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To cite this version:

Nicola Maggiore, Marco Picollo. Let us twist again: N = 2 super-YangMills theory coupled to matter. Classical and Quantum Gravity, 2011, 28 (8), pp.85022. 10.1088/0264-9381/28/8/085022 . hal-00690078

HAL Id: hal-00690078

https://hal.science/hal-00690078v1

Submitted on 21 Apr 2012

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GEF-TH-4/2010

Let's Twist Again: N=2 Super Yang-Mills Theory Coupled To Matter

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`Abstract` : We give the twisted version of N=2 Super Yang-Mills theory coupled to matter, including quantum fields, supersymmetry transformations, action and algebraic structure. We show that the whole action, coupled to matter, can be written as the variation of a nilpotent operator, *modulo* field equations. An extended Slavnov-Taylor identity , collecting gauge symmetry and supersymmetry, is written, which allows to define the web of algebraic constraints, in view of the algebraic renormalization and of the extension **of the algebraic proof** of the non-renormalization theorems holding for N=2 SYM theory without matter.

`Keywords:` Topological Quantum Field Theories, Extended Supersymmetry, BRST Renormalization.

`PACS Nos:` 03.70.+k (theory of quantized fields), 11.10.Gh (renormalization), 11.30.Pb (supersymmetry), 12.60.Jv (supersymmetric models).

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1 Introduction

Before 1994, the reasons to discard N=2 Super Yang-Mills (SYM) theory prevailed over those for studying it. This is testified by the weak occurrence in the literature before then, of theories with extended supersymmetries.

Concerning phenomenology, the presence of the so called "mirror particles" eliminates every possible physical interest: fermions of opposite chirality, *but in the same representation* of the gauge group, unavoidably appear in the theory, which hence is not chiral, and consequently not realistic, if one wants to include the particles of the Standard Model [1].

From the Quantum Field Theory (QFT) point of view, on the other hand, theories with extended supersymmetry represent a real challenge, as explained in [2].

In fact, while for N=1 SYM the superspace formalism based on unconstrained superfields allowed to perform the algebraic quantum extension of the theory [3], the superfield approach to theories with extended supersymmetry is troublesome for several reasons. N=2 supersymmetry can be realized by means of of N=1 superfields, but the necessary additional symmetry involving N=1 superfields in non-polynomial. On the other hand, the harmonic superspace approach [4] is possible, but a regularization scheme preserving both supersymmetry and gauge invariance, to all orders of perturbation theory, is still lacking. Despite this, the most celebrated results concerning the good renormalization properties of theories with extended supersymmetry, in particular the vanishing of the β-function above one loop, have been obtained in a superspace (N=1 and/or N=2) framework [5, 6, 7]. A review of these results, of the ways employed to get them and also of as the weaknesses of each of them, can be found in Chapter 18 of [1][1]**.**

The situation doesn't sound much better in components. The drawback of adopting the WZ gauge, is that the supersymmetry transformations are nonlinear, and the supersymmetry algebra does not close on translations, but two kinds of obstructions occur: field dependent gauge transformations and field equations of motion. This fact has two consequences: the difficulty of defining a gauge fixing term, which is invariant under both supersymmetry and BRS symmetry, and the need of an infinite

[1] **At pag 194 of [1], it is pointed out: "Here we have stressed these weaknesses not because of a mistrust in the arguments for finiteness, but to show that they are not proofs in a mathematical sense and that there is still room for further work".**

number of external sources, with increasing negative mass dimensions, in order to control the algebra [2].

After the appearance of the celebrated Seiberg-Witten papers [8, 9] on the *electric-magnetic duality* in N=2 SYM theory, which relates the weak and strong coupling regimes of that theory, the N=2 susy theories faced a kind of second youth, becoming extremely popular, and were massively reconsidered by the community.

Most of the problems described in [2] were solved, and the renormalizability of N=2 coupled to matter, by means of a non-anomalous Slavnov-Taylor identity, was rigorously established [10, 11], using a technique which has been successfully repeated since, and which we are adopting also in this paper for the classical definition of the theory (see Section 5).

More recently, the method of "shadow fields" has been introduced, which allowed to write a system of Slavnov-Taylor identities by means of which supersymmetric gauge field theories can be renormalized in a regularization independent way, permitting also to study the observables which are not scalar under supersymmetric transformations [12].

Other important goals have been reached exploiting the *twist* [13, 14]. Indeed, it was a known fact that N=2 SYM is related to topological field theories, in particular Topological Yang-Mills (TYM) theory, by means of a twist, which ultimately reduces to a linear redefinition of the quantum fields, to which the path integral defining the generating functionals is insensitive (we shall be more precise in Section 3). Consequently, the twisted-related theories are completely equivalent.

The bad news driven by this, is that the results concerning N=2,4 SYM theories can hardly be extended to the more realistic N=1 SYM theories, which are definitely not topological QFTs, having local degrees of freedom.

This drawback is partially compensated by the fact that some important facts concerning N=2 SYM can be proved through their equivalent twisted version: TYM. This has been the case, for instance, for the theorem concerning the N=2 SYM β-function, whose finiteness above one loop has been algebraically demonstrated in [15] exploiting the existence of the twist.

Later, it has also been algebraically proved by means of the shadow technique that, chosen the matter hypermultiplet in order to have a vanishing β-function at one loop, it vanishes at all orders of perturbation theory [16].

Moreover, a central role is played by the operator $\mathrm{Tr}\,\phi^2$, which, in the Seiberg-Witten supersymmetric theory, is the gauge invariant quantity parametrizing the space of vacua of the theory, and, in the twisted topological theory, is the finite operator [17] by means of which the pure gauge theory can be defined [15, 18].

So far the state of the art on which this paper stands. The program is not yet completely carried out: the non-renormalization theorems concerning the β-function are **algebraically** proved for the N=2 SYM, *in absence of matter*. We recall that matter is coupled to the pure gauge theory by means of the hypermultiplet [1], in a generic representation of the gauge group. As a consequence of the second supersymmetry, the theory, even in presence of matter, has *only one* coupling constant. It is natural to ask which is the fate of the non-renormalization theorem concerning the unique β-function in presence of matter. Related to this, it is interesting to know if, as in the pure gauge case, the whole theory can be written in terms of a single operator, which is finite to all orders of perturbation theory, and, if the answer is positive, which this operator is. Finally, the inclusion of matter allows also for taking into account N=4 SYM, which can be reached from N=2 in the particular case of matter in the adjoint, rather than generic, representation of the gauge group.

The aim of this paper, is to contribute to answer these questions. The preliminary and necessary step is to give the complete twisted version of N=2 SYM *coupled to matter*, and to achieve the whole set up for its quantum extension (gauge fixing, BRS symmetry, Slavnov-Taylor identity, algebraic structure, etc.) [19].

The paper is organized as follows. In Section 2 we recall the basics of N=2 SYM theories, with and without matter. In Section 3 we introduce the twist for the pure gauge case. The main results of this paper are contained in Sections 4 and 5, where the twisted version of the whole theory, including matter, is given, as well as the basis for the quantum implementation, which relies on the extended Slavnov-Taylor identity and on the *off shell* closed algebra. Conclusions and perspectives are summarized in Section 6.

2 The untwisted theory: N=2 SYM coupled to matter

2.1 Pure N=2 SYM

The N=2 susy algebra reads

$$\begin{aligned} \{\mathcal{Q}^i_\alpha, \overline{\mathcal{Q}}_{j\dot\alpha}\} &= \delta^i_j (\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu \\ \{\mathcal{Q}^i_\alpha, \mathcal{Q}^j_\beta\} &= \{\overline{\mathcal{Q}}_{i\dot\alpha}, \overline{\mathcal{Q}}_{j\dot\beta}\} = 0 \ , \end{aligned} \tag{2.1}$$

where $(\mathcal{Q}^i_\alpha, \overline{\mathcal{Q}}_{j\dot\alpha})$ are the supersymmetry charges, indexed by $i = 1, 2$ and Weyl spinor indices $\alpha, \dot\alpha = 1, 2$. The total number of supercharges is therefore eight.

The pure N=2 SYM theory is based on the Yang-Mills (YM) multiplet [1], which belongs to the adjoint representation of the gauge group, and whose field components are $(A_\mu, \lambda^{i\alpha}, \overline{\lambda}_{i\dot\alpha}, \phi, \overline{\phi})$, where $A_\mu(x)$ is the gauge field, $\lambda^{i\alpha}(x), \overline{\lambda}_{i\dot\alpha}(x)$ are two pairs of Weyl spinors, and $\phi(x), \overline{\phi}(x)$ are two scalars.

The corresponding pure N=2 SYM action reads

$$
\begin{aligned}
S_{YM} &= \frac{1}{g^2}\mathrm{Tr}\int \mathrm{d}^4x\Big(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - 4\lambda^{i\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}D_{\mu}\overline{\lambda}^{\dot{\alpha}}_{i} - \frac{1}{2}\overline{\phi}D_{\mu}D^{\mu}\phi \\
&\quad -\overline{\phi}\left\{\overline{\lambda}^{i\dot{\alpha}},\overline{\lambda}_{i\dot{\alpha}}\right\} + \phi\left\{\lambda^{i\alpha},\lambda_{i\alpha}\right\} - \frac{1}{32}\left[\phi,\overline{\phi}\right]\left[\phi,\overline{\phi}\right]\Big) \ , \qquad (2.2)
\end{aligned}
$$

where the Trace Tr is done over the adjoint representation group.

The global symmetry group of the theory is

$$
H = SU(2)_L \times SU(2)_R \times SU(2)_I \times U(1) \ , \qquad (2.3)
$$

where $SU(2)_L \times SU(2)_R$ represents the Lorentz group, $SU(2)_I \times U(1)$ is the internal symmetry group, $SU(2)_I$ referring to the supersymmetry index $i = 1, 2$ and $U(1)$ being the rigid $\mathcal{R}$–symmetry.

