verify the matrix equation

$$\begin{bmatrix} \mathbb{A}^s(\omega) & \mathbb{C} \\ \omega^2 \mathbb{C}^T & \mathbb{A}^f(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{q}^s(\omega) \\ \mathbf{q}^f(\omega) \end{bmatrix} = \begin{bmatrix} \mathbb{F}^s(\omega) \\ \mathbb{F}^f(\omega) \end{bmatrix} \quad . \quad (2)$$

In Eq. (1),  $\mathbf{u}^s(\omega)$  and  $\mathbf{p}^f(\omega)$  are the  $\mathbb{C}^{n_s}$ -vector of the structural DOF and the  $\mathbb{C}^{n_f}$ -vector of the acoustical DOF. In Eq. (2),  $\mathbb{A}^s(\omega)$  and  $\mathbb{A}^f(\omega)$  are the generalized dynamical stiffness matrices for the structure and for the acoustic cavity which are written as

$$\mathbb{A}^s(\omega) = -\omega^2 \mathbb{M}_n^s + i\omega_n^s + \mathbb{K}_n^s \quad , \quad (3)$$

$$\mathbb{A}^f(\omega) = -\omega^2 \mathbb{M}_m^f + i\omega_m^f + \mathbb{K}_m^f \quad . \quad (4)$$

In Eq. (3),  $\mathbb{M}_{n, n}^s$  and  $\mathbb{K}_n^s$  are positive-definite symmetric real  $(n \times n)$  matrices corresponding to the generalized mass, damping and stiffness matrices. In Eq. (4) devoted to the acoustic cavity,  $\mathbb{M}_m^f$  is a positive-definite symmetric real  $(m \times m)$  matrix corresponding to the generalized "mass" matrix and,  $\mathbb{D}_m^f$  and  $\mathbb{K}_m^f$  are the positive symmetric real  $(m \times m)$  matrices corresponding to the generalized "damping" and "stiffness" matrices. Finally, in Eq. (2),  $\mathbb{C}$  is the real  $(n \times m)$  matrix corresponding to the generalized vibroacoustic coupling matrix and where  $\mathbb{f}^s(\omega)$  and  $\mathbb{f}^f(\omega)$  are the generalized structural forces and the generalized acoustical sources applied to the vibroacoustic system.

The non-parametric probabilistic approach [14][15][16] is used to construct the statistical computational vibroacoustic model in order to take into account both parameter uncertainties and model uncertainties. the implementation of this method to an automotive vehicle structure can be found in reference [12] In such an approach, the matrices of the reduced mean computational vibroacoustic model are replaced by random matrices whose mean values are equal, by construction, to the matrices of the reduced mean computational vibroacoustic model. Consequently, Eqs. (1) and (2) are replaced by the following random equations

$$\mathbf{U}^s(\omega) = \Psi \mathbf{Q}^s(\omega) \quad , \quad \mathbf{P}^f(\omega) = \Phi \mathbf{Q}^f(\omega) \quad , \quad (5)$$

in which the  $\mathbb{C}^n$ -valued random vector  $\mathbf{Q}^s(\omega)$  and the  $\mathbb{C}^m$ -valued random vector  $\mathbf{Q}^f(\omega)$  verify the random matrix equation

$$\begin{bmatrix} \mathbf{A}^s(\omega) & \mathbf{C} \\ \omega^2 \mathbf{C}^T & \mathbf{A}^f(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{Q}^s(\omega) \\ \mathbf{Q}^f(\omega) \end{bmatrix} = \begin{bmatrix} \mathbb{f}^s(\omega) \\ T \mathbb{f}^f(\omega) \end{bmatrix} \quad , \quad (6)$$

in which the random matrices  $\mathbf{A}^s(\omega)$  and  $\mathbf{A}^f(\omega)$  are written as

$$\mathbf{A}^s(\omega) = -\omega^2 \mathbf{M}_n^s + i\omega \mathbf{D}_n^s + \mathbf{K}_n^s \quad , \quad (7)$$

$$\mathbf{A}^f(\omega) = -\omega^2 \mathbf{M}_m^f + i\omega \mathbf{D}_m^f + \mathbf{K}_m^f \quad . \quad (8)$$

