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ROBUST UPDATING OF COMPUTATIONAL MODELS WITH RESPECT TO UNCERTAINTIES FOR DYNAMICAL SYSTEMS.

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Abstract. *This paper deals with the robust updating of stochastic computational models of composite sandwich panels in the context of structural dynamics in the low- and medium-frequency range, for which experimental results are available. The uncertain computational model is constructed using the nonparametric probabilistic approach which takes into account model and data uncertainties. The formulation of the robust updating problem includes the effects of uncertainties and consists in minimizing a cost function with respect to an admissible set of updating parameters. Several cost functions are constructed, validated and compared using the experimental results. The results of the robust updating problem shows that the methods proposed are efficient for updating the computational model in both low- and medium-frequency range.*

1 INTRODUCTION

In general, the updating of a computational model using experiments is performed with deterministic models [1]. The main challenge consists in including the effects of uncertainties in the updating process, that is called robust updating. In the context of structural engineering, the robust updating leads to solve a nonlinear constrained optimization problem with respect to the updating parameters which are the updating mean parameters of the mean computational model and the updating dispersion parameters which allow the uncertainty level in the computational model to be controlled. Until now, in the context of structural dynamics, most of the published works concern robust updating in the low-frequency range with respect to data uncertainties and not to model uncertainties [2]. In the present paper, several robust updating methodologies with respect to model and data uncertainties in the low- and medium- frequency range are proposed. Model uncertainties are taken into account by using the nonparametric probabilistic approach [3, 4]. The cost functions used to formulate the robust updating problem are defined from an uncertain computational model using experiments and are a function of the updating parameters. These methodologies are validated and compared in the context of the structural dynamics of composite sandwich panels in the low- and medium-frequency range for which experimental results issued from a set of 8 manufactured sandwich panels are available [5, 6]. In Section 2, the experimental results are summarized. Section 3 is devoted to the deterministic updating of the computational model using cost functions defined from the modulus and the phase of the experimental observations. This deterministic updating is viewed as a preliminary step for robust updating and is numerically validated using the experimental results described in Section 2. Section 4 proposes the construction of two cost functions defined from experiments by using an uncertain computational model in order to formulate the robust updating problem with respect to model and data uncertainties. The two methodologies are numerically validated and compared.

2 EXPERIMENTS IN THE LOW AND MEDIUM FREQUENCY RANGE

2.1 Description of the experimental data

Experimental data related to a set of $n_{exp} = 8$ multilayered sandwich panels manufactured from a designed composite sandwich panel [5, 6] is used. The designed composite sandwich panel is a free structure with rectangular shape and is made up of two thin carbon-resin skins constituted of two unidirectional plies [60/-60] and of one high stiffness closed-cell foam core. Dynamical experiments are conducted for each of the manufactured sandwich panels. The detailed description of the designed sandwich panel and of its corresponding experimental protocol can be found in [5, 6]. The frequency response function corresponding to a given out-plane point load is measured at $n_{obs} = 25$ observation points in the frequency band of analysis $\mathbb{B} = [100\,,\,4500]\,Hz$. Let $\mathbf{x}_j$ with $j \in \{1, \ldots, n_{obs}\}$ be the location of the observation point number j. Let $W_j^{exp}(\omega, \theta_k)$ be the the observation corresponding to the experimental frequency response function of the manufactured composite sandwich panel number k, measured at observation point $\mathbf{x}_j$, at a given frequency ω of frequency band $\mathbb{B}$ and expressed in terms of acceleration.

2.2 Analysis of the experimental results

Most often, the moduli of the frequency response functions are only used to carry out analysis of experiments and identification of the model. Presently, we propose to also use an average value of the phases (see below). The experimental complex-valued frequency response function $W_j^{exp}(\omega, \theta_k)$ can be written as $W_j^{exp}(\omega, \theta_k) = |W_j^{exp}(\omega, \theta_k)| \exp(-i\, \Phi_j^{exp}(\omega, \theta_k))$ in which $|W_j^{exp}(\omega, \theta_k)|$ and $\Phi_j^{exp}(\omega, \theta_k)$ are the modulus and the unwrapped phase angle. In order to analyze the experimental data, the following quantities corresponding to a spatial average of moduli and of phases are introduced as:

$$\underline{dB}_w^{exp}(\omega) = 10 \log_{10}\left(\frac{1}{n_{obs}\, n_{exp}} \sum_{k=1}^{n_{exp}} \left(\sum_{j=1}^{n_{obs}} |W_j^{exp}(\omega, \theta_k)|^2 \right)\right) \quad , \tag{1}$$

$$\underline{\phi}_w^{exp}(\omega) = \frac{1}{n_{obs}\, n_{exp}} \sum_{k=1}^{n_{exp}} \sum_{j=1}^{n_{obs}} \Phi_j^{exp}(\omega, \theta_k) \quad , \tag{2}$$

Figure 1 displays the graph $\nu \mapsto \underline{dB}_w^{exp}(\nu)$ where $\nu = \omega/(2\,\pi)$ is the circular frequency in Hz. Figure 2 displays the graph $\nu \mapsto \underline{\phi}_w^{exp}(\nu)$.

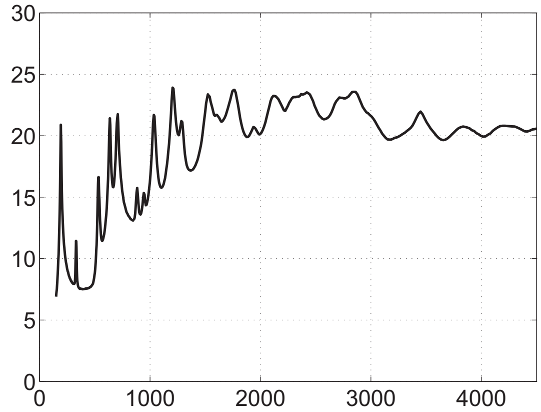


Figure 1: Graph of the experimental averaged modulus $\nu \mapsto \underline{dB}_w^{exp}(\nu)$ (thick line). Horizontal axis is frequency ν in Hz.

