

# FullSWOF a software for overland flow simulation

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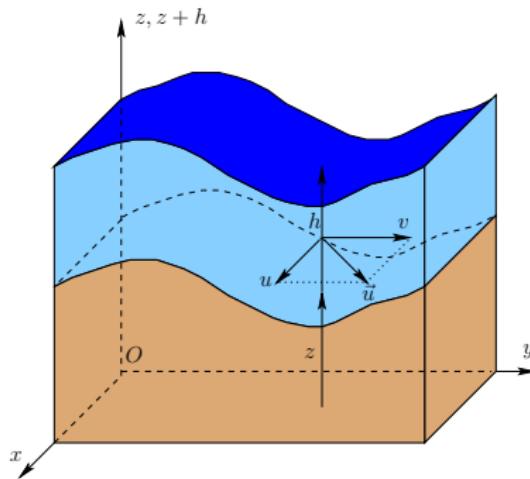
## Problem context

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# Shallow Water (Saint-Venant) system

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Data : topography  $z$ , rain  $R$ , infiltration  $I$

Unknowns : velocities  $u, v$ , water height  $h$

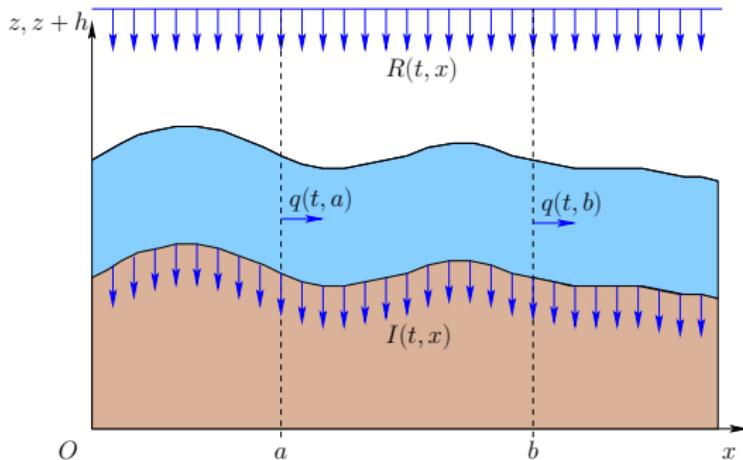
$$\begin{cases} \partial_t h + \partial_x (hu) + \partial_y (hv) = R - I \\ \partial_t (hu) + \partial_x (hu^2 + gh^2/2) + \partial_y (huv) = gh(-\partial_x z - S_{fx}) \\ \partial_t (hv) + \partial_x (huv) + \partial_y (hv^2 + gh^2/2) = gh(-\partial_y z - S_{fy}) \end{cases}$$

## Strategy

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- ▶ Properties of the 1D Shallow Water system
- ▶ Choice of the method depending on the properties
- ▶ Validation : analytical solutions and laboratory experiment
- ▶ Application : field data

# 1D Shallow Water system



Data : topography  $z$ , rain  $R$ , infiltration  $I$

Unknowns : velocities  $u$ , water height  $h$

A system of conservation laws

$$\begin{cases} \partial_t h + \partial_x(hu) = R - I \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) = gh(-\partial_x z - S_f) \end{cases} \quad (1)$$

# System properties (I) : Hyperbolicity

Setting  $q = hu$

$$\textcolor{red}{U} = \begin{pmatrix} \textcolor{red}{h} \\ q \end{pmatrix}, F(\textcolor{red}{U}) = \begin{pmatrix} \textcolor{red}{q} \\ \textcolor{red}{q}^2/\textcolor{red}{h} + gh^2/2 \end{pmatrix}, \textcolor{blue}{B} = \begin{pmatrix} \textcolor{blue}{R} - \textcolor{red}{I} \\ g\textcolor{red}{h}(-\partial_x z - S_f) \end{pmatrix},$$

compact form

$$\partial_t \textcolor{red}{U} + \partial_x F(\textcolor{red}{U}) = \partial_t \textcolor{red}{U} + F'(\textcolor{red}{U}) \partial_x \textcolor{red}{U} = \textcolor{blue}{B},$$

