Inverse problems in stochastic computational dynamics

Evangéline Capiez-Lernout, Christian Soize

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E. Capiez-Lernout, C. Soize

Université Paris-Est, Laboratoire Modélisation et Simulation Multi Echelle, MSME FRE 3160 CNRS, 5 Bd Descartes, 77455 Marne-la-Vallee, France

E-mail: Evangeline.CapiezLernout@univ-paris-est.fr

Abstract. This paper deals with robust updating of dynamical systems using stochastic computational models for which model and parameter uncertainties are taken into account by a nonparametric probabilistic approach. Such a problem is formulated as an inverse problem consisting in identifying the parameters of the mean computational model and the parameters of the probabilistic model of uncertainties. This inverse problem leads us to solve an optimization problem for which the objective function describes the capability of the uncertain computational model to best-fit the experimental data. Two objective functions are proposed. The methodology is applied in the context of the robust updating of a computational model of composite sandwich panels in the low- and medium- frequency ranges for which experimental results are available.

1. Introduction

In general, the updating of a computational model using experiments is performed with deterministic computational models [1, 2, 3]. One of the main challenges consists in including the effects of uncertainties in the updating process, that is called robust updating. This area is of interest in many industrial applications in order to reduce the variability of manufactured real systems. In the context of structural dynamics, the robust updating belongs to the class of the stochastic optimization inverse problems (see for instance [4, 5, 6]). The robust updating is formulated as an inverse problem consisting in identifying the updating parameters of the probability model used for modelling the stochastic computational model using a multi-set of experimental data. For clarity, the following terminology is introduced. Two types of uncertainties are distinguished : the parameter uncertainties and the model uncertainties. The parameter uncertainties are defined as the uncertainties on the parameters of the computational model and are modelled by random variables or stochastic fields. The model uncertainties are defined as the uncertainties which cannot be taken into account by the computational model used for representing the real dynamical system. The errors introduced by the finite element discretization and by the numerical schemes can be reduced as little as desired and can be controlled by convergence analyses. Consequently, these errors must not be considered as uncertainties. Until now, most of the published works concern robust updating with respect to parameter uncertainties (which can be carried out by using a parametric probabilistic approach) [7]. In the present paper, the stochastic computational model which has to be updated is constructed with the nonparametric probabilistic approach [8, 9] for which both model uncertainties and parameter uncertainties are taken into account. In this context, a methodology

for robust updating with respect to model uncertainties and parameter uncertainties in the low- and medium- frequency range is proposed. The stochastic inverse problem which is investigated leads to solve a nonlinear constrained optimization problem with respect to the admissible set of the updating mean parameters of the mean computational model and of the updating dispersion parameters which allow the uncertainty level in the stochastic computational model to be controlled. Two cost functions are proposed in order to formulate this stochastic inverse problem. The cost functions are defined from the stochastic computational model using experimental data and are a function of the updating parameters. This methodology is validated in the context of the structural dynamics of composite sandwich panels in the low- and medium-frequency range for which experimental results issued from a set of 8 manufactured sandwich panels are available [10].

2. Mean reduced model

The dynamical system under consideration is assumed to be linear and slightly damped. A mean computational model is constructed by discretizing the equations by using the finite element method. The equations are written in the frequency domain for which ω denotes the angular frequency. The frequency band of analysis is denoted by $\mathbb{B}$. It is assumed that the dynamical system is free and has m rigid body modes and n DOF. Let $\mathbf{r}$ be the vector-valued parameter of the mean computational model belonging to the admissible set $\mathcal{R}$. The parameter $\mathbf{r}$ represents the updating parameter of the mean computational model which has to be identified with respect to the experimental data. The mean reduced matrix model of the dynamical system is constructed by modal analysis. Since only the elastic motion of the dynamical system is investigated, a nominal mean computational model corresponding to an initial value $\mathbf{r}_0$ of the updating parameter is considered. The $(n \times N)$ real matrix $[\underline{\Phi}(\mathbf{r}_0)]$ whose columns are the $N \ll n$ elastic eigenmodes $\underline{\boldsymbol{\varphi}}_j(\mathbf{r}_0)$ related to the N positive lowest eigenfrequencies $\underline{\omega}_j(\mathbf{r}_0)$ is calculated. The mean reduced matrix model is then written as $\underline{\mathbf{w}}(\mathbf{r},\omega) = [T(\omega)]\,[\underline{\Phi}(\mathbf{r}_0)]\,\underline{\mathbf{q}}(\mathbf{r},\omega)$ in which $\underline{\mathbf{w}}(\mathbf{r},\omega)$ is the $\mathbb{C}^{n_{obs}}$-vector of the n_{obs} observations, where $[T(\omega)]$ is the $(n_{obs} \times n)$ observation matrix and where the $\mathbb{C}^N$-vector of the generalized coordinates $\underline{\mathbf{q}}(\mathbf{r},\omega)$ is solution of the matrix equation