Correspondingly, the fields belonging to the N=2 YM multiplet are assigned the following H–group quantum numbers:

$$
\begin{aligned}
A_{\mu} &: \left(\frac{1}{2},\frac{1}{2},0\right)^{0} \\
\lambda^{i\alpha} &: \left(\frac{1}{2},0,\frac{1}{2}\right)^{-1} \\
\overline{\lambda}_{i\dot{\alpha}} &: \left(0,\frac{1}{2},\frac{1}{2}\right)^{+1} \\
\phi &: (0,0,0)^{+2} \\
\overline{\phi} &: (0,0,0)^{-2} \ ,
\end{aligned} \qquad (2.4)
$$

where we adopted the notation

$$
(SU(2)_L, SU(2)_R, SU(2)_I)^{U(1)} \ . \qquad (2.5)
$$

For what concerns the supersymmetry generators, the quantum numbers are:

$$
Q_{i\alpha} = \left(\frac{1}{2},0,\frac{1}{2}\right)^{+1} \quad ; \quad \overline{Q}_{i\dot{\alpha}} = \left(0,\frac{1}{2},\frac{1}{2}\right)^{-1} \ . \qquad (2.6)
$$

The supersymmetry transformations of the pure N=2 SYM fields are:

$$
\begin{aligned}
\delta A_{\mu} &= -\sqrt{2}\xi^{\alpha j}(\sigma_{\mu})_{\alpha\dot{\alpha}}\overline{\lambda}^{\dot{\alpha}}_{j} - \sqrt{2}\overline{\xi}^{\dot{\alpha}j}(\sigma_{\mu})_{\alpha\dot{\alpha}}\lambda^{\alpha}_{i} \\
\delta\phi &= -4\sqrt{2}\overline{\xi}^{\dot{\alpha}j}\overline{\lambda}_{j\dot{\alpha}} \\
\delta\overline{\phi} &= -4\sqrt{2}\xi^{\alpha j}\lambda_{j\alpha} \qquad (2.7) \\
\delta\lambda_{i\kappa} &= \frac{\sqrt{2}}{8}\xi^{\alpha j}\varepsilon_{\alpha\kappa}\varepsilon_{ij}\left[\phi,\overline{\phi}\right] + \frac{1}{\sqrt{2}}\xi^{\alpha j}\varepsilon_{ij}(\sigma^{\mu\nu})_{\kappa\alpha}F_{\mu\nu} + \frac{1}{\sqrt{2}}\overline{\xi}^{\dot{\alpha}j}\varepsilon_{ij}(\sigma_{\mu})_{\kappa\dot{\alpha}}D^{\mu}\overline{\phi} \\
\delta\overline{\lambda}_{i\dot{\kappa}} &= \frac{1}{\sqrt{2}}\xi^{\alpha j}\varepsilon_{ij}(\sigma_{\mu})_{\alpha\dot{\kappa}}D^{\mu}\phi + \frac{\sqrt{2}}{8}\overline{\xi}^{\dot{\alpha}j}\varepsilon_{\dot{\kappa}\dot{\alpha}}\varepsilon_{ji}\left[\phi,\overline{\phi}\right] + \frac{1}{\sqrt{2}}\overline{\xi}^{\dot{\alpha}j}\varepsilon_{ji}(\overline{\sigma}^{\mu\nu})_{\dot{\kappa}\dot{\alpha}}F_{\mu\nu} \ ,
\end{aligned}
$$

where the operator δ collects the supercharges $Q_{i\alpha}$ and $Q_{i\dot{\alpha}}$ through $\delta = \xi^{\alpha j} Q_{j\alpha} + \overline{\xi}^{\dot{\alpha} j} \overline{Q}_{j\dot{\alpha}}$. Notice that in the Wess-Zumino gauge the supersymmetry transformations (2.7) are nonlinear.

The action (2.2) is susy invariant :

$$\delta S_{YM} = 0 \ . \tag{2.8}$$

2.2 N=2 SYM coupled to matter

To couple pure N=2 SYM to matter, we need the matter hypermultiplet $(q_i, \tilde{q}_i, \psi_q, \overline{\psi}_q, \psi_{\tilde{q}}, \overline{\psi}_{\tilde{q}})$ [1], formed by two pairs of scalar fields $q_i(x)$ and $\tilde{q}_i(x)$, two Weyl fermions $\psi_q(x)$ and $\psi_{\tilde{q}}(x)$ and their complex conjugates, all in a generic complex representation of the gauge group. The matter H–quantum numbers (2.5) are:

$$\begin{aligned}
q_i &: (0, 0, \frac{1}{2})^0 \\
\tilde{q}_i &: (0, 0, \frac{1}{2})^0 \\
(\psi_q)_\alpha &: (\frac{1}{2}, 0, 0)^{+1} \\
(\overline{\psi}_q)_{\dot{\alpha}} &: (0, \frac{1}{2}, 0)^{-1} \\
(\psi_{\tilde{q}})_\alpha &: (\frac{1}{2}, 0, 0)^{+1} \\
(\overline{\psi}_{\tilde{q}})_{\dot{\alpha}} &: (0, \frac{1}{2}, 0)^{-1} \ .
\end{aligned} \tag{2.9}$$

The complete N=2 SYM action is:

$$S = S_{YM} + S_{matter} \ , \tag{2.10}$$

where S_{YM} is given by (2.2), and

$$\begin{aligned}
S_{matter} &= \frac{1}{g^2} \mathrm{Tr}_m \int \mathrm{d}^4 x \Big(\frac{1}{2} \tilde{q}^i D_\mu D^\mu q_i + 2 \tilde{q}^i \overline{\lambda}_{i\dot{\alpha}} (\overline{\psi}_q)^{\dot{\alpha}} - 2 q^i \overline{\lambda}_{i\dot{\alpha}} (\overline{\psi}_{\tilde{q}})^{\dot{\alpha}} \\
&- \frac{1}{2} \tilde{q}^i \lambda_{i\alpha} (\psi_q)^\alpha - \frac{1}{2} q^i \lambda_{i\alpha} (\psi_{\tilde{q}})^\alpha + (\psi_{\tilde{q}})^\alpha (\sigma^\mu)_{\alpha\dot{\alpha}} D_\mu (\overline{\psi}_q)^{\dot{\alpha}} \\
&- (\overline{\psi}_{\tilde{q}})_{\dot{\alpha}} (\overline{\sigma}^\mu)^{\dot{\alpha}\alpha} D_\mu (\psi_q)_\alpha + \frac{1}{8} (\psi_{\tilde{q}})^\alpha \overline{\phi} (\psi_q)_\alpha - 2 (\overline{\psi}_{\tilde{q}})^{\dot{\alpha}} \phi (\overline{\psi}_q)_{\dot{\alpha}} \\
&+ \frac{1}{16} \tilde{q}^i \left\{ \phi, \overline{\phi} \right\} q_i - \frac{1}{32} \tilde{q}^i q_i \tilde{q}^j q_j \Big) \ .
\end{aligned} \tag{2.11}$$

In the previous expression, Tr_m is the Trace over the matter representation of the gauge group.

The (nonlinear) supersymmetry transformations of the matter fields are:

$$
\begin{aligned}
\delta q_i &= \sqrt{2}\varepsilon_{ji}\xi^{\alpha j}(\psi_q)_\alpha + \sqrt{2}\varepsilon_{ji}\overline{\xi}^{\dot\alpha j}(\overline{\psi}_q)_{\dot\alpha} \\
\delta \tilde{q}_i &= \sqrt{2}\varepsilon_{ji}\xi^{\alpha j}(\psi_{\tilde q})_\alpha + \sqrt{2}\varepsilon_{ji}\overline{\xi}^{\dot\alpha j}(\overline{\psi}_{\tilde q})_{\dot\alpha} \\
\delta(\psi_q)_\gamma &= \sqrt{2}\varepsilon_{\gamma\alpha}\xi^{\alpha j}\phi q_j + \frac{1}{\sqrt{2}}\overline{\xi}^{\dot\alpha j}(\sigma^\nu)_{\gamma\dot\alpha}\mathrm{D}_\nu q_j \\
\delta(\overline{\psi}_q)_{\dot\gamma} &= -\frac{1}{\sqrt{2}}\xi^{\alpha j}(\sigma^\nu)_{\alpha\dot\gamma}\mathrm{D}_\nu q_j - \frac{\sqrt{2}}{16}\varepsilon_{\dot\gamma\dot\alpha}\overline{\xi}^{\dot\alpha j}\overline{\phi} q_j \\
\delta(\psi_{\tilde q})_\gamma &= -\sqrt{2}\varepsilon_{\gamma\alpha}\xi^{\alpha j}\phi \tilde q_j + \frac{1}{\sqrt{2}}\overline{\xi}^{\dot\alpha j}(\sigma^\nu)_{\gamma\dot\alpha}\mathrm{D}_\nu \tilde q_j \\
\delta(\overline{\psi}_{\tilde q})_{\dot\gamma} &= -\frac{1}{\sqrt{2}}\xi^{\alpha j}(\sigma^\nu)_{\alpha\dot\gamma}\mathrm{D}_\nu \tilde q_j + \frac{\sqrt{2}}{16}\varepsilon_{\dot\gamma\dot\alpha}\overline{\xi}^{\dot\alpha j}\overline{\phi} \tilde q_j \ ,
\end{aligned}
\tag{2.12}
$$

and the matter action (2.11) is susy invariant:

$$
\delta S_{matter} = 0 \ , \tag{2.13}
$$

so that, finally, one has

$$
\delta S = \delta(S_{YM} + S_{matter}) = 0 \ . \tag{2.14}
$$

3 Introducing the twist: the pure N=2 SYM theory

As we said, the global symmetry group for N=2 SYM in four dimensions is given by H (2.3), and the total number of generators, including supersymmetry, are:

	$SU(2)_L \times SU(2)_R$	Susy	$SU(2)_I$	$U(1)$
generators	$P_\mu(4)$, $M_{\mu\nu}(6)$	$Q_{i\alpha}(4)$, $\overline{Q}_{i\dot\alpha}(4)$	$T^i_j(3)$	$\mathcal{R}(1)$

The nonvanishing algebraic relations are

$$
\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= -i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\mu\sigma}M_{\mu\rho}) \\
[M_{\mu\nu}, P_\rho] &= i(\eta_{\nu\rho}P_\mu - \eta_{\mu\rho}P_\nu) \\
\left[M_{\mu\nu}, Q^i_\alpha\right] &= -(\sigma_{\mu\nu})_\alpha{}^\beta Q^i_\beta \\
\{Q^i_\alpha, \overline{Q}^j_{\dot\beta}\} &= 2\sigma^\mu_{\alpha\dot\beta}P_\mu\delta^{ij} \\
\left[T^j_i, Q_{k\alpha}\right] &= -\frac{1}{2}(\delta^j_k Q_{i\alpha} - \frac{1}{2}\delta^j_i Q_{k\alpha}) \\
\left[T^j_i, T^l_k\right] &= \frac{1}{2}(\delta^l_i T^j_k - \delta^j_k T^l_i) \\
[\mathcal{R}, Q_{i\alpha}] &= Q_{i\alpha} \ ,
\end{aligned}
\tag{3.1}
$$

and their hermitian conjugates. Remember that the $SU(2)_I$ generators are traceless: $T_i^i = 0$, hence only three of them are independent.

It is now convenient to rearrange the Lorentz and translations generators $M_{\mu\nu}$ and P_μ as follows:

$$J_{\alpha\beta} := \frac{1}{2}(\sigma^{\mu\nu})_{\alpha\beta} M_{\mu\nu} \quad ; \quad \overline{J}_{\dot\alpha\dot\beta} := \frac{1}{2}(\overline{\sigma}^{\mu\nu})_{\dot\alpha\dot\beta} M_{\mu\nu} \quad ; \quad P_{\alpha\dot\beta} := (\sigma^\mu)_{\alpha\dot\beta} P_\mu \ , \tag{3.2}$$

exploiting the isomorphism ρ between the Minkowski space M_4 and the space $H(2,C)$ of 2×2 hermitian matrices:

$$\rho : \ M_4 \to H \quad , \quad \rho(x_\mu) = x_\mu \sigma^\mu \tag{3.3}$$

$$\rho^{-1} : \ H \to M_4 \quad , \quad \rho^{-1}(h) = \frac{1}{2}\mathrm{Tr}\ [h\overline{\sigma}^\mu] \ . \tag{3.4}$$

The twisting procedure, introduced by Witten in [13, 14], simply consists into a redefinition of the internal group indices i as lefthanded spinorial indices α:

$$i \stackrel{twist}{\longrightarrow} \alpha \ . \tag{3.5}$$

This is possible thanks to the fact that both the spinorial indices $\{\alpha, \dot\alpha\}$ and the susy index i run from 1 to 2. The Lorentz group generators $J_{\alpha\beta}$ are correspondingly redefined through a linear combination $J'_{\alpha\beta}$ with the $SU(2)_I$ internal group generators, which, after the twist, are written as T_α^β:

$$J'_{\alpha\beta} := J_{\alpha\beta} + kT_{\alpha\beta} \ , \tag{3.6}$$

where k is a constant to be fixed by requiring that $[J', J'] = [J, J]$. Since both J and T are symmetric in (α, β), the same holds also for J'. Notice that lefthandedness is a possibility, the twist defined through the identification of i and $\dot\alpha$ being equally legitimate.