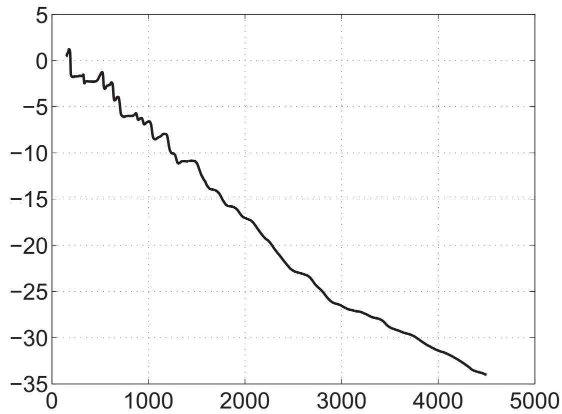


Figure 2: Graph of the experimental averaged phase $\nu \mapsto \underline{\phi}_w^{exp}(\nu)$ (thick line). Horizontal axis is frequency ν in Hz.

In Figure 2, it should be noted that the low-frequency range and the medium-frequency range can be easily identified from each other. The low-frequency range is characterized by the low-frequency band $\mathbb{B}_L$ for which $\underline{\phi}_w^{exp}(\nu)$ is a non monotonous function of the frequency

ν, showing discontinuities when crossing an isolated resonance. Analyzing Figure 2 (but also Figure 1) yields $\mathbb{B}_L = [0\,,\,1200]\,Hz$. As the frequency grows, the modal density increases and the medium-frequency range corresponds to the medium-frequency band $\mathbb{B}_M$ for which $\underline{\phi}_w^{exp}(\nu)$ is a smooth function of ν. In Figure 2, it can be seen that $\mathbb{B}_M = [1200\,,\,4500]\,Hz$.

3 UPDATING METHOD FOR THE MEAN MECHANICAL MODEL OF THE DYNAMICAL SYSTEM USING THE EXPERIMENTAL FREQUENCY RESPONSE FUNCTIONS

3.1 Motivation and strategy

The mean computational model related to the designed sandwich panel is constructed by the finite element method. The updating method has to be efficient both in the low-frequency range and in the medium-frequency range. It is assumed that the conservatory part (mass and stiffness) has been already updated [5, 6] and in this paper, we propose to update the damping model in the medium-frequency range. In general, damping varies with the frequency in the medium-frequency range. Consequently, the damping model used has to take into account this phenomenon. In this Section, a damping model controlled by four updating parameters defined on an admissible set of updating parameters is introduced in order to update the mean computational model with respect to the experimental data. For the updating of the mean computational model with respect to the admissible set of the updating parameters, first, we present a usual updating method consisting in minimizing the cost function defined as a distance between a spatial average of moduli of the frequency response function and the corresponding experimental data. Secondly, we present a new approach consisting in using for constructing the cost function, an average value of the phases.

3.2 Description of the mean finite element model

The mean finite element model of the designed sandwich panel which has to be updated is a laminated composite thin plate in bending mode. Its middle plane occupies the domain $[0\,,\,0.4] \times [0\,,\,0.3]\,m$ in the plane (Ox, Oy) of a cartesian coordinate system $(O\,x\,y\,z)$. The out-plane displacements are only considered. The laminated composite thin plate is constituted of five layers, each one made up of an orthotropic elastic material. The first two layers are two unidirectional plies in a $[-60/60]$ layup with width $0.00017\,m$, mass density $1600\,kg.m^{-3}$ and whose elasticity constants expressed in the local coordinate system $(0\,X\,Y\,z)$ are given by $E_X = 101\,GPa$, $E_Y = 6.2\,GPa$, $\nu_{XY} = 0.32$, $G_{XY} = G_{XZ} = G_{YZ} = 2.4\,GPa$. The third layer is a closed-cell foam with thickness $0.01\,m$, mass density $80\,Kg.m^{-3}$ and elasticity constants $E_x = E_y = 60\,MPa$, $\nu_{xy} = 0$, $G_{xy} = G_{xz} = G_{yz} = 30\,MPa$. The last two layers are two unidirectional plies in a $[60/-60]$ lay-up with the same characteristic as the first two layers. The laminated thin plate is a free structure. The finite element mesh is constituted of 64×64 rectangular four nodes elements and has $n = 12\,288$ DOF. The mean finite element model is submitted to a deterministic unit transverse load constant in frequency band $\mathbb{B}$ with amplitude 1 and located at the node with coordinates $(0.187, 0.103, 0)$. In the present case, the updating concerns the model used for modeling the damping in the composite panel. Let $\mathbf{r}$ be the vector of the updating parameters. Vector $\mathbf{r}$ belongs to an admissible set $\mathcal{R}$ corresponding to a given family of damping models that is defined in the next Subsection. Assuming the designed sandwich panel to be linear and slightly damped, for fixed $\mathbf{r}$ belonging to $\mathcal{R}$ and for fixed ω belonging to $\mathbb{B}$, the mean finite element matrix equation of the sandwich panel is written as

$$(-\omega^2\,[\underline{M}] \,+\, i\,\omega\,[\underline{D}(\mathbf{r})] \,+\, [\underline{K}])\,\underline{\mathbf{u}}(\mathbf{r},\omega) \,=\, \underline{\mathbf{f}}(\omega) \quad , \tag{3}$$

in which $\underline{\mathbf{u}}(\mathbf{r},\omega)$ is the $\mathbb{C}^n$-vector of the n DOF and where $\underline{\mathbf{f}}(\omega)$ is the $\mathbb{C}^n$-vector induced by the external forces. Since the sandwich panel has a free boundary, the mean mass matrix $[\underline{M}]$ is a positive-definite symmetric $(n \times n)$ real matrix and the mean damping and stiffness matrices $[\underline{D}(\mathbf{r})]$ and $[\underline{K}]$ are positive semi-definite symmetric $(n \times n)$ real matrices. It should be noted that the rank of mean matrices $[\underline{D}(\mathbf{r})]$ and $[\underline{K}]$ is $n-3$ (presence of three rigid body modes). For j belonging to $\{1,\ldots,n_{obs}\}$, the frequency response functions expressed in terms of acceleration at point $\mathbf{x}_j$ are denoted by $\underline{w}_j(\mathbf{r},\omega)$ and are stored in the $\mathbb{C}^{n_{obs}}$-vector $\underline{\mathbf{w}}(\mathbf{r},\omega) \,=\, (\underline{w}_1(\mathbf{r},\omega),\ldots,\underline{w}_{n_{obs}}(\mathbf{r},\omega))$ such that $\underline{\mathbf{w}}(\mathbf{r},\omega) \,=\, [T(\omega)]\,\underline{\mathbf{u}}(\mathbf{r},\omega)$, in which $[T(\omega)]$ is the $(n_{obs} \times n)$ observation matrix.