$$\Big(-\omega^2\,[\underline{\mathcal{M}}(\mathbf{r})] + i\,\omega\,[\underline{\mathcal{D}}(\mathbf{r})] + [\underline{\mathcal{K}}(\mathbf{r})] \Big)\,\underline{\mathbf{q}}(\mathbf{r},\omega) = \underline{\boldsymbol{\mathcal{F}}}(\mathbf{r},\omega) \quad . \tag{1}$$

In Eq. (1), the $\mathbb{C}^N$-vector $\underline{\boldsymbol{\mathcal{F}}}(\mathbf{r},\omega)$ is the complex vector of the generalized loads and the matrices $[\underline{\mathcal{M}}(\mathbf{r})]$, $[\underline{\mathcal{D}}(\mathbf{r})]$ and $[\underline{\mathcal{K}}(\mathbf{r})]$ are positive-definite symmetric $(N \times N)$ real matrices. Note that observation matrix $[T(\omega)]$ has been chosen in order to analyze the acceleration frequency response functions corresponding to the n_{obs} observation points.

3. Stochastic computational model

It is assumed that the mean computational model of the dynamical system contains model uncertainties and parameter uncertainties. The probabilistic model used for modelling the uncertainties in the computational model is the nonparametric probabilistic approach for which the complete developments can be found in [8, 11, 9]. Applying the nonparametric probabilistic approach yields the random matrix equation

$$\Big(-\omega^2\,[\boldsymbol{\mathcal{M}}(\mathbf{r},\delta_M)] + i\,\omega\,[\boldsymbol{\mathcal{D}}(\mathbf{r},\delta_D)] + [\boldsymbol{\mathcal{K}}(\mathbf{r},\delta_K)] \Big)\,\mathbf{Q}(\mathbf{r},\boldsymbol{\delta},\omega) = \underline{\boldsymbol{\mathcal{F}}}(\mathbf{r},\omega) \quad , \tag{2}$$

in which $\boldsymbol{\delta} = (\delta_M\,,\,\delta_D\,,\,\delta_K)$ is the $\mathbb{R}^3$-vector of the updating dispersion parameter defined on the admissible set $\mathcal{A}$, which allows the amount of uncertainty in the stochastic computational model to be controlled. In Eq. 2, the random matrices $[\boldsymbol{\mathcal{M}}(\mathbf{r},\delta_M)]$, $[\boldsymbol{\mathcal{D}}(\mathbf{r},\delta_D)]$ and $[\boldsymbol{\mathcal{K}}(\mathbf{r},\delta_K)]$ are (N,N)

random matrices for which the probability distribution is known and such that $\mathcal{E}\{[\boldsymbol{\mathcal{M}}(\mathbf{r},\delta_M)]\} = [\underline{\mathcal{M}}(\mathbf{r})]$, $\mathcal{E}\{[\boldsymbol{\mathcal{D}}(\mathbf{r},\delta_D)]\} = [\underline{\mathcal{D}}(\mathbf{r})]$ and $\mathcal{E}\{[\boldsymbol{\mathcal{K}}(\mathbf{r},\delta_K)]\} = [\underline{\mathcal{K}}(\mathbf{r})]$ in which $\mathcal{E}$ is the mathematical expectation. Note that all the details concerning the construction of the probability model of the matrices can be found in [8, 11, 9]. The $\mathbb{C}^{n_{obs}}$-valued random vector of the n_{obs} observations is denoted by $\mathbf{W}(\mathbf{r},\boldsymbol{\delta},\omega)$ and is written as $\mathbf{W}(\mathbf{r},\boldsymbol{\delta},\omega) = [T(\omega)]\,[\underline{\Phi}(\mathbf{r}_0)]\,\mathbf{Q}(\mathbf{r},\boldsymbol{\delta},\omega)$.