If $SU(2)_L$ is the group associated to the generators $J_{\alpha\beta}$, the redefinition (3.6) corresponds to twisting the Lorentz group $SU(2)_L \times SU(2)_R$ into $SU(2)'_L \times SU(2)_R$, where $SU(2)'_L$ is the diagonal sum of $SU(2)_L$ and $SU(2)_I$.

The new, twisted, global symmetry group H' is

$$H \stackrel{twist}{\longrightarrow} H' = SU(2)'_L \times SU(2)_R \times U(1) \ . \tag{3.7}$$

The supersymmetry charges become:

$$Q_{i\alpha} \stackrel{twist}{\longrightarrow} Q_{\beta\alpha} \quad \text{and} \quad \overline{Q}_{i\dot\alpha} \stackrel{twist}{\longrightarrow} \overline{Q}_{\beta\dot\alpha} \ . \tag{3.8}$$

The twisted supercharges under H' transform as $Q_{\beta\alpha} = (0,0)^{+1} \oplus (1,0)^{+1}$ and $\overline{Q}_{\beta\dot\alpha} = (\frac{1}{2}, \frac{1}{2})^{-1}$, or, more explicitly, the four supercharges $Q_{\beta\alpha}$ under the twist can be rearranged into a scalar δ_W and an anti-selfdual tensor $\delta_{\mu\nu}$, while the other four $\overline{Q}_{\beta\dot\alpha}$ become a vector operator δ_μ:

$$\begin{aligned} Q_{\beta\alpha} &\stackrel{twist}{\longrightarrow} \delta_W := \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta} Q_{\beta\alpha} \oplus \delta_{\mu\nu} := \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta} Q_{\beta\alpha} \\ \overline{Q}_{\beta\dot\alpha} &\stackrel{twist}{\longrightarrow} \delta_\mu := \frac{1}{\sqrt{2}}\overline{Q}_{\beta\dot\alpha}(\overline{\sigma}_\mu)_\beta^{\dot\alpha} \ , \end{aligned} \tag{3.9}$$

and $\delta_{\mu\nu}$ is selfdual

$$\delta_{\mu\nu} = \tilde{\delta}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\delta^{\rho\sigma} \ . \tag{3.10}$$

The subalgebra formed by the eight twisted supercharges $\delta_W, \delta_\mu, \delta_{\mu\nu}$ and the $\mathcal{R}$ symmetry, reads:

$$\begin{aligned}
\{\delta_W, \delta_W\} &= 2\delta_W^2 = 0 \\
\{\delta_W, \delta_\mu\} &= \partial_\mu \\
\{\delta_\mu, \delta_\nu\} &= 0 \\
\{\delta_\mu, \delta_{\rho\sigma}\} &= -\left(\varepsilon_{\mu\rho\sigma\nu}\partial^\nu + g_{\mu\rho}\partial_\sigma - g_{\mu\sigma}\partial_\rho\right) \\
\{\delta_W, \delta_{\mu\nu}\} &= 0 \\
[\mathcal{R}, \delta_W] &= +\delta_W \\
[\mathcal{R}, \delta_\mu] &= -\delta_\mu \\
[\mathcal{R}, \delta_{\mu\nu}] &= +\delta_{\mu\nu} \ ,
\end{aligned} \tag{3.11}$$

where $g_{\mu\nu} = diag(+,+,+,+)$ is the euclidean flat space metric.

A few remarks are in order:

1. The operator δ_W, which coincides with the "fermionic symmetry" introduced by Witten in [13, 14], is nilpotent

$$\delta_W^2 = 0 \ . \tag{3.12}$$

   In the Wess Zumino gauge its realization is nonlinear, as we shall see, and δ_W will turn out to be nilpotent *modulo* (field-dependent) gauge transformations and field equations, as usual in supersymmetry algebras.

2. The operators δ_W and δ_μ form a subalgebra which closes on translations. This is a common, and somehow defining, feature of topological models [20], and remarkably suggests that the twist has deeply to do with topological quantum field theories and their algebraic structure. In fact the common feature of all topological field theories, is the existence of three operators $\delta, \delta_\mu, \partial_\mu$ satisfying the following algebra [19]

$$\delta^2 = 0 \quad , \quad \{\delta, \delta_\mu\} = \partial_\mu \quad , \quad \{\delta_\mu, \delta_\nu\} = 0 \ . \tag{3.13}$$

   In other words, it is not surprising at all that, twisting N=2 SYM, a topological quantum field theory is recovered.

3. The twist does not change the mass dimensions of the supersymmetry charges, which is $\frac{1}{2}$. The $\mathcal{R}$-charge is $+1$ for δ_W and $\delta_{\mu\nu}$, and -1 for the vector symmetry δ_μ.

4. The following table summarizes the effect of the twist on the global group, its generators and on the supersymmetry charges:

	UNTWISTED				TWISTED		
group	$SU(2)_L$	$SU(2)_R$	$SU(2)_I$			$SU(2)'_L$	$SU(2)_R$
generators	$J_{\alpha\beta}(3)$	$\overline{J}_{\dot\alpha\dot\beta}(3)$	$T_{ij}(3)$			$J'_{\alpha\beta}(3)$	$\overline{J}_{\dot\alpha\dot\beta}(3)$
$Q_{i\alpha}$	1/2	0	1/2		δ_W	0	0
					$\delta_{\mu\nu}$	1	0
$\overline{Q}_{i\dot\alpha}$	0	1/2	1/2		δ_μ	1/2	1/2

3.1 Twisted fields

The fields belonging to the YM multiplet concerned by the twist are the fermionic fields $\lambda^{i\alpha}(x)$ and $\overline{\lambda}_{i\dot\alpha}(x)$, *i.e.* those carrying the internal supersymmetry index i, which, like the supercharges $Q_{i\alpha}$ and $\overline{Q}_{i\dot\alpha}$, are twisted as follows

$$\lambda^{i\alpha} \to \lambda_{\beta\alpha}(\frac{1}{2},0,\frac{1}{2})^{-1} \quad \to \quad \eta(0,0)^{-1} \oplus \chi_{\mu\nu}(1,0)^{-1} \tag{3.14}$$

$$\overline{\lambda}_{i\dot\alpha} \to \overline{\lambda}_{\beta\dot\alpha}(0,\frac{1}{2},\frac{1}{2})^{+1} \quad \to \quad \psi_\mu(\frac{1}{2},\frac{1}{2})^{+1} \ . \tag{3.15}$$

The field $\lambda(x)$ is twisted into a scalar field $\eta(x)$ and an antiselfdual antisymmetric tensor $\chi_{\mu\nu}(x)$, while $\overline{\lambda}(x)$ yields a vector field $\psi_\mu(x)$:

$$\begin{aligned} \lambda_{\beta\alpha} \quad &\to \quad \eta := \varepsilon^{\alpha\beta}\lambda_{[\beta\alpha]} \ \oplus \ \chi_{\mu\nu} := \frac{1}{4}(\sigma_{\mu\nu})^{\alpha\beta}\lambda_{(\beta\alpha)} \\ \overline{\lambda}_{\beta\dot\alpha} \quad &\to \quad \psi_\mu := \overline{\lambda}_{\beta\dot\alpha}(\overline{\sigma}_\mu)^{\dot\alpha}_{\beta} \ , \end{aligned} \tag{3.16}$$

with

$$\chi_{\mu\nu} = \tilde{\chi}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\chi^{\rho\sigma} \ . \tag{3.17}$$

The bi-spinor $\lambda_{\beta\alpha}(x)$ is thus decomposed into its symmetric and antisymmetric part: $\lambda_{\beta\alpha} = \frac{1}{2}(\lambda_{[\beta\alpha]} + \lambda_{(\beta\alpha)})$.

Summarizing, the effect of the twist on the fields of the YM multiplet is

$$(A_\mu, \lambda_{i\alpha}, \overline{\lambda}_{i\dot\alpha}, \phi, \overline{\phi}) \stackrel{twist}{\longrightarrow} (A_\mu, \psi_\mu, \chi_{\mu\nu}, \eta, \phi, \overline{\phi}) \ , \tag{3.18}$$

which, not by chance, coincides with the field content of the Donaldson-Witten topological QFT [13, 21].

3.2 Twisted action

The twisting procedure changes the action (2.2) accordingly. It is important to stress that the twist, as far as quantum fields are concerned, is simply a

linear rearrangement, which does not modify the path integral defining the functional generators. The partition function is not affected by the twist, hence the two theories, the untwisted and the twisted one, are completely equivalent. This means, in particular, that the physical observables should be the same in the two theories, that the finiteness properties must be preserved, and that the susy invariance should reflect into invariance under the twisted operators δ_W, δ_μ and $\delta_{\mu\nu}$.

In order to verify this latter property, let us twist the pure N=2 SYM action (2.2):

$$\begin{aligned} S_{YM} \stackrel{twist}{\longrightarrow} S_{TYM} &= \frac{1}{g^2}\mathrm{Tr}\int \mathrm{d}^4x\Big(\frac{1}{2}F^+_{\mu\nu}F^{+\mu\nu} - \chi^{\mu\nu}\left(D_\mu\psi_\nu - D_\nu\psi_\mu\right)^+ \\ &+ \eta D_\mu\psi^\mu - \frac{1}{2}\overline{\phi}D_\mu D^\mu\phi + \frac{1}{2}\overline{\phi}\{\psi^\mu,\psi_\mu\} \\ &- \frac{1}{2}\phi\{\chi^{\mu\nu},\chi_{\mu\nu}\} - \frac{1}{8}\left[\phi,\eta\right]\eta - \frac{1}{32}\left[\phi,\overline{\phi}\right]\left[\phi,\overline{\phi}\right]\Big)\ . \end{aligned} \tag{3.19}$$

It is evident that the twisted N=2 SYM theory coincides with the Topological Yang-Mills (TYM) theory, as expected [13, 14].

Of course, under the gauge transformations

$$\begin{aligned} \delta^g_\epsilon A_\mu &= -D_\mu\epsilon \\ \delta^g_\epsilon\lambda &= [\epsilon,\lambda]\ , \quad \lambda = \chi,\psi,\eta,\phi,\overline{\phi}\ , \end{aligned} \tag{3.20}$$

$\epsilon(x)$ being the local infinitesimal gauge parameter, the action S_{TYM} is invariant

$$\delta^g_\epsilon S_{TYM} = 0. \tag{3.21}$$

As we already said, we expect that S_{TYM} keeps memory of susy invariance through its invariance under the twisted operators δ_W, δ_μ and $\delta_{\mu\nu}$. Notice that it would have been a very difficult task to identify *a priori* the symmetries of the TYM action (3.19). This, on the contrary, turns out to be quite natural thanks to the twisting procedure. Before verifying that the twisted operators are indeed symmetries of the TYM action, we have to write down their action on the twisted fields.

