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NONLINEAR STOCHASTIC DYNAMICS OF TWO COUPLED UNCERTAIN DYNAMICAL SYSTEMS

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Abstract. *This paper deals with a reduction method of models composed of a linear behaviour subsystem which has a high number of eigenmodes in the range of analysis and of a nonlinear behaviour subsystem. Each subsystem has model uncertainties and data uncertainties. Those uncertainties are taken into account using the usual parametric probabilistic approach and the non parametric probabilistic approach. We present a numerical example constituted of a simple system owning all the properties of the systems we are interested in and which validates the proposed methodology.*

1 INTRODUCTION

We are interested in predicting the nonlinear dynamical response of two subsystems coupled in a fixed frequency range. The first subsystem is constituted of a structure with a linear behavior coupled with localized nonlinearities (so, the behavior of this subsystem is globally nonlinear). The second subsystem is constituted of a structure with a linear behavior having several quasi-symmetries; its numerical model has a high number of degrees of freedom. A finite element model of each subsystem is available. The FE model of the second subsystem (about 100 000 DOF for the considered applications) has a high number of eigenmodes in the range of analysis [0 , 1000] Hz (about 10 000 modes due to the quasi-symmetries). As a consequence, those eigenmodes cannot be used in order to reduce the second subsystem and the usual methods, that are based on mode synthesis methods (see [4] [5] [6] [7] [9]) or more generally that solve an eigenvalue problem cannot be used (see [1] [2] [8] [10]). In addition, each subsystem has model uncertainties and data uncertainties. This kind of structure corresponds for example to fuel assemblies of Pressurized Water Reactors. We first propose here a new time domain dynamical condensation method of the second subsystem on its coupling interface DOFs with the nonlinear subsystem, allowing such numerical computation to be performed in presence of a probabilistic model allowing uncertainties to be taken into account. Concerning the implementation of the probabilistic model of uncertainties in the mean numerical model of each subsystem, the adopted strategy is based on : (1) the nonparametric approach [3] for the positive linear operators (mass, damping, stiffness) of the linear part of the nonlinear subsystem to take into account the model uncertainties and data uncertainties, (2) the usual parametric probabilistic approach for the uncertain parameters of the localized nonlinearities of the nonlinear dynamical subsystem. After the presentation of the theory and the numerical aspect, we present a numerical example constituted of a simple system owning all the properties of the systems we are interested in and which validates the proposed methodology.

2 Equation of of the mean model for the coupled system

We consider a three dimentional damped structure. Let Ω be the bounded open domain of $\mathbb{R}^3$ composed of two subdomains, Ω_A which corresponds to the nonlinear behaviour first subsystem and Ω_B which corresponds to the linear behaviour second subsystem. The two subsytems are dynamically coupled on the coupling interface Γ_C . The boundary Γ_B of the domain Ω_B is

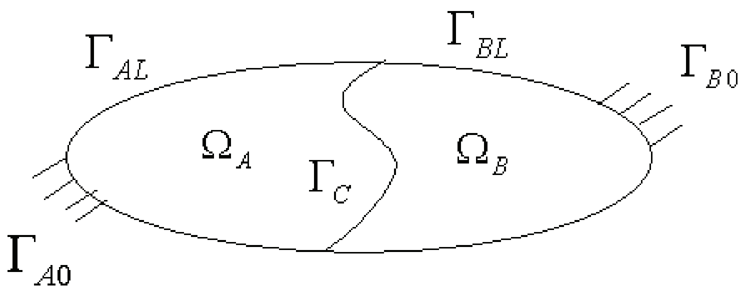

Figure 1: Decomposition of the domain Ω.