4. Formulation of the robust updating

It is assumed that experimental data related to a set of n_{exp} dynamical systems manufactured from a given design are available. The experimental data are constituted of n_{obs} frequency response functions corresponding to a given point load and measured at n_{obs} observation points. The observation corresponding to the experimental frequency response function related to manufactured dynamical system number k and measured at observation point number j at a given frequency ω of frequency band $\mathbb{B}$ is denoted as $W_j^{exp}(\omega,\theta_k)$. Let us recall that these manufactured dynamical systems are uncertain and are modeled by the stochastic computational model introduced in the previous Section. The robust updating problem consists in the calibration of the parameters of the stochastic computational model with respect to the experiments. These parameters are constituted of the updating parameter $\mathbf{r}$ of the mean computational model and the dispersion parameter $\boldsymbol{\delta}$ allowing the amount of uncertainty in the stochastic computational model to be controlled. The robust updating problem is solved by optimizing a cost function constructed from the stochastic computational model and using the experimental data with respect to the admissible set of updating parameters $\mathcal{R}\times\mathcal{A}$. Two formulations corresponding to two different cost functions are investigated.

4.1. Formulation of the first robust updating problem

The first cost function proposed is denoted as $j_1(\mathbf{r},\boldsymbol{\delta})$ and is a multi-objective function defined as the sum of (1) the bias between the mean value of the stochastic computational model and the mean value of the experiment and (2) the variance of the stochastic computational model [12]. The mean value of the experiment is denoted by the $\mathbb{C}^{n_{obs}}$-vector $\mathbf{m}_{\mathbb{W}}^{exp}(\omega) = (m_{\mathbb{W},1}^{exp}(\omega)),\ldots,m_{\mathbb{W},n_{obs}}^{exp}(\omega))$ such that $m_{\mathbb{W},j}^{exp}(\omega) = \dfrac{1}{n_{exp}}\displaystyle\sum_{k=1}^{n_{exp}}\mathbb{W}_j^{exp}(\omega,\theta_k)$ where $\mathbb{W}_j^{exp}(\omega,\theta_k) = 20\,log_{10}(|W_j^{exp}(\omega,\theta_k)|)$. Let $\underline{\mathbb{w}}_j(\mathbf{r},\omega) = 20\,log_{10}(|\underline{w}_j(\mathbf{r},\omega)|)$. Cost function $j_1(\mathbf{r},\boldsymbol{\delta})$ is then written as

$$j_1(\mathbf{r},\boldsymbol{\delta}) = \gamma\,||\mathbf{m}_{\mathbb{W}}(\mathbf{r},\boldsymbol{\delta},\cdot) - \mathbf{m}_{\mathbb{W}}^{exp}||_{\mathbb{B}}^2 + (1-\gamma)\,|||\mathbb{W}(\mathbf{r},\boldsymbol{\delta},\cdot) - \mathbf{m}_{\mathbb{W}}(\mathbf{r},\boldsymbol{\delta},\cdot)|||^2 \quad , \tag{3}$$

in which $\mathbf{m}_{\mathbb{W}}(\mathbf{r},\boldsymbol{\delta},\omega) = \mathcal{E}\{\mathbb{W}(\mathbf{r},\boldsymbol{\delta},\omega)\} \in \mathbb{C}^{n_{obs}}$, where $\mathbb{W}(\mathbf{r},\boldsymbol{\delta},\omega) = (\mathbb{W}_1(\mathbf{r},\boldsymbol{\delta},\omega),\ldots,\mathbb{W}_{n_{obs}}(\mathbf{r},\boldsymbol{\delta},\omega))$ with $\mathbb{W}_j(\mathbf{r},\boldsymbol{\delta},\omega) = 20\,log_{10}(|W_j(\mathbf{r},\boldsymbol{\delta},\omega)|)$ and where $||\mathbf{g}||_{\mathbb{B}}^2 = \int_{\mathbb{B}}||\mathbf{g}(\omega)||^2\,d\omega$ with $||\mathbf{g}(\omega)||$ the Hermitian norm of $\mathbf{g}(\omega)$. In Eq. (3), the norm $|||\mathbb{X}|||$ is defined by $|||\mathbb{X}|||^2 = E\{||\mathbb{X}||_{\mathbb{B}}^2\}$, where $\{\mathbb{X}(\omega),\omega\in\mathbb{B}\}$ is a stochastic process indexed by $\mathbb{B}$. The scalar γ is a weighting factor that controls the weight of the bias term relative to the variance term in the optimization process. Any value γ belonging to $]0,0.5[$ means that more importance is attributed to the minimization of the variance term which characterizes the width of the confidence region related to the stochastic computational model. The value $\gamma = 0.5$ corresponds to the usual case for which each term is equally considered. Any value γ belonging to $]0.5,1[$ means that more importance is attributed to the minimization of the bias term. Consequently, large confidence regions can be obtained which is inconsistent with the present context of robust updating because it is expected that the experimental data are included in the smallest possible confidence region. Consequently, the value of scalar γ has to belong to $]0,0.5[$. The robust updating problem leads to a non-linear constrained optimization problem which consists in minimizing the cost function