3.3 Description of the mean reduced matrix model

The mean reduced matrix model of the sandwich panel is constructed by modal analysis. Since we are interested in the elastic motion of the structure, we introduce the $(n \times N)$ real matrix $[\underline{\Phi}]$ whose columns are the $N \ll n$ eigenvectors $\underline{\boldsymbol{\varphi}}_j$ related to the N positive lowest eigenfrequencies $\underline{\lambda}_j \,=\, \underline{\omega}_j^2$. The mean reduced matrix model is then written as $\underline{\mathbf{w}}(\mathbf{r},\omega) \,=\, [T(\omega)]\,[\underline{\Phi}]\,\underline{\mathbf{q}}(\mathbf{r},\omega)$ in which $\underline{\mathbf{q}}(\mathbf{r},\omega)$ is the $\mathbb{C}^N$-vector of the generalized coordinates which is solution of the matrix equation

$$\Big(-\,\omega^2\,[\underline{\mathcal{M}}] \,+\, i\,\omega\,[\underline{\mathcal{D}}(\mathbf{r})] \,+\, [\underline{\mathcal{K}}]\Big)\,\underline{\mathbf{q}}(\mathbf{r},\omega) \,=\, \underline{\boldsymbol{\mathcal{F}}}(\omega) \tag{4}$$

In Eq. (4), the $\mathbb{C}^N$-vector $\underline{\boldsymbol{\mathcal{F}}}(\omega)$ is written as $\underline{\boldsymbol{\mathcal{F}}}(\omega) \,=\, [\underline{\Phi}]^T\,\underline{\mathbf{f}}(\omega)$ and the matrices $[\underline{\mathcal{M}}]$ and $[\underline{\mathcal{K}}]$ are the positive-definite symmetric $(N \times N)$ real diagonal matrices such that $[\underline{\mathcal{M}}]_{jk} \,=\, \mu_j\,\delta_{jk}$ and $[\underline{\mathcal{K}}]_{jk} \,=\, \underline{\mu}_j\,\underline{\omega}_j^2\,\delta_{jk}$ in which $\underline{\mu}_j$ is the generalized mass related to eigenmode $\underline{\boldsymbol{\varphi}}_j$ and where δ_{jk} denotes the Kronecker symbol. The mean reduced damping matrix $[\underline{\mathcal{D}}(\mathbf{r})]$ (which is a positive-definite symmetric $(N \times N)$ real matrix) is then introduced such that $[\underline{\mathcal{D}}(\mathbf{r})]_{jk} \,=\, 2\,\underline{\mu}_j\,\underline{\omega}_j\,\underline{\xi}_j(\mathbf{r})\,\delta_{jk}$ in which $\underline{\xi}_j(\mathbf{r})$ is the mean modal damping rate related to eigenmode $\underline{\boldsymbol{\varphi}}_j$ defined as $\underline{\xi}_j(\mathbf{r}) \,=\, f(\underline{\omega}_j,\mathbf{r})$.

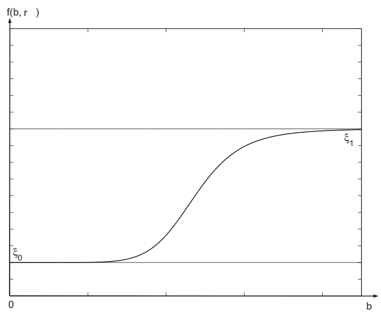


Figure 3: Graph of function $b \mapsto f(b,\mathbf{r})$

Let $\mathbf{r} \,=\, \{\xi_0,\xi_1,\alpha,\beta\}$ be the $\mathbb{C}^4$-vector of the updating parameters belonging to the admissible set $\mathcal{R}$ defined as $\mathcal{R} \,=\, \Big\{\{\xi_0,\xi_1,\alpha,\beta\},\xi_1 \geq \xi_0 > 0; \alpha > 0; \beta > 0\Big\}$. For $\mathbf{r}$ fixed in $\mathcal{R}$, the function $b \mapsto f(b,\mathbf{r})$ from $\mathbb{R}^+$ into $\mathbb{R}^+$ is defined by

$$f(b,\mathbf{r}) \,=\, \xi_0 \,+\, (\xi_1 \,-\, \xi_0)\,\frac{b^\alpha}{b^\alpha \,+\, 10^\beta} \quad , \tag{5}$$

and its graph is displayed in Figure 3.

3.4 Updating the mean computational model of the designed sandwich panel with experiments.

In this Section, two formulations are proposed to update the mean computational model of the sandwich panel with respect to the experiments. Similarly to Eqs. (6) and (7), the following observations are introduced

$$\underline{dB}_w(\mathbf{r},\omega) \,=\, 10\,\log_{10}\left(\frac{1}{n_{obs}}\left(\sum_{j=1}^{n_{obs}} |\underline{w}_j(\mathbf{r},\omega)|^2\right)\right) \quad , \tag{6}$$

$$\underline{\phi}_w(\mathbf{r},\omega) \,=\, \frac{1}{n_{obs}}\sum_{j=1}^{n_{obs}} \underline{\phi}_j(\mathbf{r},\omega) \quad , \tag{7}$$

in which $|\underline{w}_j(\mathbf{r},\omega)|$ and $\underline{\phi}_j(\mathbf{r},\omega)$ are the modulus and the unwrapped phase angle of $\underline{w}_j(\mathbf{r},\omega)$. Two formulations are then proposed to update the mean computational model of the sandwich panel with respect to parameter $\mathbf{r}$. The first cost function is written as

$$\underline{j}_{mod}(\mathbf{r}) \,=\, \frac{||\underline{dB}_w(\mathbf{r},.) \,-\, \underline{dB}_w^{exp}||^2_{\mathbb{B}}}{||\underline{dB}_w^{exp}||^2_{\mathbb{B}}} \quad , \tag{8}$$

in which $||g||^2_{\mathbb{B}} \,=\, \int_{\mathbb{B}} |g(\omega)|^2\, d\omega$. The second cost function is written as

$$\underline{j}_{pha}(\mathbf{r}) \,=\, \frac{||\underline{\phi}_w(\mathbf{r},.) \,-\, \underline{\phi}_w^{exp}||^2_{\mathbb{B}}}{||\underline{\phi}_w^{exp}||^2_{\mathbb{B}}} \quad . \tag{9}$$