3.3 Twisted supersymmetry transformations on twisted fields

Recalling the definition of the twisted operators (3.9) and of the twisted fields (3.16), after a little algebra, one gets

δ_W transformations on twisted fields

$$\begin{aligned} \delta_W A_\mu &= \psi_\mu \\ \delta_W\psi_\mu &= -D_\mu\phi \\ \delta_W\phi &= 0 \end{aligned}$$

$$
\begin{aligned}
\delta_W \chi_{\mu\nu} &= F^+_{\mu\nu} \\
\delta_W \overline{\phi} &= 2\eta \\
\delta_W \eta &= \frac{1}{2}\left[\phi, \overline{\phi}\right]
\end{aligned}
\tag{3.22}
$$

δ_μ **transformations on twisted fields**

$$
\begin{aligned}
\delta_\mu A_\nu &= \frac{1}{2}\chi_{\mu\nu} + \frac{1}{8} g_{\mu\nu}\eta \\
\delta_\mu \psi_\nu &= F_{\mu\nu} - \frac{1}{2}F^+_{\mu\nu} - \frac{1}{16} g_{\mu\nu}\left[\phi, \overline{\phi}\right] \\
\delta_\mu \eta &= \frac{1}{2} D_\mu \overline{\phi} \\
\delta_\mu \chi_{\sigma\tau} &= \frac{1}{8}\left(\varepsilon_{\mu\sigma\tau\nu} D^\nu \overline{\phi} + g_{\mu\sigma} D_\tau \overline{\phi} - g_{\mu\tau} D_\sigma \overline{\phi}\right) \\
\delta_\mu \phi &= -\psi_\mu \\
\delta_\mu \overline{\phi} &= 0
\end{aligned}
\tag{3.23}
$$

$\delta_{\mu\nu}$ **transformations on twisted fields**

$$
\begin{aligned}
\delta_{\mu\nu} A_\sigma &= -\left(\varepsilon_{\mu\nu\sigma\tau}\psi^\tau + g_{\mu\sigma}\psi_\nu - g_{\nu\sigma}\psi_\mu\right) \\
\delta_{\mu\nu} \psi_\sigma &= -\left(\varepsilon_{\mu\nu\sigma\tau} D^\tau \phi + g_{\mu\sigma} D_\nu \phi - g_{\nu\sigma} D_\mu \phi\right) \\
\delta_{\mu\nu} \phi &= 0 \\
\delta_{\mu\nu} \overline{\phi} &= 8\chi_{\mu\nu} \\
\delta_{\mu\nu} \eta &= -4F^+_{\mu\nu} \\
\delta_{\mu\nu} \chi_{\sigma\tau} &= \frac{1}{8}\left(\varepsilon_{\mu\nu\sigma\tau} + g_{\mu\sigma} g_{\nu\tau} - g_{\mu\tau} g_{\nu\sigma}\right)\left[\phi, \overline{\phi}\right] \\
&\quad + \left(F^+_{\mu\sigma} g_{\nu\tau} - F^+_{\nu\sigma} g_{\mu\tau} - F^+_{\mu\tau} g_{\nu\sigma} + F^+_{\nu\tau} g_{\mu\sigma}\right) \\
&\quad + \left(\varepsilon_{\mu\nu\sigma}{}^\alpha F^+_{\tau\alpha} - \varepsilon_{\mu\nu\tau}{}^\alpha F^+_{\sigma\alpha} + \varepsilon_{\sigma\tau\mu}{}^\alpha F^+_{\nu\alpha} - \varepsilon_{\sigma\tau\nu}{}^\alpha F^+_{\mu\alpha}\right) \ .
\end{aligned}
\tag{3.24}
$$

The following tables summarize the quantum numbers of the twisted fields

twisted fields	A_μ	$\chi_{\mu\nu}$	ψ_μ	η	ϕ	$\overline{\phi}$
dim.	1	3/2	3/2	3/2	1	1
$\mathcal{R}-$ charge.	0	-1	1	-1	2	-2
statistics	*comm*	*ant*	*ant*	*ant*	*comm*	*comm*

and of the twisted operators

twisted operators	δ_W	δ_μ	$\delta_{\mu\nu}$
dim.	1/2	1/2	1/2
$\mathcal{R}-$ charge.	1	1	-1
statistics	*ant*	*ant*	*ant*

where *comm* and *ant* stand for *commuting* and *anticommuting* respectively.

Long but straightforward calculations confirm that, indeed, the twisted operators are symmetries of the twisted action:

$$\delta_W S_{TYM} = \delta_\mu S_{TYM} = \delta_{\mu\nu} S_{TYM} = 0 \ . \tag{3.25}$$

It is important to stress that the fermionic, nilpotent, Witten's δ_W symmetry does not completely fix the coefficients of every term appearing in S_{TYM}. In other words, S_{TYM} *is not* the most general action invariant under δ_W. In order to fix completely all the terms by means of *a unique* coupling constant, the role of the vector δ_μ symmetry is crucial. On the other hand, the three $\delta_{\mu\nu}$ twisted symmetries are automatically satisfied, therefore, under this respect, they seem to be redundant.

3.4 Twisted algebra

Let us see what becomes the twisted supersymmetry algebra in the Wess-Zumino gauge, where the symmetries are nonlinearly realized. The following algebraic relations hold:

$$\delta_W^2 = \delta_\phi^g \ + \ (\text{field equations}) \ , \tag{3.26}$$

where δ_ϕ^g is a gauge transformation whose gauge parameter is the field $\phi(x)$. The operator δ_W is therefore *on shell* nilpotent in the space of gauge invariant local functionals. The cohomology in this constrained functional space defines the so called Witten observables [13, 14];

$$\{\delta_\mu, \delta_\nu\} = -\frac{1}{8} g_{\mu\nu} \delta_{\overline{\phi}}^g \ + (\text{field equations}), \tag{3.27}$$

where $\delta_{\overline{\phi}}^g$ is a field dependent gauge transformation, with the field $\overline{\phi}(x)$ as gauge parameter;

$$\{\delta_W, \delta_\mu\} = \partial_\mu + \delta_{A_\mu}^g + (\text{field equations}), \tag{3.28}$$

where $\delta_{A_\mu}^g$ is a field dependent gauge transformation, with the field $A_\mu(x)$ as gauge parameter.

Finally, the algebraic relations involving $\delta_{\mu\nu}$ are:

$$\{\delta_W, \delta_{\mu\nu}\} = (\text{gauge transformation}) + (\text{field equations}); \tag{3.29}$$

$$\{\delta_{\mu\nu}, \delta_{\rho\sigma}\} = (\text{gauge transformation}) + (\text{field equations}); \tag{3.30}$$

$$\{\delta_\mu, \delta_{\rho\sigma}\} = -(\varepsilon_{\mu\rho\sigma\nu}\partial^\nu + g_{\mu\rho}\partial_\sigma - g_{\mu\sigma}\partial_\rho) \\ +(\text{gauge transformation}) + (\text{field equations}). \tag{3.31}$$

The above algebraic structure is typical of the supersymmetry in the Wess-Zumino gauge. Two kind of obstructions to the closure of the algebra on translations occur: field equations and *field dependent* gauge transformations. The canonical way to proceed (see, for instance, [1]), is to take care of the first type of obstructions, namely the field equations, introducing auxiliary fields, whose transformations coincide with the field equations. Still, the other kind of obstruction, namely the field dependent gauge transformations, remains, and the algebra is *open*, needing an infinite number of external fields. This problem has been exhaustively treated in [2], where the non-renormalizability of theories with extended supersymmetry is discussed. The problem has nonetheless been solved, turning the situation the other way around [10, 11], as we shall see.

4 The twisted theory: N=2 SYM coupled to matter

Let us now apply the twisting procedure, described in the previous section, to the complete N=2 SYM theory, coupled to matter. Besides the pure YM multiplet, belonging to the adjoint representation of the gauge group, the field content of the theory is completed by the hypermultiplet, in a generic representation of the gauge group. The global symmetry group does not change, and the twist goes the same way:

$$H \stackrel{twist}{\longrightarrow} H' \ , \tag{4.1}$$

where H and H' are defined in (2.3) and (3.7) respectively.

In this section, we shall find out the twisted matter fields, the complete twisted action, the twisted operators and the corresponding twisted algebra. We shall moreover verify that the twisted operators are still symmetries of the twisted theory. The result should not be taken for granted, since the topological character of the twist is spoiled by the introduction of matter, and therefore we do not expect that the twisted theory is topological. Hence, the algebraic topological structure, which we shall find for a non-topological field theory, comes somehow as a surprise.

4.1 Twisted hypermultiplet

The matter hypermultiplet is $(q_i, \tilde{q}_i, \psi_q, \overline{\psi}_q, \psi_{\tilde{q}}, \overline{\psi}_{\tilde{q}})$. Only the bosonic fields $q_i(x)$ and $\tilde{q}_i(x)$, which have a nonvanishing $SU(2)_I$ quantum number, will be

twisted, the other fields remaining unchanged. The action of the twist is as follows, and we rename the fields in order to simplify notations:

$$
\begin{aligned}
q_i \stackrel{twist}{\longrightarrow} q_\alpha(0,0,\frac{1}{2})^0 &\rightarrow \mathrm{H}_\alpha(\frac{1}{2},0)^0 \\
\tilde{q}_i \stackrel{twist}{\longrightarrow} \tilde{q}_\alpha(0,0,\frac{1}{2})^0 &\rightarrow \overline{\mathrm{H}}_\alpha(\frac{1}{2},0)^0 \\
(\psi_q)_\alpha(\frac{1}{2},0,0)^{+1} &\rightarrow u_\alpha(\frac{1}{2},0)^{+1} \\
(\overline{\psi}_q)_{\dot\alpha}(0,\frac{1}{2},0)^{-1} &\rightarrow v_{\dot\alpha}(0,\frac{1}{2})^{-1} \\
(\psi_{\tilde{q}})_\alpha(\frac{1}{2},0,0)^{+1} &\rightarrow \overline{u}_\alpha(\frac{1}{2},0)^{+1} \\
(\overline{\psi}_{\tilde{q}})_{\dot\alpha}(0,\frac{1}{2},0)^{-1} &\rightarrow \overline{v}_{\dot\alpha}(0,\frac{1}{2})^{-1} \ .
\end{aligned}
\tag{4.2}
$$

Notice that, while in the pure N=2 SYM the twist gets rid of the spinorial fields, this does not happen for the hypermultiplet, whose twisted version, on the contrary, is entirely formed by spinors.

4.2 Twisted matter action

Twisting the matter N=2 SYM action (2.11), we get

$$
S_{matter} \stackrel{twist}{\longrightarrow} S_{Tmatter} \ , \tag{4.3}
$$

with

$$
\begin{aligned}
S_{Tmatter} \ = \ & \frac{1}{g^2}\mathrm{Tr_m}\int \mathrm{d}^4x\Big(\frac{1}{2}\overline{\mathrm{H}}^\gamma D_\mu D^\mu \mathrm{H}_\gamma + \overline{\mathrm{H}}^\gamma(\sigma^\mu)_{\gamma\dot\gamma}\psi_\mu v^{\dot\gamma} \\
& -\overline{v}_{\dot\gamma}(\overline{\sigma}^\mu)^{\dot\gamma\gamma}\psi_\mu \mathrm{H}_\gamma + \frac{1}{8}\overline{\mathrm{H}}^\gamma \eta u_\gamma + \frac{1}{8}\overline{\mathrm{H}}^\gamma(\sigma^{\mu\nu})_{\gamma\beta}\chi_{\mu\nu}u^\beta + \frac{1}{8}\overline{u}^\gamma\eta\mathrm{H}_\gamma \\
& -\frac{1}{8}\overline{u}^\gamma(\sigma^{\mu\nu})_{\gamma\beta}\chi_{\mu\nu}\mathrm{H}^\beta + \overline{u}^\gamma(\sigma^\mu)_{\gamma\dot\gamma}D_\mu v^{\dot\gamma} - \overline{v}_{\dot\gamma}(\overline{\sigma}^\mu)^{\dot\gamma\gamma}D_\mu u_\gamma \\
& +\frac{1}{8}\overline{u}^\gamma\overline{\phi}u_\gamma - 2\overline{v}^{\dot\gamma}\phi v_{\dot\gamma} + \frac{1}{16}\overline{\mathrm{H}}^\gamma\{\phi,\overline{\phi}\}\mathrm{H}_\gamma - \frac{1}{32}\overline{\mathrm{H}}^\gamma\mathrm{H}_\gamma\overline{\mathrm{H}}^\delta\mathrm{H}_\delta\Big)
\end{aligned}
\tag{4.4}
$$

