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Criteria to measure the quality of TVAR estimation for audio signals

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Definition

Audio signal = Correlation + Non-stationarity

Classically

Audio signal is modeled by an AutoRegressive process (AR)

1 Advantage

To take account of the correlation of the signal

2 Drawback

Non stationary process

Recently

Generalization of the AR model to the non-stationary context:
TVAR model (Time-Varying AutoRegressive)

⇒ How to evaluate the estimation of the TVAR model?

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⇒ How to evaluate the estimation of the TVAR model?

- 1 TVAR model and parameter estimation
 - TVAR model
 - Parameter estimation
- 2 Evaluation of the TVAR estimation
 - Cepstral distance
 - Signal to Noise Ration (SNR)
 - Classical statistical criteria
 - Proposed approach
- 3 Experimentation validation

Part I

TVAR model and parameter estimation

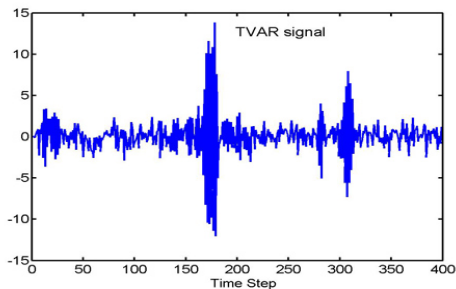
TVAR model

Definition

The output, x_t , of a p order TVAR process is:

$$x_t = \sum_{i=1}^p a_{i,t} x_{t-i} + \sigma_{e_t} e_t,$$

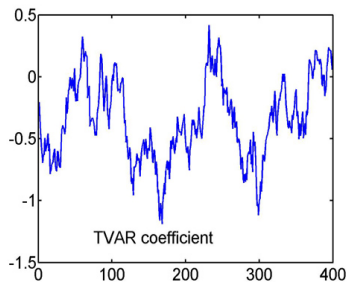
Example: Synthetic 2nd order TVAR signal



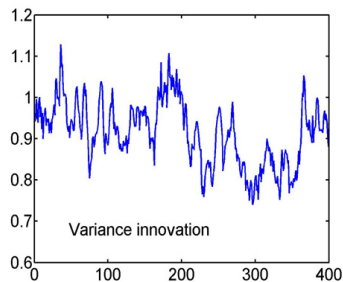
TVAR model

$a_{i,t}, \log \sigma_{e_t}^2$: first order Markov process

- Example: TVAR coefficient

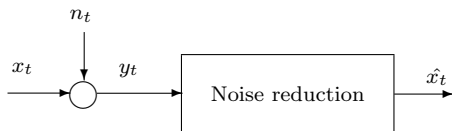


- Example: Variance innovation



Adaptative estimation of audio signal

- **Problem:** Estimation of audio signal from noisy observation



- State System:

$$\begin{cases} x_t = A_t(\theta_t)X_{t-1} + E_t e_t \\ y_t = C_t(\theta_t)X_t + N_t n_t \end{cases}$$

$$\theta_t = (a_t, \log \sigma_{e_t}^2, \log \sigma_{n_t}^2)$$

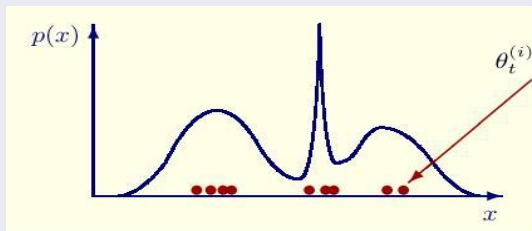
- Estimation objective:

- 1 First stage: estimation of θ_t by **Particle Filter**
- 2 Second stage: estimation of \hat{x} by **Kalman Filter**

Particle Filter

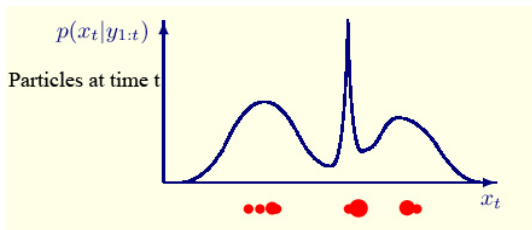
Principle

- *Suboptimal filters perform sequential Monte Carlo (SMC) estimation based on weighted particles.*