The updating of the mean computational model is then performed by solving the optimization problem:

$$\text{find } \mathbf{r}^{mod} \in \mathcal{R} \text{ such that } \underline{j}_{mod}(\mathbf{r}^{mod}) \leq \underline{j}_{mod}(\mathbf{r}) \text{ for all } \mathbf{r} \in \mathcal{R} \quad ,$$

for the first cost function or the following one for the second cost function

$$\text{find } \mathbf{r}^{pha} \in \mathcal{R} \text{ such that } \underline{j}_{pha}(\mathbf{r}^{pha}) \leq \underline{j}_{pha}(\mathbf{r}) \text{ for all } \mathbf{r} \in \mathcal{R} \quad .$$

Each constrained optimization problem can be solved numerically by using the sequential quadratic optimization algorithm [8, 9]. Moreover, it should be noted that the gradient and the Hessian of cost functions $\underline{j}_{mod}(\mathbf{r})$ and $\underline{j}_{pha}(\mathbf{r})$ can be easily algebraically constructed.

3.5 Numerical Results

First, a convergence analysis is performed with respect to the reduced order model N. The convergence analysis is performed by analyzing the function $N \,\mapsto\, 10\,log_{10}\,(||\underline{\mathbf{w}}||^2_{\mathbb{B}})$. Since updating parameters only concern damping, convergence with respect to N weakly depends on $\mathbf{r}$ and can be neglected in the convergence analysis. It has been verified that convergence is reached for $N \,=\, 120$ [5]. We are interested in comparing the two updated mean computational models. The optimization of cost function $\underline{j}_{mod}(\mathbf{r})$ yields optimal updating parameters $\mathbf{r}^{mod} \,=$

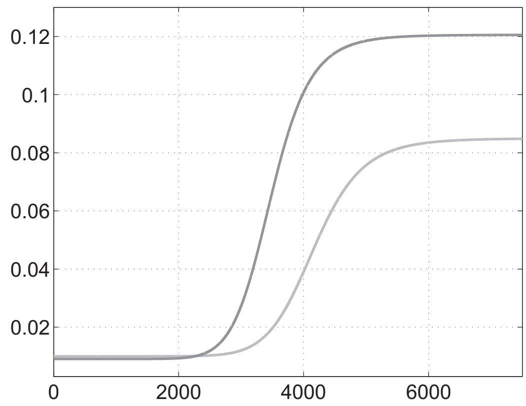


Figure 4: (Graph of function $\nu \mapsto f(\nu, \mathbf{r}^{mod})$ (thick dark gray line) and $\nu \mapsto f(\nu, \mathbf{r}^{pha})$ (thick light gray line) corresponding to the updated mean damping model. Horizontal axis is frequency ν in Hz.

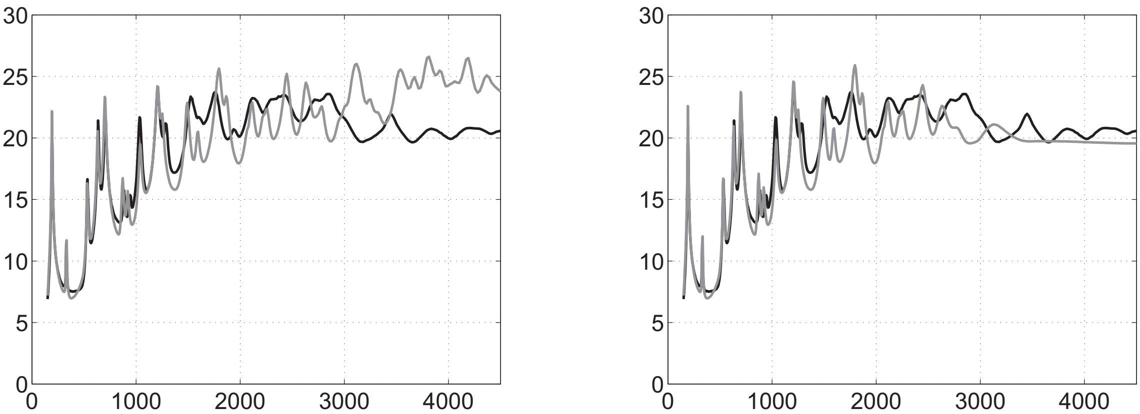
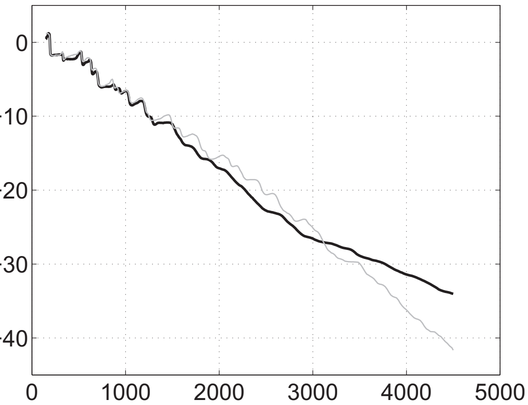


Figure 5: (a) Graph of function $\nu \mapsto \underline{dB}_w(\nu, \mathbf{r}^{ini})$ (thick dark gray line) and $\nu \mapsto \underline{dB}_w^{exp}(\nu)$ (thick black line), (b) Graph of function $\nu \mapsto \underline{dB}_w(\nu, \mathbf{r}^{mod})$ (thick dark gray line) and $\nu \mapsto \underline{dB}_w^{exp}(\nu)$ (thick black line). Horizontal axis is frequency ν in Hz.

$\{0.0091, 0.1206, 10.6856, 46.2361\}$ whereas the optimization of cost function $\underline{j}_{pha}(\mathbf{r})$ yields optimal updating parameters $\mathbf{r}^{pha} = \{0.0099, 0.08495, 10.5867, 46.6657\}$.