4.3 Twisted supersymmetry transformations on twisted fields

The action of the twisted operators $(\delta_W, \delta_\mu, \delta_{\mu\nu})$ on the twisted matter fields, is as follows

δ_W transformations on twisted matter fields

$$
\begin{aligned}
\delta_W \mathrm{H}_\gamma &= \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}Q_{\beta\alpha}\mathrm{H}_\gamma = \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}(\sqrt{2}u_\alpha\varepsilon_{\beta\gamma}) = u_\gamma \\
\delta_W \overline{\mathrm{H}}_\gamma &= \overline{u}_\gamma
\end{aligned}
$$

$$
\begin{aligned}
\delta_W u_\gamma &= \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}(\sqrt{2}\varepsilon_{\gamma\alpha}\phi \mathrm{H}_\beta) = +\phi \mathrm{H}_\gamma \\
\delta_W \overline{u}_\gamma &= -\phi \overline{\mathrm{H}}_\gamma \\
\delta_W v_{\dot\gamma} &= \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}\left(-\frac{1}{\sqrt{2}}(\sigma^\nu)_{\alpha\dot\gamma}D_\nu \mathrm{H}_\beta\right) = -\frac{1}{2}(\sigma^\nu)_{\alpha\dot\gamma}D_\nu \mathrm{H}^\alpha \\
\delta_W \overline{v}_{\dot\gamma} &= -\frac{1}{2}(\sigma^\nu)_{\alpha\dot\gamma}D_\nu \overline{\mathrm{H}}^\alpha
\end{aligned}
\tag{4.5}
$$

δ_μ transformations on twisted matter fields

$$
\begin{aligned}
\delta_\mu \mathrm{H}_\gamma &= \frac{1}{\sqrt{2}}(\overline{\sigma}_\mu)^{\dot\alpha\beta}\left(\sqrt{2}v_{\dot\alpha}\varepsilon_{\beta\alpha}\right) = (\sigma_\mu)_{\gamma\dot\alpha}v^{\dot\alpha} \\
\delta_\mu \overline{\mathrm{H}}_\gamma &= (\sigma_\mu)_{\gamma\dot\alpha}\overline{v}^{\dot\alpha} \\
\delta_\mu u_\gamma &= \frac{1}{\sqrt{2}}(\overline{\sigma}_\mu)^{\dot\alpha\beta}\left(\frac{1}{\sqrt{2}}(\sigma_\nu)_{\gamma\dot\alpha}D^\nu \mathrm{H}_\beta\right) = \frac{1}{2}D_\mu \mathrm{H}_\gamma - \frac{1}{2}(\sigma_{\mu\nu})_\gamma{}^\beta D^\nu \mathrm{H}_\beta \\
\delta_\mu \overline{u}_\gamma &= \frac{1}{2}D_\mu \overline{\mathrm{H}}_\gamma - \frac{1}{2}(\sigma_{\mu\nu})_\gamma{}^\beta D^\nu \overline{\mathrm{H}}_\beta \\
\delta_\mu v_{\dot\gamma} &= \frac{1}{\sqrt{2}}(\overline{\sigma}_\mu)^{\dot\alpha\beta}\left(\frac{\sqrt{2}}{16}\varepsilon_{\dot\gamma\dot\alpha}\overline{\phi}\mathrm{H}_\beta\right) = -\frac{1}{16}(\overline{\sigma}_\mu)_{\dot\gamma\beta}\overline{\phi}\mathrm{H}^\beta \\
\delta_\mu \overline{v}_{\dot\gamma} &= \frac{1}{16}(\overline{\sigma}_\mu)_{\dot\gamma\beta}\overline{\phi}\overline{\mathrm{H}}^\beta
\end{aligned}
\tag{4.6}
$$

$\delta_{\mu\nu}$ transformations on twisted matter fields

$$
\begin{aligned}
\delta_{\mu\nu} \mathrm{H}_\gamma &= \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}\left(\sqrt{2}u_\alpha\varepsilon_{\beta\gamma}\right) = -(\sigma_{\mu\nu})_\gamma{}^\alpha u_\alpha \\
\delta_{\mu\nu} \overline{\mathrm{H}}_\gamma &= -(\sigma_{\mu\nu})_\gamma{}^\alpha \overline{u}_\alpha \\
\delta_{\mu\nu} u_\gamma &= \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}\left(\sqrt{2}\varepsilon_{\gamma\alpha}\phi \mathrm{H}_\beta\right) = (\sigma_{\mu\nu})_\gamma{}^\beta \phi \mathrm{H}_\beta \\
\delta_{\mu\nu} \overline{u}_\gamma &= -(\sigma_{\mu\nu})_\gamma{}^\beta \phi \overline{\mathrm{H}}_\beta \\
\delta_{\mu\nu} v_{\dot\gamma} &= \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}\left(-\frac{1}{\sqrt{2}}(\sigma^\lambda)_{\alpha\dot\gamma}D_\lambda \mathrm{H}_\beta\right) = \frac{1}{2}(\sigma_{\mu\nu})_\beta{}^\alpha(\sigma_\lambda)_{\alpha\dot\gamma}D^\lambda \mathrm{H}^\beta \\
&= \frac{1}{2}(\sigma_\mu)_{\beta\dot\gamma}D_\nu \mathrm{H}^\beta - \frac{1}{2}(\sigma_\nu)_{\beta\dot\gamma}D_\mu \mathrm{H}^\beta - \frac{1}{2}\varepsilon_{\mu\nu\lambda\tau}(\sigma^\tau)_{\beta\dot\gamma}D^\lambda \mathrm{H}^\beta \\
&= \frac{1}{2}\left[(\sigma_\mu)_{\beta\dot\gamma}D_\nu \mathrm{H}^\beta - (\sigma_\nu)_{\beta\dot\gamma}D_\mu \mathrm{H}^\beta\right]^+ \\
\delta_{\mu\nu} \overline{v}_{\dot\gamma} &= \frac{1}{2}\left[(\sigma_\mu)_{\beta\dot\gamma}D_\nu \overline{\mathrm{H}}^\beta - (\sigma_\nu)_{\beta\dot\gamma}D_\mu \overline{\mathrm{H}}^\beta\right]^+ \ .
\end{aligned}
\tag{4.7}
$$

The following table summarizes the quantum number and the statistics of the twisted matter fields:

twisted hypermultiplet	H	$\overline{\mathrm{H}}$	u	$\overline{u}$	v	$\overline{v}$
dim.	1	1	3/2	3/2	3/2	3/2
$\mathcal{R}-$ charge.	0	0	+1	+1	−1	−1
statistics	*comm.*	*comm.*	*ant.*	*ant.*	*ant.*	*ant.*

4.4 Twisted algebra

Once we have the twisted action (4.4) and the twisted field transformations (4.5) and (4.6), we can verify that the algebra formed by the twisted operators $(\delta_W, \delta_\mu, \delta_{\mu\nu})$ is the same of the pure gauge case, *i.e.* it is a topological, supersymmetric algebra which closes on translations, *modulo* gauge dependent field transformations and equations of motion. Notice that the fields on which the gauge transformations depend, are the same as in the pure gauge case.

The complete N=2 SYM theory contains interactions terms between the two supermultiplets, the pure gauge and the matter one. Hence, the field equations of motion for the vector supermultiplet change. In order to preserve the algebra, which depends on the field equations, we must modify the transformations of the gauge multiplet. Let us see how this can be done. The field equations appearing in the algebra as obstructions, are those concerning the fields $\eta(x), \chi(x)$ and $\psi(x)$, which appear in the interaction terms of the complete action. Their transformations under the twisted operators are those to be changed. Let us see in detail how, for example, the transformation $\delta_W \chi_{\mu\nu}$ must be modified.

Since

$$\begin{aligned} \delta_W^2 \chi_{\mu\nu} &= [\phi, \chi_{\mu\nu}] - g^2 \frac{\delta S_{TYM}}{\delta \chi^{\mu\nu}} \\ &= [\phi, \chi_{\mu\nu}] - [\phi, \chi_{\mu\nu}] + (D_\mu \psi_\nu - D_\nu \psi_\mu)^+ \\ &\quad - \frac{1}{8} \overline{\mathrm{H}}^\gamma (\sigma^{\mu\nu})_{\gamma\beta} u^\beta - \frac{1}{8} \overline{u}^\gamma (\sigma^{\mu\nu})_{\gamma\beta} \mathrm{H}^\beta \ , \end{aligned} \tag{4.8}$$

it must be

$$\delta_W \chi_{\mu\nu} = F^+_{\mu\nu} - \frac{1}{8} \overline{\mathrm{H}}^\gamma (\sigma_{\mu\nu})_{\gamma\beta} \mathrm{H}_\gamma \ . \tag{4.9}$$

Analogously, by analyzing the whole set of algebraic relations, from (3.26) to (3.31), we can infer the modified transformations of the fields belonging to the YM multiplet, when coupled to the matter hypermultiplet:

$$\begin{aligned} \delta_W \chi_{\mu\nu} &= F^+_{\mu\nu} - \frac{1}{8} \overline{\mathrm{H}}^\gamma (\sigma_{\mu\nu})_{\gamma\beta} \mathrm{H}_\gamma \\ \delta_\mu \psi_\nu &= F^-_{\mu\nu} - \frac{1}{16} g_{\mu\nu} \left[\phi, \overline{\phi}\right] + \frac{1}{16} \overline{\mathrm{H}}^\alpha (\sigma_{\mu\nu})_{\alpha\beta} \mathrm{H}^\beta \\ \delta_{\mu\nu} \eta &= -4 F^+_{\mu\nu} + \frac{1}{2} \overline{\mathrm{H}}^\gamma (\sigma_{\mu\nu})_{\gamma\beta} \mathrm{H}^\beta \\ \delta_{\mu\nu} \chi_{\rho\sigma} &= \frac{1}{8} \left(\varepsilon_{\mu\nu\sigma\tau} + g_{\mu\sigma} g_{\nu\tau} - g_{\mu\tau} g_{\nu\sigma}\right) \left[\phi, \overline{\phi}\right] \\ &\quad + \left(F^+_{\mu\sigma} g_{\nu\tau} - F^+_{\nu\sigma} g_{\mu\tau} - F^+_{\mu\tau} g_{\nu\sigma} + F^+_{\nu\tau} g_{\mu\sigma}\right) \\ &\quad + \left(\varepsilon_{\mu\nu\sigma}{}^\alpha F^+_{\tau\alpha} - \varepsilon_{\mu\nu\tau}{}^\alpha F^+_{\sigma\alpha} + \varepsilon_{\sigma\tau\mu}{}^\alpha F^+_{\nu\alpha} - \varepsilon_{\sigma\tau\nu}{}^\alpha F^+_{\mu\alpha}\right) \\ &\quad - \frac{1}{16} [g_{\mu\sigma} (\sigma_{\rho\nu})_\gamma^{\ \beta} - g_{\mu\rho} (\sigma_{\sigma\nu})_\gamma^{\ \beta} \end{aligned} \tag{4.10}$$

$$+g_{\rho\nu}(\sigma_{\sigma\nu})_{\gamma}^{\ \beta} + g_{\sigma\nu}(\sigma_{\mu\rho})_{\gamma}^{\ \beta}] \left(\overline{\mathrm{H}}_{\beta}\mathrm{H}^{\gamma} + \overline{\mathrm{H}}^{\gamma}\mathrm{H}_{\beta}\right) \ ,$$

all the other transformations remaining unchanged.