Hyperbolicity if  $\textcolor{red}{h} > 0$  :

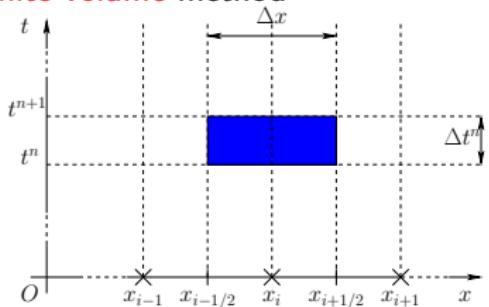
$$\lambda_-(\textcolor{red}{U}) = \textcolor{red}{u} - \sqrt{gh}, \quad \lambda_+(\textcolor{red}{U}) = \textcolor{red}{u} + \sqrt{gh}$$

Saint-Venant	gaz dynamic
Froude number $Fr = \frac{ \textcolor{red}{u} }{c}$ $c = \sqrt{gh}$ free surface waves celerity	Mach number $\frac{ \textcolor{red}{u} }{c}$ $c = \sqrt{p'(\rho)}$ sound speed <sup>1</sup>
<u>subcritical</u> $Fr < 1$	subsonic
<u>supercritical</u> $Fr > 1$	supersonic

1.  $p(\rho) = \rho RT$  perfect gaz

# Numerical method (I)

Finite volume method



we get

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} [F_{i+1/2}^n - F_{i-1/2}^n],$$

with the interface flux approximation

$$F_{i+1/2}^n = \mathcal{F}(U_i^n, U_{i+1}^n) \sim \frac{1}{\Delta t} \int_{t^n}^{t^{n+1}} F(U(t, x_{i+1/2})) dt.$$

We integrate  
 $\partial_t \mathbf{U} + \partial_x F(\mathbf{U}) = 0$   
on the volume  
 $[t^n, t^{n+1}] \times [x_{i-1/2}, x_{i+1/2}]$ ,  
and we set  
 $U_i^n = \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} U(t^n, x) dx$

## Numerical method (I)

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- ▶ For each choice of  $\mathcal{F}(U_G, U_D)$  we have a different **finite volume scheme** :  
HLL, kinetic, Rusanov, VF Roe-ncv, (suliciu, ...)
- ▶ second Order
  - ▶ in space : MUSCL, ENO, modified ENO, (UNO, WENO, DG, ...)
  - ▶ in time : Heun/RK2 TVD (RK3, RK4, ...)

## Numerical method (I)

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- ▶ second Order
  - ▶ in space : MUSCL, ENO, modified ENO, (UNO, WENO, DG, ...)
  - ▶ in time : Heun/RK2 TVD (RK3, RK4, ...)
- ▶ Coupling with the source term (topography  $\partial_x z$ )  
Necessity : compatibility with steady states

## System properties (II) : Steady states

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## System properties (II) : Steady state

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$$\begin{cases} \partial_t h + \partial_x(hu) = R - I \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) = gh(-\partial_x z - S_f) \end{cases} \quad (2)$$