with respect to the admissible set $\mathcal{R} \times \mathcal{A}$. The solution $(\mathbf{r}_1^{opt}, \boldsymbol{\delta}_1^{opt})$ of the robust updating problem is written as

$$(\mathbf{r}_1^{opt}, \boldsymbol{\delta}_1^{opt}) = \arg \min_{(\mathbf{r},\boldsymbol{\delta}) \in \{\mathcal{R} \times \mathcal{A}\}} j_1(\mathbf{r}, \boldsymbol{\delta}) \quad . \tag{4}$$

4.2. Formulation of the second robust updating problem

The second cost function is based on the criterion that the experimental observations have to belong to the confidence region of the stochastic computational model [12, 13] and that the confidence region has to be the smallest as possible. Let $\underline{\mathbb{w}}_j^{exp,+}(\omega)$ (resp. $\underline{\mathbb{w}}_j^{exp,-}(\omega)$) be defined as the extreme values statistics over the n_{exp} samples of the manufactured dynamical system defined as $\underline{\mathbb{w}}_j^{exp,+}(\omega) = \max_k \mathbb{W}_j^{exp}(\omega, \theta_k)$ and $\underline{\mathbb{w}}_j^{exp,-}(\omega) = \min_k \mathbb{W}_j^{exp}(\omega, \theta_k)$. For a given observation point number j, for a given frequency $\omega \in \mathbb{B}$ and for a given value of the updating parameter $(\mathbf{r}, \boldsymbol{\delta}) \in \{\mathcal{R} \times \mathcal{A}\}$, if experimental observations $\underline{\mathbb{w}}_j^{exp,\pm}(\omega)$ belong to the confidence region of the stochastic computational model defined for a given probability level P_c, no contribution is added in the cost function. In the opposite case, a local contribution related to the distance between $\underline{\mathbb{w}}_j^{exp,\pm}(\omega)$ and the upper/lower envelope of the confidence region is added in the cost function. Let $\underline{\mathbb{w}}_j^{+}(\mathbf{r}, \boldsymbol{\delta}, \omega)$ (resp. $\underline{\mathbb{w}}_j^{-}(\mathbf{r}, \boldsymbol{\delta}, \omega)$) be the upper (resp. lower) envelope of the confidence region of observation $\mathbb{W}_j(\mathbf{r}, \boldsymbol{\delta}, \omega)$ obtained with a given probability level P_c [14]. The cost function $j_2(\mathbf{r}, \boldsymbol{\delta})$ is then defined as

$$j_2(\mathbf{r}, \boldsymbol{\delta}) = ||\boldsymbol{\Delta}^+(\mathbf{r}, \boldsymbol{\delta}, \cdot)||_{\mathbb{B}}^2 + ||\boldsymbol{\Delta}^-(\mathbf{r}, \boldsymbol{\delta}, \cdot)||_{\mathbb{B}}^2 \quad , \tag{5}$$

in which $\boldsymbol{\Delta}^+(\mathbf{r}, \boldsymbol{\delta}, \omega)$ and $\boldsymbol{\Delta}^-(\mathbf{r}, \boldsymbol{\delta}, \omega)$ are the $\mathbb{C}^{n_{obs}}$-vector whose component j is defined as

$$\Delta_j^+(\mathbf{r}, \boldsymbol{\delta}, \omega) = \{\underline{\mathbb{w}}_j^{+}(\mathbf{r}, \boldsymbol{\delta}, \omega) - \underline{\mathbb{w}}_j^{exp,+}(\omega)\} \{1 - H(\underline{\mathbb{w}}_j^{+}(\mathbf{r}, \boldsymbol{\delta}, \omega) - \underline{\mathbb{w}}_j^{exp,+}(\omega))\} \quad , \tag{6}$$

$$\Delta_j^-(\mathbf{r}, \boldsymbol{\delta}, \omega) = \{\underline{\mathbb{w}}_j^{-}(\mathbf{r}, \boldsymbol{\delta}, \omega) - \underline{\mathbb{w}}_j^{exp,-}(\omega)\} \{H(\underline{\mathbb{w}}_j^{-}(\mathbf{r}, \boldsymbol{\delta}, \omega) - \underline{\mathbb{w}}_j^{exp,-}(\omega))\} \quad . \tag{7}$$