In Eq. (7),  $\mathbf{M}_n^s$ ,  $\mathbf{D}_n^s$  and  $\mathbf{K}_n^s$  are random matrices with values in the set of all the positive-definite symmetric real  $(n \times n)$  matrices. In Eq. (8),  $\mathbf{M}_m^f$  is a random matrix with values in the set of all the positive-definite symmetric real  $(m \times m)$  matrices and,  $\mathbf{D}_m^f$  and  $\mathbf{K}_m^f$  are random matrices with values in the set of all the positive symmetric real  $(m \times m)$  matrices. Finally, in Eq. (6),  $\mathbf{C}$  is a random matrix with values in the set of all the real  $(n \times m)$  matrices. The probability distributions of these seven random matrices are completely defined in the non-parametric probabilistic approach and a generator of independent realizations of these random matrices is explicitly known(see [14][15][16]). It should be noted that, in this random matrix theory, the statistical fluctuation level of each random matrix is controlled by a dispersion parameter  $\delta > 0$ . If  $\delta = 0$  (deterministic case) the random matrix is equal to its mean value. The larger the value of  $\delta$ , the larger is the uncertainty level.

### 3 PRESENTATION OF THE ENERGY METHOD

The Random equations are solved using the Monte Carlo method and the random response of each realization is obtained on the physical degrees of freedom. Let  $\mu = n^s + n^f$  be the total number of DOF. One will only use the subset  $\{j_1, \dots, j_\alpha, \dots, j_\nu\}$  of the  $\nu$  observed and excited DOF of the vibroacoustic system. In general, one has  $\nu \ll \mu$ . Note that the excited DOF are the same as the observed DOF. The excited DOF correspond to external forces applied to the structure and/or to external acoustic sources in the acoustic cavity. For  $\alpha$  fixed in  $\{1, \dots, \nu\}$ , let  $t \mapsto \mathbf{f}^\alpha(t)$  be the function from  $\mathbb{R}$  into  $\mathbb{R}^\nu$  representing the excitation vector relative to the DOF  $j_\alpha$  which is written as  $\mathbf{f}^\alpha(t) = \{0, \dots, f_\alpha^\alpha(t), \dots, 0\}$  and which is such that  $\mathbf{f}^\alpha(-t) = \mathbf{f}^\alpha(t)$ . It is assumed that  $\mathbf{f}^\alpha$  is square integrable on  $\mathbb{R}$ . Let  $\mathbf{f}^\alpha(\omega) = \int_{\mathbb{R}} e^{-i\omega t} \mathbf{f}^\alpha(t) dt$  be its Fourier transform which is real function such that  $\mathbf{f}^\alpha(-\omega) = \mathbf{f}^\alpha(\omega)$ . Consequently, we have  $\mathbf{f}^\alpha(\omega) = \{0, \dots, f_\alpha^\alpha(\omega), \dots, 0\}$ .

finally it will be assumed that the support of  $\omega \mapsto \mathbf{f}^\alpha(\omega)$  is the bounded interval  $\overline{B} \cup B$  in which  $\overline{B} = [-\omega_{max}, -\omega_{min}]$ . Let  $\mathbb{Z}(\omega)$  be the  $(\mu \times \mu)$  complex random matrix such that

$$\mathbb{Z}(\omega) = \begin{bmatrix} \Psi & 0 \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} \mathbf{A}^s(\omega) & \mathbf{C} \\ \omega^2 \mathbf{C}^T & \mathbf{A}^f(\omega) \end{bmatrix}^{-1} \begin{bmatrix} \Psi^T & 0 \\ 0 & \Phi^T \end{bmatrix} \quad , \quad (9)$$

which exists for all  $\omega$  in  $B$ . Let  $\mathbf{Z}(\omega)$  be the  $(\nu \times \nu)$  complex random matrix such that, for all  $\alpha$  and  $\beta$  in  $\{1, \dots, \nu\}$ , one has

$$\mathbf{Z}_{\alpha\beta}(\omega) = \mathbb{Z}_{j_\alpha j_\beta} \quad . \quad (10)$$