$$\partial_t h = \partial_t u = \partial_t q = 0$$

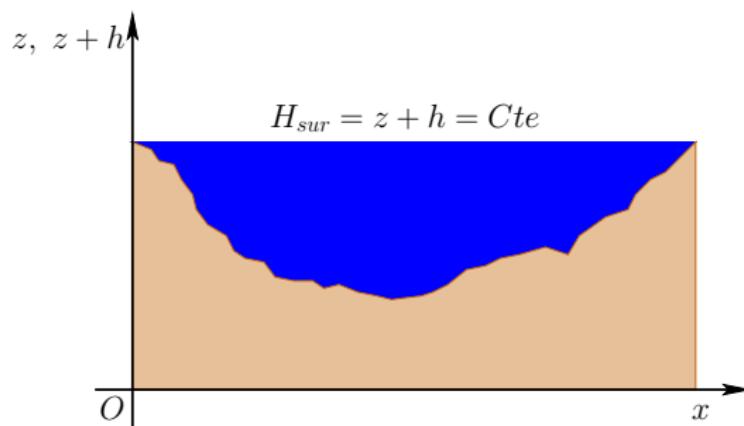
$$\begin{cases} \partial_x hu = R - I \\ \partial_x(hu^2 + gh^2/2) = gh(-\partial_x z - S_f) \end{cases} .$$

## System properties (II) : Steady states

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Lac at rest equilibrium

$$\begin{cases} u = 0 \\ g(\textcolor{red}{h+z}) = Cst \end{cases} .$$



## Hydrostatic reconstruction (II) [Audusse et al., 2004]

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We define

$$z^* = \max(z_G, z_D)$$

and

$$\begin{cases} U_G^* = (h_G^*, h_G^* u_G), \quad U_D^* = (h_D^*, h_D^* u_D) \\ h_G^* = \max(\textcolor{red}{h_G + z_G - z^*}, 0) \\ h_D^* = \max(\textcolor{red}{h_D + z_D - z^*}, 0) \end{cases} .$$

Thus, we have

$$\begin{cases} \mathcal{F}_G(U_G, U_D, \Delta Z) = \mathcal{F}(U_G^*, U_D^*) + \begin{pmatrix} 0 \\ g(h_G^2 - (h_G^*)^2)/2 \end{pmatrix} \\ \mathcal{F}_D(U_G, U_D, \Delta Z) = \mathcal{F}(U_G^*, U_D^*) + \begin{pmatrix} 0 \\ g(h_D^2 - (h_D^*)^2)/2 \end{pmatrix} \end{cases} ,$$

where  $\mathcal{F}(U_G, U_D)$  is the numerical flux.

## Friction treatment [Bristeau and Coussin, 2001]

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Shallow Water system with friction  $f$

$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) + h\partial_x z = -hf, \end{cases} \quad (3)$$

$f = f(h, u)$  friction force (on the bottom)

Several friction laws possible

- ▶ Manning :  $f = n^2 \frac{u|u|}{h^{4/3}}$
- ▶ Darcy-Weisbach :  $f = F \frac{u|u|}{8gh}$

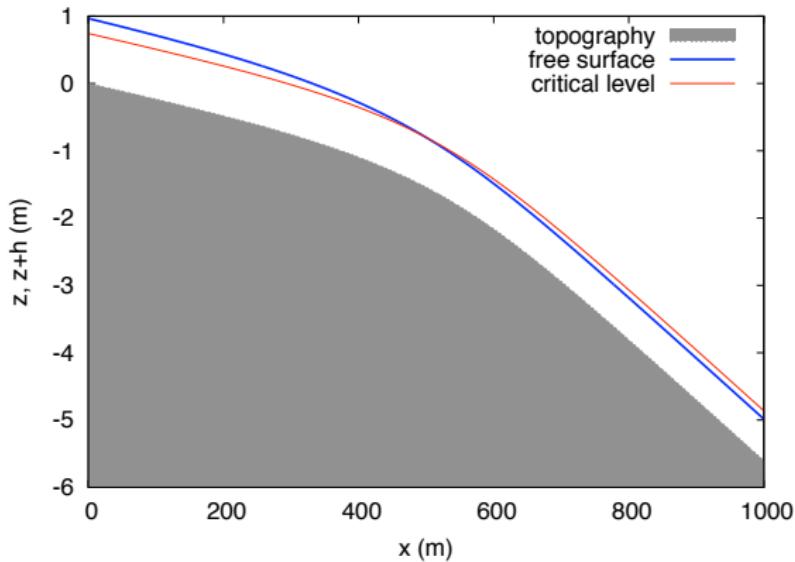
Semi-implicit treatment [Bristeau and Coussin, 2001]

$$q_i^{n+1} + F \frac{|q_i^n| q_i^{n+1}}{8h_i^n h_i^{n+1}} \Delta t = q_i^n - \frac{\Delta t}{\Delta x} (F_{i+1/2} - F_{i-1/2})$$

