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A sequent calculus with procedure calls

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Abstract
In this paper, we extend the sequent calculus $\text{LKF}$ \cite{LM09} into a calculus $\text{LK}(\mathcal{T})$, allowing calls to a decision procedure. We prove cut-elimination of $\text{LK}(\mathcal{T})$.

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1 The sequent calculus \( \text{LK}(T) \)

The sequent calculus \( \text{LK}(T) \) manipulates the formulae of first-order logic, with the specificity that every predicate symbol is classified as either positive or negative, and boolean connectives come in two versions: positive and negative.

**Definition 1 (Formulae)** Literals are predicates (a predicate symbol applied to a list of first-order terms) or negations of predicates. Literals are equipped with the obvious involutive negation, and the negation of a literal \( l \) is denoted \( l^\perp \).

Let \( P \) be the set of literals that are either predicates with positive predicate symbols, or negations of predicates with negative predicate symbols.

Positive formulae

\[
P ::= p \mid A \land^+ B \mid A \lor^+ B \mid \exists x A
\]

Negative formulae

\[
N ::= p^\perp \mid A \land^− B \mid A \lor^− B \mid \forall x A
\]

Formulae

\[
A, B ::= P \mid N
\]

where \( p \) ranges over \( P \).

**Definition 2 (Negation)** Negation is extended from literals to all formulae:

- \( (p)^\perp := p^\perp \)
- \( (A \land^+ B)^\perp := A^\perp \land^− B^\perp \)
- \( (A \lor^+ B)^\perp := A^\perp \lor^− B^\perp \)
- \( (\exists x A)^\perp := \forall x A^\perp \)

2 Admissible rules

**Definition 4 (Assumptions on the procedure)**

We assume that the procedure calls satisfy the following properties:

- **Weakening** If \( T(S) \) then \( T(S, S') \).
- **Contraction** If \( T(S, A, A) \) then \( T(S, A) \).
- **Instantiation** If \( T(S) \) then \( T(\{x\} S) \).
- **Consistency** If \( T(S, p) \) and \( T(S, p^\perp) \) then \( T(S) \).

where \( S \) is a set of literals.

**Lemma 1 (Admissibility of weakening and contraction)**

The following rules are admissible in \( \text{LK}(T) \).

\[
\begin{align*}
\Gamma \vdash [B] & \quad \Gamma, A \vdash [B] \\
\Gamma, A \vdash [B] & \quad \Gamma, A \vdash \Delta \\
\Gamma, A \vdash [B] & \quad \Gamma, A \vdash \Delta \\
\end{align*}
\]

**Proof:** By induction on the derivation of the premiss. \( \square \)
Synchronous rules

\[
\vdash [A] \quad \vdash [B] \\
\vdash [A \land B] \\
\vdash [A \lor B] \\
\vdash [\exists x A] \\
\vdash [\forall x A]
\]

Γ, p ⊢ [p] p positive literal

\[
\vdash (\text{lit}(\Gamma), p^+) \\
\vdash [p] p positive literal
\]

Γ ⊢ N N negative

Aynchronous rules

\[
\vdash A, \Delta \quad \vdash B, \Delta \\
\vdash A \land B, \Delta \\
\vdash A \lor B, \Delta \\
\vdash (\forall x A), \Delta \\
\vdash A^+, \Delta \\
\vdash A, \Delta \\
\vdash (\text{lit}(\Gamma))
\]

Γ ⊢ A positive or literal

Structural rules

\[
\vdash P^+ [P] \\
\vdash P^+ [P] \\
\vdash T(\text{lit}(\Gamma)) \\
\vdash -
\]

Figure 1: System LK(T)

Lemma 2 (Admissibility of instantiation) The following rules are admissible in LK(T).

\[
\vdash [B] \\
\vdash \Delta \\
\vdash \{s\} \Gamma \vdash \{s\} B \\
\vdash \{s\} \Gamma \vdash \{s\} \Delta
\]

Proof: By induction on the derivation of the premiss. □

3 Invertibility of the asynchronous phase

Lemma 3 (Invertibility of asynchronous rules)
All asynchronous rules are invertible in LK(T).

Proof: By induction on the derivation proving the conclusion of the asynchronous rule considered.