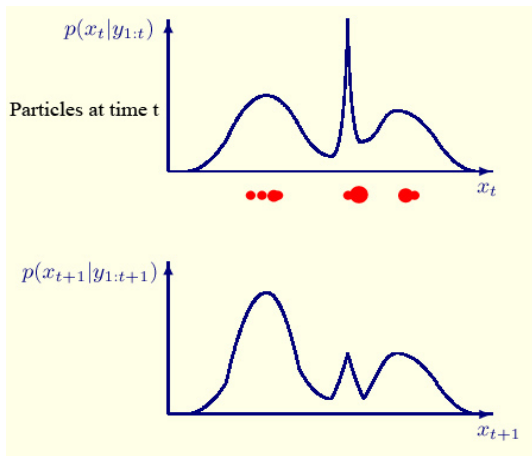


- *Monte Carlo estimation: $\hat{\theta}_t \approx \sum_{i=1}^N \theta_t^{(i)} \tilde{w}_t^{(i)}$*

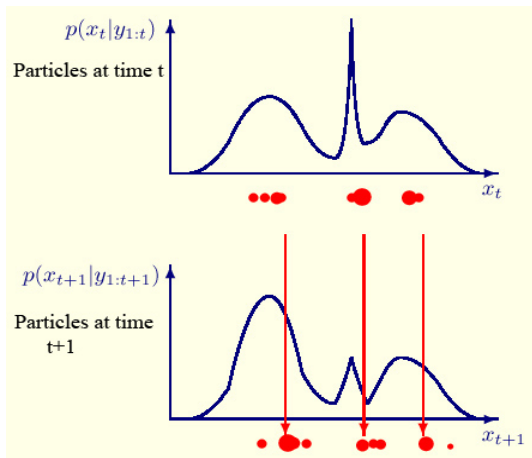
Particle Filter



Particle Filter

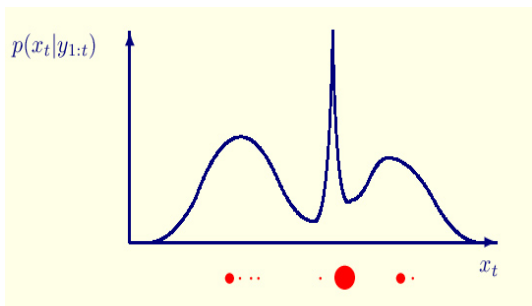


Particle Filter



Particle Filter

⇒ After some iterations: particles degenerate



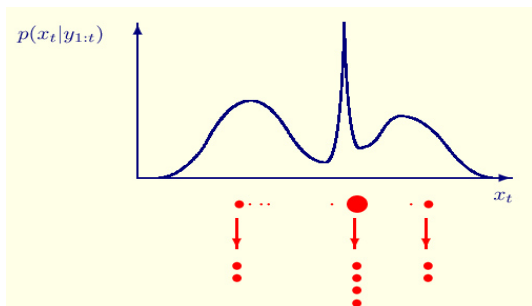
⇒ Divergence of the filter: **resampling is needed**

Particle Filter

Resampling stage:

⇒ Duplicating the particles of strong weight

⇒ Eliminating the particles of weak weight



Part II

Evaluation of TVAR estimation

Direct criteria: Cepstral distance

Comparison between the original and estimated parameters
 \Rightarrow **Cepstral Distance**

Definition

- **Cepstrum coefficients**

$$c_t(i) = -a_t(i) - \sum_{n=1}^{i-1} \left(1 - \frac{n}{i}\right) a_t(n) c_t(i-n).$$

- **Cepstral distance**

$$d_t^c = \sqrt{\sum_{i=1}^p (c_t(i) - \hat{c}_t(i))^2}.$$

Drawbacks

Not available for natural signals

\Rightarrow **Undirect criteria**

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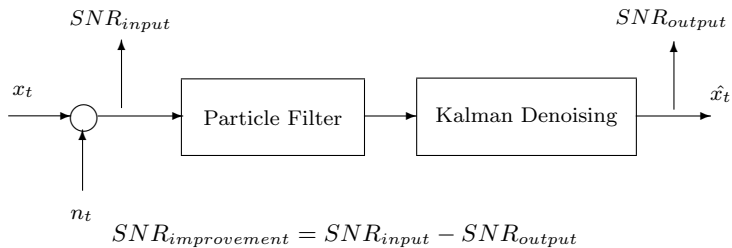
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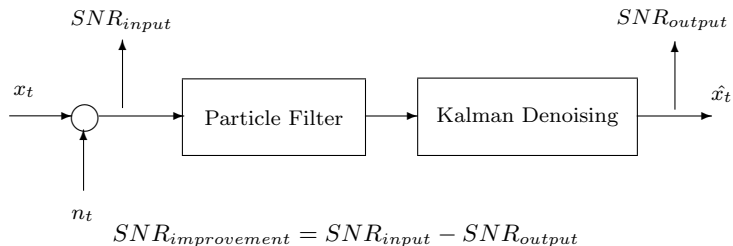
Not available for natural signals