# Validation on analytical solutions – SWASHES

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Semi-implicit (subcritical-supercritical)



## Summary of the chosen numerical method

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- ▶ Numerical flux : HLL
- ▶ Second order scheme : MUSCL
- ▶ Friction : semi-implicit treatment
- ▶ Shallow Water system with rain  $R$

$$\begin{cases} \partial_t h + \partial_x(hu) = R \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) + h\partial_x z = -hf \end{cases} \quad (4)$$

time splitting/explicit treatment

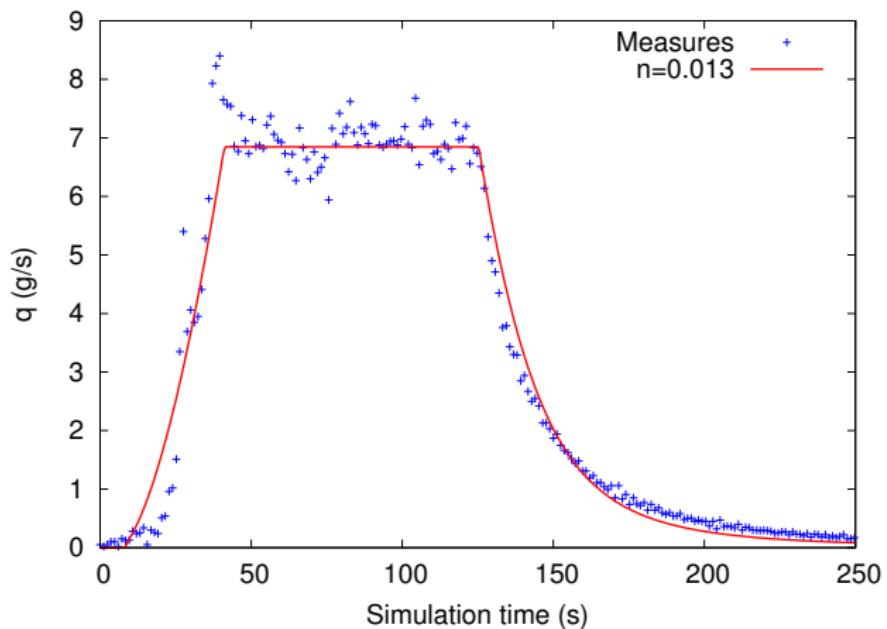
# Validation on experiments – INRA rain simulator

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# A simulation result (Manning)

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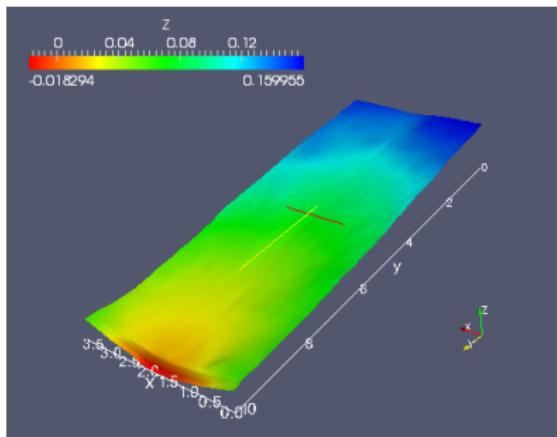