made of the part Γ_C, the fixed part Γ_{B0} and of Γ_{BL}. The finite element model of the coupled system defined on frequency domain in the range analysis $B = [-\omega_{max}, \omega_{max}]$ is written with respect to the internal n_p DOFs $\underline{\mathbf{u}}_p^B(\omega)$ and the n_C coupling DOFs $\underline{\mathbf{u}}_c^B(\omega)$

$$\begin{bmatrix} \underline{A}_{pp}^B(\omega) & \underline{A}_{pc}^B(\omega) \\ \underline{A}_{cp}^B(\omega) & \underline{A}_{cc}^B(\omega) \end{bmatrix} \begin{bmatrix} \underline{\mathbf{u}}_p^B(\omega) \\ \underline{\mathbf{u}}_c^B(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{F}}_p^B(\omega) \\ \underline{\mathbf{F}}_c^B(\omega) + \underline{\mathbf{F}}_{coupl}^B(\omega) \end{bmatrix} \tag{1}$$

in which $\left[\underline{A}^B(\omega)\right]$ is the dynamic stiffness matrix such that:

$$\left[\underline{A}^B(\omega)\right] = -\omega^2 \left[\underline{M}^B\right] + i\omega \left[\underline{D}^B\right] + \left[\underline{K}^B\right] \tag{2}$$

where $\left[\underline{M}^B\right]$, $\left[\underline{D}^B\right]$ and $\left[\underline{K}^B\right]$ are respectively the positive definite mass, damping and stiffness matrices. $\underline{\mathbf{F}}_c^B(\omega)$ and $\underline{\mathbf{F}}_{coupl}^B(\omega)$ represent respectively the external load applied on the coupling interface Γ_C and the coupling force vector. The dynamical condensation with respect to the coupling DOFs leads to the equation

$$\begin{aligned} \underline{\mathbf{F}}_{coupl}^B(\omega) = \left[\underline{A}_{cc}^B(\omega) - \underline{A}_{cp}^B(\omega)(\underline{A}_{pp}^B(\omega))^{-1}\underline{A}_{pc}^B(\omega)\right]\underline{\mathbf{u}}_c^B(\omega) \\ +\underline{A}_{cp}^B(\omega)(\underline{A}_{pp}^B(\omega))^{-1}\underline{\mathbf{F}}_p^B(\omega) - \underline{\mathbf{F}}_c^B(\omega) \end{aligned} \tag{3}$$

The inverse Fourier transform of the equation 3 and the finite element model of the nonlinear first subsystem model lead to the matricial following equation for the nonlinear dynamical coupled system

$$\begin{aligned} & \begin{bmatrix} \underline{M}_{pp}^A & \underline{M}_{pc}^A \\ \underline{M}_{cp}^B & \underline{M}_{cc}^A + \underline{M}_{cc}^B \end{bmatrix} \begin{bmatrix} \ddot{\underline{\mathbf{u}}}_p^B(t) \\ \ddot{\underline{\mathbf{u}}}_c^B(t) \end{bmatrix} + \begin{bmatrix} \underline{D}_{pp}^A & \underline{D}_{pc}^A \\ \underline{D}_{cp}^B & \underline{D}_{cc}^A + \underline{D}_{cc}^B \end{bmatrix} \begin{bmatrix} \dot{\underline{\mathbf{u}}}_p^B(t) \\ \dot{\underline{\mathbf{u}}}_c^B(t) \end{bmatrix} \\ + & \begin{bmatrix} \underline{K}_{pp}^A & \underline{K}_{pc}^A \\ \underline{K}_{cp}^B & \underline{K}_{cc}^A + \underline{K}_{cc}^B \end{bmatrix} \begin{bmatrix} \underline{\mathbf{u}}_p^B(t) \\ \underline{\mathbf{u}}_c^B(t) \end{bmatrix} \\ = & \begin{bmatrix} \underline{\mathbf{F}}_p^{NL}(t, \underline{\mathbf{u}}^A, \dot{\underline{\mathbf{u}}}^A, \underline{\mathbf{w}}) + \underline{\mathbf{F}}_p^A(t) \\ \underline{\mathbf{F}}_c^{NL}(t, \underline{\mathbf{u}}^A, \dot{\underline{\mathbf{u}}}^A, \underline{\mathbf{w}}) + \underline{\mathbf{F}}_c^A(t) + \underline{\mathbf{F}}_c^B(t) + \tilde{\underline{\mathbf{F}}}(t) \end{bmatrix} \\ + & \begin{bmatrix} \mathbf{0} \\ \int_0^t \underline{B}_U(\tau)\underline{\mathbf{u}}_c^A(t-\tau)d\tau + \int_0^t \underline{B}_{\dot{U}}(\tau)\dot{\underline{\mathbf{u}}}_c^A(t-\tau)d\tau + \int_0^t \underline{B}_{\ddot{U}}(\tau)\ddot{\underline{\mathbf{u}}}_c^A(t-\tau)d\tau \end{bmatrix} \end{aligned} \tag{4}$$