For all  $\omega$  fixed in  $B$ , let  $\mathbf{T}(\omega)$  be the  $(\nu \times \nu)$  complex random matrix defined by

$$\mathbf{T}(\omega) = i\omega \mathbf{Z}(\omega) \quad . \quad (11)$$

The function  $\omega \mapsto \mathbf{T}(\omega)$  is called the matrix-valued random FRF related to the excited and the observed DOF. It should be noted that  $\mathbf{T}(-\omega) = \overline{\mathbf{T}(\omega)}$ . Let  $\mathbf{V}^\alpha(\omega)$  be the  $\nu$  complex random vector of the velocity responses for the observed DOF  $\{j_1, \dots, j_\nu\}$ . One then has

$$\mathbf{V}^\alpha(\omega) = \mathbf{T}(\omega) \mathbf{f}^\alpha(\omega) \quad . \quad (12)$$

It can be shown that the system represented by equation 12 can be represented in terms of input and output power density functions as follows,

$$\boldsymbol{\pi}^R(\omega) = \mathcal{E}(\omega) \boldsymbol{\pi}_{in}(\omega) \quad . \quad (13)$$

where  $\boldsymbol{\pi}_{in}$  and  $\boldsymbol{\pi}^R(\omega)$  are the input and response power density functions vectors of the vibroacoustic system respectively. The random input power density vector of the vibroacoustic system induced by the excitation  $\mathbf{f}^\alpha$  is defined by

$$\boldsymbol{\pi}_{in} = (\pi_{in}^1, \dots, \pi_{in}^\alpha, \dots, \pi_{in}^\nu) \quad (14)$$

it can be shown that  $\boldsymbol{\pi}_{in}(\omega)$  can be written as

$$\boldsymbol{\pi}_{in}(\omega) = 2\mathbf{Y}(\omega) \mathbf{s}^f(\omega) \quad . \quad (15)$$

where  $\mathbf{s}^f(\omega)$  is the input forces spectral density vector defined by  $\mathbf{s}^f = (s_1^f(\omega), \dots, s_\nu^f(\omega))$  where one can write,

$$s_\alpha^f(\omega) = (1/2\pi) f_\alpha^\alpha(\omega)^2 \quad , \quad (16)$$

and  $\mathbf{Y}(\omega)$  is the diagonal mobility matrix defined by,

$$\mathbf{Y}_{\alpha\beta}(\omega) = \begin{cases} \text{Re}(\mathbf{T}_{\alpha\alpha}(\omega)) & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases} \quad . \quad (17)$$

Similarly the Random spectral density function  $\mathbf{s}^v$  of the velocity responses  $\mathbf{V}^1, \dots, \mathbf{V}^\nu$  is such that

$$\mathbf{s}^v(\omega) = (s_1^v(\omega), \dots, s_\nu^v(\omega)) \quad . \quad (18)$$

where  $s_\alpha^v(\omega) = \frac{1}{\pi} \|\mathbf{V}^\alpha(\omega)\|^2$  is the spectral density function of the response velocity  $\mathbf{V}^\alpha$ .  $\mathbf{s}^v(\omega)$  can be related to the power density function using the equation,

$$\mathbf{s}^v(\omega) = \mathbf{Y}(\omega) \boldsymbol{\pi}^R(\omega) \quad , \quad (19)$$

Also in Eq. (13)  $\mathcal{E}(\omega)$  is the dimensionless FRF which can be defined as a normalisation of the usual FRF with respect to the input and output mobilities of the system [17] such that,

$$\mathcal{E}(\omega) = \mathbf{Y}(\omega)^{-1} \mathbf{H}(\omega) \mathbf{Y}(\omega)^{-1} \quad . \quad (20)$$

where  $\mathbf{H}(\omega)$  is the term to term square of the FRF such that  $\mathbf{H}_{\beta\alpha}(\omega) = |\mathbf{T}_{\beta\alpha}(\omega)|^2$ . The spectral density function of the output velocity can then be related to the spectral density function of the input forces in terms of mobility matrix and dimensionless FRF and using the following equation

$$\mathbf{s}^v(\omega) = 2 \mathbf{Y}(\omega) \mathcal{E}(\omega) \mathbf{Y}(\omega) \mathbf{s}^f(\omega) \quad . \quad (21)$$