Figure 4 shows the graph $\nu \mapsto f(\nu, \mathbf{r}^{mod})$ and $\nu \mapsto f(\nu, \mathbf{r}^{pha})$ related to the two updated damping models. It is seen that the two cost functions yield similar function f in $[100\,,\,2000]\,Hz$ but are different for higher frequencies. In particular, it can be seen that the updated mean model solution with cost function $\underline{j}_{mod}(\mathbf{r})$ is more damped than the updated mean model solution with cost function $\underline{j}_{pha}(\mathbf{r})$. Let $\mathbf{r}^{ini} = \{0.01, 0.01, \alpha, \beta\}$ be the value of the updating parameter corresponding to the computational model obtained by updating the conservative part [5, 6]. Figure 5 compares (a) the graphs $\nu \mapsto \underline{dB}_w(\nu, \mathbf{r}^{ini})$ and $\nu \mapsto \underline{dB}_w^{exp}(\nu)$ with (b) the graphs $\nu \mapsto \underline{dB}_w(\nu, \mathbf{r}^{mod})$ and $\nu \mapsto \underline{dB}_w^{exp}(\nu)$ related to the optimization of cost function $\underline{j}_{mod}(\mathbf{r})$. Figure 6 shows (a) the graph $\nu \mapsto \underline{\phi}_w(\nu, \mathbf{r}^{ini})$ and $\nu \mapsto \underline{\phi}_w^{exp}(\nu)$ with (b) the graph $\nu \mapsto \underline{\phi}_w(\nu, \mathbf{r}^{pha})$ and $\nu \mapsto \underline{\phi}_w^{exp}(\nu)$ related to the optimization of cost function $\underline{j}_{pha}(\mathbf{r})$.

In figure 5 and 6, it can be seen that both cost functions used yield a computational model which improves the updating in the medium-frequency range. In Figure 5, it can be seen that the optimization of cost function $\underline{j}_{mod}(\mathbf{r})$ yields an updated computational model which



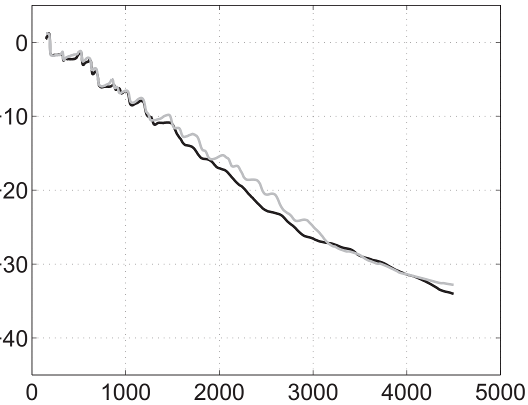


Figure 6: (a) Graph of function $\nu \mapsto \underline{\phi}_w(\nu, \mathbf{r}^{ini})$ (thick light gray line) and $\nu \mapsto \underline{\phi}_w^{exp}(\nu)$ (thick black line), (b) Graph of function $\nu \mapsto \underline{\phi}_w(\nu, \mathbf{r}^{pha})$ (thick light gray line) and $\nu \mapsto \underline{\phi}_w^{exp}(\nu)$ (thick black line). Horizontal axis is frequency ν in Hz.

matches relatively well with the experiment. Nevertheless, it should be noted that the dynamical behaviour of the sandwich panel is not well represented in $[3000\,,\,4500]\,Hz$ because the computational model does not yield any resonance peakings in this frequency range. In Figure 6, it can be seen that the optimization of cost function $\underline{j}_{pha}(\mathbf{r})$ yields an updated mean computational model for which there is a good agreement with respect to the experiments in both low and medium-frequency range. We are then interested by comparing the frequency response functions obtained by the two updating methods with respect to the experimental frequency response functions. Let $\mathbf{m}_{\mathbb{w}}^{exp}(\omega) = (m_{\mathbb{w},1}^{exp}(\omega)), \ldots, m_{\mathbb{w},n_{obs}}^{exp}(\omega))$ be the $\mathbb{C}^{n_{obs}}$-vector such that $m_{\mathbb{w},j}^{exp}(\omega) = \frac{1}{n_{exp}} \sum_{k=1}^{n_{exp}} \mathbb{W}_j^{exp}(\omega, \theta_k)$ where $\mathbb{W}_j^{exp}(\omega, \theta_k) = 20\,log_{10}(|W_j^{exp}(\omega, \theta_k)|)$. Let $\underline{\mathbb{w}}_j(\mathbf{r}, \omega) = 20\,log_{10}(|\underline{w}_j(\mathbf{r}, \omega)|)$. Figure 7 shows the graph of functions $\nu \mapsto \underline{\mathbb{w}}_1(\nu, \mathbf{r}^{mod})$, $\nu \mapsto \underline{\mathbb{w}}_1(\nu, \mathbf{r}^{pha})$ and $\nu \mapsto m_{\mathbb{w},1}^{exp}(\nu)$ in which subscript 1 corresponds to observation point number 1 located at $\mathbf{x}_1 = (0.337, 0.103, 0)$. In Figure 7, it can be seen that both updating methods are efficient in the low-frequency band $[100\,,\,1200]Hz$ and give satisfactory results in $[1200\,,\,3000]Hz$. In $[3000\,,\,4500]\,Hz$, it is clearly seen that the updated mean model obtained from cost function $\underline{j}_{pha}(\mathbf{r})$ yields a better agreement with respect to the experiment than the updated mean model obtained from cost function $\underline{j}_{mod}(\mathbf{r})$. Based on this observation, it can be deduced that the information contained in cost function $\underline{j}_{pha}(\mathbf{r})$ is more rich in the context of the updating in the medium-frequency range than the information contained in cost function $\underline{j}_{mod}(\mathbf{r})$.

4 ROBUST UPDATING METHOD OF THE DYNAMICAL SYSTEM WITH RESPECT TO MODEL UNCERTAINTIES

4.1 Description of the random matrix model

As explained in the Introduction, the objective of this paper is to include the effects of data uncertainties and model uncertainties in the formulation of the updating problem. In this Section, the nonparametric probabilistic approach of uncertainties [3, 4] is briefly summarized. It is assumed that the mean computational model of the sandwich panel contains model uncertainties and data uncertainties. The methodology of the nonparametric probabilistic approach consists in replacing matrices $[\underline{\mathcal{M}}]$, $[\underline{\mathcal{D}}(\mathbf{r})]$, $[\underline{\mathcal{K}}]$ by random matrices $[\boldsymbol{\mathcal{M}}]$, $[\boldsymbol{\mathcal{D}}(\mathbf{r})]$ and $[\boldsymbol{\mathcal{K}}]$ such