with the initial conditions,

$$\underline{\mathbf{u}}(0) = \dot{\underline{\mathbf{u}}}(0) = 0 \tag{5}$$

where $\underline{\mathbf{F}}^{NL}(t, \underline{\mathbf{u}}^A, \dot{\underline{\mathbf{u}}}^A, \underline{\mathbf{w}})$ is the nonlinear localized forces vector in which $\underline{\mathbf{w}}$ is an n_c dimension vector of the uncertain parameters describing thoses nonlinearities.The functions $\underline{B}_U$, $\underline{B}_{\dot{U}}$ and $\underline{B}_{\ddot{U}}$ are with values in the set of the (n_c, n_c) dimension symmetric complex matrices, where n_c is the number of coupling DOFs, and are such that

$$\widehat{\underline{B}}_U(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t}\underline{B}_U(t)dt = \underline{K}_{cp}^B(\underline{A}_{pp}^B(\omega))^{-1}\underline{K}_{pc}^B \tag{6}$$

$$\widehat{\underline{B}}_{\dot{U}}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t}\underline{B}_{\dot{U}}(t)dt = 2sym(\underline{D}_{cp}^B(\underline{A}_{pp}^B(\omega))^{-1}\underline{K}_{pc}^B) \tag{7}$$

$$\begin{aligned} \widehat{\underline{B}}_{\ddot{U}}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t}\underline{B}_{\ddot{U}}(t)dt &= 2sym(\underline{M}_{cp}^B(\underline{A}_{pp}^B(\omega))^{-1}\underline{K}_{pc}^B) + \omega\underline{D}_{cp}^B(\underline{A}_{pp}^B(\omega))^{-1}\underline{D}_{pc}^B \\ &+ 2\omega sym(\underline{M}_{cp}^B(\underline{A}_{pp}^B(\omega))^{-1}\underline{D}_{pc}^B) + \omega^2\underline{M}_{cp}^B(\underline{A}_{pp}^B(\omega))^{-1}\underline{M}_{pc}^B \end{aligned} \tag{8}$$

$\underline{\tilde{\mathbf{F}}}(t)$ is an n_c dimension vector reprensenting the condesation of the external load applied on the internal DOFs of the linear subsystem and is such that

$$\underline{\tilde{\mathbf{F}}}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} \underline{A}_{cp}^{B}(\omega)(\underline{A}_{pp}^{B}(\omega)\underline{\mathbf{F}}_{p}^{B}(\omega) d\omega \tag{9}$$

The equation 4 can be reduced using the Craig Bampton method for the linear part of the first subsystem.

3 Modelisation of the uncertainties

We suppose here that there aren't uncertainties for the linear second subsystem condensed on the coupling interface. To take into account model uncertainties and data uncertainties we use a mixt parametric nonparametric probabilistic approach. The parametric probabilistic approach is used to take into account the uncertainties on the parameters describing the models of the localized nonlinearities. It consists in modelising $\underline{\mathbf{w}}$ with a random variable $\mathbf{W}$ with values in $\mathbb{R}^{n_p}$. The nonparametric probabilistic approach is used to take into account model uncertainties and data uncertainties in the linear part of the nonlinear first subsystem. It consist in modelising with random matricies the mass, damping and stiffness matrices of the linear part of the first subytem in physical or generalised coordinates.The displacement vector $\underline{\mathbf{u}}^A(t)$ of the first subsystem is then modelised with a second order stochastic process $\mathbf{U}^A(t)$ whose vectorial values are solution of the following differential nonlinear stochastic equation :