Taking into account the above transformations, now the whole algebra formed by the twisted operators $(\delta_W, \delta_\mu, \delta_{\mu\nu})$ is closed, *modulo* field dependent gauge transformations **and field equations, for the whole theory, including matter.**

4.5 Symmetries of the complete twisted action

Lengthy and uninstructive computations lead us to claim that the complete, twisted action

$$S_T = S_{TYM} + S_{Tmatter} \ , \tag{4.11}$$

where S_{TYM} and $S_{Tmatter}$ are given in (3.19) and (4.4) respectively, is invariant under the twisted supercharges:

$$\delta_W S_T = \delta_\mu S_T = \delta_{\mu\nu} S_T = 0 \ . \tag{4.12}$$

As for the twisted pure gauge N=2 SYM action, the total S_T action is univocally determined by the two symmetries δ_W and δ_μ, δ_W alone being not sufficient. Only one coupling constant is left, as expected.

Starting from the untwisted N=2 SYM theory coupled to matter, through the twisting procedure we got an action, gauge invariant, and invariant as well under two symmetries δ_W and δ_μ, remnant of five over the eight supercharges, three of them turning out to be redundant. The resulting action is equivalent to the starting, untwisted supersymmetric action, and the twist revealed an algebraic topological structure.

The twisted N=2 sym action coupled to matter appears to be a Witten-type topological action, since it can be written as the variation of a nilpotent operator (δ_W, in our case), *modulo* field equations:

$$S_T = \delta_W \Delta + \widehat{\Delta} \ , \tag{4.13}$$

where

$$\begin{aligned} \Delta \ &= \ \mathrm{Tr} \int d^4x \Big(\frac{1}{2}\chi^{\mu\nu} F^{+}_{\mu\nu} - \frac{1}{2}\overline{\phi} D^{\mu}\psi_{\mu} + \frac{1}{16}\eta \left[\phi, \overline{\phi}\right] \\ &\quad + \frac{1}{16}\overline{\mathrm{H}}^{g}(\sigma_{\mu\nu})_{\gamma}^{\beta}\chi^{\mu\nu}\mathrm{H}_{\beta} + \frac{1}{16}\overline{u}^{\gamma}\overline{\phi}\mathrm{H}_{\gamma} + \frac{1}{16}u_{\gamma}\overline{\phi\mathrm{H}}^{\gamma}\Big) \ , \end{aligned} \tag{4.14}$$

and $\widehat{\Delta}$ is a contact term

$$\widehat{\Delta} = \frac{1}{2}\chi^{\mu\nu}\frac{\delta S_T}{\delta\chi^{\mu\nu}} + v^{\dot{\gamma}}\frac{\delta S_T}{\delta v^{\dot{\gamma}}} + \overline{v}^{\dot{\gamma}}\frac{\delta S_T}{\delta \overline{v}^{\dot{\gamma}}} \ . \tag{4.15}$$

We recall that δ_W is nilpotent only *on shell*, and the above relation is not yet an *exact* relation, in the cohomological sense. But, still, this last result is quite remarkable. It suggests indeed to consider the operator δ_W – which, we stress again, is not sufficient, alone, to completely determine the action – as the starting point towards the identification of a nilpotent operator under which the total action is *off shell* exact.

5 Towards quantum extension

In the previous sections, we managed in order to treat a known, though complex, situation. Namely we are now dealing with the gauge invariant action S_T (4.11), invariant also under the scalar operator δ_W and the vector operator δ_μ. The underlying supersymmetry algebra closes on translations, *modulo* field equations and gauge dependent field transformations. The situation is similar to that encountered in topological field theories (like Chern-Simons theory or BF models) and in supersymmetric field theories (N=1 and N=2 SYM). We shall adopt in this case the same technique successfully used there, to define the classical theory and to proceed towards the algebraic renormalization.

The study of the divergences of a quantum field theory and of the possible quantum extension of its classical symmetries requires the usual renormalizations tools. In the case of supersymmetric field theories, so far it is not known a completely satisfactory regularization scheme which preserves at the same time BRS symmetry and supersymmetry. The algebraic renormalization [19], which does not rely on any regularization scheme, is, hence, a mandatory choice. **The first algebraic study of the renormalizability of a supersymmetric QFT, has been completely performed, for the N=1 case, using the superspace formalism (see [3] and references therein, in particular [22]), and a class of N=1 SYM theories has been shown to have no coupling constant renormalization at all [23, 24, 25, 26, 27].**

For what concerns N=2 SYM, with and without matter, the first algebraic approach **to the study of counterterms and anomalies** has been given in [10, 11].

In this Section, we set the standard for the quantum extension of the twisted N=2 TYM, all the results obtained previously thanks to the twist being valid at the classical level only. The basic steps of the procedure are the construction of an invariant gauge fixing term, the definition of a classical action, including gauge fixing and source-dependent terms, which satisfies all the symmetries of the theory through an extended Slavnov-Taylor identity, which resumes both gauge symmetries and supersymmetries. The key point in our reasoning is the closure of the algebra *off shell*.

5.1 TYM and the extended BRS operator

Our starting point is the classical action S_T (4.11), equivalent to the classical N=2 SYM coupled to matter.

Besides being gauge invariant, the action S_T is invariant also under a set of global transformations, whose generators $\delta_W, \delta_\mu, \delta_{\mu\nu}$ commute with the gauge transformations δ^g_ϵ (3.20), and satisfy the following algebra:

$$\begin{aligned}
[\delta_W, \delta^g_\epsilon] &= [\delta_\mu, \delta^g_\epsilon] = [\delta_{\mu\nu}, \delta^g_\epsilon] = 0 \\
\delta^2_W &= \delta^g_\phi \ + \ (\text{field eq.}) \\
\{\delta_\mu, \delta_\nu\} &= -\frac{1}{8} g_{\mu\nu} \delta^g_{\bar{\phi}} \ + (\text{field eq.}) \\
\{\delta_W, \delta_\mu\} &= \partial_\mu + \delta^g_{A_\mu} + (\text{field eq.}) \\
\{\delta_W, \delta_{\mu\nu}\} &= (\text{gauge transf.}) + (\text{field eq.}) \\
\{\delta_{\mu\nu}, \delta_{\rho\sigma}\} &= (\text{gauge transf.}) + (\text{field eq.}) \\
\{\delta_\mu, \delta_{\rho\sigma}\} &= -(\varepsilon_{\mu\rho\sigma\nu}\partial^\nu + g_{\mu\rho}\partial_\sigma - g_{\mu\sigma}\partial_\rho) + (\text{gauge transf.}) + (\text{field eq.}) \ .
\end{aligned} \tag{5.1}$$

The quantum extension of susy, or susy-like, theories presents some serious difficulties, as explained in [2]:

gauge fixing term: In absence of supersymmetry, the gauge fixing term is a BRS variation, hence it is BRS invariant by construction, being the BRS operator nilpotent. In presence of supersymmetry, instead, because of the algebra, which in the WZ gauge does not simply close on translations, such a term is not susy invariant. The usual way to add a gauge fixing term cannot be applied for supersymmetric QFT.

open algebra: In the Wess Zumino gauge, the susy transformations (and hence their twisted versions) are not linear. The algebra closes only *on shell* and *modulo* field dependent gauge transformations. The standard way to deal with this kind of algebras is to introduce auxiliary fields in order to get rid of the field equations, but still the algebra does not close, and an infinite number of external fields is needed, which renders the quantum extension of the theory meaningless.

A solution is to define an extended BRS operator which collects all the symmetries of the theory [10, 11], but, before doing that, let us write the usual BRS operator s, promoting the gauge parameter $\epsilon(x)$ to a ghost field $c(x)$, so that the gauge transformation δ^g_ϵ becomes the BRS operator s:

$$\epsilon^a(x) \to c^a(x) \quad , \quad \delta^g_\epsilon \to s \ . \tag{5.2}$$

In addition, we introduce an antighost field $\bar{c}(x)$ and a Lagrange multiplier $b(x)$, always in the adjoint representation of the gauge group, so that the

BRS operator

$$
\begin{aligned}
sA_\mu &= -D_\mu c \\
s\psi_\mu &= \{c, \psi_\mu\} \\
s\chi_{\mu\nu} &= \{c, \chi_{\mu\nu}\} \\
s\eta &= \{c, \eta\} \\
s\phi &= [c, \phi] \\
s\overline{\phi} &= \left[c, \overline{\phi}\right] \\
sc &= c^2 = \frac{1}{2} f^{abc} c^b c^c \\
s\overline{c} &= b \\
sb &= 0 \\
s\mathrm{H} &= [c, \mathrm{H}] \\
s\overline{\mathrm{H}} &= \left[c, \overline{\mathrm{H}}\right] \\
su &= \{c, u\} \\
s\overline{u} &= \{c, \overline{u}\} \\
sv &= \{c, v\} \\
s\overline{v} &= \{c, \overline{v}\}
\end{aligned}
\tag{5.3}
$$

is nilpotent

$$
s^2 = 0 \ . \tag{5.4}
$$

Let us now introduce *global* ghosts ω, ϵ^μ and v^μ, coupled respectively to δ_W, δ_μ and to the translations ∂_μ

$$
\omega \leftrightarrow \delta_{\mathcal{W}} \ , \quad \varepsilon^\mu \leftrightarrow \delta_\mu \ , \quad v^\mu \leftrightarrow \partial_\mu \ , \tag{5.5}
$$

in order to define the *extended* BRS operator as

$$
\mathcal{Q} = s + \omega\delta_{\mathcal{W}} + \varepsilon^\mu \delta_\mu + v^\mu \partial_\mu - \omega\varepsilon^\mu \frac{\partial}{\partial v^\mu} \ . \tag{5.6}
$$

The mass dimensions, $\mathcal{R}$-charge, ghost number and the statistics of the ghosts (both global and local), of the antighost and of the Lagrange multiplier, are summarized in the following table

	ω	ε^μ	v^μ	c	$\bar{c}$	b
dim	$-1/2$	$-1/2$	-1	0	2	2
$\mathcal{R}-$charge	-1	1	0	0	0	0
ghost number	1	1	1	1	-1	0
statistics	*comm*	*comm*	*ant*	*ant*	*ant*	*comm*

The extended BRS operator $\mathcal{Q}$ has ghost number $+1$, zero $\mathcal{R}$-charge and mass dimensions, is a symmetry of the action S_T and it is nilpotent *on shell*:

$$
\begin{aligned}
\mathcal{Q}\, S_T &= 0 \ , \\
\mathcal{Q}^2 &= \text{field equations} \ .
\end{aligned}
\tag{5.7}
$$

The $\mathcal{Q}$-invariance of the action is obvious, since S_T does not depend on the ghosts and it is invariant under translations. On the other hand, the nilpotency of $\mathcal{Q}$ is obtained defining the action of the twisted operators δ_W, δ_μ (and hence of $\mathcal{Q}$) on the ghosts $(c(x), \omega, \varepsilon^\mu, v^\mu)$ suitably.