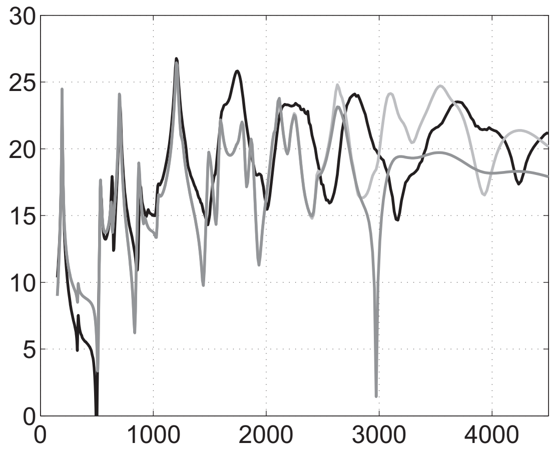


Figure 7: Graph of $\nu \mapsto \underline{\mathbb{w}}_1(\nu, \mathbf{r}^{mod})$ (thick light gray line), $\nu \mapsto \underline{\mathbb{w}}_1(\nu, \mathbf{r}^{pha})$ (thick dark gray line) and $\nu \mapsto m^{exp}_{\mathbb{w},1}(\nu)$ (thick black line). Horizontal axis is frequency ν in Hz.

that $\mathcal{E}\{[\boldsymbol{\mathcal{M}}]\} = [\underline{\mathcal{M}}]$, $\mathcal{E}\{[\boldsymbol{\mathcal{D}}(\mathbf{r})]\} = [\underline{\mathcal{D}}(\mathbf{r})]$ and $\mathcal{E}\{[\boldsymbol{\mathcal{K}}]\} = [\underline{\mathcal{K}}]$ in which $\mathcal{E}$ is the mathematical expectation and for which the probability distribution is known. The random matrices $[\boldsymbol{\mathcal{M}}]$, $[\boldsymbol{\mathcal{D}}(\mathbf{r})]$ and $[\boldsymbol{\mathcal{K}}]$ are written as $[\boldsymbol{\mathcal{M}}] = [\underline{L}_M]^T\,[\mathbf{G}_M]\,[\underline{L}_M]$, $[\boldsymbol{\mathcal{D}}(\mathbf{r})] = [\underline{L}_D(\mathbf{r})]^T\,[\mathbf{G}_D]\,[\underline{L}_D(\mathbf{r})]$ and $[\boldsymbol{\mathcal{K}}] = [\underline{L}_K]^T\,[\mathbf{G}_K]\,[\underline{L}_K]$ in which $[\underline{L}_M]$, $[\underline{L}_D(\mathbf{r})]$ and $[\underline{L}_K]$ are $N \times N$ real diagonal matrices such that $[\underline{\mathcal{M}}] = [\underline{L}_M]^T\,[\underline{L}_M]$, $[\underline{\mathcal{D}}(\mathbf{r})] = [\underline{L}_D(\mathbf{r})]^T\,[\underline{L}_D(\mathbf{r})]$ and $[\underline{\mathcal{K}}] = [\underline{L}_K]^T\,[\underline{L}_K]$ and where $[\mathbf{G}_M]$, $[\mathbf{G}_D]$ and $[\mathbf{G}_K]$ are full random matrices with value in the set of all the positive-definite symmetric $N \times N$ matrices. The probability model of random matrices $[\mathbf{G}_M]$, $[\mathbf{G}_D]$ and $[\mathbf{G}_K]$ is constructed by using the maximum entropy principle with the available information. All the details concerning the construction of this probability model can be found in [3, 4]. The dispersion of each random matrix $[\mathbf{G}_M]$, $[\mathbf{G}_D]$ and $[\mathbf{G}_K]$ is controlled by one real positive parameter δ_M, δ_D and δ_K called the dispersion parameter. In addition, there exists an algebraic representation of this random matrix useful to the Monte Carlo numerical simulation. Let $\boldsymbol{\delta} = (\delta_M\,,\,\delta_D\,,\,\delta_K)$ be the $\mathbb{R}^3$-vector of the dispersion parameters defined on the admissible set $\mathcal{D} = \{[0\,,\,\sqrt{\frac{N+1}{N+5}}]\}^3$. In coherence with the notation of Section 3.2, let $\mathbf{W}(\mathbf{r},\boldsymbol{\delta},\omega) = (W_1(\mathbf{r},\boldsymbol{\delta},\omega),\ldots,W_{n_{obs}}(\mathbf{r},\boldsymbol{\delta},\omega))$ be the $\mathbb{C}^{n_{obs}}$-valued random vector of the n_{obs} observations. The equations of the stochastic reduced matrix system constructed with the nonparametric approach of uncertainties are given by $\mathbf{W}(\mathbf{r},\boldsymbol{\delta},\omega) = [T(\omega)]\,[\underline{\Phi}]\,\mathbf{Q}(\mathbf{r},\boldsymbol{\delta},\omega)$, where $\mathbf{Q}(\mathbf{r},\boldsymbol{\delta},\omega)$ is the $\mathbb{C}^N$-valued random vector of the generalized coordinates, which is solution of the random matrix equation

$$\Big(-\omega^2\,[\boldsymbol{\mathcal{M}}] + i\,\omega\,[\boldsymbol{\mathcal{D}}(\mathbf{r})] + [\boldsymbol{\mathcal{K}}]\Big)\,\mathbf{Q}(\mathbf{r},\boldsymbol{\delta},\omega) = \underline{\boldsymbol{\mathcal{F}}}(\omega) \quad . \tag{10}$$

4.2 Formulation for the robust updating problem.

In this Section, the robust updating problem is formulated with respect to model uncertainties and data uncertainties using the nonparametric probabilistic approach described above. This robust updating problem consists in minimizing a cost function with respect to the updating parameters of the stochastic system, that is to say (1) the updating parameter $\mathbf{r}$ of the mean computational model and (2) the updating parameter $\boldsymbol{\delta}$ which allows the amount of uncertainty in the computational model to be controlled. Contrary to the previous Section, the cost function is constructed with an uncertain computational model and describes the performance of the stochastic dynamical system. Concerning the formulation of the cost function, the same observations related to the experiments are used. Nevertheless, the norm used in the cost function is different. The norm used in the previous Section is constructed from a spatial average of the observation. In the context of robust updating (using a computational model with uncertainties), the use of a similar norm is too rough for representing with sufficient accuracy the random responses. Two formulations are proposed in order to update the stochastic computational model. The first formulation consists in minimizing the sum of (1) the bias between the mean value of the stochastic computational model and the mean value of the experiment and (2) the variance of the stochastic computational model. The cost function $j_{BV}(\mathbf{r},\boldsymbol{\delta})$ is then defined as