# Thies parcel – Senegal ([Tatard et al., 2008], IRD)

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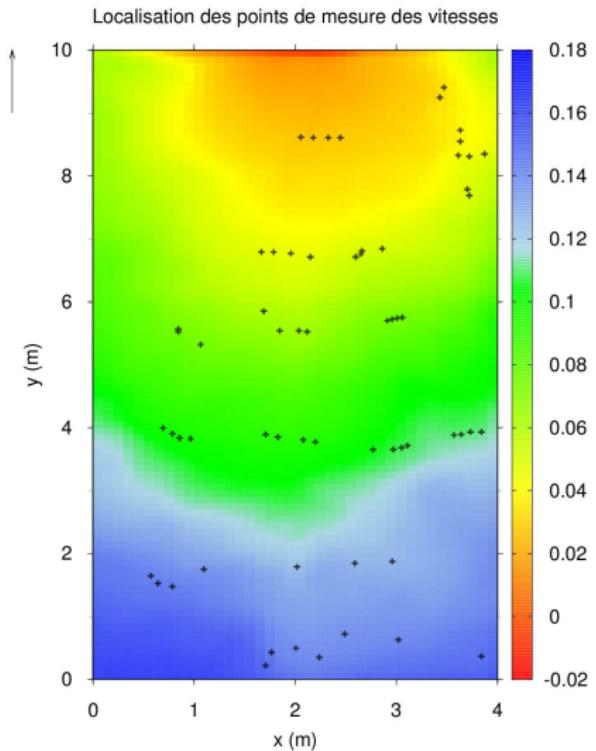
SVG velocity measure [Planchon et al., 2005]

Number of meshes :  $40 \times 100$  ( $4\text{ m} \times 10\text{ m}$ )