it should be noted that the mobility matrix is a diagonal matrix due to the assumption of uncorrelated forces. This hypothesis might lead to loss of information in case of correlated forces. Thus, in order to avoid this loss of information the system is projected on the local coordinates system defined by the mean local mobilities. In these local coordinates the mobility matrix is diagonal by definition. To calculate the local coordinates system let  $\mathbf{T}_p(\omega)$  be the random matrix with values in the set of all the symmetric complex ( $3 \times 3$ ) matrices and corresponding to the translational DOF of the random FRF at a given point  $p$  of the structure (note that the rotational DOF are not considered here). One then introduces the mean value  $E\{\mathbf{T}_p(\omega)\}$  of the random matrix  $\mathbf{T}_p(\omega)$  in which  $E$  denotes the mathematical expectation. Let  $\mathbf{T}_p(\omega)$  be the symmetric real ( $3 \times 3$ ) matrix such that  $\mathbf{T}_p(\omega) = \Re\{E\{\mathbf{T}_p(\omega)\}\}$ . The representation of the random matrix  $\mathbf{T}_p(\omega)$  in the local coordinates attached to the given point and defined by the principal direction of the mean local mobility, is the random matrix denoted by  $\mathbf{T}_p^{loc}(\omega)$  and is written as

$$\mathbf{T}_p^{loc}(\omega) = \mathbb{X}_p(\omega)^T \mathbf{T}_p(\omega) \mathbb{X}_p(\omega) \quad . \quad (22)$$

One can now consider the Eq. (20) in the local coordinates for all the local DOF of the structure at points  $p$  and all the the global DOF of the acoustic cavity all together which can then be rewritten

$$\mathcal{E}^{loc}(\omega) = \mathbf{Y}^{loc}(\omega)^{-1} \mathbf{H}^{loc}(\omega) \mathbf{Y}^{loc}(\omega)^{-1} \quad . \quad (23)$$

All other equations of the energy method still hold true in the local coordinates. Thus, these equations are going to be used in what follows with a subscript or a superscript *loc* to refer to values in these coordinates.

#### 4 CONSTRUCTION OF THE SIMPLIFIED MODEL AND SUB-STRUCTURING METHOD

The construction of the simplified model is based on the mean values of the energy method parameters. The mean value of a parameter is calculated using statistical averaging which is defined as the mathematical expectation of this parameter. In what follows an underlined parameter represents a mean value. Taking the mathematical expectation of Eq. (21) in the local coordinates leads us to the calculation of what we call the reference mean value of the spectral density function of the output velocity calculated without any approximation,

$$\underline{\mathbf{s}}_{loc}^v(\omega)^{ref} = 2 E\{\mathbf{Y}^{loc}(\omega) \mathcal{E}^{loc}(\omega) \mathbf{Y}^{loc}(\omega) \mathbf{s}_{loc}^f(\omega)\} \quad . \quad (24)$$

Now, let  $J$  and  $O$  be the set of excitation and observation DOF respectively such that  $J = \{k_q, q = 1, \dots, \mu\}$  and  $O = \{j_p, p = 1, \dots, \nu\}$ , where  $\mu$  and  $\nu$  are the number of excitation and number of observation DOF respectively as illustrated in Fig.(1).

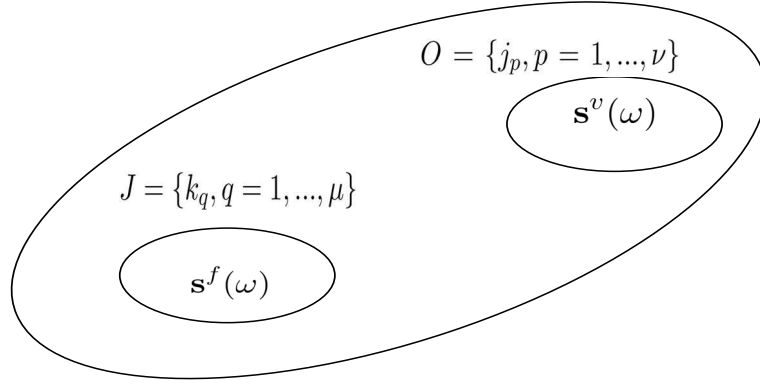


Figure 1. Schematic presentation of the sets of excitation and observation points.