It must be

$$\begin{aligned} \mathcal{Q}c &= c^2 - \omega^2\phi - \omega\varepsilon^\mu A_\mu + \frac{\varepsilon^2}{16}\overline{\phi} + v^\mu\partial_\mu c \\ \mathcal{Q}\omega &= 0 \\ \mathcal{Q}\varepsilon^\mu &= 0 \\ \mathcal{Q}v^\mu &= -\omega\varepsilon^\mu \ . \end{aligned} \tag{5.8}$$

The antighost $\overline{c}(x)$ and the Lagrange multiplier $b(x)$ form a $\mathcal{Q}$-doublet:

$$\begin{aligned} \mathcal{Q}\overline{c} &= b + v^\mu\partial_\mu\overline{c} \ , \\ \mathcal{Q}b &= \omega\varepsilon^\mu\partial_\mu\overline{c} + v^\mu\partial_\mu b \ , \end{aligned} \tag{5.9}$$

with

$$\mathcal{Q}^2\overline{c} = \mathcal{Q}^2 b = 0 \ . \tag{5.10}$$

At this point we are able to define a gauge fixing term, as the $\mathcal{Q}$-variation of the usual "gauge fermion":

$$\begin{aligned} S_{gf} &= Q \ \mathrm{Tr} \int d^4x \ \overline{c}\partial A \qquad (5.11) \\ &= \mathrm{Tr} \int d^4x \ \left(b\partial^\mu A_\mu + \overline{c}\partial^\mu D_\mu c - \omega\overline{c}\partial^\mu\psi_\mu - \frac{\varepsilon^\nu}{2}\overline{c}\partial^\mu\chi_{\nu\mu} - \frac{\varepsilon^\mu}{8}\overline{c}\partial_\mu\eta \right) \ . \end{aligned}$$

Since the extended BRS operator $\mathcal{Q}$ is strictly nilpotent on the fields appearing in S_{gf}, the gauge fixed action $\mathcal{S}$ is $\mathcal{Q}$-invariant by construction:

$$\mathcal{Q}\left(\mathcal{S}\right) = \mathcal{Q}\left(S_T + S_{gf}\right) = 0 \ . \tag{5.12}$$

The gauge fixing procedure takes into account not only the local pure gauge symmetry, but also the twisted symmetries δ_W and δ_μ, as can be seen by the presence in the gauge fixing term (5.11) of the global ghosts ω and ϵ^μ. The absence of v^μ is due to the translation invariance.

Therefore, the action of the extended BRS operator $\mathcal{Q}$ on the whole set of fields and ghosts, is:

$$\begin{aligned} \mathcal{Q}A_\mu &= -D_\mu c + \omega\psi_\mu + \frac{\varepsilon^\nu}{2}\chi_{\nu\mu} + \frac{\varepsilon_\mu}{8}\eta + v^\nu\partial_\nu A_\mu \\ \mathcal{Q}\psi_\mu &= \{c, \psi_\mu\} - \omega D_\mu\phi + \varepsilon^\nu\left(F_{\nu\mu} - \frac{1}{2}F^+_{\nu\mu}\right) - \frac{\varepsilon_\mu}{16}[\phi, \overline{\phi}] \\ &\quad + v^\nu\partial_\nu\psi_\mu + \frac{1}{16}\overline{\mathrm{H}}^\gamma(\sigma_{\nu\mu})_{\gamma\beta}\mathrm{H}^\beta\varepsilon^\nu \\ \mathcal{Q}\chi_{\sigma\tau} &= \{c, \chi_{\sigma\tau}\} + \omega F^+_{\sigma\tau} + \frac{\varepsilon^\mu}{8}(\varepsilon_{\mu\sigma\tau\nu} + g_{\mu\sigma}g_{\nu\tau} - g_{\mu\tau}g_{\nu\sigma})D^\nu\overline{\phi} \end{aligned}$$

$$
\begin{aligned}
& +v^\nu\partial_\nu\chi_{\sigma\tau} - \frac{\omega}{8}\overline{\mathrm{H}}^\gamma(\sigma_{\sigma\tau})_{\gamma\beta}\mathrm{H}^\beta \\
\mathcal{Q}\eta &= \{c,\eta\} + \frac{\omega}{2}[\phi,\overline{\phi}] + \frac{\varepsilon^\mu}{2}D_\mu\overline{\phi} + v^\nu\partial_\nu\eta \\
\mathcal{Q}\phi &= [c,\phi] - \varepsilon^\mu\psi_\mu + v^\nu\partial_\nu\phi \\
\mathcal{Q}\overline{\phi} &= \left[c,\overline{\phi}\right] + 2\omega\eta + v^\nu\partial_\nu\overline{\phi} \\
\mathcal{Q}c &= c^2 - \omega^2\phi - \omega\varepsilon^\mu A_\mu + \frac{\varepsilon^2}{16}\overline{\phi} + v^\nu\partial_\nu c \\
\mathcal{Q}\omega &= 0 \\
\mathcal{Q}\varepsilon^\mu &= 0 \\
\mathcal{Q}v^\mu &= -\omega\varepsilon^\mu \\
\mathcal{Q}\overline{c} &= b + v^\mu\partial_\mu\overline{c} \\
\mathcal{Q}b &= \omega\varepsilon^\mu\partial_\mu\overline{c} + v^\mu\partial_\mu b \\
\mathcal{Q}\mathrm{H}_\gamma &= [c,\mathrm{H}_\gamma] + \omega u_\gamma + \varepsilon^\mu(\sigma_\mu)_{\gamma\dot\alpha}v^{\dot\alpha} + v^\mu\partial_\mu\mathrm{H}_\gamma \\
\mathcal{Q}\overline{\mathrm{H}}_\gamma &= \left[c,\overline{\mathrm{H}}_\gamma\right] + \omega\overline{u}_\gamma + \varepsilon^\mu(\sigma_\mu)_{\gamma\dot\alpha}\overline{v}^{\dot\alpha} + v^\mu\partial_\mu\overline{\mathrm{H}}_\gamma \\
\mathcal{Q}u_\gamma &= [c,u_\gamma] + \omega\phi\mathrm{H}_\gamma + \varepsilon^\mu\left(\frac{1}{2}D_\mu\mathrm{H}_\gamma - \frac{1}{2}(\sigma_{\mu\nu})_\gamma^\beta D^\nu\mathrm{H}_\beta\right) + v^\mu\partial_\mu u_\gamma \\
\mathcal{Q}\overline{u}_\gamma &= [c,\overline{u}_\gamma] + \omega\phi\overline{\mathrm{H}}_\gamma + \varepsilon^\mu\left(\frac{1}{2}D_\mu\overline{\mathrm{H}}_\gamma - \frac{1}{2}(\sigma_{\mu\nu})_\gamma^\beta D^\nu\overline{\mathrm{H}}_\beta\right) + v^\mu\partial_\mu\overline{u}_\gamma \\
\mathcal{Q}v_{\dot\gamma} &= [c,v_{\dot\gamma}] - \frac{1}{2}\omega(\sigma^\nu)_{\alpha\dot\gamma}D_\nu\mathrm{H}^\alpha - \frac{1}{16}\varepsilon^\mu(\overline{\sigma}_\mu)_{\dot\gamma\beta}\overline{\phi}\mathrm{H}^\beta + v^\mu\partial_\mu v_{\dot\gamma} \\
\mathcal{Q}\overline{v}_{\dot\gamma} &= [c,\overline{v}_{\dot\gamma}] - \frac{1}{2}\omega(\sigma^\nu)_{\alpha\dot\gamma}D_\nu\overline{\mathrm{H}}^\alpha - \frac{1}{16}\varepsilon^\mu(\overline{\sigma}_\mu)_{\dot\gamma\beta}\overline{\phi}\overline{\mathrm{H}}^\beta + v^\mu\partial_\mu\overline{v}_{\dot\gamma}\ ,
\end{aligned}
\tag{5.13}
$$

with

$$
\mathcal{Q}^2 = 0 \quad \text{on} \quad \left(A,\phi,\overline{\phi},\eta,\mathrm{H},\overline{\mathrm{H}},c,\omega,\varepsilon,v,\overline{c},b\right)\ , \tag{5.14}
$$

and

$$
\begin{aligned}
\mathcal{Q}^2\psi_\sigma &= \frac{g^2}{4}\omega\varepsilon^\mu\frac{\delta\mathcal{S}}{\delta\chi^{\mu\sigma}} \\
&\quad + \frac{g^2}{32}\varepsilon^\mu\varepsilon^\nu\left(g_{\mu\sigma}\frac{\delta\mathcal{S}}{\delta\psi^\nu} + g_{\nu\sigma}\frac{\delta\mathcal{S}}{\delta\psi^\mu} - 2g_{\mu\nu}\frac{\delta\mathcal{S}}{\delta\psi^\sigma}\right)
\end{aligned}
\tag{5.15}
$$

$$
\begin{aligned}
\mathcal{Q}^2\chi_{\sigma\tau} &= -\frac{g^2}{2}\omega^2\frac{\delta\mathcal{S}}{\delta\chi^{\sigma\tau}} \\
&\quad + \frac{g^2}{8}\omega\varepsilon^\mu\left(\varepsilon_{\mu\sigma\tau\nu}\frac{\delta\mathcal{S}}{\delta\psi_\nu} + g_{\mu\sigma}\frac{\delta\mathcal{S}}{\delta\psi^\tau} - g_{\mu\tau}\frac{\delta\mathcal{S}}{\delta\psi^\sigma}\right)
\end{aligned}
\tag{5.16}
$$

$$
\mathcal{Q}^2 u_\gamma = \frac{g^2}{2}\Big(\omega\varepsilon^\mu(\sigma_\mu)_{\gamma\dot\gamma}\frac{\delta\mathcal{S}}{\delta\overline{v}^{\dot\gamma}} + \varepsilon^2\frac{\delta\mathcal{S}}{\delta\overline{u}^\gamma}\Big) \tag{5.17}
$$

$$
\mathcal{Q}^2\overline{u}_\gamma = \frac{g^2}{2}\Big(\omega\varepsilon^\mu(\sigma_\mu)_{\gamma\dot\gamma}\frac{\delta\mathcal{S}}{\delta v^{\dot\gamma}} + \varepsilon^2\frac{\delta\mathcal{S}}{\delta u^\gamma}\Big) \tag{5.18}
$$

$$\mathcal{Q}^2 v_{\dot\gamma} = \frac{g^2}{2}\Big(\omega^2 \frac{\delta\mathcal{S}}{\delta\overline{v}_{\dot\gamma}} - \omega\varepsilon^\mu(\sigma_\mu)_{\beta\dot\gamma}\frac{\delta\mathcal{S}}{\delta\overline{u}_\beta}\Big) \tag{5.19}$$

$$\mathcal{Q}^2 \overline{v}_{\dot\gamma} = \frac{g^2}{2}\Big(\omega^2 \frac{\delta\mathcal{S}}{\delta v_{\dot\gamma}} - \omega\varepsilon^\mu(\sigma_\mu)_{\beta\dot\gamma}\frac{\delta\mathcal{S}}{\delta u_\beta}\Big) \ . \tag{5.20}$$

5.2 The Slavnov-Taylor identity

For the functional implementation of the extended BRS operator $\mathcal{Q}$, we must couple external sources $\Phi^{\star i}(x)$ to the nonlinear $\mathcal{Q}$-transformations of the fields $\Phi^i(x)$ (5.13):

$$\begin{array}{ccc} L \to c & , & X^\gamma \to \mathrm{H}_\gamma \\ D \to \phi & , & \overline{X}^\gamma \to \overline{\mathrm{H}}_{\dot\gamma} \\ \Omega^\mu \to A_\mu & , & U^\gamma \to u_\gamma \\ \xi^\mu \to \psi_\mu & , & \overline{U}^\gamma \to \overline{u}_\gamma \\ \rho \to \overline{\phi} & , & V^{\dot\gamma} \to v_{\dot\gamma} \\ \tau \to \eta & , & \overline{V}^{\dot\gamma} \to \overline{v}_{\dot\gamma} \\ B^{\mu\nu} \to \chi_{\mu\nu} & & , \end{array}$$

so that we can add to $\mathcal{S} = S_T + S_{gf}$ the "external" term

$$S_{ext} = \mathrm{Tr}\int d^4x\ \Phi^{\star i}\mathcal{Q}\Phi^i \ , \tag{5.21}$$

where we collectively denoted with $\Phi^i(x)$ all the fields transforming nonlinearly under $\mathcal{Q}$, and with $\Phi^{\star i}(x)$ the corresponding external sources, whose quantum numbers and statistics are