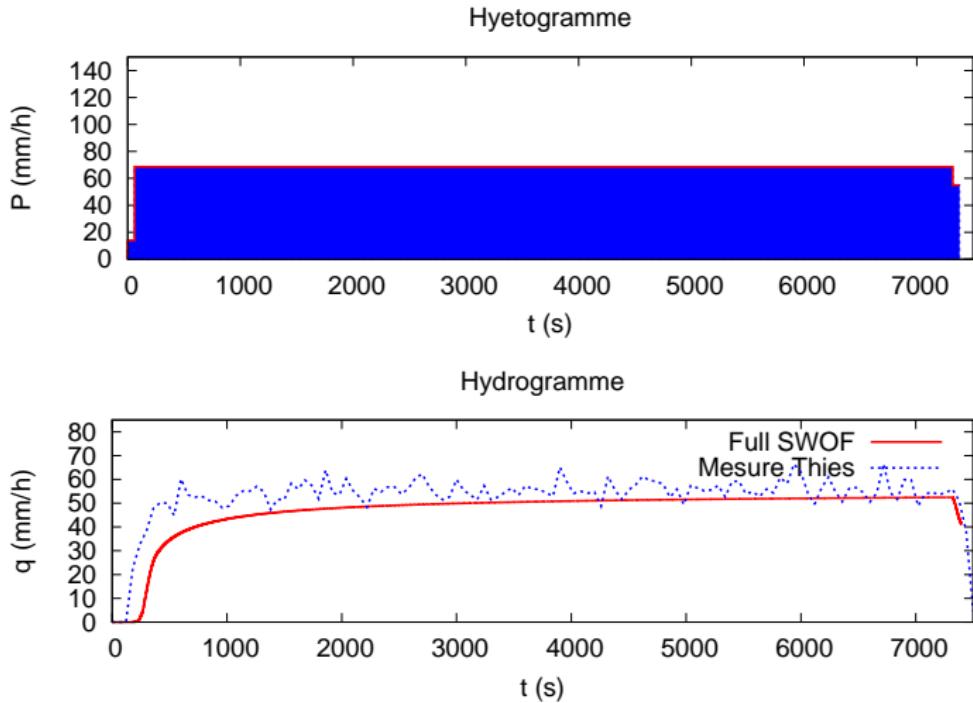
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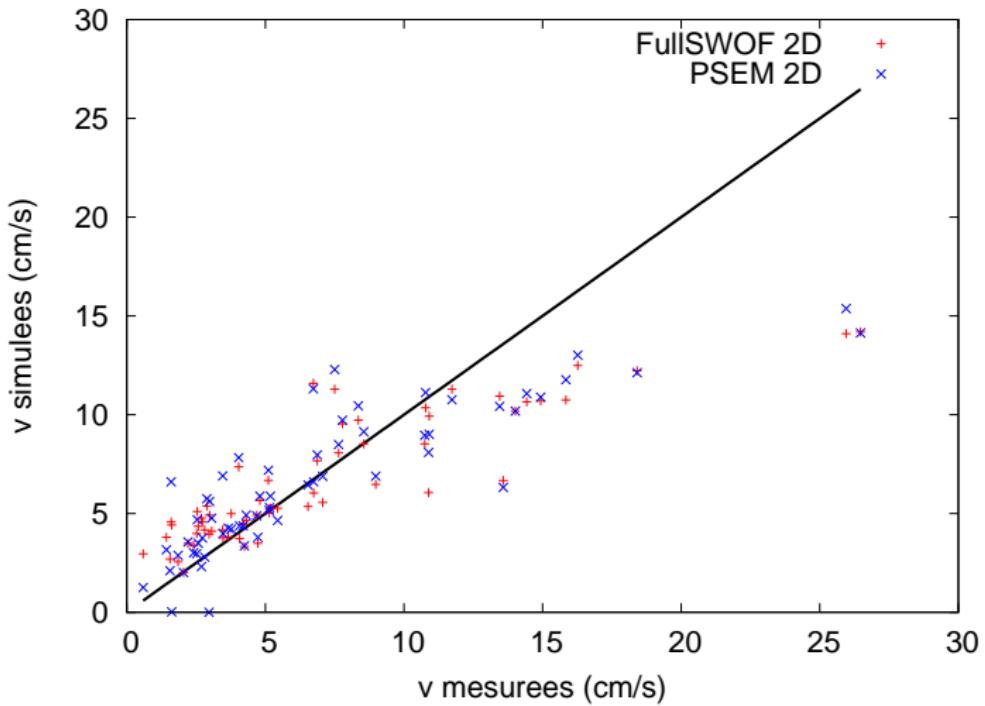
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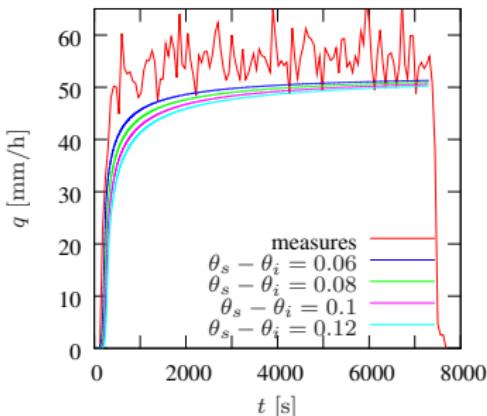
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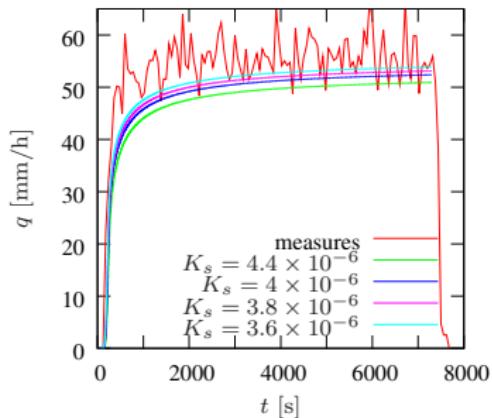
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## Green-Ampt infiltration parameters

- ▶  $\theta_i$  (resp.  $\theta_s$ ) initial (saturation) volumetric water content
- ▶  $K_s$  soil saturated hydraulic conductivity



(a)

