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FullSWOF:
A SOFTWARE FOR OVERLAND FLOW SIMULATION
FullSWOF : un logiciel pour la simulation du ruissellement

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Le ruissellement sur les terres agricoles peut avoir des effets indésirables tels que l'érosion des sols, les inondations et le transport de polluants. Afin de mieux comprendre ce phénomène et d'en limiter les conséquences, nous avons développé un code à l'aide de méthodes numériques récentes : FullSWOF (Full Shallow Water equations for Overland Flow) un code orienté objet écrit en C++. Il est libre et peut être téléchargé à partir de <http://www.univ-orleans.fr/mapmo/sofi/FullSWOF/>. Le modèle est basé sur le système de Saint-Venant. Les difficultés numériques viennent des nombreuses transitions sec/mouillé et de la topographie très variable rencontré dans un champ. Il intègre

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le ruissellement, les entrées de précipitations, l'infiltration (Green-Ampt modifié), la friction (les lois de Darcy-Weisbach et de Manning).

Nous présentons d'abord la méthode numérique pour la résolution des équations en eaux peu profondes intégrée dans FullSWOF_2D (la version en deux dimensions). Cette méthode est basée sur le schéma de reconstruction hydrostatique, couplée à un traitement semi-implite du terme de friction. FullSWOF_2D a déjà été validé à l'aide des solutions analytiques de la bibliothèque SWASHES. Enfin, FullSWOF_2D est exécuté sur des données de terrain acquises sur une parcelle située à Thiès (Sénégal). Les résultats de la simulation sont comparés avec les données mesurées. Ce banc d'essai expérimental permet de démontrer les capacités de FullSWOF à simuler l'écoulement de surface. FullSWOF pourrait également être utilisé pour d'autres problèmes environnementaux, tels que les inondations fluviales et les ruptures de barrage.

Overland flow on agricultural fields may have some undesirable effects such as soil erosion, flood and pollutant transport. To better understand this phenomenon and limit its consequences, we developed a code using state-of-the-art numerical methods: FullSWOF (Full Shallow Water equations for Overland Flow), an object oriented code written in C++. It has been made open-source and can be downloaded from <http://www.univ-orleans.fr/mapmo/soft/FullSWOF/>. The model is based on the classical system of Shallow Water (SW) (or Saint-Venant system). Numerical difficulties come from the numerous dry/wet transitions and the highly-variable topography encountered inside a field. It includes runoff and rainfall inputs, infiltration (modified Green-Ampt equation), friction (Darcy-Weisbach and Manning formulas).

First we present the numerical method for the resolution of the Shallow Water equations integrated in FullSWOF_2D (the two-dimension version). This method is based on hydrostatic reconstruction scheme, coupled with a semi-implicit friction term treatment. FullSWOF_2D has been previously validated using analytical solutions from the SWASHES library (Shallow Water Analytic Solutions for Hydraulic and Environmental Studies). Finally, FullSWOF_2D is run on a real topography measured on a runoff plot located in Thies (Senegal). Simulation results are compared with measured data. This experimental benchmark demonstrate the capabilities of FullSWOF to simulate adequately overland flow. FullSWOF could also be used for other environmental issues, such as river floods and dam-breaks.

Key words

Rainfall runoff, Shallow Water system, well-balanced scheme, hydrostatic reconstruction, finite volumes, dry/wet transition, Green-Ampt model.