\Rightarrow Undirect criteria

Undirect criteria: Signal to Noise Ratio (SNR)

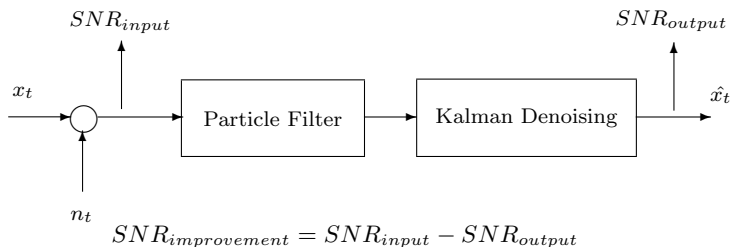


Undirect criteria: Signal to Noise Ratio (SNR)



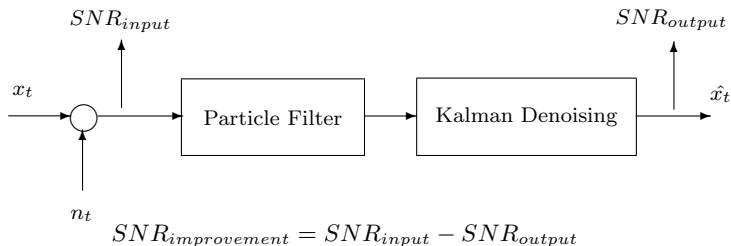
+ Appropriate for denoising purposes

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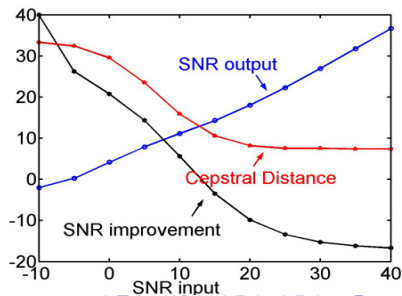


- + Appropriate for denoising purposes
- Depending on the denoising process

Undirect criteria: Signal to Noise Ratio (SNR)



- + Appropriate for denoising purposes
- Depending on the denoising process
- Not relevant for strong input SNR



Undirect criteria: conventional statistical tests

Principle

- *Test based on u_t :*

$$u_t = p(Y_t \leq y_t | y_{1:t-1})$$

- *TVAR model is correct if $v_t = \phi^{-1}(u_t)$ is i.i.d according to $\mathcal{N}(0, 1)$, ϕ is the standard normal cumulative distribution function.*
- *Statistical tests:*
 - *Normality Index: Bowman-Shenton Test*
 - *Whiteness Index: Ljung-Box Test*

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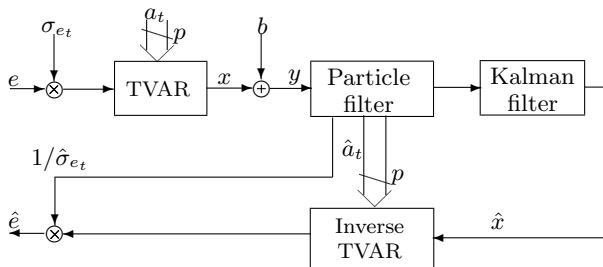
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Proposed approach

Principle

The estimation is good if there exists a stationary gaussian white noise that can produce x by exciting the estimated TVAR model



$$\hat{e}(t) = \frac{1}{\hat{\sigma}_{e_t}} \left(\hat{x}(t) - \sum_{i=1}^p \hat{a}_{i,t} \hat{x}(t-i) \right).$$

Tests validation: Whiteness, Normality, **Stationarity**

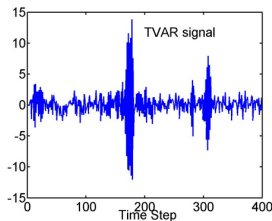
Part III

Experimental validation

Experimental validation: Simulation context

1 Test signal:

- Synthetic TVAR signal of order 2.
- $\Delta_a = 10^{-2} I_2$ for the TVAR coefficients.
- $\delta_e^2 = \delta_n^2 = 10^{-3}$ for the log-variance.