Assuming that the excitation and observation DOF  $J$  and  $O$  are sufficiently distant from each, let  $e_{OJ}$  be the real number such that for each  $\omega$  in the frequency band  $B$ , One can assume that,

$$(\underline{\mathcal{E}}^{loc}(\omega))_{j_p k_q} \simeq e_{OJ}(\omega) \quad . \quad (25)$$

In order that this last assumption be verified, the quantity  $e_{OJ}(\omega)$  is computed using the reference mean value of the output velocity such that,

$$e_{OJ} = \frac{\sum_{p=1}^{\nu} (E\{\mathbf{s}_{loc}^v(\omega)\})_{j_p}}{E\{\pi_{in}^{loc}\} \sum_{p=1}^{\nu} (\mathbf{Y}^{loc}(\omega))_{j_p j_p}} \quad , \quad (26)$$

The associated error due to this hypothesis can then be evaluated by defining  $\varepsilon_{\mathcal{E}}(\omega)_{j_p k_q}$  such that

$$\varepsilon_{\mathcal{E}}(\omega)_{j_p k_q} = |\text{dB}(\omega)_{j_p k_q} - \text{dB}(\omega)_{j_p k_q}^{app}| \quad , \quad (27)$$

$$\text{dB}(\omega)_{j_p k_q} = 10 \log_{10} \underline{\mathcal{E}}^{loc}(\omega)_{j_p k_q} \quad , \quad \text{dB}(\omega)_{j_p k_q}^{app} = 10 \log_{10} \mathbb{E}_{OJ}(\omega)_{j_p k_q}^{app} \quad , \quad (28)$$

where  $\mathbb{E}_{OJ}(\omega)_{j_p k_q}^{app} = e_{OJ}(\omega)$  and which measures the accuracy of the calculation of  $\mathbb{E}_{OJ}(\omega)_{j_p k_q}^{app}$ .

## 5 NUMERICAL VALIDATION

The energy density field approach was applied to a numerical model of a production vehicle. The finite element models of the numerical model of the structure and its internal cavity are shown in fig. 2

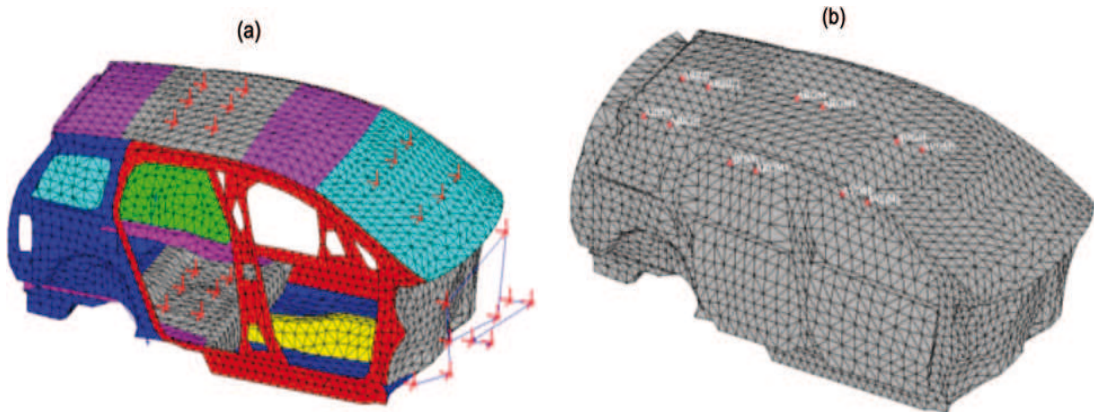


Figure 2. Finite element mesh of the structure (a) and of the acoustic cavity (b)

The model consists of 1 042 851 DOF for the structure, 9157 DOF for the acoustic cavity. The frequency band of analysis was chosen to be  $B= [50,350]$  Hz. Only translational DOF were considered for the structure. In the frequency band of analysis the structure has 1958 modes including the 6 rigid body modes and the acoustic cavity has 160 modes including the zero pressure mode. Forces were applied to twenty eight points of the structure including the motor support and the front suspension while twelve acoustic sources were placed in the acoustic cavity for a total of 96 DOF. Observation points were assigned to each excitation point in order to obtain a square FRF matrix. Six hundred realizations are necessary for the convergence of the Monte Carlo method for both the structure and the acoustic cavity. fig 3 shows the mean values of all the Monte Carlo realizations of the usual FRF and the dimensionless FRF corresponding to an excitation in the three local directions at one point of the structure and observation at a DOF on another point.