I INTRODUCTION

Rain on agricultural fields can yield rainfall runoff. At field scale, overland flow may have some undesirable effects such as soil erosion and pollutant transport. Downstream the watersheds, roads and houses may be damaged. Some control measures can be taken such as using grass strips. We have to know how the water is moving in order to put these developments. In overland flow prediction, several methods are used from black box models to physically based models. Two physical models are often used to model overland flow: Kinematic (KW) and Diffusive Wave (DW) equations [12,13]. But following [20,8,9], we choose to use the Shallow Water (de Saint Venant [16]) physical model. Indeed KW and DW models may give poor results in terms of water heights and velocities in case of mixed subcritical and supercritical flow. In spite of computational time SW is mandatory. MacCormack scheme is widely used to solve SW equations [20,8,9]. But it neither guarantees the positivity of water depths at the wet/dry transitions, nor preserves steady states (not well-balanced) [17]. In industrial codes (ISIS, Canoe, HEC-RAS, MIKE11...), SW equations are often solved under non-conservative form [13] with either Preissmann scheme or Abbott-Ionescu scheme. Thus transcritical flows and hydraulic jump are not solved properly. In order to cope with all these problems, we choose to use the hydrostatic reconstruction. This positive preserving well-balanced finite volume scheme is integrated in FullSWOF_2D. In what follows, we present the physical model, then the numerical methods. In the end, FullSWOF_2D is applied on a real event measured in Thiès by IRD [19].

II THE MODEL

II.1 The shallow water equations

As in [8,9], we consider the 2D Shallow Water equations (SW2D) which write (see **Figure 1 a**)

$$\begin{aligned}
 \partial_t h + \partial_x(hu) + \partial_y(hv) &= R - I \\
 \partial_t(hu) + \partial_x(hu^2 + gh^2/2) + \partial_y(huv) &= gh(S_{0x} - S_{fx}) , \\
 \partial_t(hv) + \partial_x(huv) + \partial_y(hv^2 + gh^2/2) &= gh(S_{0y} - S_{fy})
 \end{aligned} \tag{1}$$

where the unknowns are the velocities $u(x,y,t)$ and $v(x,y,t)$ [m/s] and the water height $h(x,y,t)$ [m]. The subscript x (respectively y) stands for the x -direction (resp. the y -direction): $S_{0x} = -\partial_x z(x,y)$ and $S_{0y} = -\partial_y z(x,y)$ are the ground slopes, S_{fx} and S_{fy} are friction terms. $R(x,y,t)$ [m/s] is the rainfall intensity and $I(x,y,t)$ [m/s] is the infiltration rate. As in [8], we use the Darcy-Weisbach friction law which writes:

$$S_{fx} = f \frac{u\sqrt{u^2+v^2}}{8gh}, \quad S_{fy} = f \frac{v\sqrt{u^2+v^2}}{8gh} . \tag{2}$$