2 Experimental objectives:

- Evaluating the correlation between whiteness/ normality/ stationarity indices and cepstral distance.
- Compare these correlations for v_t and \hat{e}_t

3 Experiment:

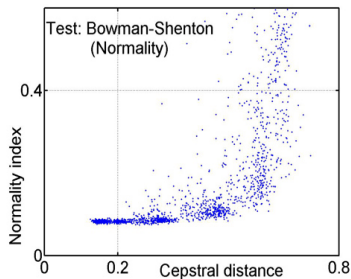
For various SNR ($-10 : 2 : 40$ dB) and for various number of particles ($N = 10 : 10 : 150$ particles), plot set of point (d^c, I) for v_t and \hat{e}_t .

⇒ Correlation analysis

Experimental validation: Results

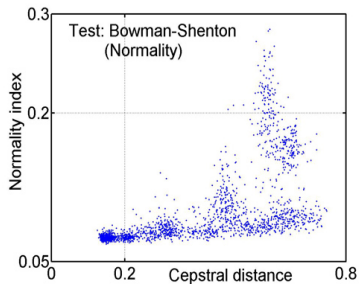
⇒ Normality index

- Estimate excitation: \hat{e}



Correlation coefficient: 0.6

- Residual time series: v



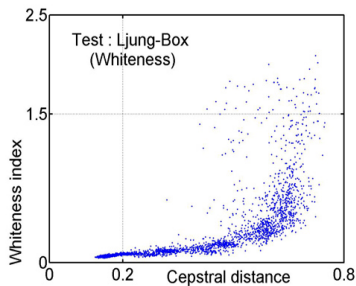
Correlation coefficient: 0.8

each point $(d^C, I) \sim 1$ experiment with a particular (SNR, N)

Experimental validation: Results

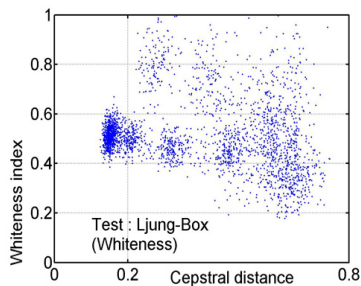
⇒ Whiteness index

- Estimate excitation: \hat{e}



Correlation coefficient: 0.9

- Residual time series: v



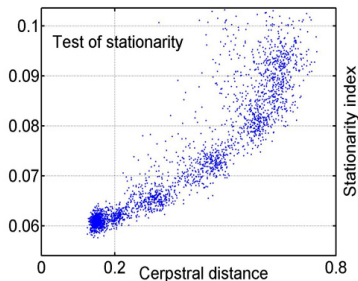
Correlation coefficient: 0.06

each point $(d^C, I) \sim 1$ experiment with a particular (SNR, N)

Experimental validation: Results

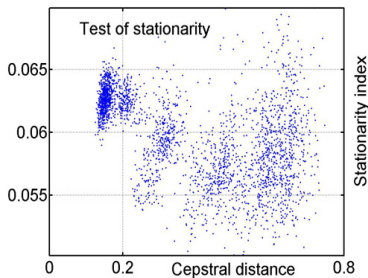
⇒ **Stationarity index**

● Estimate excitation: \hat{e}



Correlation coefficient: 0.95

● Residual time series: v



Correlation coefficient: 0.7

each point $(d^C, I) \sim 1$ experiment with a particular (SNR, N)

Summary

- Statistical indices are well correlated to cepstral distance for \hat{e}

	Whiteness	Normality	Stationarity
\hat{e}	0.93	0.6	0.95
v	0.06	0.80	-0.7

- Less complex than classical method
- Not need any knowledge of the original model \rightarrow appropriate for natural signals

Part IV

Conclusion

Summary

- 1 A new evaluation method of TVAR estimation:
 - appropriate to natural signals: not need of the original model, neither of the original signal.
 - simple: just inverse the estimated TVAR system.
 - better result than classical methods: indices well correlated to cepstral distance.
- 2 Not aim at performing binary validation of TVAR model: quantitative measure of the quality of the TVAR model estimation

Further investigations

- 1 What happen in case of undermodeling?
- 2 Take into account the gaussian assumption of noise excitation.

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