In Eq. (6) and (7), $x \mapsto H(x)$ is the Heaviside function. The solution $(\mathbf{r}^{opt}, \boldsymbol{\delta}^{opt})$ of the robust updating problem is written as

$$(\mathbf{r}_2^{opt}, \boldsymbol{\delta}_2^{opt}) = \arg \min_{(\mathbf{r},\boldsymbol{\delta}) \in \{\mathcal{R} \times \mathcal{A}\}} j_2(\mathbf{r}, \boldsymbol{\delta}) \quad . \tag{8}$$

4.3. Numerical aspects

4.3.1. Optimization with the first formulation The first formulation uses a differentiable cost function. Consequently, a sequential quadratic optimization algorithm [15, 16] coupled with the Monte Carlo numerical simulation is used in order to solve the robust updating problem. It should be noted that the cost function is not convex and that such an algorithm yields in general in a local optimum. The strategy used consists (1) in solving a deterministic updating problem with updating dispersion parameter set to the value $\boldsymbol{\delta} = 0$ (The optimal value of parameter $\mathbf{r}$ is then denoted as $\mathbf{r}^0$), (2) in identifying the updated dispersion parameter by solving Eq. (4) with updating parameter $\mathbf{r}$ set to $\mathbf{r}^0$ (the optimal value of parameter $\boldsymbol{\delta}$ is then denoted as $\boldsymbol{\delta}^0$), (3) in solving Eq. (4) around the point $(\mathbf{r}^0, \boldsymbol{\delta}^0)$.

4.3.2. Optimization with the second formulation The second formulation uses a cost function which is not a differentiable cost function. Consequently, a genetic algorithm [17] is used in order to solve the robust updating formulation defined by Eqs. (5) and (8).

For the two formulations, once the optimal parameters are calculated, it should be noted that the calculations are made a second time with these optimal parameters using a large number of numerical simulations in order to analyze the results.

5. Numerical Validation

The numerical validation concerns the updating of a stochastic computational model of a composite sandwich panel. Experimental data related to a set of $n_{exp} = 8$ multilayered sandwich panels manufactured from a designed composite sandwich panel are used. The frequency response functions corresponding to a given out-plane point load and measured at $n_{obs} = 24$ observation points in the frequency band of analysis $\mathbb{B} = [100\,,\,4500]\,Hz$ are available. The detailed description of the designed sandwich panel and of its corresponding experimental protocol can be found in [10]. The mean computational model of the sandwich panel is constructed by the finite element method with 12 288 DOFs. It is assumed that this mean computational model has already been updated with respect to the conservative parameters and is taken as an initial mean computational model. The mean reduced matrix model has $N = 120$ modes (a convergence analysis has been carried out) and is constructed with a constant damping rate $\xi = 0.01$. Let $\underline{\mathbb{w}}^{ini}(\omega)$ be the $\mathbb{C}^{n_{obs}}$-vector of the predictions corresponding to this initial mean computational model. Let $\nu = \omega/(2\pi)$. Figure 1 compares the graph of $\nu \mapsto \underline{\mathbb{w}}_3^{ini}(\nu)$ and $\nu \mapsto \underline{\mathbb{w}}_{17}^{ini}(\nu)$ with the graphs $\nu \mapsto \mathbb{W}_3^{exp}(\nu, \theta_k)$ and $\nu \mapsto \mathbb{W}_{17}^{exp}(\nu, \theta_k)$ with $k = \{1, \ldots, 8\}$. Note that these two observation points are respectively located far and near from the excitation point. It is seen that the quality of the initial mean computational model with respect to the experiments is acceptable in the low-frequency range $\mathbb{B}_L = [100\,,\,1500]\,Hz$ but has to be improved in the medium-frequency range $\mathbb{B}_M = [1500\,,\,4500]\,Hz$. Consequently, one proposes to update the computational model with respect to the damping parameters of the mean computational model and to the dispersion parameters of the stochastic computational model. In this case, the modal matrix, the mean reduced mass and stiffness matrices do not depend on the updating mean parameters. The model used for the mean reduced damping matrix $[\underline{\mathcal{D}}(\mathbf{r})]$ is introduced such that $[\underline{\mathcal{D}}(\mathbf{r})]_{jk} = 2\,\underline{\mu}_j\,\underline{\omega}_j\,\underline{\xi}_j(\mathbf{r})\,\delta_{jk}$ in which $\underline{\mu}_j$ is the modal mass, $\underline{\omega}_j$ is the eigenfrequency and $\underline{\xi}_j(\mathbf{r})$ is the mean modal damping rate related to eigenmode $\underline{\boldsymbol{\varphi}}_j$ defined as $\underline{\xi}_j(\mathbf{r}) = f(\underline{\omega}_j, \mathbf{r})$. Let $\mathbf{r} = \{\xi_0, \xi_1, \alpha, \beta\}$ be the $\mathbb{C}^4$-vector of the updating mean parameters belonging to the admissible set $\mathcal{R}$ defined as $\mathcal{R} = \Big\{\{\xi_0, \xi_1, \alpha, \beta\}, \xi_1 \geq \xi_0 > 0; \alpha > 1; \beta > 0\Big\}$. For $\mathbf{r}$ fixed in $\mathcal{R}$, the function $b \mapsto f(b, \mathbf{r})$ is defined from $\mathbb{R}^+$ into $\mathbb{R}^+$ by