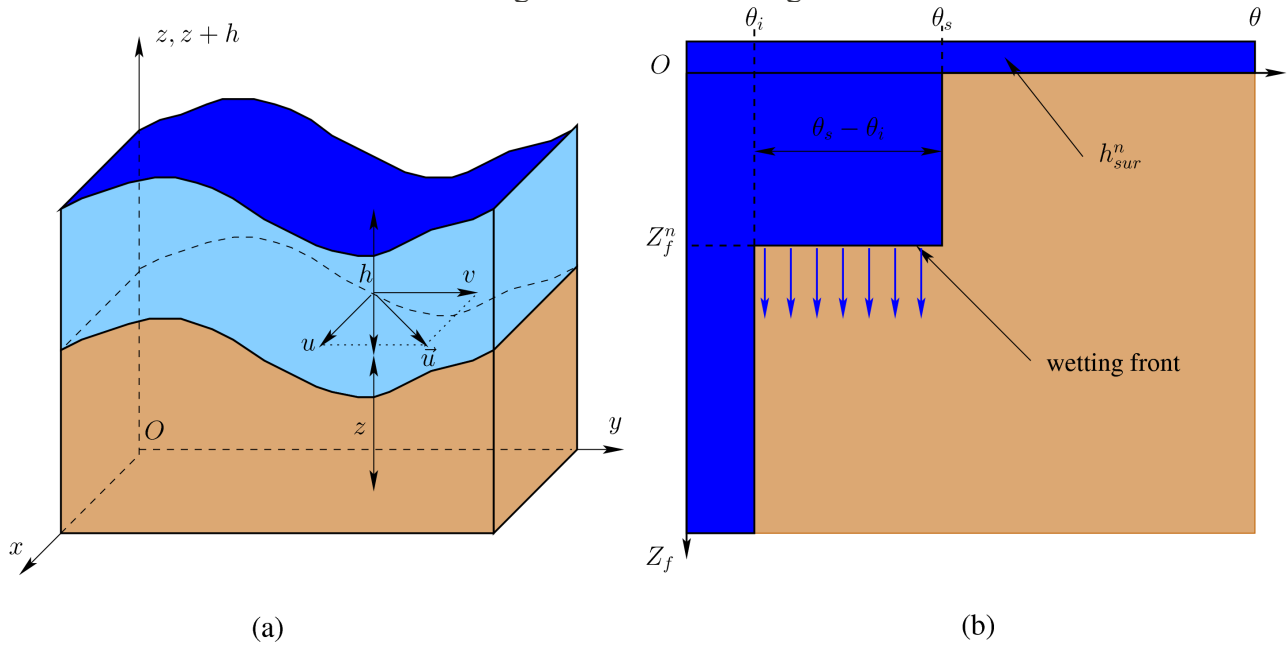


Figure 1: Illustration of variables of (a) Shallow Water equations (SW2D) and (b) Green-Ampt infiltration model.

II.2 The Green-Ampt infiltration model

Infiltration is computed at each cell using a Green-Ampt model [10,15,7]. With this model, the movement of water in soil is assumed to be in the form of an advancing wetting front (located at Z_f^n [m]) that separates a zone still at the initial soil moisture θ_s (see **Figure 1 b**). At the moment $t=t_n$, the infiltration capacity I_C^n [m/s] is calculated thanks to

$$I_C^n = K_s \left(1 + \frac{h_f - h_{sur}^n}{Z_f^n} \right) \quad \text{where} \quad Z_f^n = \frac{V_{inf}^n}{\theta_s - \theta_i} , \tag{3}$$

where h_f is the wetting front capillary pressure head, K_s the hydraulic conductivity at saturation, h_{sur}^n the water height and V_{inf}^n the infiltrated water volume. Thus we have the infiltration rate

$$I^n = \frac{\min(h_{sur}^n, \Delta t \cdot I_C^n)}{\Delta t} , \tag{4}$$

and the infiltrated volume

$$V_{inf}^{n+1} = V_{inf}^n + \Delta t \cdot I^n , \tag{5}$$

where Δt is the time step. In the case of a two-layer soil, we consider a modification of this model (see [5,8]).

III THE NUMERICAL METHOD

The scheme will be presented in one dimension (SW1D)

$$\begin{aligned} \partial_t h + \partial_x(hu) &= R - I \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) &= gh(S_{0x} - S_{fx}) \end{aligned} \quad (6)$$

the extension to SW2D on structured grid is straightforward and is integrated in an object oriented code in C++: FullSWOF_2D (for details about the code see [4,5]). In what follows, we note the discharge $q=hu$ [m²/s] and the vector of conservative variables $U = (h \quad hu)^t$.

III.1 Convective step

A finite volume discretization of SW1D writes

$$U_i^* = U_i^n - \frac{\Delta t}{\Delta x} [F_{i+1/2L}^n - F_{i-1/2R}^n - Fc_i^n] \quad (7)$$

with Δx the space step and

$$\begin{aligned} F_{i+1/2L}^n &= F_{i+1/2}^n + S_{i+1/2L}^n \\ F_{i-1/2R}^n &= F_{i-1/2}^n + S_{i-1/2R}^n \end{aligned} \quad (8)$$

the left (respectively right) modification of the numerical flux F_l for the homogeneous problem (see section III.3)