- Inversion of \(A \land B\): by case analysis on the last rule actually used

\[
\vdash A \land B, C, \Delta' \quad \vdash A \land B, D, \Delta' \\
\vdash A \land B, C \land D, \Delta' \\
\vdash A \land B, C \lor D, \Delta' \\
\vdash A \land B, (\forall x C), \Delta' \quad x \notin \text{FV}(\Gamma, \Delta)
\]

By induction hypothesis we get

\[
\vdash A, C, \Delta' \\
\vdash A, D, \Delta' \\
\vdash B, C, \Delta' \\
\vdash B, D, \Delta' \\
\vdash B, (\forall x C), \Delta' \\
\vdash B, C \land D, \Delta' \\
\vdash B, C \lor D, \Delta'
\]

- Inversion of \(A \lor B\): by case analysis on the last rule actually used

\[
\vdash A \land B, C, \Delta' \\
\vdash A \land B, C \land D, \Delta' \\
\vdash A \land B, C \lor D, \Delta' \\
\vdash A \land B, (\forall x C), \Delta' \quad x \notin \text{FV}(\Gamma, \Delta, A \land B)
\]

By induction hypothesis we get

\[
\vdash A, C, \Delta' \\
\vdash A, D, \Delta' \\
\vdash B, C, \Delta' \\
\vdash B, D, \Delta' \\
\vdash B, (\forall x C), \Delta' \\
\vdash B, C \land D, \Delta' \\
\vdash B, C \lor D, \Delta'
\]

3
By induction hypothesis we get

\[ \Gamma \vdash A, C, \Delta', x \notin \text{FV}(\Gamma, \Delta', A) \quad \text{and} \quad \Gamma \vdash B, C, \Delta' \quad x \notin \text{FV}(\Gamma, \Delta', B) \]

\[ \Gamma, C \vdash A \wedge B, \Delta' \quad \text{C positive or literal} \]

By induction hypothesis we get

\[ \Gamma \vdash A, \forall x C, \Delta' \]

\[ \Gamma, C \vdash A \wedge B, \Delta' \]

\[ \Gamma \vdash A, C, \Delta' \quad \text{C positive or literal} \quad \text{and} \quad \Gamma, C \vdash B, \Delta' \quad \text{C positive or literal} \]

**Inversion of \( AV^{-}B \)**

\[ \Gamma \vdash AV^{-}B, C, \Delta' \quad \Gamma \vdash AV^{-}B, D, \Delta' \]

By induction hypothesis we get

\[ \Gamma \vdash A, B, C, \Delta' \quad \Gamma \vdash A, B, D, \Delta' \]

\[ \Gamma \vdash AV^{-}B, C, \Delta' \]

By induction hypothesis we get

\[ \Gamma \vdash A, B, C, D, \Delta' \quad \Gamma \vdash A, B, \forall x C, \Delta' \]

\[ \Gamma \vdash AV^{-}B, C, \Delta' \quad x \notin \text{FV}(\Gamma, \Delta') \]

By induction hypothesis we get

\[ \Gamma \vdash A, B, C, D, \Delta' \]

\[ \Gamma \vdash AV^{-}B, C \wedge D, \Delta' \]

By induction hypothesis we get

\[ \Gamma \vdash A, B, C \wedge D, \Delta' \]

**Inversion of \( \forall x A \)**

\[ \Gamma \vdash (\forall x A), C, \Delta' \quad \Gamma \vdash (\forall x A), D, \Delta' \]

By induction hypothesis we get

\[ \Gamma \vdash A, C, \Delta' \quad x \notin \text{FV}(\Gamma, \Delta') \quad \text{and} \quad \Gamma \vdash A, D, \Delta' \]

\[ \Gamma \vdash (\forall x A), C, D, \Delta' \]

By induction hypothesis we get

\[ \Gamma \vdash A, C, D, \Delta' \]

\[ \Gamma \vdash (\forall x A), C \wedge D, \Delta' \]

By induction hypothesis we get

\[ \Gamma \vdash A, C \wedge D, \Delta' \]

**Inversion of literals and positive formulae** \((A)\)

\[ \Gamma \vdash A, C, \Delta' \quad \Gamma \vdash A, D, \Delta' \]

By induction hypothesis we get

\[ \Gamma, A \vdash C, \Delta' \quad \Gamma, A \vdash D, \Delta' \]

\[ \Gamma, A \vdash C \wedge D, \Delta' \]
4 Cut-elimination

Theorem 4 (cut₁ and cut₂) The following rules are admissible in \( LK(T) \).

\[
\begin{array}{c}
\frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp)}{\Gamma \vdash \Delta} \quad \frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash [B]}{\Gamma \vdash [B]} \\
\text{cut₁} & \text{cut₂}
\end{array}
\]

Proof: By simultaneous induction on the derivation of the right premise.