$$f(b, \mathbf{r}) = \xi_0 + (\xi_1 - \xi_0)\,\frac{b^\alpha}{b^\alpha + 10^\beta} \quad . \tag{9}$$

Note that the chosen function is a usual function (see for instance [18]). In the present context, this function is used in order to model the damping as an increasing function of the frequency.

The admissible set $\mathcal{R}$ of the updating mean parameter is defined by the following constraints $0.0095 \leq \xi_0 \leq 0.0105$, $0.05 \leq \xi_1 \leq 0.15$, $5 \leq \alpha \leq 20$ and $30 \leq \beta \leq 50$ and the admissible set $\mathcal{A}$ of the updating dispersion parameter is defined by $\mathcal{A} = [0.05\,,\,0.5]^3$. First, the optimization is carried out by using the differentiable cost function defined by Eq. (3) with parameter $\gamma = 0.25$. The optimization is carried out with respect to admissible set $\mathcal{R} \times \mathcal{A}$ by using the sequential quadratic optimization algorithm around $(\mathbf{r}^0, \boldsymbol{\delta}^0) = \{(0.0099, 0.08495, 10.5867, 46.6657), (0.30, 0.019, 0.09)\}$ [12] for which the value of cost function is normalized such that $j_1(\mathbf{r}^0, \boldsymbol{\delta}^0) = 1$. The optimization of cost function $j_1(\mathbf{r}, \boldsymbol{\delta})$ with respect to the admissible set $\mathcal{R} \times \mathcal{A}$ yields optimal updating parameters $\mathbf{r}_1^{opt} = (0.01, 0.0850, 10.7144, 46.1018)$ and $\boldsymbol{\delta}_1^{opt} = (0.31, 0.20, 0.14)$ which corresponds to $j_1(\mathbf{r}_1^{opt}, \boldsymbol{\delta}_1^{opt}) = 0.8248$. Secondly, the optimization is carried out by using the non-differentiable cost function defined by Eq. (5) and yields optimal updating parameters $\mathbf{r}_2^{opt} = (0.01, 0.081, 10.9, 47)$ and $\boldsymbol{\delta}_2^{opt} = (0.23, 0.07, 0.24)$. Note that the two optimization methods yield different values of the dispersion parameter. This can be explained by the fact that a computational model can be few sensitive to the damping dispersion parameter (see [19]).