$$\begin{aligned}
& \begin{bmatrix} M_{pp}^{A} & M_{pc}^{A} \\ M_{cp}^{B} & M_{cc}^{A} + \underline{M}_{cc}^{B} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}_{p}^{B}(t) \\ \ddot{\mathbf{U}}_{c}^{B}(t) \end{bmatrix} + \begin{bmatrix} D_{pp}^{A} & D_{pc}^{A} \\ D_{cp}^{B} & D_{cc}^{A} + \underline{D}_{cc}^{B} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{U}}_{p}^{B}(t) \\ \dot{\mathbf{U}}_{c}^{B}(t) \end{bmatrix} \\
+ \quad & \begin{bmatrix} K_{pp}^{A} & K_{pc}^{A} \\ K_{cp}^{B} & K_{cc}^{A} + \underline{K}_{cc}^{B} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{p}^{B}(t) \\ \mathbf{U}_{c}^{B}(t) \end{bmatrix} \\
= \quad & \begin{bmatrix} \mathbf{F}_{p}^{NL}(t, \mathbf{U}^{A}, \dot{\mathbf{U}}^{A}, \mathbf{W}) + \underline{\mathbf{F}}_{p}^{A}(t) \\ \mathbf{F}_{c}^{NL}(t, \mathbf{U}^{A}, \dot{\mathbf{U}}^{A}, \mathbf{W}) + \underline{\mathbf{F}}_{c}^{A}(t) + \underline{\mathbf{F}}_{c}^{B}(t) + \underline{\tilde{\mathbf{F}}}(t) \end{bmatrix} \\
+ \quad & \begin{bmatrix} \mathbf{0} \\ \int_0^t \underline{B}_{U}(\tau)\mathbf{U}_{c}^{A}(t-\tau)d\tau + \int_0^t \underline{B}_{\dot{U}}(\tau)\dot{\mathbf{U}}_{c}^{A}(t-\tau)d\tau + \int_0^t \underline{B}_{\ddot{U}}(\tau)\ddot{\mathbf{U}}_{c}^{A}(t-\tau)d\tau \end{bmatrix}
\end{aligned} \tag{10}$$

with the initial conditions,

$$\mathbf{U}(0) = \dot{\mathbf{U}}(0) = 0 \tag{11}$$

The probability density function of the variable $\mathbf{W}$ is constructed using the maximum entropy principle taking into account the available information and the probablisity density functions of each mass, damping and stiffness random matrices is constructed using [3]. The dispersion level of those matrices is controled by a dispersion parameter δ_M, δ_D and δ_K respectively for the masse, the dissipation and the stiffness.

4 Validation application

The mean model is constituted of an Euler beam fixed at its two bounds. This beam has a constant circular section with radius=0.5 m, thickness=0.2 m, lengh=20 m, mass density=500 kg/m^3, Young's modulus=450 N/mm^2 and damping rate=0.02. This system is divided into deux subsystems. The nonlinear first subsystem is the beam A (see figure 2) of 12 m lengh, free on its left bound (coupling interface) and fixed on its right bound. The linear second one is the beam B, it is condensed on the coupling interface which is free, its right bound is fixed. The nonlinear beam A has an elastic stop localized at 8 m from the fixed bound, with gap=10^{-5} m and choc stiffness=10^8 m. We are interested in calculating the time domaine response of the nonlinear coupled system excited by two external loads $\mathbf{F}^A(t)$ et $\mathbf{F}^B(t)$ localised at 4 m from each fixed bound and whose Fourier Transform has a constant modulus=1 N/Hz on its support range [-150, 150] Hz.

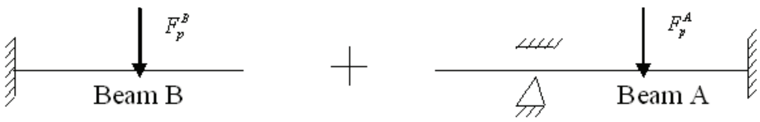

Figure 2: Description of the two subsystems Ω.