$$j_{BV}(\mathbf{r},\boldsymbol{\delta}) = ||\mathbf{m}_{\mathbb{W}}(\mathbf{r},\boldsymbol{\delta},\cdot) - \mathbf{m}_{\mathbb{W}}^{exp}||_{\mathbb{B}}^2 + |||\mathbb{W}(\mathbf{r},\boldsymbol{\delta},\cdot) - \mathbf{m}_{\mathbb{W}}(\mathbf{r},\boldsymbol{\delta},\cdot)|||^2 \quad , \tag{11}$$

in which $\mathbf{m}_{\mathbb{W}}(\mathbf{r},\boldsymbol{\delta},\omega) = \mathcal{E}\{\mathbb{W}(\mathbf{r},\boldsymbol{\delta},\omega)\} \in \mathbb{C}^{n_{obs}}$, where $\mathbb{W}(\mathbf{r},\boldsymbol{\delta},\omega) = (\mathbb{W}_1(\mathbf{r},\boldsymbol{\delta},\omega),\ldots, \mathbb{W}_{n_{obs}}(\mathbf{r},\boldsymbol{\delta},\omega))$ with $\mathbb{W}_j(\mathbf{r},\boldsymbol{\delta},\omega) = 20\,log_{10}(|W_j(\mathbf{r},\boldsymbol{\delta},\omega)|)$ and where $||\mathbf{g}||_{\mathbb{B}}^2 = \int_{\mathbb{B}} ||\mathbf{g}(\omega)||^2\, d\omega$ with $||\mathbf{g}(\omega)||$ the Hermitian norm of $\mathbf{g}(\omega)$. In Eq. (11), the norm $|||\mathbb{X}|||$ is defined by $|||\mathbb{X}|||^2 = E\{||\mathbb{X}||_{\mathbb{B}}^2\}$, where $\{\mathbb{X}(\omega),\omega \in \mathbb{B}\}$ is a stochastic process indexed by $\mathbb{B}$. The second formulation consists in minimizing the contributions of the experiments which are outside the confidence region constructed with the stochastic computational model. Let $\mathbb{W}_j^+(\mathbf{r},\boldsymbol{\delta},\omega)$ (resp. $\mathbb{W}_j^-(\mathbf{r},\boldsymbol{\delta},\omega)$) and $\mathbb{W}_j^{exp,+}(\omega)$ (resp. $\mathbb{W}_j^{exp,-}(\omega)$) be the upper (resp. lower) envelope of the confidence region of observation $\mathbb{W}_j(\mathbf{r},\boldsymbol{\delta},\omega)$ obtained with a probability level $\alpha = 0.95$ [7] and the upper (resp. lower) envelope of experiments $\mathbb{W}_j^{exp}(\omega)$. The cost function $j_{CR}(\mathbf{r},\boldsymbol{\delta})$ is then defined as

$$j_{CR}(\mathbf{r},\boldsymbol{\delta}) = ||\boldsymbol{\Delta}^+(\mathbf{r},\boldsymbol{\delta},\cdot)||_{\mathbb{B}}^2 + ||\boldsymbol{\Delta}^-(\mathbf{r},\boldsymbol{\delta},\cdot)||_{\mathbb{B}}^2 \quad , \tag{12}$$

in which $\boldsymbol{\Delta}^+(\mathbf{r},\boldsymbol{\delta},\omega)$ and $\boldsymbol{\Delta}^-(\mathbf{r},\boldsymbol{\delta},\omega)$ are the $\mathbb{C}^{n_{obs}}$-vector whose component j is defined as

$$\Delta_j^+(\mathbf{r},\boldsymbol{\delta},\omega) = \{\mathbb{W}_j^+(\mathbf{r},\boldsymbol{\delta},\omega) - \mathbb{W}_j^{exp,+}(\omega)\}\,\{1 - H(\mathbb{W}_j^+(\mathbf{r},\boldsymbol{\delta},\omega) - \mathbb{W}_j^{exp,+}(\omega))\} \quad , \tag{13}$$

$$\Delta_j^-(\mathbf{r},\boldsymbol{\delta},\omega) = \{\mathbb{W}_j^-(\mathbf{r},\boldsymbol{\delta},\omega) - \mathbb{W}_j^{exp,-}(\omega)\}\{H(\mathbb{W}_j^-(\mathbf{r},\boldsymbol{\delta},\omega) - \mathbb{W}_j^{exp,+}(\omega))\} \quad . \tag{14}$$

In Eq. (13) and (14), $x \mapsto H(x)$ is the Heaviside function. The robust updating problem consists in solving the optimization problem

$$\text{find } (\mathbf{r}^{BV},\boldsymbol{\delta}^{BV}) \in \{\mathcal{R}\times\mathcal{D}\} \text{ such that } j_{BV}(\mathbf{r}^{BV},\boldsymbol{\delta}^{BV}) \leq j_{BV}(\mathbf{r},\boldsymbol{\delta}) \quad , \quad \forall(\mathbf{r},\boldsymbol{\delta}) \in \{\mathcal{R}\times\mathcal{D}\}.$$

for the first cost function or the following for the second cost function

$$\text{find } (\mathbf{r}^{CR},\boldsymbol{\delta}^{CR}) \in \{\mathcal{R}\times\mathcal{D}\} \text{ such that } j_{CR}(\mathbf{r}^{CR},\boldsymbol{\delta}^{CR}) \leq j_{CR}(\mathbf{r},\boldsymbol{\delta}) \quad , \quad \forall(\mathbf{r},\boldsymbol{\delta}) \in \{\mathcal{R}\times\mathcal{D}\}.$$