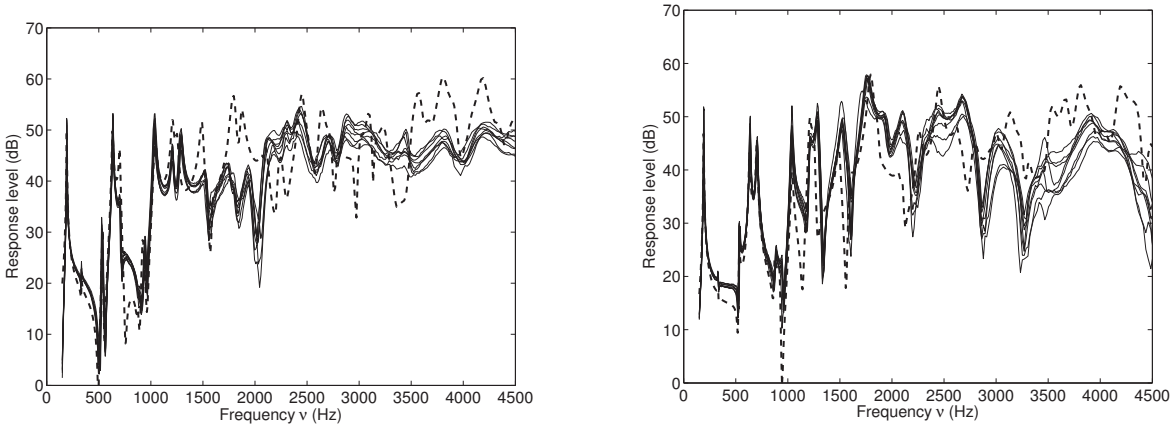

Figure 1. Comparison of the 8 experimental measurements (8 thin solid lines) for observation points number 3 (left figure) and number 17 (right figure) with the initial mean computational model.

Furthermore, the global dispersion level for the mass and stiffness term defined by $\sqrt{(\delta_M^2 + \delta_K^2)}$ are quite similar for both formulations. Figure 2 compares the updated damping models obtained with the two cost functions.

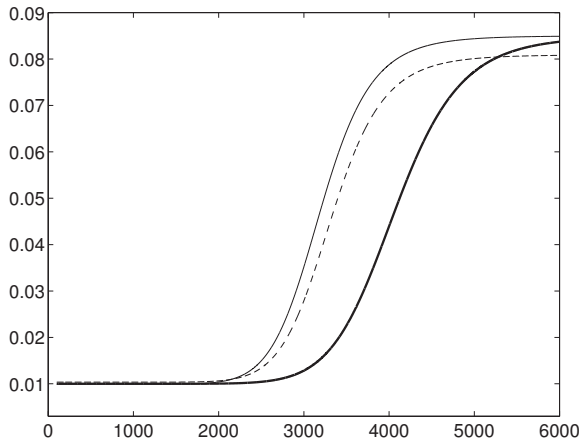

Figure 2. Damping model. Graphs of $b \mapsto f(b, \mathbf{r}^0)$ (thick solid line), $b \mapsto f(b, \mathbf{r}_1^{opt})$ (thin solid line), of $b \mapsto f(b, \mathbf{r}_2^{opt})$ (thin dashed line).

Figure 3 compares the experimental frequency response functions $\mathbb{W}_3^{exp}(\nu, \theta_k)$ and $\mathbb{W}_{17}^{exp}(\nu, \theta_k)$ with the deterministic responses $\underline{\mathbb{w}}_3^{ini}(\nu)$ and $\underline{\mathbb{w}}_{17}^{ini}(\nu)$ of the initial mean computational model, with the deterministic responses $\underline{\mathbb{w}}_3(\mathbf{r}_1^{opt}, \nu)$ and $\underline{\mathbb{w}}_{17}(\mathbf{r}_1^{opt}, \nu)$ of the updated mean computational model and with the confidence region of the random response $\mathbb{W}_3(\mathbf{r}_1^{opt}, \boldsymbol{\delta}_1^{opt}, \nu)$ and $\mathbb{W}_{17}(\mathbf{r}_1^{opt}, \boldsymbol{\delta}_1^{opt}, \nu)$ of the updated stochastic computational model obtained with a probability level $P_c = 0.96$ and corresponding to the updating method related to the differentiable cost function. Figure 4 displays similar quantities related to the updating method related to the non-differentiable cost function. It can be seen that both methods increase the quality of the

computational model in the medium-frequency band $\mathbb{B}_M$. Furthermore, it is seen that the use of the non-differentiable cost function for the optimization improves the updating relative to the use of the differentiable cost function.