We reduce cut₂ by case analysis on the last rule used to prove the right premise.

\[
\frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash B, \Delta}{\Gamma \vdash B, \Delta} \quad \frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash C, \Delta}{\Gamma \vdash C, \Delta} \quad \frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash B \land C, \Delta}{\Gamma \vdash B \land C, \Delta} \quad \text{cut₁}
\]

reduces to

\[
\frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash B_1, \Delta \quad \Gamma, p \vdash B_2, \Delta}{\Gamma \vdash B_1 \lor B_2, \Delta} \quad \frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash \forall x B, \Delta}{\Gamma \vdash \forall x B, \Delta} \quad \frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash B^+, \Delta}{\Gamma \vdash B^+, \Delta} \quad \text{cut₁}
\]

reduces to

\[
\frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash B_1, B_2, \Delta}{\Gamma \vdash B_1 \land B_2, \Delta} \quad \frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash B^+, \Delta}{\Gamma \vdash B^+, \Delta} \quad \text{cut₁}
\]

We have \( \mathcal{T}(\text{lit}(\Gamma), p^\perp, B^+) \) as we assume the procedure to satisfy weakening.

If \( p^\perp \in (\Gamma, p) \),

\[
\frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash [P]}{\Gamma \vdash [P]} \quad \text{cut₁}
\]

reduces to

\[
\frac{\mathcal{T}(\text{lit}(\Gamma), p^\perp) \quad \Gamma, p \vdash [P]}{\Gamma \vdash [P]} \quad \text{cut₂}
\]

as \( p^\perp \in (\Gamma) \).

\[\Box\]
By simultaneous induction on the following lexicographical measure:

**Proof:**

We reduce $\text{cut}_2$ again by case analysis on the last rule used to prove the right premiss.

\[
\frac{T(\text{lit}(\Gamma), p^\perp)}{\Gamma \vdash \text{cut}_1}
\]

reduces to

\[
\frac{T(\text{lit}(\Gamma), p^\perp), \Gamma, p \vdash \text{cut}_2}{\Gamma \vdash [B]}
\]

\[
\frac{T(\text{lit}(\Gamma), p^\perp)}{\Gamma \vdash \text{cut}_2}
\]

Finally,

\[
\frac{T(\text{lit}(\Gamma), p^\perp, p^\perp)}{\Gamma \vdash \text{cut}_2}
\]

since weakening gives $T(\text{lit}(\Gamma), p^\perp, p^\perp)$ and consistency then gives $T(\text{lit}(\Gamma), p^\perp)$.

\[\square\]

**Theorem 5 (\text{cut}_1, \text{cut}_4 and \text{cut}_5):** The following rules are admissible in $\mathcal{LK}(T)$.

\[
\frac{\Gamma \vdash [A]}{\Gamma \vdash A^\perp, \Delta} \quad \text{cut}_3
\]

\[
\frac{\Gamma \vdash N \quad \Gamma, N \vdash \Delta}{\Gamma \vdash \Delta} \quad \text{cut}_4
\]

\[
\frac{\Gamma \vdash N \quad \Gamma, N \vdash [B]}{\Gamma \vdash [B]} \quad \text{cut}_5
\]

**Proof:** By simultaneous induction on the following lexicographical measure:

- the size of the cut-formula ($A$ or $N$)
- the fact that the cut-formula ($A$ or $N$) is positive or negative
  - (if of equal size, a positive formula is considered smaller than a negative formula)
- the height of the derivation of the right premiss
Weakenings and contractions (as they are admissible in the system) are implicitly used throughout this proof.

In order to eliminate \( \text{cut}_3 \), we analyse which rule is used to prove the left premiss. We then use invertibility of the negative phase so that the last rule used in the right premiss is its dual one.

\[
\begin{align*}
\Gamma \vdash [A] & \quad \Gamma \vdash [B] & \quad \Gamma \vdash A^\perp, B^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [A \wedge B] & \quad \Gamma \vdash A^\perp \wedge B^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [A_1] & \quad \Gamma \vdash A_1^\perp, \Delta \quad \Gamma \vdash A_2^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [A_1 \vee A_2] & \quad \Gamma \vdash A_1^\perp \vee A_2^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [\forall x.A] & \quad \Gamma \vdash \forall x.A^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [\exists x.A] & \quad \Gamma \vdash \exists x.A^\perp, \Delta \quad \text{cut}_3 \\
\end{align*}
\]