Finally, the robust updating problem is solved by using the sequential quadratic optimization algorithm [8, 9] coupled with the Monte Carlo numerical simulation. Since the cost function is a strong nonconvex function of updating parameter $(\mathbf{r},\boldsymbol{\delta})$, the robust updating problem is solved around the solution of the deterministic updating problem describes in Section 3. Let $(\mathbf{r}^i\,,\,\boldsymbol{\delta}^i)$ denote the iteration number i of the updating parameter. First, the initial value of the updating parameter is chosen as $(\mathbf{r}^0\,,\,\boldsymbol{\delta}^0) = (\mathbf{r}^{pha},\mathbf{0})$. The optimization process is then splitted in two optimization steps which consist (1) in solving the optimization problem $\boldsymbol{\delta}^i = min_{\boldsymbol{\delta}\in\mathcal{D}}\, j(\mathbf{r}^{i-1},\boldsymbol{\delta})$ and (2) in solving the optimization problem $\mathbf{r}^i = min_{\mathbf{r}\in\mathcal{R}}\, j(\mathbf{r},\boldsymbol{\delta}^i)$, in which $j(\mathbf{r},\boldsymbol{\delta})$ denotes $j_{BV}(\mathbf{r},\boldsymbol{\delta})$ or $j_{CR}(\mathbf{r},\boldsymbol{\delta})$. The optimization process is then repeated until $||\boldsymbol{\delta}^i - \boldsymbol{\delta}^{i-1}|| < \epsilon_{\boldsymbol{\delta}}$ or $||\mathbf{r}^i - \mathbf{r}^{i-1}|| < \epsilon_{\mathbf{r}}$. It should be noted that the random germs of the random matrices do not depend on the updating parameter $\mathbf{r}$. Consequently, the gradient and the Hessian of the cost function with respect to parameter $\mathbf{r}$ can be algebraically constructed, that improves the precision of the optimization algorithm [10].

4.3 Numerical results

First, a convergence analysis is performed with respect to the number N of eigenmodes and the number n_s of realizations for the Monte Carlo numerical simulation. A convergence analysis shows that convergence is reasonably reached for $n_s = 750$ and $N = 300$ [5]. We are interested in comparing the two robust updating methods. The optimization of cost function $j_{BV}(\mathbf{r}, \boldsymbol{\delta})$ with respect to vector $\boldsymbol{\delta}$ yields $\boldsymbol{\delta} = (0.3, 0.19, 0.09)$ whereas the optimization of cost function $j_{CR}(\mathbf{r}, \boldsymbol{\delta})$ with respect to vector $\boldsymbol{\delta}$ yields $\boldsymbol{\delta} = (0.11, 0.09, 0.18)$. Figures 8 compare the experiments with the confidence region of the random response $\nu \mapsto \mathbb{W}_1(\mathbf{r}^{pha}, \boldsymbol{\delta}^{BV}, \nu)$ and the confidence region of the random response $\nu \mapsto \mathbb{W}_1(\mathbf{r}^{pha}, \boldsymbol{\delta}^{CR}, \nu)$ obtained with a probability level $\alpha = 0.95$.

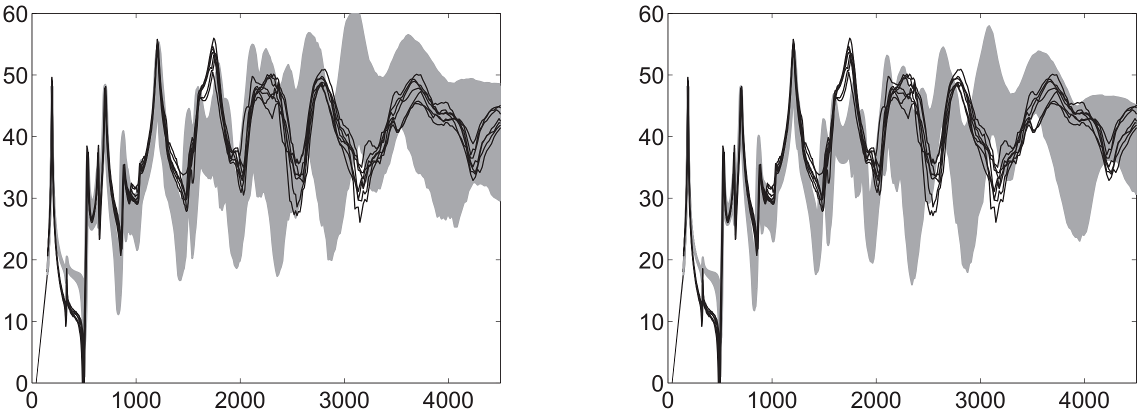


Figure 8: (a) Graph of the experiments $\nu \mapsto \mathbb{W}_1^{exp}(\nu, \theta_k)$ (thin black lines) and of the confidence region of the random response $\nu \mapsto \mathbb{W}_1(\mathbf{r}^{pha}, \boldsymbol{\delta}^{BV}, \nu)$ (light grey region). (b) Graph of the confidence region of the random response $\nu \mapsto \mathbb{W}_1^{exp}(\nu, \theta_k)$ (thin black lines) and of the confidence region of the random response $\nu \mapsto \mathbb{W}_1(\mathbf{r}^{pha}, \boldsymbol{\delta}^{CR}, \nu)$ (light grey region). Horizontal axis is frequency ν in Hz.

In Figure 8, it can be seen that both robust updating methods yield an updated computational model for which there is a good agreement with the experiments and which stays robust with respect to model and data uncertainties in the low-frequency band $\mathbb{B} = [100\,,\,1200]\,Hz$. For higher frequencies, both robust updating methods yield a confidence region which contains relatively well the experiment. In that sense, it can be deduced that both robust methods are valuable. Nevertheless, it can be seen in the medium-frequency range that the updated computational model obtained with cost function $j_{CR}(\mathbf{r}, \boldsymbol{\delta})$ represents better the experiment than the one obtained with cost function $j_{BV}(\mathbf{r}, \boldsymbol{\delta})$. From these observations, it is then deduced that although both robust updating methods yield different updating parameters $\boldsymbol{\delta}$, both robust updating methods are efficient in the low-frequency range. Nevertheless, in the medium-frequency range, the use of a cost function defined from the confidence region yield an updated computational model which not only agrees with the experiment but also is more robust with respect to model and data uncertainties.

5 CONCLUSIONS

Two updating methodologies have been proposed for the robust updating problem in the context of structural dynamics in the low- and medium-frequency range. These methodologies are validated from experimental results issued from a set of 8 manufactured composite sandwich panels. Concerning the deterministic updating (no uncertainties in the dynamical system), it is shown that the use of a cost function defined from the phase of experimental frequency response function is particularly adapted for updating the computational model in the low- and medium-frequency range. Concerning the robust updating (using a computational model with model and data uncertainties), it is shown that the use of a cost function defined from the confidence region of the experimental frequency response functions yields an updated computational model particularly robust with respect to model and data uncertainties in the low- and medium-frequency range.

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