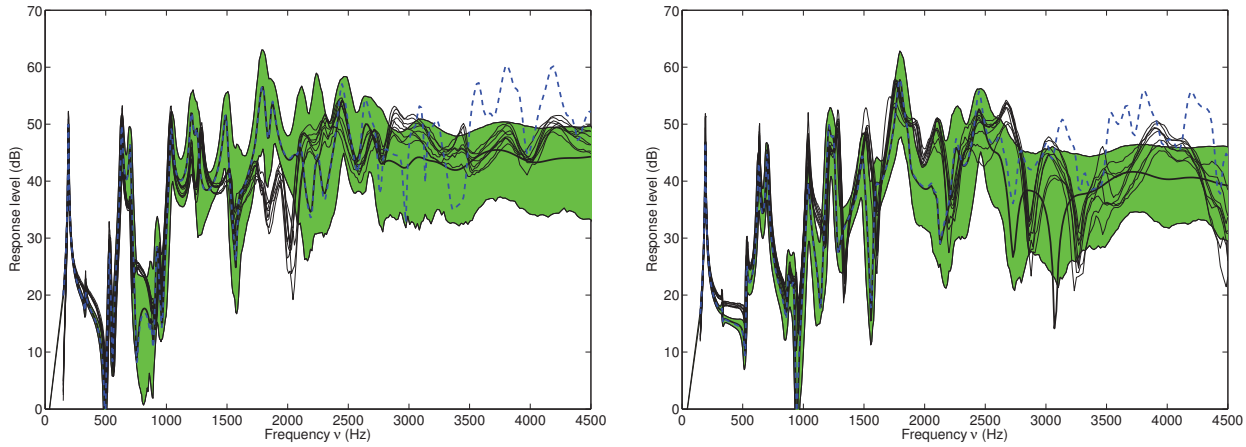

Figure 3. Differentiable objective function, observation point number 3 (left figure) and 17 (right figure). Comparison of the 8 experimental measurements (8 thin solid lines) with (1) the initial mean computational model (dotted line), (2) the optimal mean computational model (thick solid line), (3) the confidence region of the optimal stochastic computational model (grey region).

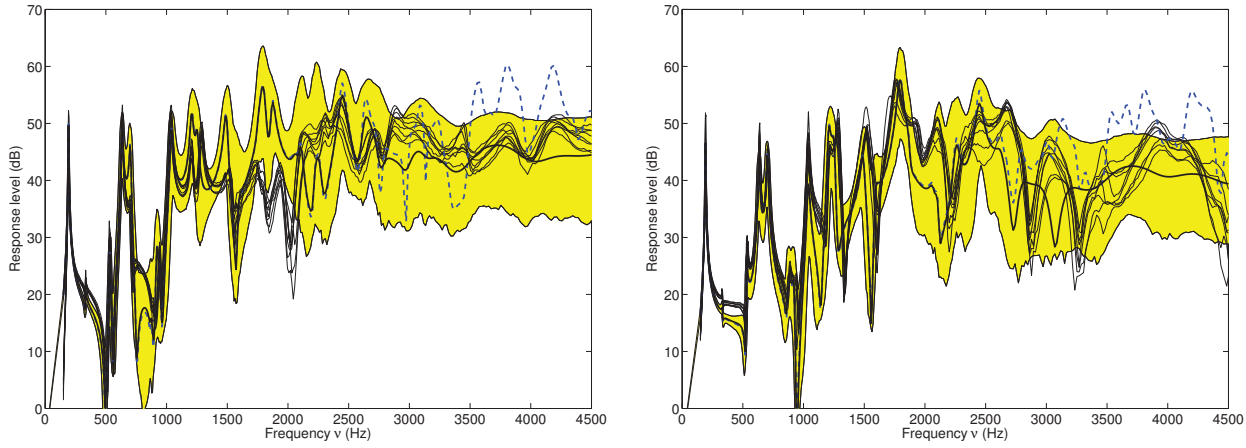

Figure 4. Non differentiable objective function, observation point number 3 (left figure) and 17 (right figure). Comparison of the 8 experimental measurements (8 thin solid lines) with (1) the initial mean computational model (dotted line), (2) the optimal mean computational model (thick solid line), (3) the confidence region of the optimal stochastic computational model (grey region).

6. Conclusions

Two updating formulations have been proposed for the robust updating problem in the context of structural dynamics for which parameter uncertainties and model uncertainties are taken into account by the nonparametric probabilistic approach. These two formulations are used in order to perform the robust updating of a composite sandwich panel for which experimental results issued from a set of 8 manufactured composite sandwich panels are available. It can be seen that both methods increase the quality of the computational model. Furthermore, it is seen that the use of the non-differentiable cost function defined from the confidence region particularly improves the updating in both low- and medium-frequency ranges.

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