We will describe below how \( \text{cut}_4 \) is reduced.

\[
\begin{align*}
\Gamma \vdash [A] & \quad \Gamma \vdash [B] & \quad \Gamma \vdash A^\perp, B^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [A] & \quad \Gamma \vdash [B] & \quad \Gamma \vdash A^\perp, B^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [A_1] & \quad \Gamma \vdash [A_2] & \quad \Gamma \vdash A_1^\perp, A_2^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [A_1 \lor A_2] & \quad \Gamma \vdash A_1^\perp \lor A_2^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [\forall x.A] & \quad \Gamma \vdash [\exists x.A] & \quad \Gamma \vdash \forall x.A^\perp, \Delta \quad \Gamma \vdash \exists x.A^\perp, \Delta \quad \text{cut}_3 \\
\end{align*}
\]

using the admissibility of instantiation.

\[
\begin{align*}
\Gamma \vdash N & \quad \Gamma, N^\perp, \Delta \quad \text{cut}_3 \\
\Gamma \vdash [N] & \quad \Gamma \vdash (N^\perp), \Delta \quad \text{cut}_3 \\
\end{align*}
\]

We will describe below how \( \text{cut}_4 \) is reduced.

\[
\begin{align*}
\Gamma, p, p \vdash [p] & \quad \Gamma, p, p \vdash (p^\perp), \Delta \quad \text{cut}_3 \\
\Gamma, p \vdash [p] & \quad \Gamma, p \vdash (p^\perp), \Delta \quad \text{cut}_3 \\
\end{align*}
\]

using the admissibility of contraction.

\[
\begin{align*}
\Gamma \vdash [p] & \quad \Gamma \vdash (p^\perp), \Delta \quad \text{cut}_3 \\
\end{align*}
\]

In order to reduce \( \text{cut}_4 \), we analyse which rule is used to prove the right premiss.

\[
\begin{align*}
\Gamma \vdash [\text{lit}(\Gamma)] & \quad \Gamma \vdash \text{lit}(\Gamma), p^\perp, \Delta \quad \text{cut}_4 \\
\Gamma \vdash [\text{lit}(\Gamma)] & \quad \Gamma \vdash \text{lit}(\Gamma), p^\perp, \Delta \quad \text{cut}_4 \\
\Gamma \vdash \text{lit}(\Gamma) & \quad \Gamma \vdash \text{lit}(\Gamma), p^\perp, \Delta \quad \text{cut}_4 \\
\end{align*}
\]

if \( N \) is not an literal (hence, it is not passed on to the procedure).

\[
\begin{align*}
\Gamma, p \vdash [\text{lit}(\Gamma), p^\perp] & \quad \Gamma \vdash \text{lit}(\Gamma), p^\perp, \Delta \quad \text{cut}_4 \\
\end{align*}
\]

if \( p^\perp \) is a literal passed on to the procedure.

\[
\begin{align*}
\Gamma, p^\perp \vdash [\text{lit}(\Gamma), p^\perp] & \quad \Gamma \vdash \text{lit}(\Gamma), p^\perp, \Delta \quad \text{cut}_4 \\
\end{align*}
\]
\[ \Gamma \vdash N \quad \Gamma, N \vdash C, \Delta \]
\[ \Gamma, N \vdash B, \Delta \quad \Gamma, N \vdash C, \Delta \]
\[ \Gamma \vdash \Delta \quad \text{cut}_4 \]

reduces to
\[ \Gamma \vdash N \quad \Gamma, N \vdash B, \Delta \]
\[ \Gamma \vdash B, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash C, \Delta \]
\[ \Gamma \vdash C, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash B \land \lnot C, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash B, C, \Delta \]
\[ \Gamma \vdash B, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash \forall x B, \Delta \]
\[ \Gamma \vdash \forall x B, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash B, \Delta \]
\[ \Gamma \vdash B, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash B, \Delta \]
\[ \Gamma \vdash B, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash B, C, \Delta \]
\[ \Gamma \vdash B, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash \forall x B, \Delta \]
\[ \Gamma \vdash \forall x B, \Delta \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash B, \Delta \]
\[ \Gamma \vdash B, \Delta \quad \text{cut}_4 \]

using weakening, and if \( B \) is positive or a negative literal.