	L	D	Ω^μ	ξ^μ	ρ	τ	$B^{\mu\nu}$
dim.	4	3	3	5/2	3	5/2	5/2
$\mathcal{R}-$ charge	0	-2	0	-1	2	1	1
ghostnumber	-2	-1	-1	-1	-1	-1	-1
statistics	*comm*	*ant*	*ant*	*comm*	*ant*	*comm*	*comm*

	X	$\overline{X}$	U	$\overline{U}$	V	$\overline{V}$
dim.	3	3	5/2	5/2	5/2	5/2
$\mathcal{R}-$ charge	0	0	-1	-1	1	1
ghostnumber	-1	-1	-1	-1	-1	-1
statistics	*ant*	*ant*	*comm*	*comm*	*comm*	*comm*

In order to write a Slavnov-Taylor identity, the last step is to add to the action $S_T + S_{gf} + S_{ext}$ a fourth term S_{quad} which takes into account the fact the the extended BRS operator $\mathcal{Q}$ is nilpotent *on shell*, according to the Batalin-Vilkokisky procedure [28, 29]. Such a term must be quadratic in the external sources $\Phi^{*i}(x)$

$$S_{quad} = \mathrm{Tr}\int \mathrm{d}^4x\left(\Omega_{ij}\Phi^{*i}\Phi^{*j}\right) \ , \tag{5.22}$$

where $\Omega_{ij}(x)$ are coefficients which, in general, may depend on the quantum fields and on the global ghosts. They are determined by imposing the validity of the Slavnov-Taylor identity, which we shall write shortly.

The result is the following:

$$\begin{aligned} S_{quad} &= g^2 \mathrm{Tr} \int d^4x \Big(\frac{1}{8}\omega^2 B^{\mu\nu} B_{\mu\nu} - \frac{1}{4}\omega B^{\mu\nu}\varepsilon_\mu \xi_\nu - \frac{1}{32}\varepsilon^\mu \varepsilon^\nu \xi_\mu \xi_\nu + \frac{1}{32}\varepsilon^2 \xi^2 \\ &\quad - \frac{1}{2}\varepsilon^2 U_\gamma \overline{U}^\gamma + \frac{1}{2}\omega^2 V_{\dot\gamma}\overline{V}^{\dot\gamma} - \frac{1}{2}\omega\varepsilon^\mu \overline{U}^\alpha (\sigma_\mu)_{\alpha\dot\gamma} V^{\dot\gamma} \Big) \ . \end{aligned} \tag{5.23}$$

With this choice, the complete classical action

$$\Sigma = S_T + S_{gf} + S_{ext} + S_{quad} \tag{5.24}$$

satisfies the Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = 0 \ , \tag{5.25}$$

where

$$\begin{aligned} \mathcal{S}(\Sigma) &= \mathrm{Tr} \int d^4x \left(\frac{\delta\Sigma}{\delta\Phi^{\star i}} \frac{\delta\Sigma}{\delta\Phi^i} + (b + v^\mu \partial_\mu \overline{c}) \frac{\delta\Sigma}{\delta\overline{c}} + (\omega\varepsilon^\mu \partial_\mu \overline{c} + v^\mu \partial_\mu b) \frac{\delta\Sigma}{\delta b} \right) \\ &\quad - \omega\varepsilon^\mu \frac{\partial\Sigma}{\partial v^\mu} \ . \end{aligned} \tag{5.26}$$

We can go further, introducing the translation operator

$$\mathcal{P}_\mu \Sigma = \mathrm{Tr} \int d^4x \left(\partial_\mu \Phi^i \frac{\delta\Sigma}{\delta\Phi^i} + \partial_\mu \Phi^{*i} \frac{\delta\Sigma}{\delta\Phi^{*i}} \right) = 0 \ , \tag{5.27}$$

which obviously is a symmetry of the theory.

We observe that, since $\mathcal{P}_\mu$ acts linearly on all the fields, the dependence of the action Σ on the global ghost for translations v_μ, is fixed by the following identity

$$\frac{\partial\Sigma}{\partial v^\mu} = \Delta^{cl}_\mu \ , \tag{5.28}$$

where

$$\begin{aligned} \Delta^{cl}_\mu &= \mathrm{Tr} \int d^4x (\ L\partial_\mu c - D\partial_\mu \phi - \Omega^\nu \partial_\mu A_\nu + \xi^\nu \partial_\mu \psi_\nu \\ &\quad - \rho\partial_\mu \overline{\phi} + \tau\partial_\mu \eta + B^{\nu\sigma}\partial_\mu \chi_{\nu\sigma} - X^\gamma \partial_\mu \mathrm{H}_\gamma - \overline{X}^\gamma \partial_\mu \overline{\mathrm{H}}_\gamma \\ &\quad + U^\gamma \partial_\mu u_\gamma + \overline{U}^\gamma \partial_\mu \overline{u}_\gamma + V^{\dot\gamma} \partial_\mu v_{\dot\gamma} + \overline{V}^{\dot\gamma} \partial_\mu \overline{v}_{\dot\gamma} \ ) \ , \end{aligned} \tag{5.29}$$

being linear in the quantum fields, is present only at the classical level [19]. We can therefore get rid of the global ghost v_μ, introducing the "reduced" classical action $\widehat{\Sigma}$

$$\Sigma = \widehat{\Sigma} + v^\mu \Delta^{cl}_\mu \ , \tag{5.30}$$

with

$$\frac{\partial\widehat{\Sigma}}{\partial v^\mu} = 0 \ . \tag{5.31}$$

It is easily verified that $\widehat{\Sigma}$ satisfies the modified ST identity

$$\mathcal{S}(\widehat{\Sigma}) = \omega\varepsilon^\mu\Delta^{cl}_\mu \ , \tag{5.32}$$

where

$$\mathcal{S}(\widehat{\Sigma}) = \mathrm{Tr}\int d^4x\left(\frac{\delta\widehat{\Sigma}}{\delta\Phi^{\star i}}\frac{\delta\widehat{\Sigma}}{\delta\Phi^i} + b\frac{\delta\widehat{\Sigma}}{\delta\overline{c}} + \omega\varepsilon^\mu\partial_\mu\overline{c}\frac{\delta\widehat{\Sigma}}{\delta b}\right) \ . \tag{5.33}$$

The (classically broken) ST identity (5.32) is the one which must be used to determine the quantum extension of the theory. The corresponding linearized ST operator

$$\mathcal{B}_{\widehat{\Sigma}} = \mathrm{Tr}\int d^4x\left(\frac{\delta\widehat{\Sigma}}{\delta\Phi^i}\frac{\delta}{\delta\Phi^{\star i}} + \frac{\delta\widehat{\Sigma}}{\delta\Phi^{\star i}}\frac{\delta}{\delta\Phi^i} + b\frac{\delta\widehat{\Sigma}}{\delta\overline{c}} + \omega\varepsilon^\mu\partial_\mu\overline{c}\frac{\delta\widehat{\Sigma}}{\delta b}\right) \ , \tag{5.34}$$

is not nilpotent. In fact, it holds

$$\mathcal{B}_{\widehat{\Sigma}}\mathcal{B}_{\widehat{\Sigma}} = \omega\varepsilon^\mu\mathcal{P}_\mu \ , \tag{5.35}$$

that is, $\mathcal{B}_{\widehat{\Sigma}}$ is nilpotent *modulo* a total derivative. It follows that the operator $\mathcal{B}_{\widehat{\Sigma}}$ is nilpotent in the space of *integrated* local functionals, which, actually, is the case we are interested in.

Summarizing, we handled the problem in order to be able to deal with the usual web of symmetries and constraints which constitutes the basis for the quantum extension of the model and for the study of its algebraic renormalizability (determination of local counterterms and study of anomalies) [19]:

- **ST identity (5.32)** ;
- **Landau gauge fixing condition**

$$\frac{\delta\widehat{\Sigma}}{\delta b} = \partial^\mu A_\mu \ ; \tag{5.36}$$

- **Anti-ghost equation**

$$\frac{\delta\widehat{\Sigma}}{\delta\overline{c}} + \partial_\mu\frac{\delta\widehat{\Sigma}}{\delta\Omega_\mu} = 0 \ ; \tag{5.37}$$

- **Landau gauge ghost equation**

$$\mathrm{Tr}\int d^4x\left(\frac{\delta\widehat{\Sigma}}{\delta c} + \left[\overline{c}, \frac{\delta\widehat{\Sigma}}{\delta b}\right]\right) = \Delta^{cl}_c \ , \tag{5.38}$$

with Δ^{cl}_c linear classical breaking

$$\begin{aligned}\Delta^{cl}_c \ &= \ \mathrm{Tr}\int d^4x\Big([c,L] - [A,\Omega] - [\phi,D] + [\psi,\xi] - [\overline{\phi},\rho] + [\eta,\tau] \\ &\quad +[\chi,B] - [\mathrm{H},X] - [\overline{\mathrm{H}},\overline{X}] + [u,U] + [\overline{u},\overline{U}] + [v,V] + [\overline{v},\overline{V}]\Big) \ .\end{aligned} \tag{5.39}$$

6 Conclusions

In this paper we studied the twisted version of N=2 SYM theory coupled to matter.

The twist, being simply a linear redefinition of the quantum fields, does not affect the partition function, and hence two **twisted-related theories are completely equivalent. This means, in particular, that they have the same physical content, the same observables and the same coupling constant(s) β-function(s).**

In this paper we included matter into the game. We twisted the hypermultiplet, which became entirely spinorial, we modified the (twisted) supersymmetries in order to have a close off-shell algebra, and we achieved the complete off-shell set up by means of a unique Slavnov-Taylor identity, which collects both BRS symmetry and supersymmetries of the theory.

As it is well known [13, 14], pure N=2 SYM is twisted to a topological quantum field theory: TYM. An interesting and new result presented in this article is the fact that TYM theory coupled to matter have the same set of invariances of the same theory without matter. Since the theory with matter is not topological, the presence of these symmetries contradicts the common belief that they are peculiar to topological theories.[2]

The twisted version of the whole theory, including matter, is the necessary step towards the study of the β-function, for which a well known non-renormalization theorem holds, and which has been algebraically proved only in absence of matter [15]. We stress also that we never specified to which representation of the gauge group the matter hypermultiplet belongs: in the particular case of matter in the adjoint representation, N=4 SYM is recovered.

In general, the introduction of matter spoils the topological character of the theory. In our case, the relation (4.13) suggests that matter might enter in the theory simply through an extended BRS variation, and this result strongly induces to suppose that matter does not alter neither the physical sector of observables nor the finite, or protected, operators of the theory [17]. In other terms, the presence of matter should not spoil the AdS/CFT duality between non-conformal N=2 theories and string theories [30]. It is

[2] We thank one of the referees for this remark.

also natural to expect that the whole action can be written, as in the pure gauge case, in terms of a unique, and probably finite, operator, which in the pure gauge case is $\mathrm{Tr}\,\phi^2$, whose relevance for the algebraic proof of the non-renormalization theorem of the β-function has been discussed in [15].

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