The figure 3 validates the proposed appproach concerning the condensation method. It compares for the transversal displacement at the excitation point of the first subsystem (Beam A), in the frequency domain (modulus of the Fourier Transform of the time domain response of the nonlinear coupled system), the reference response obtained by direct resolution without condensation of the second subsystem (Beam B) on the coupling interface, with the response obtained with the condensation method.

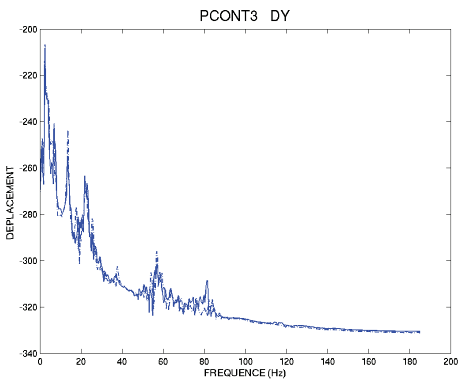

Figure 3: Comparison between the reference response at the excitation point of the beam A (dashed line) and the response obtained by condensation(solid line).

The figures 4 and 5 concern the analysis of the nonlinear coupled system with the mixt parametric/nonparametric probabilistic modelisation of the uncertainties. The random variable **W** is scalar and represents the gap. Its dispersion is $\delta_W = 0.1$. The dispersion parameters of the mass, dissipation and stiffness random matrices of the beam A are $\delta_M = 0.1$, $\delta_D = 0.1$ and $\delta_K = 0.1$.The figure 4 shows the graph of the convergence of the mean of the stochastic response in function of the number of samples of the Monte Carlo numeric simulation. The figure 5 shows the transversal displacement at the excitation point of the first subsystem (Beam A) in the frequency domain. It shows the response of the nonlinear coupled mean model, the stochastic mean of the stochastic nonlinear coupled system and the region of confidence associated with a probability level=0.95.

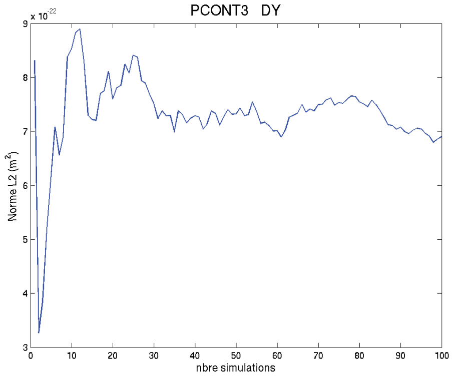

Figure 4: Convergence of the mean of response in function of of the number of sample.

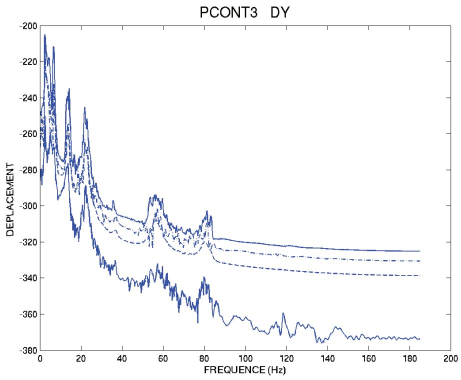

Figure 5: Region of confidence of the response at the excitation point (solid line); mean of the stochastic response (dashed line) and response of the mean model (mixt line).

5 CONCLUSIONS

We have proposed a reduction method for models of systems constituted of a linear subsystem having a huge number of eigenmodes in the range analysis with a nonlinear behaviour subsytem; indeed, usual methods don't match for this situation. We have presented a validation on a simple example. The objective is to use this model to identify with inverse methods and experimental data, external loads aplied on such systems (for example, loads resulting from a turbulent flow). The developement has been carried out in order to implement a mixt parametric/nonparametric model of the uncertainties so that we can carry out a robust identification of the external loads. We have presented on the example the first step corresponding to the stochastic response of the coupled system with uncertainties.

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