We have reduced all cases of \( \text{cut}_4 \); we now reduce the cases for \( \text{cut}_5 \) (again, by case analysis on the last rule used to prove the right premise).

\[ \Rightarrow \]
\[ \Gamma \vdash N \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash [B] \quad \Gamma, N \vdash [C] \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash [B \land ^+ C] \quad \text{cut}_5 \]
\[ \Gamma \vdash [B \land ^+ C] \quad \text{cut}_5 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash [B_1] \quad \Gamma, N \vdash [B_1 \lor ^+ B_2] \]
\[ \Gamma \vdash [B_1 \lor ^+ B_2] \quad \text{cut}_5 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash [\exists x B] \quad \Gamma, N \vdash [\exists x B] \]
\[ \Gamma \vdash [\exists x B] \quad \text{cut}_5 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash [N'] \quad \Gamma, N \vdash [N'] \]
\[ \Gamma \vdash [N'] \quad \text{cut}_4 \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash [p] \quad \Gamma \vdash [p] \]
\[ \Gamma \vdash [p] \quad \text{cut}_5 \]

since \( p \) has to be in \( \Gamma \).

\[ \Rightarrow \]
\[ \Gamma \vdash N \quad \Gamma, N \vdash [p] \]
\[ \Gamma \vdash [p] \quad \text{cut}_5 \]
\[ \Gamma \vdash [p] \quad \text{cut}_5 \]

\[ \square \]
Theorem 6 (cut_6, cut_7, cut_8, and cut_9) The following rules are admissible in LK(T).
\[
\begin{align*}
\Gamma \vdash N, \Delta & \quad \Gamma, N \vdash \Delta \quad \text{cut}_6 \quad \Gamma \vdash \Delta \\
\Gamma, l \vdash \Delta & \quad \Gamma, l^+ \vdash \Delta \quad \text{cut}_8 \quad \Gamma \vdash \Delta \\
\Gamma, l_1, \ldots, l_n \vdash \Delta & \quad \Gamma, (l_1^+ \lor \cdots \lor l_n^+) \vdash \Delta \quad \text{cut}_9 \quad \Gamma \vdash \Delta \\
\end{align*}
\]

Proof: cut_6 is proved admissible by induction on the multiset \(\Delta\): the base case is the admissibility of cut_4, and the other cases just require the inversion of the connectives in \(\Delta\).

For cut_7, we can assume without loss of generality (swapping \(A\) and \(A^\bot\)) that \(A\) is negative. Applying inversion on \(\Gamma \vdash A^\bot, \Delta\) gives a proof of \(\Gamma, A \vdash \Delta\), and cut_7 is then obtained by cut_6:
\[
\begin{align*}
\Gamma \vdash A^\bot, \Delta & \quad \Gamma, A \vdash \Delta \quad \text{cut}_6 \quad \Gamma \vdash \Delta \\
\end{align*}
\]

cut_8 is obtained as follows:
\[
\begin{align*}
\Gamma, l^+ \vdash \Delta & \quad \Gamma, l \vdash \Delta \\
\Gamma \vdash l, \Delta & \quad \Gamma \vdash l^+, \Delta \quad \text{cut}_7 \\
\Gamma \vdash \Delta \\
\end{align*}
\]

cut_9 is obtained as follows:
\[
\begin{align*}
\Gamma, (l_1^+ \lor \cdots \lor l_n^+) \vdash \Delta & \quad \Gamma, l_1, \ldots, l_n \vdash \Delta \\
\Gamma \vdash l_1^+, \ldots, l_n^+, \Delta & \quad \Gamma \vdash (l_1^+ \lor \cdots \lor l_n^+), \Delta \\
\Gamma \vdash \Delta \\
\end{align*}
\]

5 Conclusion

It is worth noting that an instance of such a theory is the theory where \(T(S)\) holds if and only if there is a literal \(p \in S\) such that \(p^\bot \in S\).

We proved the admissibility of cut_4 and cut_9 as they are used to simulate the DPLL(T) procedure [NOT06] as the proof-search mechanism of LK(T).

Further work will consist in using the cut-admissibility results to:
- show that changing the polarities of the connectives and predicates that are present in a sequent, does not change the provability of that sequent in LK(T);
- prove the completeness of LK(T) with respect to the standard notion of provability in first-order logic, working in a particular theory \(T\) for which we have a (sound and complete) decision procedure;
- show how the DPLL(T) procedure can be simulated in LK(T) (with backtracking as well as with backjumping and lemma learning).

References