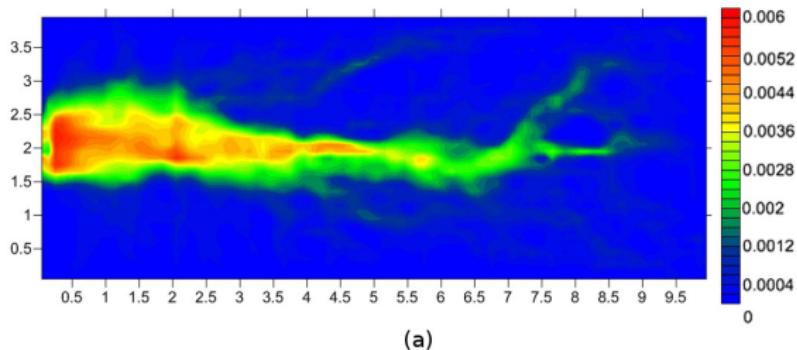


(b)

# Thies parcel – Senegal ([Tatard et al., 2008], IRD)

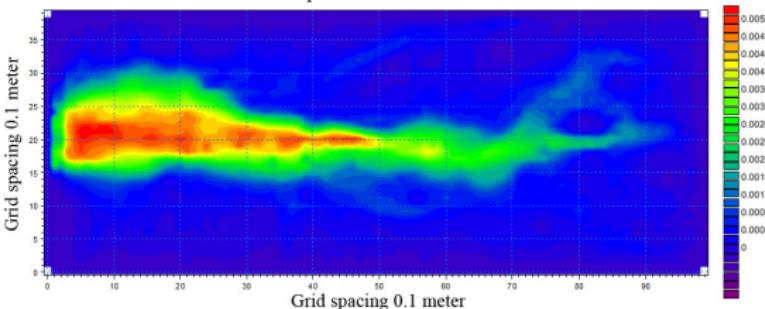
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(a) FullSWOF\_2D



(a)

depth of overland water



(b)

## FullSWOF & conclusion

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- ▶ code in C++ under CecCILL-V2 (GPL compatible) license and free
- ▶ sources available from <https://sourcesup.cru.fr/projects/>  
possibilities to modify the code, add numerical methods, physical models,  
customize inputs/outputs, ...
- ▶ object and inheritance
- ▶ variables encapsulation
- ▶ vector class (2d)
- ▶ objects "distributor"
- ▶ fixed CFL and fixed  $\Delta t$
- ▶ Doxygen documentation : html, latex, ... documentations
- ▶ GUI
- ▶ Watersheds & FullSWOF\_parallel

Thank you

## HLL flux

$$\mathcal{F}(U_G, U_D) = \begin{cases} F(U_G) & \text{if } 0 < c_1 \\ F(U_D) & \text{if } c_2 < 0 \\ \frac{c_2 F(U_G) - c_1 F(U_D)}{c_2 - c_1} + \frac{c_1 c_2 (U_D - U_G)}{c_2 - c_1} & \text{else} \end{cases},$$

with two parameters

$$c_1 < c_2.$$

For  $c_1$  and  $c_2$ , we take

$$c_1 = \inf_{U=U_G, U_D} \left( \inf_{j \in \{1, 2\}} \lambda_j(U) \right) \text{ and } c_2 = \sup_{U=U_G, U_D} \left( \sup_{i \in \{1, 2\}} \lambda_i(U) \right).$$

with  $\lambda_1(U) = u - \sqrt{gh}$  and  $\lambda_2(U) = u + \sqrt{gh}$ .

▶ retour

-  Audusse, E., Bouchut, F., Bristeau, M.-O., Klein, R., and Perthame, B. (2004).  
A fast and stable well-balanced scheme with hydrostatic reconstruction for shallow water flows.  
SIAM J. Sci. Comput., 25(6) :2050–2065.
-  Bristeau, M.-O. and Coussin, B. (2001).  
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Technical Report 4282, INRIA.
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Earth Surface Processes and Landforms, 30(7) :833–844.
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Measurement and modelling of high-resolution flow-velocity data under simulated rainfall on a low-slope sandy soil.  
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