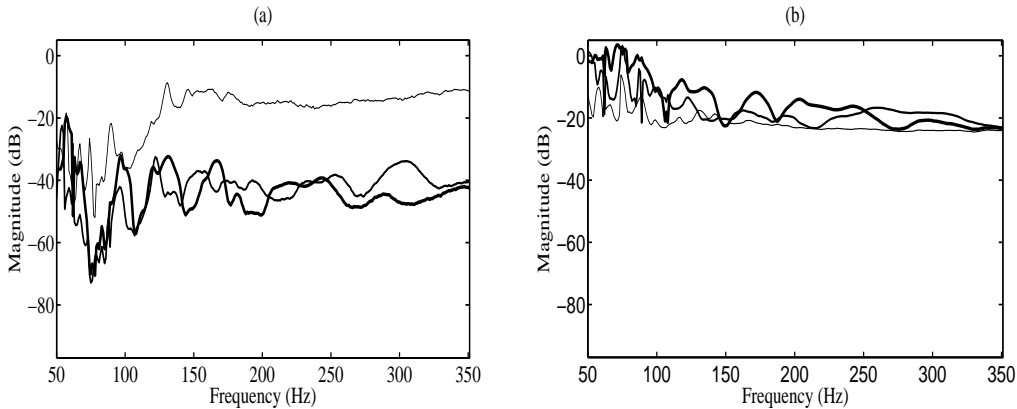


Figure 3: Mean values of the usual FRF (a) and dimensionless FRF (b). Excitation: at 3 structural DOF of the right engine mount, observation: maximum mobility direction on the roof.

It can be seen that the dimensionless FRF show less fluctuations versus frequency than the usual FRF. Moreover, when exciting in the three local DOF the resulting dimensionless FRF have very close values and tend to converge towards an asymptotic value, which is not the case for the usual FRF. This shows that, above a given frequency, these dimensionless FRF are slightly dependent of the direction of excitation. Reciprocally, they are also nearly independent of the observation direction. This property could lead to a significant reduction of the number of FRF to be treated for such a complex model. In addition, the dimensionless FRF have been shown to be much less sensitive to uncertainties than the usual FRF as shown in Fig.4. The proposed formulation is thus more robust regarding the model and parameters uncertainties.

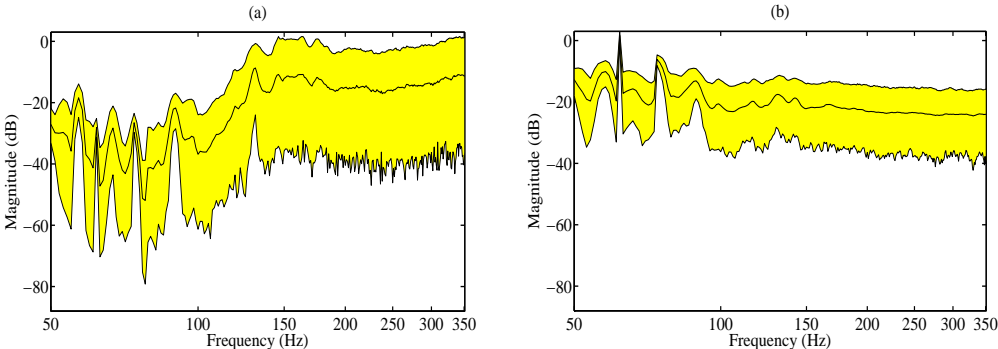


Figure 4: Confidence regions of the dimensional FRF (a) and dimensionless FRF (b). Excitation: right engine mount in its maximum mobility direction, observation: single point on the roof in its maximum mobility direction.



one may say that the vibrational FRF between two points may be separated into local effects described by the diagonal mobility matrices, and a global robust effect characterized by the dimensionless FRF matrix. Similar results are observed for acoustic and vibroacoustic FRF. Figs. 5 From Fig. 5, it can be seen that, at low frequency the error between  $\mathbb{E}_{OJ}(\omega)$  and  $\underline{\mathcal{E}}_{loc}(\omega)$  is high at most of the matrix elements. At higher frequencies the error decreases away from the diagonal terms, that is to say when the excitation and observation points are far enough from each others confirming the starting hypothesis used for the construction of the simplified model. Such a behavior seems to hold true for frequencies higher than 170Hz. Starting at this frequency each block on the diagonal of the matrix corresponds to the DOF's located on the same part of the structure. We can then conclude that the positive real value  $\underline{e}_{OJ}(\omega)$  can be used to approximate  $\underline{\mathcal{E}}_{loc}(\omega)_{jpkq}$  see Eq. (25) between zones of the structure for which the corresponding error function is small. The scalar value  $\underline{e}_{OJ}(\omega)$  is then calculated for DOF located in each of these zones corresponding to functional parts. the number of DOF's is increased gradually and the error function is estimated until the zone limits are obtained.