$$F_{i+1/2}^n = Fl(U_{i+1/2L}^n, U_{i+1/2R}^n) \quad (9)$$

The values $U_{i+1/2L}$ and $U_{i+1/2R}$ are obtained thanks to two consecutive reconstructions. Firstly a MUSCL reconstruction [2,5] is performed on u , h and $h+z$ in order to get a second order scheme in space. This gives us the reconstructed values (U_-, z_-) and (U_+, z_+) . Secondly we apply the hydrostatic reconstruction [1,2] on the water height which allows us to get a positive preserving well-balanced scheme (in the sense it preserves at least steady state at rest)

$$\begin{aligned} h_{i+1/2L} &= \max(h_{i+1/2-} + z_{i+1/2-} - \max(z_{i+1/2-}, z_{i+1/2+}), 0) \\ U_{i+1/2L} &= (h_{i+1/2L}, h_{i+1/2L} u_{i+1/2-}) \\ h_{i+1/2R} &= \max(h_{i+1/2+} + z_{i+1/2+} - \max(z_{i+1/2-}, z_{i+1/2+}), 0) \\ U_{i+1/2R} &= (h_{i+1/2R}, h_{i+1/2R} u_{i+1/2+}) \end{aligned} \quad (10)$$

We introduce

$$S_{i+1/2L}^n = \begin{pmatrix} 0 \\ \frac{g}{2}(h_{i+1/2-}^2 - h_{i+1/2L}^2) \end{pmatrix}, \quad S_{i-1/2R}^n = \begin{pmatrix} 0 \\ \frac{g}{2}(h_{i-1/2+}^2 - h_{i-1/2R}^2) \end{pmatrix} \quad (11)$$

and a centered source term is added to preserve consistency and well-balancing (see [1,2])

$$Sc_i = \begin{pmatrix} 0 \\ -g \frac{h_{i-1/2+} + h_{i+1/2-}}{2} (z_{i+1/2-} - z_{i-1/2+}) \end{pmatrix} \quad (12)$$

The rain and the infiltration are treated explicitly (for details see [5]).

III.2 Friction treatment

In this step, the friction term is taken into account with the following system

$$\partial_t U = \begin{pmatrix} 0 \\ -ghS_f \end{pmatrix} \quad (13)$$

This system is solved thanks to a semi-implicit method (as in [3,9])

$$h^{n+1} = h^* \quad (14)$$

$$q^{n+1} = \frac{q^*}{1 + \Delta t \frac{f}{8} \frac{|q^n|}{h^n h^{n+1}}} .$$

where h^* , q^* and u^* are the variables from the convective step. This method allows to preserve stability (under a classical CFL condition) and steady states at test. Finally, these two steps are combined in a second order TVD Runge Kutta method which is the Heun's predictor-corrector method. It writes

$$\begin{aligned} U^* &= U^n + \Delta t \Phi(U^n) \\ U^{**} &= U^* + \Delta t \Phi(U^*) \\ U^{n+1} &= \frac{U^n + U^{**}}{2} \end{aligned} \quad , \quad (15)$$

where Φ is the right part of (7).

III.3 Numerical flux

We use the HLL flux which writes

$$Fl(U_L, U_R) = \begin{cases} F(U_L) & \text{if } 0 < c_1 \\ \frac{c_2 F(U_L) - c_1 F(U_R)}{c_2 - c_1} + \frac{c_1 c_2}{c_2 - c_1} (U_R - U_L) & \text{if } c_1 < 0 < c_2 , \\ F(U_R) & \text{if } c_2 < 0 \end{cases} \quad (16)$$

with two parameters $c_1 < c_2$ given by

$$c_1 = \min_{U=U_L, U_R} (\min_{j \in \{1,2\}} \lambda_j(U)) , \quad c_2 = \max_{U=U_L, U_R} (\max_{j \in \{1,2\}} \lambda_j(U)) \quad , \quad (17)$$

where $\lambda_1(U) = u - \sqrt{gh}$ and $\lambda_2(U) = u + \sqrt{gh}$ are the eigenvalues of SW1D. In practice, we use a CFL condition $n_{CFL} = 0.5$ at second order and $n_{CFL} = 1$ at first order, with

$$\Delta t \leq n_{CFL} \frac{\Delta x}{\max_{i \in \{1, J\}} (|u_i| + \sqrt{gh_i})} \quad , \quad (18)$$

where J is the number of space cells. At second order, variables (h_i, u_i) in (18) are replaced by the reconstructed values $(h_{i+1/2-}, u_{i+1/2-})$ and $(h_{i+1/2+}, u_{i+1/2+})$ (detailed in next section).

III.4 MUSCL-reconstruction

We define the MUSCL reconstruction of a scalar function $s \in \mathbb{R}$ by

$$s_{i-1/2+} = s_i - \frac{\Delta x}{2} Ds_i , \quad s_{i+1/2-} = s_i + \frac{\Delta x}{2} Ds_i \quad , \quad (19)$$

with the operator

$$Ds_i = \minmod \left(\frac{s_i - s_{i-1}}{\Delta x} , \frac{s_{i+1} - s_i}{\Delta x} \right) \quad , \quad (20)$$

and the *minmod* limiter

$$\minmod(x, y) = \begin{cases} \min(x, y) & \text{if } x, y \geq 0 \\ \max(x, y) & \text{if } x, y \leq 0 \\ 0 & \text{else} \end{cases} \quad . \quad (21)$$

As mentioned previously, the MUSCL reconstruction is performed on u , h and $h+z$ then we deduce the reconstruction of z . In order to keep the discharge conservation, the reconstruction of the velocity has to be modified as what follows

$$u_{i-1/2+} = u_i - \frac{h_{i+1/2-}}{h_i} \frac{\Delta x}{2} Du_i , \quad u_{i+1/2-} = u_i + \frac{h_{i-1/2+}}{h_i} \frac{\Delta x}{2} Du_i \quad . \quad (22)$$

IV VALIDATION

FullSWOF_2D has already been validated on analytical solutions integrated in SWASHES a library of analytical solutions [6]. The purpose of this section is to confront FullSWOF_2D to a real system: the plot of Thies, Senegal [19]. The results presented below are the results preliminary to a more detailed study. They simply aim at illustrating the ability of FullSWOF_2D to simulate a dynamic real runoff.

IV.1 Experimental data

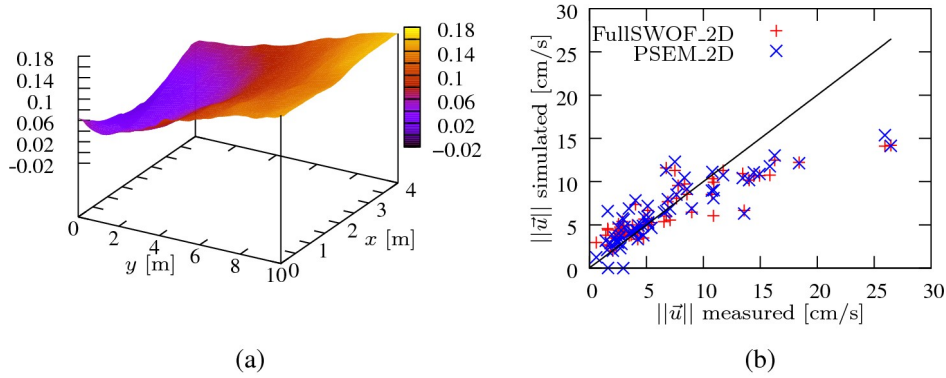


Figure 2: (a) The topography of the experimental plot and (b) comparison between measured and simulated velocities.