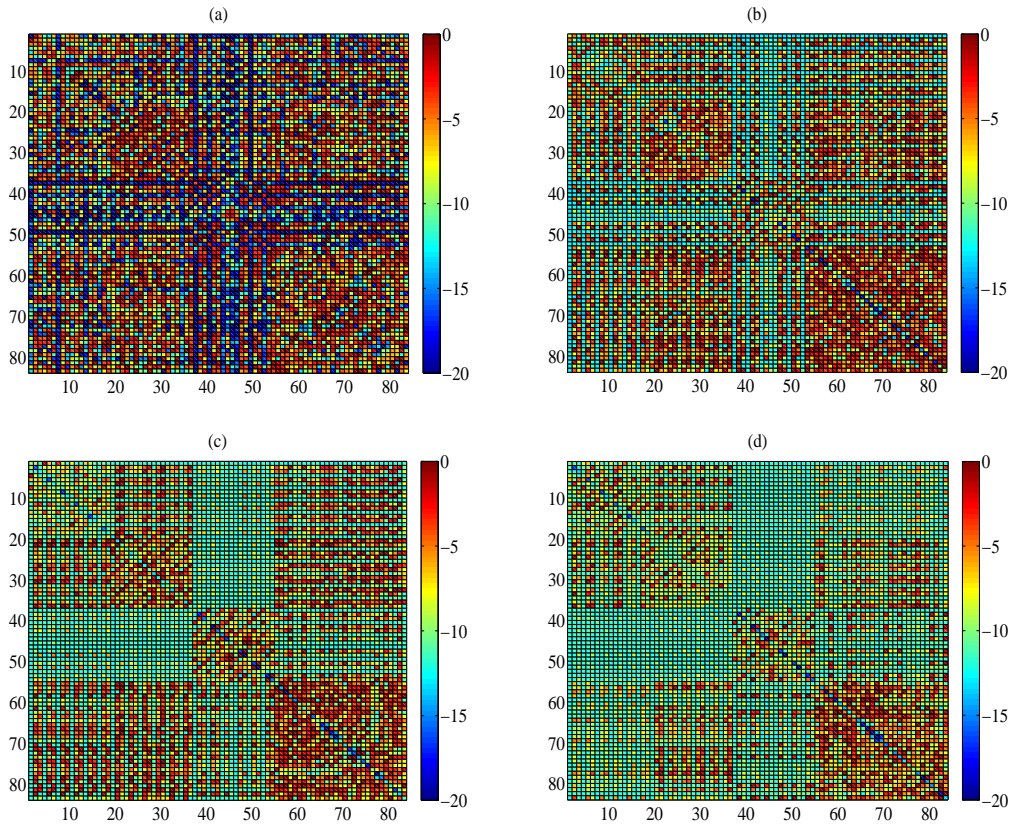


Figure 5. Color plots of  $\varepsilon_{\mathcal{E}}(\omega)$  at 70 Hz (a), 170 Hz (b), 270 Hz (c) and 350 Hz (d).

## 6 CONCLUSION

In this paper, an automatic sub-structuring method based on an energy approach has been presented. The energy method is derived based on a probabilistic computational model constructed using the non parametric probabilistic approach of model and data uncertainties. In the energy method the random vibroacoustic system is represented using input and response power density functions related using a dimensionless FRF. This dimensionless FRF is independent of the excitation or observation directions and less dispersed than the usual FRF. It has been seen that these dimensionless FRF behaves similarly for neighbor DOF's lying in the same zone. A simplified model was constructed and a formula for obtaining a positive scalar considered as an indicator for

sub-structuring was obtained. Using this sub-structuring indicator and the error induced by the model simplification the boundaries of each zone are obtained.

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