This is the plot of an experimental system instrumented by IRD [19] in the project PNRH RIDES. This parcel is 10 meters long and 4 meters wide (**Figure 2 a**). It has the classical configuration of the open book of Wooding: a slope of 1% both along the axe (Ox) and (Oy). The soil consists of a sandy type. Several experiments have been carried out to test the Salt Velocity Gauge [14] (a new measurement technique) and to study the dynamics of runoff and erosion. The set of measures considered consists of flow measurement at the outlet and the speed measurement in 63 parts of the parcel. These measures were used to compare different computer codes: NCF and MAHLERAN (based on KW), RillGrow (based on DW) and PSEM_2D (based on SW2D with MacCormack scheme). In the following, we have used the set of parameters obtained with PSEM_2D and we have observed the sensitivity to infiltration parameters.

IV.2 Numerical results

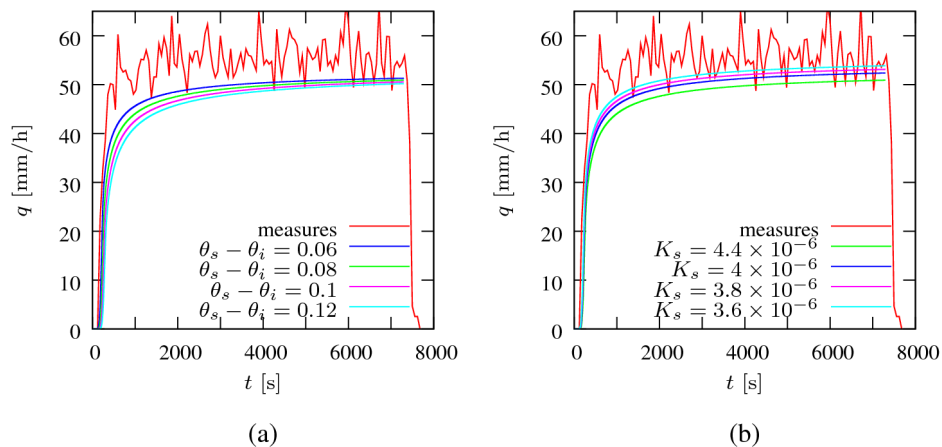


Figure 3: Comparison between simulated hydrographs and measures (a) for several values of $\theta_s - \theta_i$ and (b) K_s .

For all the simulations, we have used $\Delta x = \Delta y = 0.1 m$ as space steps and $f = 0.26$ as Darcy Weisbach friction coefficient. The average rain intensity was $70 mm/h$ during two hours. Infiltration parameters used with PSEM_2D [18] were $h_f = 0.06 m$, $\theta_s - \theta_i = 0.12$ and $K_s = 4.4e-6 m/s$. We

tried this set of parameters. We notice that the simulated velocities with FullSWOF_2D are closed to those obtained with PSEM_2D (**Figure 2 b**). Small flow-velocities are well caught, excepted for the bigger values. In these cases, values are underestimated both by FullSWOF_2D and PSEM_2D. This means that the friction law is not adapted to do this simulation. Moreover we notice that the simulated hydrograph is under the measured one. Thus we have tried other values for $\theta_s - \theta_i$ and K_s . We kept h_f and $\theta_s - \theta_i$ (respectively K_s) values and we changed K_s (resp. $\theta_s - \theta_i$). We notice that decreasing $\theta_s - \theta_i$ improves mainly the beginning of the hydrograph (**Figure 3 a**), while decreasing K_s improves the entire hydrogram (**Figure 3 b**). We notice that the saturated hydraulic conductivity K_s is the most influent one, as observed in [16] a study on the impact of this parameter on the surface runoff.

V CONCLUSIONS

FullSWOF is an object oriented code designed for overland flow on agricultural fields. This code is based on the Shallow Water system. The numerical difficulties such as dry/wet transitions and steady states are dealt thanks to adapted numerical methods. FullSWOF gave good results on real data. Nevertheless, the physical model has to be improved to match more accurately with the measured velocities.

VI ACKNOWLEDGMENTS AND THANKS

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